ABSTRAK

Transform Laplace dari suatu fungsi f(t) adalah fungsi F(s) yang dinyatakan oleh $\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt$. Transform Laplace dapat digunakan untuk menyelesaikan masalah nilai awal sistem persamaan diferensial linear simultan. Bentuk umum sistem persamaan diferensial linear simultan orde pertama dengan koefisien konstan dari dua persamaan diferensial dengan dua fungsi yang tidak diketahui adalah sebagai berikut :

$$a_{11}\frac{dx}{dt} + a_{12}\frac{dy}{dt} + a_{13}x + a_{14}y = f_1(t),$$

$$a_{21}\frac{dx}{dt} + a_{22}\frac{dy}{dt} + a_{23}x + a_{24}y = f_2(t)$$

dimana a_{11}, a_{12}, ... , a_{23}, a_{24} merupakan koefisien yang berupa konstanta dan sistem tersebut juga memenuhi kondisi awal $x(0) = c_1$ dan $y(0) = c_2$. Untuk menyelesaikan masalah nilai awal sistem persamaan diferensial linear simultan tersebut pertama-tama kita kerjakan Transform Laplace pada masing-masing persamaan diferensial, yaitu

$$\mathcal{L}\left\{a_{11}\frac{dx}{dt} + a_{12}\frac{dy}{dt} + a_{13}x + a_{14}y\right\} = \mathcal{L}\{f_1(t)\},$$

$$\mathcal{L}\left\{a_{21}\frac{dx}{dt} + a_{22}\frac{dy}{dt} + a_{23}x + a_{24}y\right\} = \mathcal{L}\{f_2(t)\}.$$

Kemudian kita gunakan sifat linearitas Transform Laplace, diperoleh

$$a_{11}\,\mathcal{L}\left\{\frac{dx}{dt}\right\} + a_{12}\,\mathcal{L}\left\{\frac{dy}{dt}\right\} + a_{13}\,\mathcal{L}\{x(t)\} + a_{14}\,\mathcal{L}\{y(t)\} = \mathcal{L}\{f_1(t)\},$$

$$a_{21}\,\mathcal{L}\left\{\frac{dx}{dt}\right\} + a_{22}\,\mathcal{L}\left\{\frac{dy}{dt}\right\} + a_{23}\,\mathcal{L}\{x(t)\} + a_{24}\,\mathcal{L}\{y(t)\} = \mathcal{L}\{f_2(t)\}.$$

Berdasarkan teorema Transform Laplace dari turunan dan kondisi awalnya didapat

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = sX(s) - x(0) = sX(s) - c_1$$

$$\mathcal{L}\left\{\frac{dy}{dt}\right\} = sY(s) - y(0) = sY(s) - c_2$$

Akibatnya diperoleh sistem persamaan aljabar

$a_{11}(sX(s) - c_1) + a_{12}(sY(s) - c_2) + a_{13}X(s) + a_{14}Y(s) = F_1(s)$,

$a_{21}(sX(s) - c_1) + a_{22}(sY(s) - c_2) + a_{23}X(s) + a_{24}Y(s) = F_2(s)$.

Selanjutnya sistem persamaan aljabar tersebut kita selesaikan misalnya dengan menggunakan eliminasi untuk memperoleh Transform Laplace X(s) dan Y(s). Terakhir untuk mendapatkan penyelesaian dari sistem persamaan diferensial linear simultan tersebut kita gunakan invers Transform Laplace yang didefinisikan

sebagai berikut $f(t) = \mathcal{L}^{-1}\{F(s)\}$. Dalam kasus ini kita mencari $x(t) = \mathcal{L}^{-1}\{X(s)\}$ dan $y(t) = \mathcal{L}^{-1}\{Y(s)\}$.

