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Neutrino Masses and Effective Majorana Mass from a Cobimaximal Neutrino Mixing Matrix

Asan Damanik*

Department of Physics Education, Sanata Dharma University
Kampus III USD Paingan, Maguwoharjo, Sleman, Yogyakarta, Indonesia

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Abstract: Now, we have confidence that neutrino has a tiny mass and mixing does exist among the neutrino flavors as one can see from the experimental data that have already been reported by many collaborations. Based on the experimental facts, we derive a neutrino mass matrix from cobimaximal neutrino mixing matrix. By constraining the obtained neutrino mass matrix with texture zero, we evaluate its predictions on the neutrino mass relation, the neutrino masses, and the effective Majorana mass. By using the advantages of the experimental data of neutrino oscillations, then we obtain neutrino masses in normal hierarchy: $m_1 = 0.02403$ eV, $m_2 = 0.02554$ eV, and $m_3 = 0.04957$ eV, and the effective Majorana mass: $\langle m_{\beta\beta}\rangle = 0.02271$ eV that can be tested in the future neutrinoless double beta decay experiments.

Keywords: Neutrino Masses; Cobimaximal Mixing Matrix; Effective Majorana Mass
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1 Introduction

One of the unsolved long standing problems in neutrino physics till today is the explicit form of the neutrino mixing matrix and the absolute values of neutrino masses that can be used to explain the recent experimental data of neutrino oscillations. We have already known three types of neutrino mixing matrix i.e. bimaximal mixing, tribimaximal mixing, and democratic mixing, but now all of them become incompatible anymore when confronted to the recent experimental data, especially about the fact that mixing angle $\theta_{13} \neq 0$ as one can read from the results of the experimental data reported by T2K [1] and Daya Bay [2] collaborations. In order to obtain a mixing matrix which can proceed a consistent predictions with the experimental data, Ma [3] proposed a new

* Email: asandamanik11@gmail.com

mixing matrix which is known as the cobimaximal mixing (CBM) by assuming the mixing angle $\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, and the Dirac phase $\delta = \pm\pi/2$. In Ref. [3] also claimed that the cobimaximal neutrino mixing matrix is achieve rigorously in a renormalizable model of radiative charged-lepton and neutrino masses.

To explain the evidence of nonzero and relatively large mixing angle θ_{13}, some models and methods have already been proposed by several authors. The simple way to accommodate nonzero mixing angle θ_{13} is to modify the neutrino mixing matrix by introducing a perturbation matrix into known mixing matrix such that it can proceeds the nonzero mixing angle θ_{13} [4, 5, 6, 7, 8] and the other is to build the model by using some discrete symmetries [9, 10]. The nonzero mixing angle θ_{13} is also known relate to the Dirac phase δ as one can see in the standard parameterization of the neutrino mixing matrix. Thus, nonzero mixing angle θ_{13} gives a clue to the possible determination of CP violation in neutrino sector. Perturbation of neutrino mixing matrix in order to accommodate both nonzero mixing angle θ_{13} and CP violation have also been reported [11, 12, 13, 14, 15, 16].

In this paper, we evaluate the neutrino mass matrix which is obtained from a cobimaximal neutrino mixing matrix. The obtained neutrino mass matrix to be used to predict the neutrino masses and an the effective Majorana mass that can be tested in the future neutrinoless double beta decaya experiments. According to the aim of this paper as stated previously, then in section 2 we derive the neutrino mass matrix from a cobimaximal neutrino mixing matrix. In section 3, we evaluate the power prediction of the resulted neutrino mass matrix, which is constrained by texture zero, on the neutrino masses and the effective Majorana mass of neutrinoless double beta decay. Finally, the section 4 is devoted for conclusions.

2 Neutrino mass matrix from a CBM

Theoretically, the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ relate to the neutrino mass eigenstates (ν_1, ν_2, ν_3) via a neutrino mixing matrix V as follow

$$\nu_\alpha = V_{\alpha\beta}\nu_\beta, \tag{1}$$

where the indexes $\alpha = e, \mu, \tau$, $\beta = 1, 2, 3$, and $V_{\alpha\beta}$ are the elements of the neutrino mixing V. The standard parameterization of the mixing matrix (V) read [17]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2}$$

where c_{ij} is the $\cos\theta_{ij}$, s_{ij} is the $\sin\theta_{ij}$, θ_{ij} are the mixing angles, and δ is the Dirac CP-violating phase.

