Journal of Physics: Conference Series

PAPER • OPEN ACCESS

European option pricing by using a mixed fractional Brownian motion

IOP Conf. Series: Journal of Physics: Conf. Series **1097** (2018) 012081 doi:10.1088/1742-6596/1097/1/012081

European option pricing by using a mixed fractional Brownian motion

C E Murwaningtyas [1,2], S H Kartiko [1], Gunardi [1] and H P Suryawan [3]

[1] Department of Mathematics, Universitas Gadjah Mada, Indonesia,
[2] Department of Mathematics Education, Universitas Sanata Dharma, Indonesia,
[3] Department of Mathematics, Universitas Sanata Dharma, Indonesia.

Corresponding author: enny@usd.ac.id

Abstract. Financial modeling is conventionally based on a Brownian motion (Bm). A Bm is a semimartingale process with independent and stationary increments. However, some financial data do not support this assumption. One of the models that can overcome this problem is a fractional Brownian motion (fBm). In fact, the main problem in option pricing by implementing an fBm is not arbitrage-free. This problem can be handled by using a mixed fBm (mfBm) to model stock prices. The mfBm is a linear combination of an fBm and an independent Bm. The aim of this paper is to find European option pricing by using the mfBm based on Fourier transform method and quasi-conditional expectations. The main result of this research is a closed form formula for calculating the price of European call options.

1. Introduction

The Black-Scholes formula is a formula for calculating option prices based on geometric Bm. A Bm is a centered and continuous Gaussian process with independent and stationary increments. The existence of long-range dependence in stock returns has been an essential topic of both empirical and theoretical research. If stock returns show long-range dependence, the time series is said to depend on time to time for a long lag. This is the case of an fBm. Long-range dependence in stock returns has been tested in a number of studies, for example [1–6].

Kolmogorov introduced an fBm in 1940. Mandelbrot and Van Ness gave a representation theorem for Kolmogorov's process and introduced the name of fBm in [7]. The fBm has further been developed by Hurst in [8]. Currently, an fBm has an important part in assorted fields of study such as hydrology [8,9], insurance [10,11] and finance [12–14].

The stochastic integral in an fBm is different from the classical Itô integral because the fBm is not a martingale. Duncan et al [15] introduced a Wick product for the fractional Itô's formula. They also introduced Girsanov's theorem under the fBm. The option model under the fBm is arbitrage-free [16,17], if the Wick product is applied on the definition of stochastic integration. Hu and Oksendal [16] obtain a pricing formula for a European call option at $t=0$. Necula [18] extended the formula in [16] to $t\in[0,T]$. Moreover, Necula proved some results regarding quasi-conditional expectations by using Fourier transform.

The European call option pricing formula obtained in [16] is an arbitrage-free and complete market. However, Bender and Elliott [19] and Bjork and Hult [20] still saw a possibility of arbitrage opportunities in the resulting model in [16]. Cheridito [21] and Bender et al. [22] proposed an mfBm to reduce arbitrage opportunities. An mfBm is a linear combination of an fBm and an independent BM. Cheridito [23] has proven that an mfBm is equivalent to a Bm for $H \in \left(\frac{3}{4}, 1\right)$, therefore it can be said that the option model under an mfBm is an arbitrage-free. The aim of this paper is to obtain the pricing formula for European call options where a stock return is modeled an mfBm.

2. Mixed fractional Brownian motions

Let H be a constant belonging to (0, 1). An fBm $B^H = \left(B_t^H; t \geq 0\right)$ of Hurst index H is a continuous and centered Gaussian process with covariance function,

$$\mathbb{E}\left[B_t^H B_s^H\right] = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right), \tag{1}$$

for all $t, s \geq 0$, see [24]. Here $\mathbb{E}[\cdot]$ denotes an expectation with respect to a probability measure $\mathbb{P}^H$. Properties of fBm, see [24], are

- mean of an fBm is 0;
- variance of an fBm is t^{2H} for $t \geq 0$;
- an fBm has stationary increments, i.e., $B_{t+s}^H - B_s^H \overset{d}{=} B_t^H$ for all $t, s \geq 0$;
- an fBm is H-self similar, i.e., $B_{\alpha t}^H \overset{d}{=} \alpha^H B_t^H$ for $t \geq 0$;
- an fBm has continuous trajectories.

