Neutrino Mass Hierarchy from Modified BM with Constraint $\mu-\tau$ Symmetry

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Abstract – *The bimaximal (BM) neutrino mixing matrix was formulated in order to accommodate the data of the experimental results which indicate that both solar and atmospheric neutrino oscillation in vacuum are near maximal. But, after the T2K and Daya Bay Collaborations reported that the mixing angle θ_{13} is nonzero and relatively large, many authors have modified the neutrino mixing matrix in order to accommodate experimental data. We modified the BM mixing matrix by introducing a simple perturbation matrix into BM mixing matrix. The modified BM mixing matrix can proceed the mixing angles which are compatible with the globat fit analysis data and by imposing the $\mu-\tau$ symmetry into mass matrix from modified BM, we have the neutrino mass in normal hierarchy: $m_1 < m_2 < m_3$.*

Keywords: *neutrino mass hierarchy, modified BM, $\mu-\tau$ symmetry*

I. INTRODUCTION

Neutrino oscillation phenomena can be explained if neutrino has a nonzero mass and some mixing angles exist in neutrino sector. The mixing angles for three neutrinos can be formulated in mixing matrix. The mixing matrix relate the neutrino eigen states in flavor basis (ν_e, ν_μ, ν_τ) into neutrino eigen states in mass basis (ν_1, ν_2, ν_3) as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{1}$$

where V is the mixing matrix. The standard parameterization of the mixing matrix reads

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}e^{i\delta} & c_{23}s_{13} \end{pmatrix} \tag{2}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and δ is the Dirac phase.

There are three well-known of neutrino mixing matrices, i.e. bimaximal mixing (BM), tribimaximal mixing (TBM), and democratic mixing (DM). All of the mixing matrices predict the mixing angle $\theta_{13} = 0$. Recently, the experimental results showed that the mixing angle $\theta_{13} \neq 0$ and relatively large. Several attempt have been done theoretically to accommodate the nonzero and relatively large mixing angle θ_{13} by modifying the neutrino mixing matrix including the Dirac phase δ in relation to the CP-violation in neutrino sector. Another unsolved problem in neutrino physics today is the hierarchy of neutrino mass. Experimental results showed that we have two possibilities for neutrino mass hierarchies: normal and inverted hierarchies. We have no clue in order to decide theoretically the neutrino mass hierarchy.

The neutrino masses can be determined from neutrino mass matrix (M_ν). The neutrino mass matrix are related to the neutrino mixing matrix via equation

$$M_\nu = VMV^T, \tag{3}$$

where M is the neutrino mass matrix in mass basis and V is the mixing matrix. From Eq. (3), we can see that the neutrino mass pattern in flavor basis depend on the pattern of the neutrino mixing matrix. We also belief that the pattern of the neutrino mass matrix should be related to one of the unique underlying symmetry. One of the interesting symmetry is the $\mu-\tau$ symmetry because it can reduce the number of parameters in neutrino mass matrix and can predict qualitatively the neutrino masses that compatible with the experimental results.

In this paper, we determine the neutrino mass hierarchy from the neutrino mass matrix that obtained from the modified neutrino mixing BM with additional constraint: $\mu-\tau$ symmetry as the underlying symmetry of the resulted neutrino mass matrix. In section II, we modified BM by introducing a simple perturbation matrix and derive the corresponding neutrino mass matrix. In section III, we evaluate the neutrino mass hierarchy from the modified BM and discuss some phenomenological consequences. Finally, section IV is devoted for conclusions.

II. MODIFIED BM MIXING MATRIX

The bimaximal (BM) mixing matrix was formulated to accommodate the facts that both solar and atmospheric data can be described by maximal mixing vacuum oscillation with the relevant mass scale and it imply that there is a unique mixing matrix which is then called bimaximal mixing matrix. The BM mixing matrix (V_{BM}) reads [1]

$$V_{BM}=\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}}\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}\end{pmatrix}. \qquad (4)$$

As one can see from Eq. (4) that $\sin\theta_{13}e^{-i\delta}=0$, which implies that the BM mixing matrix leads to mixing angle $\theta_{13}=0$ which is incompatible with the recent experimental results of T2K Collaboration [2]

$$5^o\le\theta_{13}\le 16^o, \qquad (5)$$

for neutrino mass in normal hierarchy (NH), and

$$5.8^o\le\theta_{13}\le 17.8^o, \qquad (6)$$

for inverted hierarchy (IH) with Dirac phase $\delta=0$. The nonzero value of mixing angle θ_{13} was also confirmed by Daya Bay Collaboration [3] as

$$\sin^2 2\theta_{13}=0.092\pm 0.016(\text{stat})\pm 0.005(\text{syst}). \qquad (7)$$

From Eq. (4) we also see that the mixing angle $\theta_{12}=\theta_{23}=\pi/4$ (bimaximal mixing).