If we put the values for the mixing angle $\theta_{23} = \pi/4$ and the Dirac phase $\delta = \pi/2$, then the neutrino mixing matrix of Eq. (2) reads

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & is_{13} \\ -\frac{\sqrt{2}}{2}\left(s_{12} - ic_{12}s_{13}\right) & \frac{\sqrt{2}}{2}\left(c_{12} + is_{12}s_{13}\right) & \frac{\sqrt{2}}{2}c_{13} \\ \frac{\sqrt{2}}{2}\left(s_{12} + ic_{12}s_{13}\right) & -\frac{\sqrt{2}}{2}\left(c_{12} - is_{12}s_{13}\right) & \frac{\sqrt{2}}{2}c_{13} \end{pmatrix}, \tag{3}$$

which is known as a cobimaximal neutrino mixing matrix. In the basis where the charged lepton mass matrix is already diagonalized, the neutrino mass matrix defined by the mass term in the Lagrangian is given by

$$M_\nu = VMV^T, \tag{4}$$

where M is neutrino mass matrix in mass basis

$$M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{5}$$

By inserting Eqs. (3) and (5) into Eq. (4) we have the following neutrino mass matrix [18]

$$M_\nu = \begin{pmatrix} a & b + i\beta & -(b - i\beta) \\ b + i\beta & c - i\gamma & d \\ -(b - i\beta) & d & c + i\gamma \end{pmatrix}, \tag{6}$$

where

$$\begin{aligned} a &= c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 - s_{13}^2 m_3, \\ b &= -\frac{1}{\sqrt{2}} s_{12}^2 c_{12}^2 c_{13}^2 \left(m_1 - m_2\right), \\ c &= \frac{1}{2}\left((s_{12}^2 - c_{12}^2 s_{13}^2)m_1 + (c_{12}^2 - s_{12}^2 s_{13}^2)m_2 + c_{13}^2 m_3\right), \\ d &= -\frac{1}{2}\left((s_{12}^2 + c_{12}^2 s_{13}^2)m_1 + (c_{12}^2 + s_{12}^2 s_{13}^2)m_2 - c_{13}^2 m_3\right), \\ \beta &= \frac{1}{\sqrt{2}} s_{13}^2 c_{13}^2 \left(c_{12}^2 m_1 + s_{12}^2 m_2 - m_3\right), \\ \gamma &= -s_{12}c_{12}s_{13}\left(m_1 - m_2\right). \end{aligned} \tag{7}$$

It is clear from Eq. (7) if we put $b = 0$ or $\gamma = 0$, then we have the neutrino mass matrix with $\mu - \tau$ symmetry. We cannot put $b = 0$ or $\gamma = 0$ because it implies $m_1 = m_2$ which is contrary to the experimental fact that $\Delta m_{21}^2 > 0$.

3 Neutrino masses and effective Majorana mass

By knowing the explicit form of the obtained neutrino mas matrix from a cobimaximal mixing matrix as shown in Eq. (6), now we are in position to evaluate the neutrino mass matrix predictions on neutrino masses and the effective Majorana mass. To reduce the number of parameters in neutrino mass matrix, we impose the texture zero into neutrino mass matrix of Eq. (6). By inspecting the neutrino mass matrix in Eq. (6) one can see that the realistic neutrino mass matrix with texture zero is by putting the elements of neutrino mass matrix as follow

$$M_\nu(1,1) = a = 0, \tag{8}$$

and

$$M_\nu(2,3) = M_\nu(3,2) = d = 0, \tag{9}$$

By imposing the texture zero into neutrino mass matrix of Eq. (6), where the elements of neutrino mass matrix are put to be zero i.e. the elements of neutrino mass matrix as chosen in Eqs. (8) and (9), then the neutrino mass matrix reads

$$M_\nu = \begin{pmatrix} 0 & b+i\beta & -(b-i\beta) \\ b+i\beta & c-i\gamma & 0 \\ -(b-i\beta) & 0 & c+i\gamma \end{pmatrix}. \tag{10}$$

From the neutrino mass matrix of Eq. (10) we can obtain the relations of neutrino masses as function of the mixing angles and it can be used to determine the neutrino masses and its hierarchy. By using the experimental data of neutrino oscillation especially the experimental value of squared mass differences as input. From Eqs. (7), (8), and (9) we can obtain the following relations

$$s_{13}^2 m_3 = c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2, \tag{11}$$

and

$$c_{13}^2 m_3 = \left(s_{12}^2 + c_{12}^2 s_{13}^2\right) m_1 + \left(c_{12}^2 + s_{12}^2 s_{13}^2\right) m_2, \tag{12}$$

which proceed "neutrino mass sum rule"

$$m_3 = m_1 + m_2. \tag{13}$$

It is apparent from Eq. (13) that the neutrino mass hierarchy must be in normal hierarchy. The neutrino mass relation in Eq. (13) is a new result for neutrino mass relation when the obtained neutrino mass matrix from a cobimaximal mixing is constrained by two texture zero as dictated in Eq. (10).

From Eq. (13) we can have the following relation

$$m_3^2 - m_2^2 = m_1^2 + 2m_2m_1, \tag{14}$$

or

$$m_1^2 + 2m_2m_1 - \Delta m_{32}^2 = 0, \tag{15}$$

where $\Delta m_{32}^2 = m_3^2 - m_2^2$ is the squared mass difference of atmospheric neutrino. The global analysis of squared mass difference for atmospheric neutrino and solar neutrino read [19]

$$\Delta m_{32}^2 = 2.457 \times 10^{-3}\ \mathrm{eV}^2, \tag{16}$$

and

$$\Delta m_{21}^2 = 7.50 \times 10^{-5}\ \mathrm{eV}^2, \tag{17}$$

respectively.