If $H = \frac{1}{2}$, then an fBm coincides with a standard Bm. The Hurst index H determines the sign of the covariance of the future and past increments. This covariance is negative when $H \in \left(0, \frac{1}{2}\right)$, zero when $H = \frac{1}{2}$, and positive when $H \in \left(\frac{1}{2}, 1\right)$. As a consequence, for $H \in \left(0, \frac{1}{2}\right)$ it has short-range dependence and for $H \in \left(\frac{1}{2}, 1\right)$ it has long-range dependence.

An fBm is neither a semimartingale nor a Markov process unless $H = \frac{1}{2}$. When H is not equal to ½, the option model has arbitrage opportunities. An mfBm is introduced by Cheridito [23], to avoid arbitrage opportunities. An mfBm of parameter H, a, and b is a stochastic process $M^H = \left(M_t^H; t \geq 0\right)$ defined in [25] as follows

$$M_t^H = aB_t^H + bB_t$$

where B_t^H is an fBm with Hurst index H and B_t is an independent BM.

3. Quasi-conditional expectations

We will present some results regarding a quasi-conditional expectation in this section which is needed for the rest of this paper. These results were introduced by Necula [18] and then developed by Sun [14] and Xiao et al [13] for an mfBm. The proofs of theorems in this section can be seen in [13]. Let $\left(\Omega, \mathcal{F}^H, \mathbb{P}^H\right)$ be a probability space such that B_t^H is an fBm with respect to $\mathbb{P}^H$ and B_t is an independent BM.

Theorem 1 [13]. *For every* $t \in (0, T)$ *and* $\lambda, \varepsilon \in \mathbb{C}$ *we have*

$$\tilde{\mathbb{E}}\left[\exp\left(\lambda\varepsilon B_T^H + \lambda B_T\right) \middle| \mathcal{F}_t^H\right] = \exp\left(\lambda\varepsilon B_T^H + \lambda B_T + \tfrac{1}{2}\lambda^2\varepsilon^2\left(T^{2H} - t^{2H}\right) + \tfrac{1}{2}\lambda^2\left(T - t\right)\right),$$

where $\mathcal{F}_t^H$ *is a σ-algebra generated by* $\left(B_s^H; 0 \leq s \leq t\right)$ *and* $\tilde{\mathbb{E}}\left[\cdot \middle| \mathcal{F}_t^H\right]$ *denotes a quasi-conditional expectation with respect to* $\mathcal{F}_t^H$ *under a probability measure* $\mathbb{P}^H$.

Using Theorem 1, one can determine a quasi-conditional expectation of a function of an mfBm as shown in the theorem below.

Theorem 2 [13]. *Let f be a function such that $\tilde{\mathbb{E}}\left[f\left(B_T^H, B_T\right)\right] < \infty$. Then, for every $t \in (0,T)$ and $\lambda, \varepsilon \in \mathbb{R}$ we have*

$$\tilde{\mathbb{E}}\left[f\left(\lambda\varepsilon B_T^H + \lambda B_T\right)\middle|\mathcal{F}_t^H\right] = \int_{\mathbb{R}} \frac{\exp\left(\dfrac{-\left(y - \lambda\varepsilon B_t^H - \lambda B_t\right)^2}{2\left(\lambda^2\varepsilon^2\left(T^{2H} - t^{2H}\right) + \lambda^2(T-t)\right)}\right)}{\sqrt{2\pi\left(\lambda^2\varepsilon^2\left(T^{2H} - t^{2H}\right) + \lambda^2(T-t)\right)}} f(y)dy.$$

If *f* is an indicator function, $f(y) = 1_A(y)$, then we can easily obtain a corollary below.

