Variational Iteration Solution to the Gravity Wave-Model Equations

To cite this article: Sudi Mungkasi and Leo Hari Wiryanto 2018 *J. Phys.: Conf. Ser.* **1007** 012071

View the article online for updates and enhancements.

Related content

- A staggered grid finite difference method for solving the gravity wave-model equations
Catharina Mara Apriani and Sudi Mungkasi

- Performance of the Lax-Wendroff finite volume method for solving the gravity wave-model equations
Cecilia Heru Purwitaningsih and Sudi Mungkasi

- Generalized Variational Iteration Solution of Soliton for Disturbed KdV Equation
Mo Jia-Qi

Variational iteration solution to the gravity wave-model equations

Sudi Mungkasi[1*], **and Leo Hari Wiryanto**[2]

[1]Department of Mathematics, Faculty of Science and Technology, Sanata Dharma University,Mrican, Tromol Pos 29, Yogyakarta 55002, Indonesia
[2]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Bandung Institute of Technology,JalanGanesha 10, Bandung, Jawa Barat40132, Indonesia

E-mail: *sudi@usd.ac.id

Abstract. The gravity wave-model equations are considered. We solve these equations using the variational iteration method. The variational iteration solutions approximate the exact solution. The main advantage of using the variational iteration method is that we have an explicit function of the time and space variables as an approximate solution to the gravity wave-model problems.

1. Introduction

Shallow water waves occur in rivers and oceans. Water wave is considered shallow wave if the wave amplitude is smaller than the wave length. Shallow water waves can be governed by the shallow water wave equations. Furthermore, if the convective process is not significant, we can neglect the convective term in the shallow water wave equations and obtain the so called the gravity wave-model equations [1-3]. For a horizontal topography, the shallow water wave equations are

$$\frac{\partial}{\partial t} h(x,t) + \frac{\partial}{\partial x} (u(x,t)h(x,t)) = 0, \tag{1}$$

and

$$\frac{\partial}{\partial t}(u(x,t)h(x,t)) + \frac{\partial}{\partial x}\left(u^2(x,t)h(x,t) + \frac{1}{2}gh^2(x,t)\right) = 0. \tag{2}$$

Here h is the water depth;uh is the water discharge; gis the acceleration due to gravity; t is the time variable and x is the space variable. If we neglect the convective term u^2h, we obtain the gravity wave-model equations [1-3]

$$\frac{\partial}{\partial t} h(x,t) + \frac{\partial}{\partial x} (u(x,t)h(x,t)) = 0, \tag{3}$$

and

$$\frac{\partial}{\partial t}(u(x,t)h(x,t)) + \frac{\partial}{\partial x}\left(\frac{1}{2}gh^2(x,t)\right) = 0. \tag{4}$$

In this paper, we take the convention that all quantities have the SI units with the MKS system. In general, the exact solution to the gravity wave-model equations is not available until this paper is

written. Therefore, approximate solutions are usually desired. The aim of this study is to provide a way of approximating the exact solution to the gravity wave-model equations using the variational iteration method. The variational iteration method is chosen, because it is an analytical approach [4-7], so we do not need to discretise the time and space domains in order to solve the gravity wave-model problems.We note that some variants of variational iteration methods have been used to solve a number of different problems in physics [8-11], mathematics [12], computer science [13], and chemistry [14-15]. This paper is organised as follows. We provide the derivation of approximate solutions using the variational iteration method in Section 2. Results and discussion are given in Section 3. The paper is concluded in Section 4.

2. Variational iteration method

The variational iteration method works by first we construct the correction functionals for the gravity wave-model equations. Then we specify the value of the Lagrange multiplier of the correction functionals. Once we obtain the Lagrange multiplier, we substitute this Lagrange multiplier into the correction functionals to form the variational iteration formulas. The variational iteration formulas for the gravity wave-model equations are

$$h_{n+1}(x,t) = h_n(x,t) - \int_0^t \left(\frac{\partial h_n}{\partial \tau} - \frac{\partial (q_n)}{\partial x} \right) d\tau, \tag{5}$$

and

$$q_{n+1}(x,t) = q_n(x,t) - \int_0^t \left(\frac{\partial (q_n)}{\partial \tau} - \frac{\partial (\frac{1}{2}g h_n^2)}{\partial x} \right) d\tau. \tag{6}$$

Here $q_n = u_n h_n$ is the approximate water discharge function at the nth iteration level. The initialisation is set by taking the initial condition, that is, $h_0(x,t) = h(x,0)$ and $u_0(x,t) = u(x,0)$ so $q_0(x,t) = q(x,0) = u(x,0)h(x,0)$. When the variational iterations converge, they converge to the exact solution to the problem.