If we insert the value of squared mass difference of Eq. (16) into Eq. (15), then we have

$$m_1 = (-m_2 + 0.001\sqrt{m_2^2 + 2457})\ \mathrm{eV}. \tag{18}$$

From Eqs. (15), (16), (17), and (18) we can have the neutrino masses as follow

$$\begin{aligned} m_1 &= 0.02403\ \mathrm{eV}, \\ m_2 &= 0.02554\ \mathrm{eV}, \\ m_3 &= 0.04957\ \mathrm{eV}. \end{aligned} \tag{19}$$

By knowing the absolute values of neutrino masses and its hierarchy, we can evaluate the prediction of cobimaximal neutrino mixing matrix on the effective Majorana mass. The effective Majorana mass is a parameter of interest in neutrinoless double beta decay experiment because the effective Majorana mass $\langle m_{\beta\beta}\rangle$ is the combination of neutrino mass in eigenstates basis and the neutrino mixing matrix terms as follow [20]

$$\langle m_{\beta\beta}\rangle = \left|\Sigma V_{ei}^2 m_i\right|, \tag{20}$$

where V_{ei} is the i-th element of the first row of neutrino mixing matrix, and m_i is the i-th of the neutrino mass eigenstate. From Eqs. (3) and (20) we obtain the effective Majorana mass as follow

$$\langle m_{\beta\beta}\rangle = \left|c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 - s_{13}^2 m_3\right|. \tag{21}$$

For example, if we take the central values of mixing angle $\theta_{13} = 9^0$ [2] and $\theta_{12} = 35^0$ [19] and the neutrino masses as shown in Eq. (19) into Eq. (21), then we have the effective Majorana mass as follow

$$\langle m_{\beta\beta}\rangle = 0.02271\ \mathrm{eV}, \tag{22}$$

which can be tested in the future neutrinoless double beta decay experiments. The value of the effective Majorana mass in this paper is still far below the upper bound of the experimental results from isotope Xenon and Germanium experiments [21]

$$\langle m_{\beta\beta}\rangle \leq 0.3\ \mathrm{eV}, \tag{23}$$

4 Conclusions

We have used a cobimaximal neutrino mixing matrix to obtain a neutrino mass matrix. If the obtained neutrino mass matrix to be constrained by two texture zero i.e. $M_\nu(1,1) = 0$ and $M_\nu(2,3) = M_\nu(3,2) = 0$, then we can obtain the neutrino mass sum-rule: $m_3 = m_1 + m_2$ which imply that the hierarchy of neutrino mass is in normal nierarchy. By using the advantages of the experimental data of squared mass difference as input, then we obtain the neutrino masses: $m_1 = 0.02403$ eV, $m_2 = 0.02554$ eV, and $m_3 = 0.04957$ eV. By using the central values of the experimental data for mixing angle θ_{13} and θ_{23} and the obtained neutrino masses, then we obtain the effective Majorana mass: $\langle m_{\beta\beta}\rangle = 0.02271$ eV which can be tested in the future neutrinoless double beta decay experiments.

References

[1] K. Abe *et al.* (T2K Collab.), Phys. Rev. Lett. **107** (2011) 041801.

[2] F. P. An *et al.* (Daya Bay Collab.), Phys. Rev. Lett. **108** (2012) 171803.

[3] E. Ma, Phys. Lett. **B755** (2016) 348.

[4] S. Boudjemaa and S. F. King, Phys. Rev. **D79** (2009) 033001.

[5] X.-G. He and A. Zee, Phys. Rev. **D84** (2011) 053004.

[6] A. Damanik, Int. J. Mod. Phys. **A27** (2012) 1250091.

[7] A. Damanik, arXiv:1206.0987v1 [hep-ph].

[8] W. Rodejohann and H. Zhang, arXiv: 1207.1225v1 [hep-ph].

[9] Q.-H. Cao, A. Damanik, E. Ma, and D. Wegman, Phys. Rev. **D83** (2011) 093012.

[10] F. Bazzocchi and L. Merlo, arXiv: 1205.5135 [hep-ph].

[11] P. F. Harrison and W. G. Scott, Phys. Lett. **B547** (2002) 219.

[12] P. F. Harrison and W. G. Scott, Phys. Lett. **B594** (2004) 324.

[13] W. Grimus and L. Lavoura, Phys. Lett. **B579** (2004) 113.

[14] R. Friedberg and T. D. Lee, arXiv: hep-ph/0606071.

[15] Z. Z. Xing, H. Zhang, and S. Zhou, Phys. Lett. **B641** (2006) 189.

[16] S. Zhou, arXiv: 1205.0761v1 [hep-ph].

[17] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. **B530** (2002) 167.

[18] X.-G. He, Chinese J. Phys. **53** (2015) 1002021.

[19] M. Gonzales-Garcia, M. Maltoni, and T. Schwetz, JHEP **11** (2014) 052.

[20] G. Benato, Eur. Phys. J. **C75** (2015) 563.

[21] H. Pas and W. Rodejohann, New J. Phys. **17** (2015) 115010.