Corollary 3 [13]. *Let $A \in \mathcal{B}(\mathbb{R})$. Then,*

$$\tilde{\mathbb{E}}\left[1_A\left(\lambda\varepsilon B_T^H + \lambda B_T\right)\middle|\mathcal{F}_t^H\right] = \int_A \frac{\exp\left(\dfrac{-\left(y - \lambda\varepsilon B_t^H - \lambda B_t\right)^2}{2\left(\lambda^2\varepsilon^2\left(T^{2H} - t^{2H}\right) + \lambda^2(T-t)\right)}\right)}{\sqrt{2\pi\left(\lambda^2\varepsilon^2\left(T^{2H} - t^{2H}\right) + \lambda^2(T-t)\right)}} dy.$$

Let $\theta, \vartheta \in \mathbb{R}$ and $t \in [0,T]$, consider the process,

$$\theta B_t^{H*} + \vartheta B_t^* = \theta B_t^H + \theta^2 t^{2H} + \vartheta B_t + \vartheta^2 t. \tag{2}$$

From a fractional Girsanov theorem in [24], there exists a probability measure $\mathbb{P}^{H*}$ such that $\theta B_t^{H*} + \vartheta B_t^*$ is a new MFBM. We will denote $\tilde{\mathbb{E}}^*\left[\cdot\middle|\mathcal{F}_t^H\right]$ as a quasi-conditional expectation under the probability measure $\mathbb{P}^{H*}$. Now, we have defined the process

$$Z(t) = \exp\left(-\theta B_t^H - \tfrac{1}{2}\theta^2 t^{2H} - \vartheta B_t - \tfrac{1}{2}\vartheta^2 t\right), \tag{3}$$

where $t \in [0,T]$.

Theorem 4 [13]. *Let f be a function such that $\mathbb{E}\left[f\left(B_T^H, B_T\right)\right] < \infty$. Then, for every $t \in [0,T]$ we have*

$$\tilde{\mathbb{E}}^*\left[f\left(\theta B_T^H + \vartheta B_T\right)\middle|\mathcal{F}_t^H\right] = \frac{1}{Z(t)}\tilde{\mathbb{E}}\left[f\left(\theta B_T^H + \vartheta B_T\right)Z(T)\middle|\mathcal{F}_t^H\right]. \tag{4}$$

Theorem 4 illustrates a relationship between a quasi-conditional expectation $\tilde{\mathbb{E}}\left[\cdot\middle|\mathcal{F}_t^H\right]$ with respect to $\mathbb{P}^H$ and a quasi-conditional expectation $\tilde{\mathbb{E}}^*\left[\cdot\middle|\mathcal{F}_t^H\right]$ with respect to $\mathbb{P}^{H*}$. Based on Theorem 4, a discounted expectation of a function of an mfBm is calculated in the theorem below.

Theorem 5 [13]. *The price at time $t \in [0,T]$ of a bounded $\mathcal{F}_t^H$-measurable claim $V \in L^2(\mathbb{P}^H)$ is given by*

$$V_t = e^{-r(T-t)}\tilde{\mathbb{E}}\left[V_T\middle|\mathcal{F}_t^H\right], \tag{5}$$

where r is a constant riskless interest rate.

4. Results and discussion

The aim of this section is to determine a formula for calculating European call option prices. Now let us consider a mixed fractional Black-Scholes market with two investment possibilities:

- A bank account which satisfies a differential equation below

$$dA_t = rA_t dt, \qquad A_0 = 1, \qquad t \in [0,T], \tag{6}$$

where r is a constant riskless interest rate.