In order to accommodate the relatively large and nonzero mixing angle θ_{13} in accordance with BM mixing matrix, many authors have modified the BM mixing matrix [4-6]. In this paper we modified BM by introducing a simple perturbation matrix to perturb BM which is different from Refs. [4-6]. The perturbation matrix (V_p) is given by (with Dirac phase: $\delta=0$)

$$V_p=\begin{pmatrix}c_p & 0 & s_p\\ 0 & 1 & 0\\ -s_p & 0 & c_p\end{pmatrix}, \qquad (8)$$

where c_p is $\cos p$ and s_p is $\sin p$. The modified BM mixing matrix (U_{BM}) is obtained via a scenario

$$U_{BM}=V_pV_{BM}, \qquad (9)$$

which then gives

$$U_{BM}=\begin{pmatrix}\frac{s_p+\sqrt{2}c_p}{2} & \frac{s_p-\sqrt{2}c_p}{2} & \frac{\sqrt{2}s_p}{2}\\ \frac{1}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2}\\ \frac{c_p-\sqrt{2}s_p}{2} & \frac{c_p+\sqrt{2}s_p}{2} & \frac{\sqrt{2}c_p}{2}\end{pmatrix}. \qquad (10)$$

From Eq. (10) one can see that the modified BM can predicts the nonzero mixing angle θ_{13}. By comparing Eq. (10) with Eq. (2), we have

$$\tan\theta_{12}=\left|\frac{s_p-\sqrt{2}c_p}{s_p+\sqrt{2}c_p}\right|, \qquad (11)$$

$$\tan\theta_{23}=\left|\frac{1}{c_p}\right|, \qquad (12)$$

$$\sin\theta_{13}=\left|\frac{\sqrt{2}s_p}{2}\right|. \qquad (13)$$

By inspecting Eqs. (11-13), we can see that the mixing angle $\theta_{13}\ne 0$, the mixing $\theta_{12}<\pi/4$, and the mixing angle $\theta_{23}>\pi/4$. On the other hand, the modified BM in this scenario proceed "no-maximal" mixing matrix.

If we insert the central value of mixing angle θ_{13} as reported by Daya Bay Collaboration as written in Eq. (7), then we have $p=12.54^o$ and the mixing angles

$$\theta_{12}=36.07^o \text{ and } \theta_{23}=45.69^o, \qquad (14)$$

which is compatible with the global fit analysis data [7]

$$\theta_{12}=34.5\pm 1.0(^{3.2}_{-2.8})^o, \qquad (15)$$

$$\theta_{23}=42.8^{+4.5}_{-2.9}(^{+10.7}_{-7.3})^o. \qquad (16)$$

for $1\sigma(3\sigma)$ level.

III. NEUTRINO MASS HIERARCHY FROM U_{BM}

As stated in section I, one of the the unsolved problem in neutrino physics is the neutrino mass hierarchy whether it is normal or inverted. By using the Eq. (3) with neutrino mixing is the modified BM (U_{BM}) with mixing angles as shown in Eq. (14) and mass matrix (*M*) in mass basis is diagonal

$$M=\begin{pmatrix}m_1 & 0 & 0\\ 0 & m_2 & 0\\ 0 & 0 & m_3\end{pmatrix}, \qquad (17)$$

then we have the neutrino mass matrix

$$M_\nu=\begin{pmatrix}P & Q & R\\ Q & S & T\\ R & T & W\end{pmatrix}, \qquad (18)$$

where

$$P=0.638\,m_1+0.338\,m_2+0.024\,m_3, \qquad (19)$$

$$Q=-0.904\,m_1+0.084\,m_2+0.108\,m_3, \qquad (20)$$

$$R=-0.115\,m_1-0.576\,m_2+0.106\,m_3, \qquad (21)$$

$$S=0.979\,m_1+0.021\,m_2+0.499\,m_3, \qquad (22)$$

$$T=0.143\,m_1+0.143\,m_2+0.488\,m_3, \qquad (23)$$

$$W=0.021\,m_1+0.979\,m_2+0.477\,m_3. \qquad (24)$$

If we impose the $\mu-\tau$ symmetry into the resulted neutrino mass matrix (M_ν) of Eq. (18), then one must put $Q=R$ or

$$Q-R=0, \qquad (25)$$

and $S=W$ or

$$S-W=0. \qquad (26)$$

Solving simultaneously Eqs. (25) and (26), we have neutrino masses relations as

$$m_2=1.1727\,m_1 \text{ and } m_3=7.5190\,m_1 \qquad (27)$$

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which imply that

$$m_1 < m_2 < m_3. \quad (28)$$

The resulted hierarchy of neutrino mass in Eq. (28) is a normal hierarchy (IH).

IV. CONCLUSION

We have modified BM mixing matrix by introducing a simple perturbation matrix into BM that can proceed nonzero mixing angle θ_{13}. From the modified BM mixing matrix and using the central value of mixing angle θ_{13} from Daya Bay Collaboration data, we determine the mixing angles θ_{12} and θ_{23} which are compatible with the global fit analysis data. The neutrino mass matrix obtained from the modified BM mixing matrix with is constrained by $\mu-\tau$ symmetry proceed neutrino mass in normal hierarchy: $m_1 < m_2 < m_3$.

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TANYA JAWAB

Pramudita Anggraita:

? Dalam model ini neutrino partikel Dirac atau Majorana?

Asan Damanik:

√ Partikel Majorana (partikel sama dengan anti-partikelnya).

Albertus H. Panuluh (UGM):

? Apabila ada neutrino steril, bagaimana bentuk matriksnya?

Asan Damanik:

√ Bentuk matriks campuran (mixing matrix) jika ada neutrino steril, yakni matriks 4 ×4 (bukan lagi 3 ×3) kalau υ_s (neutrino steril) dianggap satu saja.

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