Neutrino masses from a cobimaximal neutrino mixing matrix

Asan Damanik*

Department of Physics Education, Sanata Dharma University
Kampus III USD Paingan Maguwoharjo Sleman Yogyakarta, Indonesia

Abstract

Recently, we have a confidence that neutrino has a tiny mass and mixing does exist among neutrino flovors as one can see from experimental data that reported by many collaborations. Based on experimental data that flavor mixing does exist in neutrino sector which imply that all three mixing angles are nonzero, we derive the neutrino mass matrix from a cobimaximal neutrino mixing matrix. We also evaluate the prediction of neutrino mass matrix with texture zero from a cobimaximal neutrino mixing matrix on neutrino masses and effective Majorana mass. By using the advantages of experimental data, the obtained neutrino masses are $m_1 = 0.028188$ eV, $m_2 = 0.029488$ eV, and $m_3 = 0.057676$ eV, and the effective Majorana mass is $\langle m_{\beta\beta} \rangle = 0.09896$ eV that can be tested in future neutrinoless double beta decay experiments

Keywords: Neutrino masses, approximate neutrino mixing matrix, nonzero θ_{13}
PACS: 14.60.Lm; 14.60.Pq

1   Introduction

One of the main problem in neutrino physics till today is the explicit form of the neutrino mixing matrix which can accommodate the recent experimental data. We have already known three well-known neutrino mixing matrices i.e. bimaximal mixing, tribimaximal mixing, and democratic mixing, but now all of the three well-known mixing matrices are inappropriate anymore when confronted to the recent experimental data, especially a nonzero mixing angle θ_{13} as reported by T2K [1] and Daya Bay [2] colloaborations. In order to obtain a mixing matrix which can proceed a consistent predictions with experimental data, Ma [3] proposed a new mixing matrix which is knowm as cobimaximal mixing by assumming mixing angle $\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, and the Dirac phase $\delta = \pm\pi/2$. In [3] also claimed that the cobimaximal neutrino mixing matrix is achieve rigourously in a renormalizable model of radiative charged-lepton and neutrino masses.

In this paper, we evaluate the neutrino mass matrix which is obtained from cobimaximal neutrino mixing matrix. The obtained neutrino mass matrix is used to predict the neutrino masses and an effective neutrino mass that can be tested in future neutrinoless double beta decaya experiments. In section II we derive the neutrino mass matrix with neutrino mixing matrix is a cobimaximal mixing. In section III, we evaluate the power prediction of neutrino mass matrix with texture zero on neutrino masses and effective Majorana mass of neutrinoless double beta decay. Finally, section IV is devoted for conclusions.

2   Neutrino mass matrix from a cobimaximal neutrino mixing matrix

Theoretically, neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ are related to neutrino mass eigenstates (ν_1, ν_2, ν_3) via a neutrino mixing matrix V as follow

$$\nu_\alpha = V_{\alpha\beta}\nu_\beta, \tag{1}$$

---

*E-mal: asandamanik11@gmail.com

where the indexes $\alpha = e, \mu, \tau$, $\beta = 1, 2, 3$, and $V_{\alpha\beta}$ are the elements of the neutrino mixing V. The standard parameterization of the mixing matrix (V) read [4]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2}$$

where c_{ij} is the $\cos\theta_{ij}$, s_{ij} is the $\sin\theta_{ij}$, θ_{ij} are the mixing angles, and δ is the Dirac CP-violating phase.

If we use the value of mixing angle $\theta_{23} = \pi/4$, and the Dirac phase $\delta = \pi/2$, then the neutrino mixing matrix in Eq. (2) has the following form