- A stock which satisfies a stochastic differential equation below

$$dS_t = \sigma S_t d\hat{B}_t^H + \sigma S_t d\hat{B}_t + \mu S_t dt, \qquad S_0 > 0, \qquad t \in [0,T], \tag{7}$$

where B_t^H is an fBm and B_t is a Bm with respect to $\hat{\mathbb{P}}^H$, μ is an appreciation rate, and σ is a volatility coefficient.

By using change of variable $\sigma B_t + \sigma B_t^H = \mu - r + \sigma \hat{B}_t + \sigma \hat{B}_t^H$, then under a risk-neutral measure, we have

$$dS_t = \sigma S_t dB_t^H + \sigma S_t dB_t + rS_t dt, \qquad S_0 > 0, \qquad t \in [0,T]. \tag{8}$$

Furthermore by using a Itô formula in [24], we obtain a solution of (8) as

$$S_t = S_0 \exp\left(\sigma B_t^H + \sigma B_t - \tfrac{1}{2}\sigma^2 t^{2H} - \tfrac{1}{2}\sigma^2 t + rt\right). \tag{9}$$

The price of a European option at time t with an expire date T and a strike price K is denoted $C(t,S_t)$. We present a formula for a call option pricing under MFBM in the theorem below.

Theorem 6. *Suppose a stock price S_t defined by* (9)*, then the price at time $t \in [0,T]$ of a European call option with an expire date T and a strike price K is given by*

$$C(t,S_t) = S_t\, N(d_1) - Ke^{-r(T-t)}\, N(d_2), \tag{10}$$

where

$$d_1 = \frac{\tfrac{1}{2}\sigma^2\left(T^{2H} - t^{2H}\right) + \tfrac{1}{2}\sigma^2\left(T-t\right) + \ln\left(\dfrac{S_t}{K}\right) + r(T-t)}{\sqrt{\sigma^2\left(T^{2H} - t^{2H}\right) + \sigma^2\left(T-t\right)}}, \tag{11}$$

$$d_2 = \frac{-\tfrac{1}{2}\sigma^2\left(T^{2H} - t^{2H}\right) - \tfrac{1}{2}\sigma^2\left(T-t\right) + \ln\left(\dfrac{S_t}{K}\right) + r(T-t)}{\sqrt{\sigma^2\left(T^{2H} - t^{2H}\right) + \sigma^2\left(T-t\right)}}, \tag{12}$$

$N(\cdot)$ is a cumulative probability function of a standard normal distribution.

Proof: Motivated from Theorem 5, the call option with an expire date T and a strike price K is theoretically equivalent to

$$\begin{aligned} C(t,S_t) &= \tilde{\mathbb{E}}\left[e^{-r(T-t)} \max\{S_T - K, 0\} \middle| \mathcal{F}_t^H\right] \\ &= e^{-r(T-t)}\tilde{\mathbb{E}}\left[S_T 1_{\{S_T > K\}} \middle| \mathcal{F}_t^H\right] - Ke^{-r(T-t)}\tilde{\mathbb{E}}\left[1_{\{S_T > K\}} \middle| \mathcal{F}_t^H\right]. \end{aligned} \tag{13}$$

Meanwhile option holders would exercise the option only when $S_T > K$. Solving (9) on this boundary, we have

$$\sigma B_T^H + \sigma B_T > \tfrac{1}{2}\sigma^2 T^{2H} + \tfrac{1}{2}\sigma^2 T + \ln\left(\frac{K}{S_0}\right) - rT.$$

Let

$$d_2^* = \tfrac{1}{2}\sigma^2 T^{2H} + \tfrac{1}{2}\sigma^2 T + \ln\left(\frac{K}{S_0}\right) - rT. \tag{14}$$

