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Variational iteration method used to solve the one-dimensional acoustic equations

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Abstract. Variational iteration method has been successful to solve a wide range of linear and nonlinear problems. This method is convergent to the exact solution, and furthermore, if there is an explicit form of the exact solution, then the method converges very rapidly. In this paper, we consider acoustic equations. Our contribution is a new application of the variational iteration method for solving the acoustic equations. Results show that the variational iteration method gives physically correct approximate solutions to the acoustic equations. We demonstrate the method solving the acoustic equations using several iterations.

1. Introduction

A mathematical model needs to be solved to find the solution to a real problem. It is generally difficult to find analytical exact solutions to mathematical models if the models contain many variables and have complex structures. In this case, an approximate approach gives a way to find solutions to real problems.

Elasticity equations are a model for wave propagation. Elasticity equations can be simplified into acoustic equation under some assumptions. Acoustics include sound waves. Acoustic equations model pressure and velocity changes of a system. One application of acoustic equations is the vibration of a cable wire that causes sound waves.

One of methods that can solve the acoustic equations is the variational iteration method. The variational iteration method was developed by Ji-Huan He [1–4]. This method has been used by many researchers to solve problems. The method is applicable for solving a large range of equations, including the acoustic equations. Solutions obtained from this method are approximations of the exact solution.

In this research, we focus on solving the acoustic equations [5–8], in particular the model discussed by LeVeque [5]. To our knowledge, our work is the first in implementing the variational iteration method to solve the acoustic equations. We construct the approximate solutions using the theory of the variational iterations. We then use the Maple software to compute the iterations and generate the plots of the approximate solutions. Several iterations are enough to show that the approximate solutions produced by the variational iteration method are physically correct.

The rest of the paper is organized as follows. First we recall how the variational iteration method works in solving a partial differential equation. Then we construct approximate solutions

to the acoustic equations. Afterward, we present some computational results. Finally some concluding remarks are drawn.

2. Variational iteration method

The variational iteration method does not need numerical discretization, as the method is an analytical approach [9, 10]. This method has been proved to be reliable, accurate and effective used to find solutions to differential equations. The method needs an initial value of the differential equations.

Let us illustrate some basic concepts of variational iteration method. Consider the following equation

$$Lu + Nu = g(x). \tag{1}$$

Here L is a linear operator, N is a nonlinear operator, $g(x)$ is a nonhomogeneous term and u is a function of x. From (1), we can construct a correction functional as follows

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda \{Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)\}d\tau, \quad n \geq 0 \tag{2}$$

with λ being a general Lagrange multiplier. The index n denotes the n-th order iteration, and then $\tilde{u}_n$ is assumed as a restriction variation, that is, $\delta\tilde{u}_n = 0$. We note that integration by parts have the following forms

$$\int \lambda(\tau)u_n'(\tau)d\tau = \lambda(\tau)u_n(\tau) - \int \lambda'(\tau)u_n(\tau)d\tau \tag{3}$$

$$\int \lambda(\tau)u_n''(\tau)d\tau = \lambda(\tau)u_n'(\tau) - \lambda'(\tau)u_n(\tau) + \int \lambda''(\tau)u_n(\tau)d\tau \tag{4}$$

To show how the variational iteration method works, let us consider the following partial differential equation (see Wazwaz [4]):

$$u_x - u_t = 0, \quad u(0,t) = t, \quad u(x,0) = x. \tag{5}$$

We construct the correction functional from (5) to obtain

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\tau)(u_{n\tau} - \tilde{u}_{nt})d\tau. \tag{6}$$

From (3) and (4) then we obtain the stationary conditions

$$\lambda'(\tau) = 0, \tag{7}$$

$$1 + \lambda(\tau)|_{\tau=t} = 0. \tag{8}$$

The Lagrange multiplier for the above equations is

$$\lambda = -1. \tag{9}$$

Then, we can substitute the Lagrange multiplier (9) into (6) to have the iteration formula

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t (u_{n\tau} - u_{nt})d\tau, \quad n \geq 0. \tag{10}$$

Iterating (10) we have

$$u_0(x,t) = t, \tag{11}$$

$$u_1(x, t) = x + t, \tag{12}$$

$$u_2(x, t) = x + t, \tag{13}$$

$$u_3(x, t) = x + t, \tag{14}$$

$$\vdots \tag{15}$$

$$u_n(x, t) = x + t. \tag{16}$$

Therefore, the exact solution to (5) is $u(x, t) = x + t$.