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & is_{13} \\ -\frac{\sqrt{2}}{2}(s_{12} - ic_{12}s_{13}) & \frac{\sqrt{2}}{2}(c_{12} + is_{12}s_{13}) & \frac{\sqrt{2}}{2}c_{13} \\ \frac{\sqrt{2}}{2}(s_{12} + ic_{12}s_{13}) & -\frac{\sqrt{2}}{2}(c_{12} - is_{12}s_{13}) & \frac{\sqrt{2}}{2}c_{13} \end{pmatrix} \tag{3}$$

which is known as a cobimaximal neutrino mixing matrix. In the basis where the charged lepton mass matrix is already diagonalized, the neutrino mass matrix defined by the mass term in the Lagrangian is given by

$$M_\nu = VMV^T, \tag{4}$$

where M is neutrino mass matrix in mass basis

$$M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{5}$$

By inserting Eqs. (3) and (20) into Eq. (4) we have the following neutrino mass matrix [5]

$$M = \begin{pmatrix} a & b+i\beta & -(b-i\beta) \\ b+i\beta & c-i\gamma & d \\ -(b-i\beta) & d & c+i\gamma \end{pmatrix}, \tag{6}$$

where

$$a = c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 - s_{13}^2 m_3,$$

$$b = -\frac{1}{\sqrt{2}} s_{12}^2 c_{12}^2 c_{13}^2 (m_1 - m_2),$$

$$c = \frac{1}{2} \left((s_{12}^2 - c_{12}^2 s_{13}^2) m_1 + (c_{12}^2 - s_{12}^2 s_{13}^2) m_2 + c_{13}^2 m_3 \right),$$

$$d = -\frac{1}{2} \left((s_{12}^2 + c_{12}^2 s_{13}^2) m_1 + (c_{12}^2 + s_{12}^2 s_{13}^2) m_2 - c_{13}^2 m_3 \right), \tag{7}$$

$$\beta = \frac{1}{\sqrt{2}} s_{13}^2 c_{13}^2 \left(c_{12}^2 m_1 + s_{12}^2 m_2 - m_3 \right),$$

$$\gamma = -s_{12} c_{12} s_{13} (m_1 - m_2).$$

3 Neutrino masses and effective Majorana mass

We are in position to impose the texture zero into neutrino mass matrix of Eq. (6). By inspecting neutrino mass matrix in Eq. (6) one can see that the realistic neutrino mass matrix with texture zero is by putting the elements of neutrino mass matrix as follow

$$M_\nu(1, 1) = a = 0, \tag{8}$$

and

$$M_\nu(2, 3) = M_\nu(3, 2) = d = 0, \tag{9}$$

By imposing the texture zero as shown in Eqs. (8) and (9), the neutrino mass matrix have the following form

$$M = \begin{pmatrix} 0 & b+i\beta & -(b-i\beta) \\ b+i\beta & c-i\gamma & 0 \\ -(b-i\beta) & 0 & c+i\gamma \end{pmatrix}. \tag{10}$$

Now, we want to derive relations of neutrino masses as function of mixing angle which then can be used to determine the neutrino masses and its hierarchy by using the experimental data of neutrino oscillation especially the experimental value of squared mass differences. From Eqs. (8), (8), and (9) we can obtain the following relation

$$s_{13}^2 m_3 = c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2, \tag{11}$$

and

$$c_{13}^2 m_3 = \left(s_{12}^2 + c_{12}^2 s_{13}^2\right) m_2 + \left(c_{12}^2 + s_{12}^2 s_{13}^2\right) m_2, \tag{12}$$

which then proceed

$$m_3 = m_1 + m_2. \tag{13}$$

The Eq. (13) is our new neutrino mass relation when the neutrino mass matrix obtained from a cobimaximal mixing constrained by two texture zero as shown in Eqs. (8) and (9).

From Eq. (13) we can have the following relation

$$m_3^2 - m_2^2 = m_1^2 + 2m_2 m_1, \tag{14}$$

or

$$m_1^2 + 2m_2 m_1 - \Delta m_{32}^2 = 0, \tag{15}$$

where $\Delta m_{32}^2 = m_3^2 - m_2^2$ is the squared mass difference of atmospheric neutrino. The global analysis of squared mass difference for atmospheric neutrino and solar neutrino read [6]

$$\Delta m_{32}^2 = 2.457 \times 10^{-3} \text{eV}^2, \tag{16}$$

and

$$\Delta m_{21}^2 = 7.50 \times 10^{-5} \text{eV}^2, \tag{17}$$

respectively.