Sistem persamaan diferensial linear simultan dapat diperoleh dari sistem pegas-massa, dimana pada sistem pegas-massa tersebut berlaku Hukum Hooke dan Hukum Newton II. Dengan menerapkan kedua hukum tersebut akan diperoleh persamaan diferensial linear orde kedua. Selanjutnya persamaan diferensial linear orde kedua tersebut kita reduksi sehingga menghasilkan sistem persamaan diferensial linear simultan orde pertama. Sehingga untuk mencari persamaan perpindahan benda sistem pegas-massa tersebut langkah-langkahnya sama seperti diatas.

ABSTRACT

Laplace Transform of function f(t) is function F(s) stated by $\mathcal{L}\{f(t)\}=$ F(s) $=\int_0^\infty e^{-st} f(t)\, dt$. Laplace Transform can be used to solve-initial value problem system of simultaneous linear differential equations. The general linear system with constant coefficients of two first-order differential equation in two unknown function is of the form

$$a_{11}\frac{dx}{dt}+a_{12}\frac{dy}{dt}+a_{13}x+a_{14}y=f_1(t),$$

$$a_{21}\frac{dx}{dt}+a_{22}\frac{dy}{dt}+a_{23}x+a_{24}y=f_2(t)$$

in which a_{11}, a_{12}, ..., a_{23}, a_{24} are constant coefficients and that satisfies the initial conditions $x(0) = c_1$ and $y(0) = c_2$. To solve the initial-value problem system of simultaneous linear differential equations, first we take the Laplace Transform of both sides of equation, that are

$$\mathcal{L}\left\{a_{11}\frac{dx}{dt}+a_{12}\frac{dy}{dt}+a_{13}x+a_{14}y\right\}=\mathcal{L}\{f_1(t)\},$$

$$\mathcal{L}\left\{a_{21}\frac{dx}{dt}+a_{22}\frac{dy}{dt}+a_{23}x+a_{24}y\right\}=\mathcal{L}\{f_2(t)\}.$$

Then we use the linear property of the Laplace Transform, we obtain

$$a_{11}\mathcal{L}\left\{\frac{dx}{dt}\right\}+a_{12}\mathcal{L}\left\{\frac{dy}{dt}\right\}+a_{13}\mathcal{L}\{x(t)\}+a_{14}\mathcal{L}\{y(t)\}=\mathcal{L}\{f_1(t)\},$$

$$a_{21}\mathcal{L}\left\{\frac{dx}{dt}\right\}+a_{22}\mathcal{L}\left\{\frac{dy}{dt}\right\}+a_{23}\mathcal{L}\{x(t)\}+a_{24}\mathcal{L}\{y(t)\}=\mathcal{L}\{f_2(t)\}.$$

Based on transform of derivatives theorem and using the initial conditions, we have

$$\mathcal{L}\left\{\frac{dx}{dt}\right\}=sX(s)-x(0)=sX(s)-c_1$$

$$\mathcal{L}\left\{\frac{dy}{dt}\right\}=sY(s)-y(0)=sY(s)-c_2$$

Consequencely, we obtain system algebraic equations

$a_{11}(sX(s)-c_1)+a_{12}(sY(s)-c_2)+a_{13}X(s)+a_{14}Y(s)=F_1(s),$

$a_{21}(sX(s)-c_1)+a_{22}(sY(s)-c_2)+a_{23}X(s)+a_{24}Y(s)=F_2(s).$

Next, we solve the algebraic equations for example using elimination to determine X(s) and Y(s). Finnaly, to get solutions of initial-value problem system of simultaneous linear differential equations, we use inverse Laplace Transform which is defined as $f(t)=\mathcal{L}^{-1}\{F(s)\}$. In this case we find

$x(t)=\mathcal{L}^{-1}\{X(s)\}$ dan $y(t)=\mathcal{L}^{-1}\{Y(s)\}$.

The system of simultaneous linear differential equations can be obtained from spring-mass system, in which the Hooke Law and the second Newton Law occur. Applying those two laws, second order of linear differential equations can be obtained. Next we reduce the second order of linear differential equations to determine the first order of simultaneous linear differential equations system. Therefore, to determine the spring-mass system of mass displacement equations, we must follow the steps above.