3. Results and discussion

In this section we provide our computational results and discussions.

3.1 Surface wave simulation

Surface wave is simulated using the initial water depth

$$h(x,0) = 0.1 \ + 0.25 sech(x) \ + \ exp(-x^2)/(1 + exp(-x^2)) \tag{7}$$

for all x, as shown in Figure 1. We take the gravitational constant $g = 1$. The simulation is supposed to find the solution at any time t. The initial fluid velocity is $u(x,0) = 0$, for all x. Using the MAPLE software, the variational solutions are computed for $h_0(x,t)$, $q_0(x,t)$ and $h_1(x,t)$, $q_1(x,t)$, as well as $h_2(x,t)$, $q_2(x,t)$. Our results are as follows. Note that the star " $*$ " sign is the multiplication operator in the MAPLE software. For the initialisation, we have

$$h_0(x,t) = 0.1 + 0.25 * sech(x) + \frac{exp(-x^2)}{1 + exp(-x^2)}$$

and

$$q_0(x,t) = 0.$$

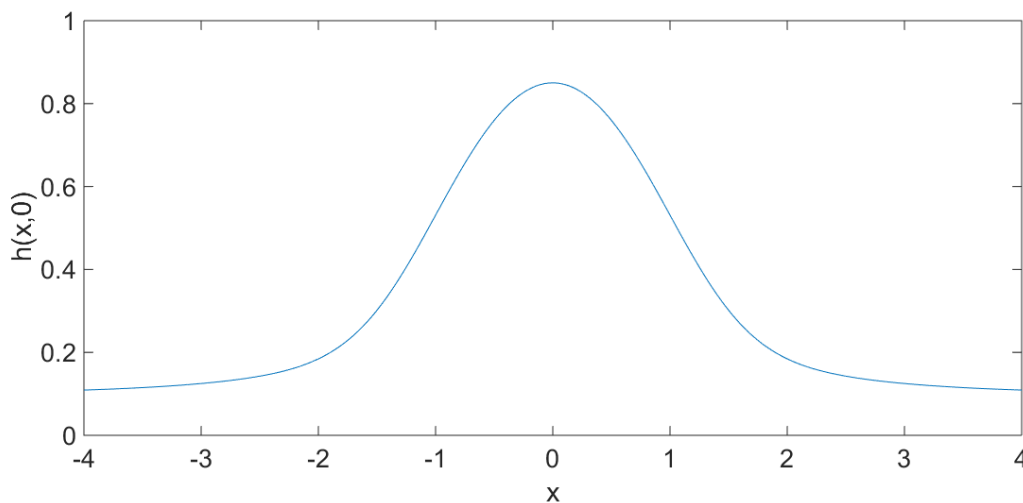The first levels of variational iteration solutions are
$$h_1(x,t) = h_0(x,t)$$
and

$$q_1(x,t) = \left(0.1 + 0.25*sech(x) + \frac{exp\,(-x^2)}{1+exp\,(-x^2)}\right)*\left(-0.25*sech(x)*tanh(x) - 2*x*\right.$$
$$\left.\frac{exp\,(-x^2)}{1+exp\,(-x^2)} + 2*\frac{exp\,(-x^2)^2}{(1+exp\,(-x^2))^2}*x\right)*t\;.$$

