Nonzero θ_{13} from Approximate Neutrino Mixing Matrix

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Abstract – *Neutrino mixing matrix is formutated by using the standard neutrino mixing matrix as a basis to formulate approximate neutrino mixing matrix by considering the experimental data as inputs. The resulted approximate neutrino mixing matrix can predict nonzero mixing angle θ_{13} which is compatible with the experimental data and the underlying symmetry of resulted neutrino mass matrix is $\mu-\tau$ symmetry when we impose: $m_1 \approx m_2$. We also obtained the nonzero mixing angle θ_{13} that related to neutrino masses, i.e. $\sin\theta_{13} = m_2/m_3$ that can be tested in future experiments*

Keywords: *nonzero θ_{13}, neutrino mixing matrix, neutrino mass matrix*

I. INTRODUCTION

Since the reported evidence of neutrino oscillations in 1998 by Kamiokande [1], the theorists have made some attempt to explain the neutrino flavor oscillations. One of the most elegant idea is that neutrino flavor eigenstate is not its own mass eigenstate. Theoretically, neutrino flavor eigenstates (ν_e,ν_μ,ν_τ) are related to neutrino mass eigenstates (ν_1,ν_2,ν_3) via a neutrino mixing matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (1)$$

where V is neutrino mixing matrix. The standard parameterization of the neutrino mixing matrix reads [2]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}s_{13} \end{pmatrix} \quad (2)$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and δ is the Dirac phase.

There are three well-known neutrino mixing matrices, i.e. bimaximal mixing (BM), tribimaximal mixing (TBM), and democratic mixing (DM). All of the mixing matrices predict the mixing angle $\theta_{13} = 0^\circ$. Recently, the experimental results showed that the mixing angle $\theta_{13} \neq 0^\circ$ and it is relatively large. Several attempt have been done theoretically to accommodate the nonzero and relatively large mixing angle θ_{13} by modifying the neutrino mixing matrix including the Dirac phase δ in relation to the CP-violation in neutrino sector. Another unsolved problem in neutrino physics till today is the hierarchy of neutrino mass. Experimental results showed that we have two possibilities for neutrino mass hierarchies: normal and inverted hierarchies. We have no clue in order to decide theoretically the neutrino mass hierarchy.

In this paper, we study sistematically the neutrino mixing matrix by using the standard neutrino mixing matrix in Eq. (1) by considering the reported neutrino oscillations experimental facts. The resulted neutrino mixing matrix to be used for predicting the mixing angle θ_{13} and its relation to neutrino masses. In section II, we show the simple basic motivations and assumptions to formulate the three well-known mixing matrices: BM, TBM, and DC which cannot proceed nonzero θ_{13}. In section III, we consider recent neutrino experimental facts to derive an approximate neutrino mixing matrix and discuss its predictions on neutrino masses. Section IV devoted to conclusions.

II. MIXING MATRICES: BM, TBM, and DC

The experimental facts from neutrino oscillation showed us that both solar neutrino mixing angle (θ_{12}) and atmospheric neutrino mixing angle (θ_{23}) nearly maximal $\theta_{12} \approx \theta_{23} \approx \pi/4$, and mixing angle $\theta_{13} \approx 0^o$. Thus, for the first approximation, we can write the neutrino mixing in Eq. (1) as

$$V_{BM} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ \frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix}, \quad (2)$$

which is known as bimaximal mixing matrix (BM). The tribimaximal mixing (TBM) also formulated in accordance with the experimental results of neutrino oscillation $\theta_{13} \approx 0^\circ$ and unitarity constraints.

Theoretical derivation of TBM can also be read in [2] as

$$V_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}, \qquad (3)$$

and the democratic (DC) mixing also with the approximation $\theta_{13} \approx 0^\circ$ reads

$$V_{DC} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} & -\sqrt{\frac{2}{3}} \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} \end{pmatrix}. \qquad (4)$$

From Eq. (2)-(4), one can see that three well-known mixing matrices predict mixing angle $\theta_{13} \approx 0^\circ$ which is incompatible with the present data of neutrino oscillations.