3. Acoustic equations

The acoustic equations, in the simplest form, are [11, 12]

$$p_t + u_x = 0, \tag{17}$$

$$u_t + p_x = 0, \tag{18}$$

where $p(x, t)$ is the pressure, $u(x, t)$ is the velocity, t is time variable and x is the space variable. We assume that the initial condition is given by $p(x, 0) = 0.1 \operatorname{sech}^2(0.2x)$ and $u(x, 0) = 0$.

We construct a correction functional to solve (17) and (18) using the variational iteration method

$$p_{n+1}(x, t) = p_n(x, t) + \int_0^t \lambda_1(\tau) \left(\frac{\partial p_n(x, \tau)}{\partial \tau} + \frac{\partial \tilde{u}_n(x, \tau)}{\partial x} \right) d\tau, \tag{19}$$

$$u_{n+1}(x, t) = u_n(x, t) + \int_0^t \lambda_2(\tau) \left(\frac{\partial u_n(x, \tau)}{\partial \tau} + \frac{\partial \tilde{p}_n(x, \tau)}{\partial x} \right) d\tau. \tag{20}$$

Here λ_1 and λ_2 are Lagrange multipliers. In (19) we make the functional to be stationary to p_n. Furthermore, in (20) we make the functional to be stationary to u_n, so we have

$$\delta p_{n+1}(x, t) = \delta p_n(x, t) + \delta \int_0^t \lambda_1(\tau) \frac{\partial p_n(x, \tau)}{\partial \tau} d\tau + \delta \int_0^t \lambda_1(\tau) \frac{\partial \tilde{u}_n(x, \tau)}{\partial x} d\tau, \tag{21}$$

$$\delta u_{n+1}(x, t) = \delta u_n(x, t) + \delta \int_0^t \lambda_2(\tau) \frac{\partial u_n(x, \tau)}{\partial \tau} d\tau + \delta \int_0^t \lambda_2(\tau) \frac{\partial \tilde{p}_n(x, \tau)}{\partial x} d\tau, \tag{22}$$

where $\tilde{u}_n$ and $\tilde{p}_n$ are assumed as restriction variations. Noticing that $\delta \tilde{p}_n$ and $\delta \tilde{u}_n = 0$, we obtain

$$\delta p_{n+1}(x, t) = \delta p_n(x, t) + \delta \int_0^t \lambda_1(\tau) \frac{\partial p_n(x, \tau)}{\partial \tau} d\tau, \tag{23}$$

$$\delta u_{n+1}(x, t) = \delta u_n(x, t) + \delta \int_0^t \lambda_2(\tau) \frac{\partial u_n(x, \tau)}{\partial \tau} d\tau. \tag{24}$$

Using integration by parts, (23) and (24) can be written as follows

$$\delta p_{n+1}(x, t) = [1 + \lambda_1(\tau)] \delta p_n(x, t) - \delta \int_0^t \lambda_1'(\tau) p_n(x, t) d\tau, \tag{25}$$

and

$$\delta u_{n+1}(x, t) = [1 + \lambda_2(\tau)] \delta u_n(x, t) - \delta \int_0^t \lambda_2'(\tau) u_n(x, t) d\tau. \tag{26}$$

Using (25), we obtain the stationary conditions

$$p_n(\tau, t) : \begin{cases} \lambda_1'(\tau) = 0, \\ 1 + \lambda_1(\tau) = 0. \end{cases} \tag{27}$$

Equation (27) leads to

$$\lambda_1(\tau) = -1. \tag{28}$$

Using (26), we obtain the stationary conditions

$$u_n(\tau, t) : \begin{cases} \lambda_2'(\tau) = 0, \\ 1 + \lambda_2(\tau) = 0. \end{cases} \tag{29}$$

Equation (29) leads to

$$\lambda_2(\tau) = -1. \tag{30}$$

Now, we obtain the value of the Lagrange multiplier $\lambda_1 = \lambda_2 = -1$. Substituting that value into (19) and (20), we find

$$p_{n+1}(x, t) = p_n(x, t) - \int_0^t \left(\frac{\partial p_n(x, \tau)}{\partial \tau} + \frac{\partial u_n(x, \tau)}{\partial x} \right) d\tau, \quad n \geq 0, \tag{31}$$

$$u_{n+1}(x, t) = u_n(x, t) - \int_0^t \left(\frac{\partial u_n(x, \tau)}{\partial \tau} + \frac{\partial p_n(x, \tau)}{\partial x} \right) d\tau, \quad n \geq 0. \tag{32}$$

Equations (31) and (32) are the variational iteration formulas for (17) and (18).