Using Corollary 3 and applying (14) on the second of the RHS in (13), we have

$$\begin{aligned} \tilde{\mathbb{E}}\left[1_{\{S_T > K\}} \middle| \mathcal{F}_t^H\right] &= \tilde{\mathbb{E}}\left[1_{\{x > d_2^*\}}\left(\sigma B_T^H + \sigma B_T\right) \middle| \mathcal{F}_t^H\right] \\ &= \int_{d_2^*}^{\infty} \frac{1}{\sqrt{2\pi\left(\sigma^2\left(T^{2H} - t^{2H}\right) + \sigma^2\left(T-t\right)\right)}} \exp\left(-\frac{\left(y - \sigma B_t^H - \sigma B_t\right)^2}{2\left(\sigma^2\left(T^{2H} - t^{2H}\right) + \sigma^2\left(T-t\right)\right)}\right) dy \end{aligned}$$

$$\tilde{\mathbb{E}}\left[1_{\{S_T>K\}}\middle|\mathcal{F}_t^H\right]=\int_{-\infty}^{\frac{\sigma B_t^H+\sigma B_t-d_2^*}{\sqrt{\left(\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2(T-t)\right)}}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)dz$$
$$=N(d_2), \tag{15}$$

where $d_2=\dfrac{\sigma B_T^H+\sigma B_T-d_2^*}{\sqrt{\left(\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2(T-t)\right)}}$. Furthermore, (9) can be written as

$$\sigma B_t^H+\sigma B_t=\tfrac{1}{2}\sigma^2t^{2H}+\tfrac{1}{2}\sigma^2t+\ln\left(\frac{S_t}{S_0}\right)-rt. \tag{16}$$

Hence, using (14) and (16) on d_2, we have

$$d_2=\frac{-\tfrac{1}{2}\sigma^2\left(T^{2H}-t^{2H}\right)-\tfrac{1}{2}\sigma^2(T-t)+\ln\left(\dfrac{S_t}{K}\right)+r(T-t)}{\sqrt{\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2(T-t)}}.$$

Let us consider a process

$$\sigma B_t^{H*}+\sigma B_t^*=\sigma B_t^H-\sigma^2t^{2H}+\sigma B_t-\sigma^2t, \tag{17}$$

for $t\in[0,T]$. The fractional Girsanov theorem assures us that there is a probability measure $\mathbb{P}^{H*}$ such that $\sigma B_t^{H*}+\sigma B_t^*$ is an new mfBm under $\mathbb{P}^{H*}$. We will denote

$$Z_t=\exp\left(\sigma B_t^H-\tfrac{1}{2}\sigma^2t^{2H}+\sigma B_t-\tfrac{1}{2}\sigma^2t\right) \tag{18}$$

By using Theorem 4 and (18) on the first of the RHS in (13), we have

$$\begin{aligned}\tilde{\mathbb{E}}\left[S_T1_{\{S_T>K\}}\middle|\mathcal{F}_t^H\right]&=\tilde{\mathbb{E}}\left[S_0\,e^{rT}e^{\left(\sigma B_T^H-\frac{1}{2}\sigma^2T^{2H}+\sigma B_T-\frac{1}{2}\sigma^2T\right)}1_{\{S_T>K\}}\middle|\mathcal{F}_t^H\right]\\
&=S_0\,e^{rT}\tilde{\mathbb{E}}\left[Z_T1_{\{S_T>K\}}\middle|\mathcal{F}_t^H\right]\\
&=S_0\,e^{rT}\tilde{\mathbb{E}}\left[Z_T1_{\{y>d_2^*\}}\left(\sigma B_T^H+\sigma B_T\right)\middle|\mathcal{F}_t^H\right]\\
&=S_0\,e^{rT}Z_t\tilde{\mathbb{E}}^*\left[1_{\{y>d_2^*\}}\left(\sigma B_T^H+\sigma B_T\right)\middle|\mathcal{F}_t^H\right]\\
&=S_0\,e^{rT}Z_t\tilde{\mathbb{E}}^*\left[1_{\{S_T>K\}}\middle|\mathcal{F}_t^H\right]\end{aligned} \tag{19}$$