The latest results from neutrino oscillation experiment reported by T2K Collaboration[3]

$$5^\circ \leq \theta_{13} \leq 16^\circ, \qquad (5)$$

for neutrino mass in normal hierarchy (NH), and

$$5.8^\circ \leq \theta_{13} \leq 17.8^\circ, \qquad (6)$$

for inverted hierarchy (IH) with Dirac phase $\delta = 0^\circ$. The nonzero value of mixing angle θ_{13} was also confirmed by Daya Bay Collaboration [4] as

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016\,(\text{stat}) \pm 0.005\,(\text{syst}). \qquad (7)$$

Thus, we should modify or reexamine the three well-known neutrino mass matrices in Eqs. (2)-(4) because it cannot accommodate the present data precisely.

III. MIXING MATRIX WITH NONZERO θ_{13}

Recently, it is apparent that the mixing angle θ_{13} is nonzero and relatively large as shown in Eqs. (5) and (6). Referring to Eq. (1) and nonzero mixing angle θ_{13} and put $\delta = 0^\circ$, we take the approximation for

$$s_{13} \approx \varepsilon, \qquad (8)$$

where $\varepsilon << 1$, and consequently

$$c_{13} = \sqrt{1-\varepsilon^2}. \qquad (9)$$

By inserting Eqs. (8) and (9) into neutrino mixing matrix of Eq. (1) we have approximate neutrino mixing matrix (V_A) as

$$V_A = \begin{pmatrix} c_{12}\sqrt{1-\varepsilon^2} & s_{12}\sqrt{1-\varepsilon^2} & \varepsilon \\ -s_{12}c_{23} - c_{12}s_{23}\varepsilon & c_{12}c_{23} - s_{12}s_{23}\varepsilon & s_{23}\sqrt{1-\varepsilon^2} \\ s_{12}s_{23} - c_{12}c_{23}\varepsilon & -c_{12}s_{23} - s_{12}c_{23}\varepsilon & c_{23}\sqrt{1-\varepsilon^2} \end{pmatrix}, \qquad (10)$$

By putting mixing angle $\theta_{23} \approx \pi/4$ (atmospheric neutrino mixing angle nearly maximal) which imply: $c_{23} = s_{23} \approx \frac{1}{2}\sqrt{2}$, then the approximate mixing angle in Eq. (10) reads

$$V_A = \begin{pmatrix} c_{12}\sqrt{1-\varepsilon^2} & s_{12}\sqrt{1-\varepsilon^2} & \varepsilon \\ -\frac{\sqrt{2}}{2}(s_{12} + c_{12}\varepsilon) & \frac{\sqrt{2}}{2}(c_{12} - s_{12}\varepsilon) & \frac{\sqrt{2}}{2}\sqrt{1-\varepsilon^2} \\ \frac{\sqrt{2}}{2}(s_{12} - c_{12}\varepsilon) & -\frac{\sqrt{2}}{2}(c_{12} + s_{12}\varepsilon) & \frac{\sqrt{2}}{2}\sqrt{1-\varepsilon^2} \end{pmatrix}. \qquad (11)$$

If we insert the experimental value of $\theta_{12} = 35^\circ$, then we can approximate

$$s_{12} \approx \frac{\sqrt{3}}{3} \text{ and } c_{12} \approx \sqrt{\frac{2}{3}}, \qquad (12)$$

and finally we have approximate neutrino mixing matrix

$$V_A = \begin{pmatrix} \sqrt{\frac{2}{3}(1-\varepsilon^2)} & \sqrt{\frac{1}{3}(1-\varepsilon^2)} & \varepsilon \\ -\frac{\sqrt{6}}{6} - \frac{\sqrt{3}}{3}\varepsilon & \frac{\sqrt{3}}{3} - \frac{\sqrt{6}}{6}\varepsilon & \sqrt{\frac{1}{2}(1-\varepsilon^2)} \\ \frac{\sqrt{6}}{6} - \frac{\sqrt{3}}{3}\varepsilon & -\frac{\sqrt{3}}{3} - \frac{\sqrt{6}}{6}\varepsilon & \sqrt{\frac{1}{2}(1-\varepsilon^2)} \end{pmatrix}. \qquad (13)$$