The second levels of variationaliteration solutions are

$$h_2(x,t) = 0.1 + 0.25*sech(x) + \frac{exp(-x^2)}{1+exp(-x^2)} - 0.5$$

$$*\left(-1\right.$$

$$*\left(-0.25*sech(x)*tanh(x) - 2*x*\frac{exp(-x^2)}{1+exp(-x^2)} + 2*\frac{exp(-x^2)^2}{(1+exp(-x^2))^2}\right.$$

$$\left.*x\right)^2 - 1*\left(0.1 + 0.25*sech(x) + \frac{exp(-x^2)}{1+exp(-x^2)}\right)$$

$$*\left(0.25*sech(x)*tanh(x)^2 - 0.25*sech(x)*(1-1*tanh(x)^2) - 2\right.$$

$$*\frac{exp(-x^2)}{1+exp(-x^2)} + 4*x^2*\frac{exp(-x^2)}{1+exp(-x^2)} - 12*x^2*\frac{exp(-x^2)^2}{(1+exp(-x^2))^2} + 8$$

$$\left.\left.*\frac{exp(-x^2)^3}{(1+exp(-x^2))^3}*x^2 + 2*\frac{exp(-x^2)^2}{(1+exp(-x^2))^2}\right)\right)*t^2$$

and

$$q_2(x,t) = \left(0.1 + 0.25*sech(x) + \frac{exp\,(-x^2)}{1+exp\,(-x^2)}\right)*\left(-0.25*sech(x)*tanh(x) - 2*x*\right.$$
$$\left.\frac{exp\,(-x^2)}{1+exp\,(-x^2)} + 2*\frac{exp\,(-x^2)^2}{(1+exp\,(-x^2))^2}*x\right)*t\;.$$


Figure 1. The initial water surface with the horizontal axis is for x variable.

3.2 Dam-break-like simulation

In this simulation we solve a dam-break-like problem. The initial water depth is

$$h(x,0) = 2 - 1/(1 + exp(-1000x)) \tag{8}$$

for all x, as shown in figure 2. Again we take the gravitational constant $g = 1$. We want to find the solution at any time t. The initial fluid velocity is again set to be $u(x,0) = 0$, for all x.
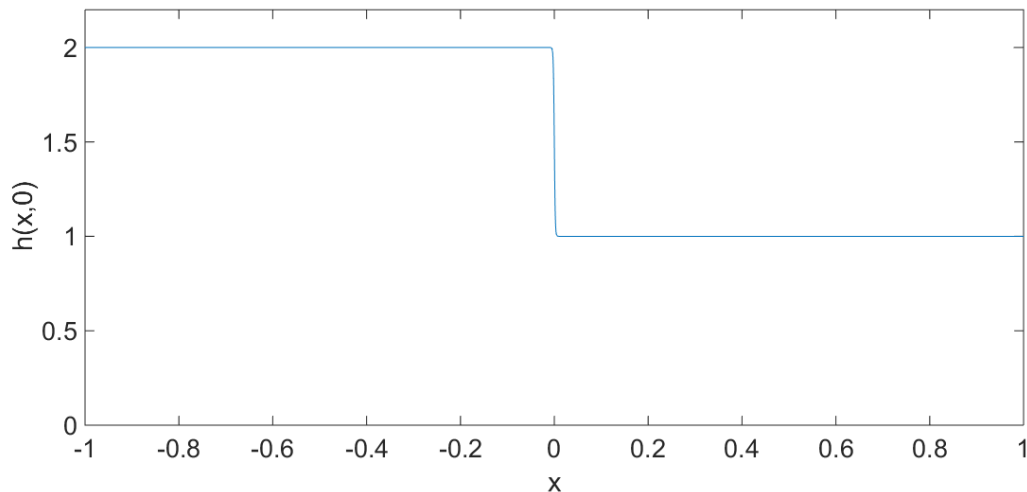
Figure 2. The initial water surface of the dam-break-like problem. The surface is actually smooth, but looks discontinuous at position x = 0.