By substituting (17) into (9), we obtain

$$S_t=S_0\exp\left(\sigma B_t^{H*}+\sigma B_t^*+\tfrac{1}{2}\sigma^2t^{2H}+\tfrac{1}{2}\sigma^2t+rt\right). \tag{20}$$

Solving (20) in time T for the boundary $S_T>K$, we have

$$\sigma B_T^{H*}+\sigma B_T^*>-\tfrac{1}{2}\sigma^2T^{2H}-\tfrac{1}{2}\sigma^2T+\ln\left(\frac{K}{S_0}\right)-rT.$$

If we denote

$$d_1^*=-\tfrac{1}{2}\sigma^2T^{2H}-\tfrac{1}{2}\sigma^2T+\ln\left(\frac{K}{S_0}\right)-rT, \tag{21}$$

we get

$$\tilde{\mathbb{E}}^*\left[1_{\{S_T>K\}}\middle|\mathcal{F}_t^H\right]=\tilde{\mathbb{E}}^*\left[1_{\{y>d_1^*\}}\left(\sigma B_T^{H*}+\sigma B_T^*\right)\middle|\mathcal{F}_t^H\right]$$

$$\tilde{\mathbb{E}}^*\left[1_{\{S_T>K\}}\left|\mathcal{F}_t^H\right.\right]=\int_{d_1^*}^{\infty}\frac{1}{\sqrt{2\pi\left(\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)\right)}}\exp\left(-\frac{\left(y-\sigma B_t^{H*}-\sigma B_t^*\right)^2}{2\left(\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)\right)}\right)dy$$

$$=\int_{-\infty}^{\frac{\sigma B_t^{H*}+\sigma B_t^*-d_1^*}{\sqrt{\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2(T-t)}}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)dz$$

$$=N(d_1), \tag{22}$$

where $d_1=\dfrac{\sigma B_t^{H*}+\sigma B_t^*-d_1^*}{\sqrt{\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)}}$. Subsequently, (20) can be written as

$$\sigma B_t^{H*}+\sigma B_t^*=-\tfrac{1}{2}\sigma^2t^{2H}-\tfrac{1}{2}\sigma^2t+\ln\left(\frac{S_t}{S_0}\right)-rt. \tag{23}$$

Substituting (21) and (23) on d_1, we get

$$d_1=\frac{\tfrac{1}{2}\sigma^2\left(T^{2H}-t^{2H}\right)+\tfrac{1}{2}\sigma^2\left(T-t\right)+\ln\left(\frac{S_t}{K}\right)+r(T-t)}{\sqrt{\sigma^2\left(T^{2H}-t^{2H}\right)+\sigma^2\left(T-t\right)}}.$$

Substitution of (22) into (19) yields

$$\begin{aligned}\tilde{\mathbb{E}}\left[S_T1_{\{S_T>K\}}\left|\mathcal{F}_t^H\right.\right]&=S_0e^{rT}Z_t\,N(d_1)\\&=S_0e^{rT}\exp\left(\sigma B_t^H+\sigma B_t-\tfrac{1}{2}\sigma^2t^{2H}-\tfrac{1}{2}\sigma^2t\right)N(d_1)\\&=e^{r(T-t)}S_t\,N(d_1).\end{aligned} \tag{24}$$

Finally, from (13), (15) and (24) we obtain

$$\begin{aligned}C\left(t,S_t\right)&=e^{-r(T-t)}e^{r(T-t)}S_t\,N(d_1)-Ke^{-r(T-t)}\,N(d_2)\\&=S_t\,N(d_1)-Ke^{-r(T-t)}\,N(d_2).\ \blacksquare\end{aligned}$$

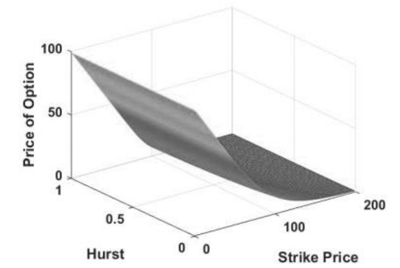

FIGURE 1. Price of option on Hurst vs strike price at T=0.25.