4. Computational results

In this section we present some computational experiment to demonstrate the calculations of the variational iteration method. Here we calculate the first, second and third iterations using the variational iteration formulas (31) and (32). All quantities are assumed to have SI units with the MKS system.

For our computational experiment, we are given the initial condition

$$p(x, 0) = 0.1 \operatorname{sech}^2(0.2x), \tag{33}$$

and

$$u(x, 0) = 0. \tag{34}$$

We take hyperbolic secant function for initial pressure and zero function for velocity because these functions are smooth, so they have continuous derivatives. Therefore, they can be used in the demonstration for our method. The amplitude and phase constant in (33) are taken to be 0.1 and 0.2, respectively.

Using the Maple software, we obtain the following results:

$$p_0(x, t) = 0.1 \operatorname{sech}^2(0.2x), \tag{35}$$

$$u_0(x, t) = 0, \tag{36}$$
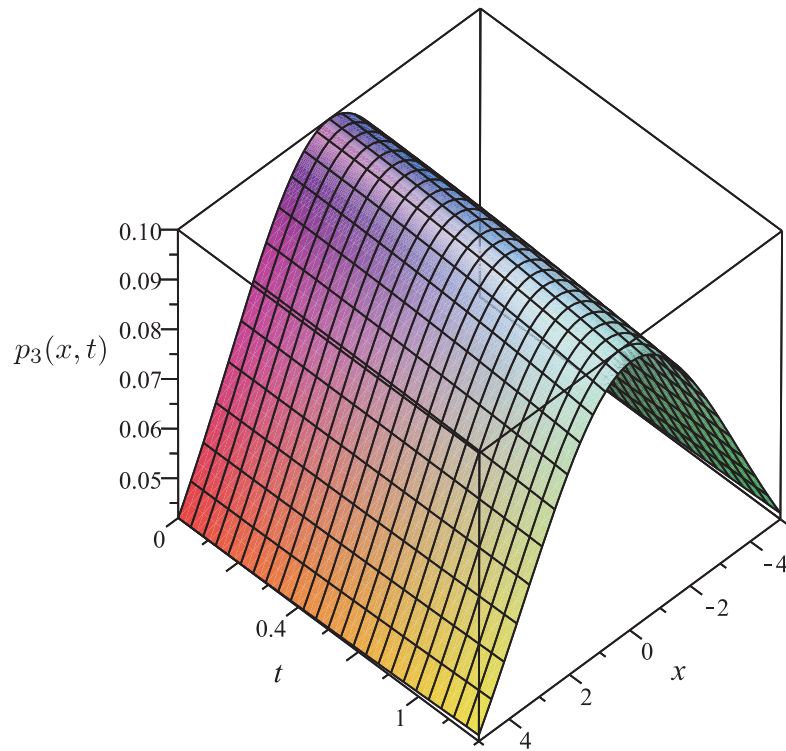
$$p_1(x, t) = 0.1 \operatorname{sech}^2(0.2x), \tag{37}$$

Figure 1. Variational iteration solution for $p_3(x,t)$.

$$u_1(x,t) = 0.04\,\mathrm{sech}(0.2x)^2\tanh(0.2x)t, \tag{38}$$

$$
\begin{aligned}
p_2(x,t) &= 0.1\,\mathrm{sech}(0.2x)^2 + 0.008\,\mathrm{sech}(0.2x)^2\tanh(0.2x)^2\,t^2 \\
&\quad - 0.02\,\mathrm{sech}(0.2x)^2\left(0.2 - 0.2\tanh(0.2x)^2\right)t^2,
\end{aligned}
\tag{39}
$$

$$u_2(x,t) = 0.04\,\mathrm{sech}(0.2x)^2\tanh(0.2x)t, \tag{40}$$

$$
\begin{aligned}
p_3(x,t) &= 0.1\,\mathrm{sech}(0.2x)^2 + 0.008\,\mathrm{sech}(0.2x)^2\tanh(0.2x)^2\,t^2 \\
&\quad - 0.02\,\mathrm{sech}(0.2x)^2\left(0.2 - 0.2\tanh(0.2x)^2\right)t^2,
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
u_3(x,t) &= 0.04\,\mathrm{sech}(0.2x)^2\tanh(0.2x)t + 0.0011\,\mathrm{sech}(0.2x)^2\tanh(0.2x)^3\,t^3 \\
&\quad - 0.0011\,\mathrm{sech}(0.2x)^2\tanh(0.2x)\left(0.2 - 0.2\tanh(0.2x)^2\right)t^3.
\end{aligned}
\tag{42}
$$