If we insert the value of squared mass difference of Eq. (16) into Eq. (15), then we have

$$m_1 = -m_2 + 0.001\sqrt{m_2^2 + 2457}. \tag{18}$$

From Eqs. (15), (16), (17), and (18) we can have the neutrino masses as follow

$$m_1 = 0.028188 \text{eV},$$
$$m_2 = 0.029488 \text{eV}, \tag{19}$$
$$m_3 = 0.057676 \text{eV}. \tag{20}$$

It is apparent from Eq. (20) that hierarchy of neutrino mass is a normal hierarcy.

After we have known the neutrino masses, we then calculate the prediction of cobimaximal mixing on effective Majorana mass because the effective Majorana mass is a parameter of interest in neutrinoless double beta decay. The effective Majorana mass $\langle m_{\beta\beta} \rangle$ is a combination of neutrino mass eigenstates and the neutrino mixing matrix terms as follow [7]

$$\langle m_{\beta\beta} \rangle = \left| \Sigma V_{ei}^2 m_i \right|, \tag{21}$$

where V_{ei} is the i-th element of the first row of neutrino mixing matrix, and m_i is the i-th of the neutrino mass eigenstate. From Eqs. (3) and (21) we then have an effective Majorana mass

$$\langle m_{\beta\beta} \rangle = \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 - s_{13}^2 m_3 \right|. \tag{22}$$

If we take the central values of mixing angle $\theta_{13} = 9^0$ [2] and $\theta_{12} = 35^0$ [6] and the neutrino masses as shown in Eq. (20), then we have the effective Majorana mass as follow

$$\langle m_{\beta\beta} \rangle = 0.09896 \text{eV}, \tag{23}$$

that can be tested in future neutrinoless double beta decay experiments.

3

4 Conclusions

We have use a cobimaximal neutrino mixing matrix to obtain a neutrino mass matrix. The obtained neutrino mass matrix is constrained by two texture zero i.e. $M_\nu(1,1) = 0$ and $M_\nu(2,3) = M_\nu(3,2) = 0$ and by using the advantages of the experimental data of squared mass difference, then we can obtain neutrino masses in normal hierarchy i.e. $m_1 = 0.028188\text{eV}$, $m_2 = 0.029488\text{eV}$, and $m_3 = 0.057676\text{eV}$. By using the central values of reported experimental mixing angle θ_{13} and θ_{23} and obtained neutrino masses, the effective Majorana mass is $\langle m_{\beta\beta} \rangle = 0.09896\text{eV}$ that can be tested in future neutrinoless double beta decay experiments.

References

[1] T2K Collab. (K. Abe *et al.*), Indication of electron neutrino apperance from an accelerator produced off-axis muon neutrino beam, *Phys. Rev. Lett.* **107** (2011), 041801.

[2] Daya Bay Collab. (F. P. An *et al.*), Observation of electron-antineutrino disappearance at Daya Bay, *Phys. Rev. Lett.* **108 (2012)**, 171803.

[3] E. Ma, Soft $A_4 \to Z_3$ symmetry breaking and cobimaximal neutrino mixing, *Phys. Lett.* **B 755** (2016), pp. 348-350.

[4] P. F. Harrison, D. H. Perkins, and W. G. Scott, Tri-Bimaximal mixing and the neutrino oscillation data, *Phys. Lett.* **B 530** (2002), 167-173.

[5] X.-G. He, Model of neutrino mass matrix with $\delta = -\pi/2$ and $\theta_{23} = \pi/4$, *Chinese. J. Phys.* **53** (2015), 100202.

[6] M. Gonzales-Garcia, M. Maltoni and J. Salvado, Updated global fit to three neutrino mixing: status of the hints of $\theta_{13} > 0$, *JHEP* **04** (2010), 056.

[7] G. Benato, Effective Majorana mass and neutrinoless double beta decay, *Eur. Phys. J.* **C 75** (2015), 563, doi:10.1140/epjc/s10052-015-3802-1