The neutrino mass matrix (M_ν) is related to the neutrino mixing matrix via equation

$$M_\nu = V_A M V_A^T, \qquad (14)$$

where M is the neutrino mass matrix in mass basis and V is the mixing matrix. If we take neutrino mass matrix in mass eigenstate (mass basis)

$$M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \qquad (15)$$

then we have neutrino mass matrix in flavor basis as

$$M_\nu = \begin{pmatrix} P & Q & R \\ Q & S & T \\ R & T & U \end{pmatrix}, \qquad (16)$$

where

$$P = \tfrac{2}{3}m_1 + \tfrac{1}{3}m_2,$$
$$Q = \left(-\tfrac{\sqrt{2}}{3}m_1 - \tfrac{\sqrt{2}}{6}m_2 + \tfrac{\sqrt{2}}{2}m_3\right)\varepsilon - \tfrac{1}{3}m_1 - \tfrac{1}{3}m_2,$$
$$R = \left(-\tfrac{\sqrt{2}}{3}m_1 - \tfrac{\sqrt{2}}{6}m_2 + \tfrac{\sqrt{2}}{2}m_3\right)\varepsilon + \tfrac{1}{3}m_1 - \tfrac{1}{3}m_2,$$
$$S = \tfrac{\sqrt{2}}{3}(m_1 - m_2)\varepsilon + \tfrac{1}{6}m_1 + \tfrac{1}{3}m_2 + \tfrac{1}{2}m_3,$$
$$T = -\tfrac{1}{6}m_1 - \tfrac{1}{3}m_2 + \tfrac{1}{2}m_3,$$
$$U = -\tfrac{\sqrt{2}}{3}(m_1 - m_2)\varepsilon + \tfrac{1}{6}m_1 + \tfrac{1}{3}m_2 + \tfrac{1}{2}m_3,$$

and we have neglected the ε^2 terms because $\varepsilon^2 << 1$.

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Due to the smallness of squared mass difference Δm_{21}^2 compared to Δm_{32}^2, we simplify neutrino mass matrix in Eq. (16) by putting $m_1 \approx m_2$ and then we have

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$$M_\nu = \begin{pmatrix} m_2 & \frac{\sqrt{2}}{2}(m_3 - m_2)\varepsilon & \frac{\sqrt{2}}{2}(m_3 - m_2)\varepsilon \\ \frac{\sqrt{2}}{2}(m_3 - m_2)\varepsilon & \frac{1}{2}(m_2 + m_3) & -\frac{1}{2}(m_2 - m_3) \\ \frac{\sqrt{2}}{2}(m_3 - m_2)\varepsilon & -\frac{1}{2}(m_2 - m_3) & \frac{1}{2}(m_2 + m_3) \end{pmatrix}, \quad (17)$$

By inspecting resulted neutrino mass matrix in Eq. (17), we can see that the symmetry of resulted neutrino mass matrix is $\mu-\tau$ symmetry. The $\mu-\tau$ and its extension (broken or partial) as an underlying symmetry for neutrino mass matrix have been studied extensively by many authors, for examples see Refs. [5-12]. If we impose zero pattern texture into resulted neutrino mass matrix in Eq. (17) in order to account the nonzero mixing angle θ_{13}, we have only the option to put $M_\nu(1,2) = M_\nu(1,,3) = 0$ and then we have

$$\varepsilon = \frac{m_2}{m_3}, \quad (18)$$

or

$$\sin\theta_{13} = \frac{m_2}{m_3}. \quad (19)$$

From Eq. (19), we can determine the value of mixing angle θ_{13} by using the advantage of experimental data [13]

$$\Delta m_{32}^2 = 2.46 \pm 0.12 \times 10^{-3}\ \text{eV}^2, \quad (20)$$

which then proceed the mixing angle

$$\theta_{13} = 9.96^\circ, \quad (21)$$

which is compatible with the experimental data reported by T2K Collaboration [3]. The relation in Eq. (19) is a new result that can be tested in future neutrino experiments.

IV. CONCLUSIONS

We have studied systematically neutrino mixing matrix by using the standard neutrino mixing matrix as a basis and considering the experimental fact to formulate approximate neutrino mixing matrix. The resulted approximate neutrino mixing matrix can predict the nonzero mixing angle θ_{13} which is compatible with the experimental data and the underlying symmetry of resulted neutrino mass matrix is $\mu-\tau$ symmetry when we impose $m_1 \approx m_2$. We also obtained the nonzero mixing angle θ_{13} related to neutrino masses, i.e. $\sin\theta_{13} = m_2/m_3$ that can be tested in future experiments.

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