In this paper, we stop our calculations up to the third iteration. The approximate solution for the pressure is $p_3(x,t)$ and the velocity is $u_3(x,t)$. The wave propagation for the pressure is
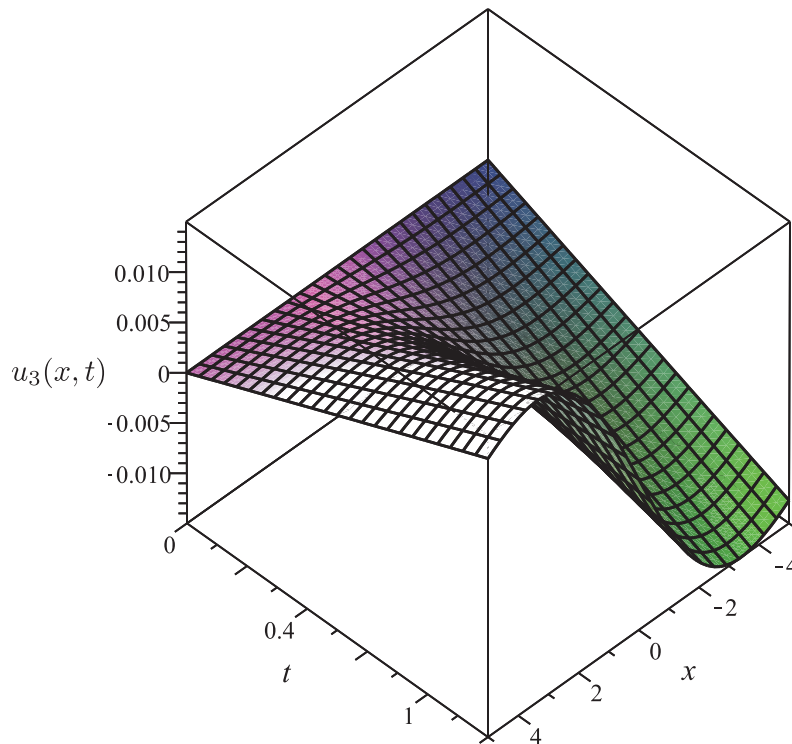
Figure 2. Variational iteration solution for $u_3(x,t)$.

illustrated in figure 1, and for the velocity in figure 2. As time evolves, the pressure from the center of the domain is spread out to the left and to the right. The pressure wave propagating to the left has negative velocity, whereas the one to the right has positive velocity. This behavior is obviously correct physically (moving to the right means that the velocity is positive, and moving to the left means the velocity is negative). In figure 2, the velocity tends to zero for large x and t.

5. Conclusion
We have solved the acoustic equations using the variational iteration method. This method is meshless, which means that it does not need any numerical discretization of the domain. To obtain more accurate results, we just need to do more iterations. Using the variational iteration method, we can compute the solution to the acoustic equations at every value of time and every position of space.

Acknowledgments

References
[1] He J H 1997 *Commun. Nonlinear Sci. Numer. Simul.* **2** 230
[2] He J H 1999 *Int. J. Non Linear Mech.* **34** 699
[3] He J H 2007 *J. Comput. Appl. Math.* **207** 3
[4] Wazwaz A M 2009 *Partial Differential Equations and Solitary Waves Theory* (Beijing: Higher Education Press and Berlin: Springer)

[5] LeVeque R J 2002 *Int. J. Numer. Methods Fluids* **40** 93
[6] LeVeque R J 1997 *J. Comput. Phys.* **131** 237
[7] Fogarty T and LeVeque R J 1998 *Proc. 4th Int. Conf. on Mathematical and Numerical Aspects of Wave Propagation* ed DeSanto J A (Golden) p 603
[8] Fogarty T and LeVeque R J 1999 *J. Acoust. Soc. Am.* **106** 17
[9] Setianingrum P S and Mungkasi S 2016 *AIP Conf. Proc.* **1746** 020057
[10] Mungkasi S and Wiryanto L H 2016 *AIP Conf. Proc.* **1707** 050010
[11] Supriyadi B and Mungkasi S 2016 *Int. J. Multiscale Comput. Eng.* **14** 479
[12] Mungkasi S and Ningrum G I J 2016 *AIP Conf. Proc.* **1746** 020056