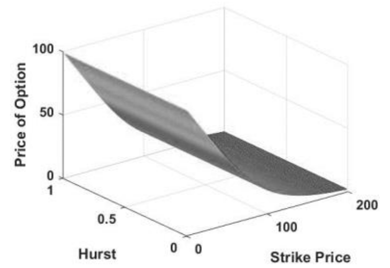

FIGURE 2. Price of option on Hurst vs strike price at T=1.

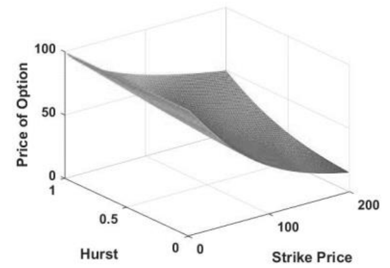

FIGURE 3. Price of option on Hurst vs strike price at T=5.

The formula in Theorem 6 allows us to determine a fair price for a European call option in terms of an expire date T, a strike price K, an initial stock price S_0, a risk-free interest rest r, and a stock volatility σ. Let $S = 100$, $K \in (0,200)$, $H \in (0,1)$, $r = 0.05$, and $\sigma = 0.3$. When $T = 0.25$, $T = 1$, and $T = 5$, we get Figure 1, 2 and 3 respectively. We see that when $K \to 200$ and $H \to 0$ the price decreases significantly in Figure 3.

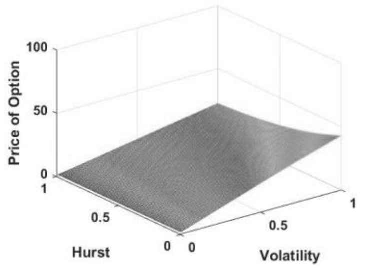

FIGURE 4. Price of option on Hurst vs volatility at T=0.25.

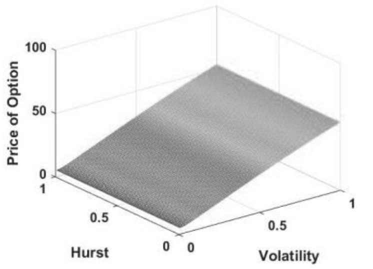

FIGURE 5. Price of option on Hurst vs volatility at T=1.

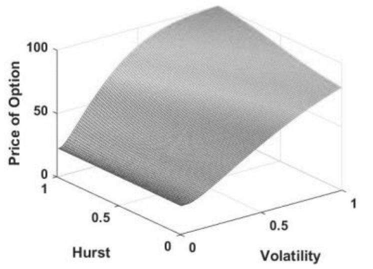

FIGURE 6. Price of option on Hurst vs volatility at T=5.

Let S = 100; K = 100; r = 0.05, $\sigma \in (0, 1)$ and $H \in (0, 1)$. If T = 0.25 we obtain Figure 4 which is concave upward. The price increases significantly when $\sigma \to 1$ and $H \to 0$. If T = 1 we obtain Figure 5 which is more linear and as the volatility increases for all Hurst parameters the price increases. If T = 5 we obtain Figure 6 which is concave down and we see that the prices increase significantly with high Hurst index and high volatility. Overall, as T, σ and H increase, the price increase in rate and magnitude.

5. Conclusion

In this paper, to exclude arbitrage opportunities in an fBm model and to capture long-range dependence, stock returns are modeled with an mfBm. By using Fourier transformation method and quasi-conditional expectation theory, we get a formula for calculating a price of European call options. This formula can be used by investors to predict option prices for stocks that have long-range dependence.

Acknowledgments

The authors are grateful to the Directorate of Research and Community Service, the Republic of Indonesia, due to the funding of Doctoral Dissertation Research Grant 2018 funds with contract number 109 / SP2H / LT / DRPM / 2018 for this work.