Again using the aid of the MAPLE software, the variational iteration solutions are computed up to the second levels $h_2(x,t)$ and $q_2(x,t)$. The zeroth levels of variational iteration solutions are

$$h_0(x,t) = 2 - \frac{1}{1 + exp(-1000 * x)}$$

and

$$q_0(x,t) = 0 .$$

The first levels of variational iteration solutions are

$$h_1(x,t) = h_0(x,t)$$

and

$$q_1(x,t) = -1000 * \frac{2 - \frac{1}{1+exp\,(-1000*x)}}{\left(1 + exp(-1000 * x)\right)^2} * exp(-1000 * x) * t .$$

The second levels of variational iteration solutions are

$$h_2(x,t) = 2 - \frac{1}{1 + exp(-1000 * x)} - 0.5$$

$$* \left(\frac{-1000000}{\left(1 + exp(-1000 * x)\right)^4} * exp(-1000 * x)^2 + 2000000 \right.$$

$$* \frac{2 - \frac{1}{1+exp(-1000*x)}}{\left(1 + exp(-1000 * x)\right)^3} * exp(-1000 * x)^2 - 1000000$$

$$\left. * \frac{2 - \frac{1}{1+exp(-1000*x)}}{\left(1 + exp(-1000 * x)\right)^2} * exp(-1000 * x) \right) * t^2$$

and

$$q_2(x,t) = -1000 * \frac{2 - \frac{1}{1+exp(-1000*x)}}{\left(1 + exp(-1000 * x)\right)^2} * exp(-1000 * x) * t.$$

Obviously we can continue our calculation to any level of iteration. More number of iterations makes the approximate solutions too long to write in this paper. Therefore, we stop our iterations at the second level. We consider it enough to demonstrate how the variational iteration method works for the gravity wave-model equations.

4. Conclusion
We have demonstrated how to solve the gravity wave-model equations using the variational iteration method.Variational iteration solutions are successfully found as approximate solutions to the exact solution. The variational iteration solutions may need enhancement for approximating very large domain of time and space which could be a future direction to research.

References
[1]   Apriani C M and Mungkasi S 2017 A staggered grid finite difference method for solving the gravity wave-model equationsJournal of Physics: Conference Series **909** 012046
[2]   Martins R, Leandro J and Djordjević S 2016 Analytical solution of the classical dam-break problemfor the gravity wave-model equations Journal of Hydraulic Engineering **142**06016003
[3]   Martins R, Leandro J and Djordjević S 2015 A well balanced Roe scheme for the local inertial equations with an unstructured meshAdvances in Water Resources **83** 351
[4]   Mungkasi S and Wiryanto L H 2016 On the relevance of a variational iteration method for solving the shallow water equations AIP Conference Proceedings **1707** 050010
[5]   Setianingrum P S and Mungkasi S 2016 Variational iteration method used to solve steady state problems of shallow water flows AIP Conference Proceedings **1746** 020057
[6]   Setianingrum P S and Mungkasi S 2016 Variational iteration method used to solve the one-dimensional acoustics equations Journal of Physics: Conference Series **856**012010
[7]   Wazwaz A M 2009 Partial Differential Equations and Solitary Waves Theory (New York: Springer)
[8]   Ghorbani A and Bakherad M 2017 A variational iteration method for solving nonlinear Lane–Emden problemsNew Astronomy **54** 1
[9]   Li Y and Yang Y 2017 Vibration analysis of conveying fluid pipe via He's variational iteration methodApplied Mathematical Modelling **43** 409

[10]   Mohyud-Din S T, Sikander W, Khan U and Ahmed N 2017 Optimal variational iteration method using Adomian's polynomials for physical problems on finite and semi-infinite intervalsThe European Physical Journal Plus **132** 236

[11]   Singh R, Das N and Kumar J 2017 The optimal modified variational iteration method for the Lane-Emden equations with Neumann and Robin boundary conditionsThe European Physical Journal Plus **132** 251

[12]   Khuri S A and Sayfy A 2017 Generalizing the variational iteration method for BVPs: Proper setting of the correction functionalApplied Mathematics Letters **68** 68

[13]   Armand A, Allahviranloo T, Abbasbandy S and Gouyandeh Z 2017 Fractional relaxation-oscillation differential equations via fuzzy variational iteration methodJournal of Intelligent & Fuzzy Systems **32** 363

[14]   Wazwaz A-M 2017 Solving the non-isothermal reaction-diffusion model equations in a spherical catalyst by the variational iteration methodChemical Physics Letters **679** 132

[15]   Wazwaz A-M 2017 The variational iteration method for solving systems of third-order Emden-Fowler type equationsJournal of Mathematical Chemistry **55** 799