References

[1] Mills T C 1993 Is there long-term memory in UK stock returns? *Appl. Financ. Econ.* **3** 303–6
[2] Cheung Y-W and Lai K S 1995 A search for long memory in international stock market returns *J. Int. Money Financ.* **14** 597–615
[3] Necula C and Radu A-N 2012 Long memory in Eastern European financial markets returns *Econ. Res. - Ekon. Istraživanja* **25** 316–77
[4] Goddard J and Onali E 2012 Short and long memory in stock returns data *Econ. Lett.* **117** 253–5
[5] Cajueiro D O and Tabak B M 2008 Testing for long-range dependence in world stock markets *Chaos, Solitons and Fractals* **37** 918–27
[6] Gyamfi E N, Kyei K and Gill R 2016 Long-memory persistence in African Stock Markets *EuroEconomica* **35** 83–91
[7] Mandelbrot B B and Van Ness J W 1968 Fractional Brownian motions, fractional noises and applications *SIAM Rev.* **10** 422–37
[8] Hurst H E 1956 Methods of using long-term storage in reservoirs. *Proc. Inst. Civ. Eng.* **5** 519–43
[9] Molz F J, Liu H H and Szulga J 1997 Fractional Brownian motion and fractional Gaussian noise in subsurface hydrology: A review, presentation of fundamental properties, and extensions*Water Resour. Res.* **33** 2273–86
[10] Zhang H Y, Bai L H and Zhou A M 2009 Insurance control for classical risk model with fractional Brownian motion perturbation *Stat. Probab. Lett.* **79** 473–80

[11] Shokrollahi F and Kılıçman A 2015 Actuarial approach in a mixed fractional Brownian motion with jumps environment for pricing currency option *Adv. Differ. Equations* **2015** 257
[12] Rostek S 2009 *Option Pricing in Fractional Brownian Markets* (Springer-Verlag Berlin)
[13] Xiao W-L, Zhang W-G, Zhang X X and Zhang X X 2012 Pricing model for equity warrants in a mixed fractional Brownian environment and its algorithm *Phys. A Stat. Mech. its Appl.* **391** 6418–31
[14] Sun L 2013 Pricing currency options in the mixed fractional Brownian motion *Phys. A Stat. Mech. its Appl.* **392** 3441–58
[15] Duncan T E, Hu Y and Pasik-Duncan B 2000 Stochastic calculus for fractional Brownian motion I. Theory *SIAM J. Control Optim.* **38** 582–612
[16] Hu Y and Øksendal B 2003 Fractional white noise calculus and applications to finance *Infin. Dimens. Anal. Quantum Probab. Relat. Top.* **06** 1–32
[17] Elliott R J and Van der Hoek J 2003 A general fractional white noise theory and applications to finance *Math. Financ.* **13** 301–30
[18] Necula C 2002 Option pricing in a fractional Brownian motion environment *Adv. Econ. Financ. Res. - DOFIN Work. Pap. Ser.* 1–18
[19] Bender C and Elliott R J 2004 Arbitrage in a discrete version of the Wick-fractional Black-Scholes market *Math. Oper. Res.* **29** 935–45
[20] Björk T and Hult H 2005 A note on Wick products and the fractional Black-Scholes model *Financ. Stochastics* **9** 197–209
[21] Cheridito P 2003 Arbitrage in fractional Brownian motion models *Financ. Stochastics* **7** 533–53
[22] Bender C, Sottinen T and Valkeila E 2007 Arbitrage with fractional Brownian motion? *Theory Stoch. Process.* **13** 23–34
[23] Cheridito P 2001 Mixed fractional Brownian motion *Bernoulli* **7** 913–34
[24] Biagini F, Hu Y, Øksendal B and Zhang T 2008 *Stochastic Calculus for Fractional Brownian Motion and Applications* (Springer)
[25] Zili M 2006 On the mixed fractional Brownian motion *J. AppliedMathematics Stoch. Anal.* **2006** 1–9