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Numerical Study On an Area of Regular Polygon as a Concept of Limit Approach for Unit Circle Using Line Integrals with MS Excel

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Abstract. This paper aims to explain the concept of the limit function presented by Varberg et al when giving motivation regarding the limit function using regular (n) terms of regular polygon made either in unit circles or outside the unit circle which will converge to the area of the unit circle. The research was carried out by calculating the area of regular rectangle, regular octagonal, regular polygon and so on. The regular polygon is created inside and outside the circle. The process of making regular polygon is done by determining the points that will be the points of the angles of the polygon and then connecting each of these points with the line segments. Furthermore, to calculate the area of the regular polygon obtained, it is used line integrals. Line integrals along a simply closed trajectory relate to a study to calculate the area of the area by using a closed trajectory and a simple calculation of the area along the path. The problem that arises is related to the number of points that are quite a lot so that it uses MS Excel to support the numerical calculation process. The results showed that the area of the regular polygon made both inside and outside the circle could be made and explained the area calculation and would converge to the area of the unit circle, that is Pi.

1. Introduction

The problem of calculating the area of a region in general can be done analytically utilizing the theory of integral of course and the simplest is to utilize the formula of area over areas that can be broken down into triangles, rectangles, and other shapes then add them together. The background of this research is to utilize the effect of Green theorem on the field to calculate the area of an area. The area in this study is limited to the area that can be made in a unit circle in the form of a regular (n) aspect. The concept of calculating the area of a regular (n) aspect of a unit circle begins with making and calculating the area of a regular rectangle, a regular number, a regular sixteen, a regular thirty-two, and finally a regular twenty-four. For the increasing number of n, the area of the regular (n) will be closer to the area of the unit circle. To help the calculation process, the study is carried out numerically using MS Excel.

In general, the results of the study are expected to provide an understanding of how to approach to calculate the area of a circle numerically using MS Excel assistance. The importance of research is to examine the use of theory in mathematics that has been studied and integrate it with the corresponding software to answer the problems that exist both in mathematical theory and problems in everyday life. This research uses fairly simple calculus, namely derivative problems with chain rules and the use of line integrals to calculate the area of an area.

2. Literature Review

The unit circle is a circle whose center (0.0) and its radius are 1 and are expressed by the equation (Sullivan [4])

$$sin^2 t + cos^2 t = 1 \quad ; \quad 0 \leq t \leq 2\pi$$

and the area is π. Varberg et al. [5] convey an idea in studying the phenomenon of limits using the area of a regular (n) facet using regular (n) facets made both inside and outside the unit circle. The greater the number of sides in terms of (n) irregularly enlarged, then the shape of the area is closer to the area of the unit circle. Furthermore, Varberg et al. [5] and Stewart [2] also said that the area can be calculated with the concept of integral lines. The study is to use the consequences of the Green Theorem (Spiegel [3]), namely the calculation of the area of an area bounded by a simple curve and closed with the requirement that the curve must be smooth.

An area S which is bounded by a trajectory C with the path condition C piecewise smooth, a simple closed curve, and the direction of its integration to keep the area at the left of the trajectory, the S area can be expressed as

$$Area(S) = \frac{1}{2} \oint_S (-y dx + x dy)$$

3. Method

The research was carried out by paying attention to the theory of the use of integral lines related to determining the area of an area. The second study is the introductory limit of a function which Varberg et al. [5] use one of the regular (n) facets to obtain the area of a circle. The research was carried out by examining the consequences of Green theorem in fields that could be used to calculate the area of regular (n) facets. Extensive calculation is carried out directly on each track (line) and adds it up. Data processing is carried out numerically by using MS Excel (Joseph Billo [1]) to calculate the regular (n) facets for n = 4, 8, 16, 32, and 64. As well as illustration images made using GeoGebra. Some things that need to be considered in the use of the Green Theorem in the field are the direction of integration and ensure that each trajectory made comes from two adjacent points.

4. Discussion and Results

The discussion begins by examining some of the rules that will be used in explaining the use of the Green Theorem in the field to calculate the area. Any two points can be made a line. Suppose that the points are (x_1, y_1), (x_2, y_2) then through these two points a line can be made

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \tag{4.1}$$

Or the equation can be expressed in the equation

$$y = y_1 + \left(\frac{x - x_1}{x_2 - x_1}\right)(y_2 - y_1) \tag{4.2}$$

Furthermore, if given a BCDE quadrilateral made in a unit circle, figure 1, then to calculate the area of quadrilateral BCDE can be done with a formula

$$Area\ (CBED) = \frac{1}{2} \oint_S (-y dx + x dy) \tag{4.3}$$

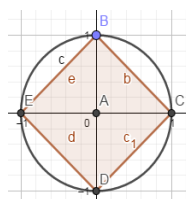On the S path, the path goes from B to C to D to E and back to B.

Figure 1. Regular Rectangles in A Unit Circle

4.1 On the BC Track.
The settlement process is done by changing equation (4.3) in the x variable and its differentiation in dx. Suppose that $B(x_1, y_1)$, and $C(x_2, y_2)$ then the equation of the line that connects BC is equation (4.2). Furthermore it can be found that differential dy is as follows:

$$dy = \frac{y_2 - y_1}{x_2 - x_1} dx \tag{4.4}$$

The next process is to substitute equation (4.2) and equation (4.4) to equation (4.3) so that it is obtained:

$$Area(CBED) = \frac{1}{2} \int_{x_1}^{x_2} \left(x_1 \frac{y_2 - y_1}{x_2 - x_1} - y_1 \right) dx \tag{4.5}$$

Solving equation (4.5) analytically on $[x_1, x_2]$, we will have:

$$Area(CBED) = \frac{1}{2} x_1 y_2 - \frac{1}{2} x_1 y_1 - \frac{1}{2} x_2 y_1 \tag{4.6}$$

On the CD track
Suppose that $C(x_2, y_2)$, and $D(x_3, y_3)$ then we have from 4.5 :

$$Area(CBED) = \frac{1}{2} x_2 y_3 - \frac{1}{2} x_2 y_2 - \frac{1}{2} x_3 y_2 \tag{4.7}$$

On the DE track
Suppose that $D(x_3, y_3)$, and $E(x_4, y_4)$ then we will have from equation 4.5:

$$Area(CBED) = \frac{1}{2} x_3 y_4 - \frac{1}{2} x_3 y_3 - \frac{1}{2} x_4 y_3 \tag{4.8}$$

On the EB track
Suppose that $E(x_4, y_4)$, and $B(x_1, y_1)$ then we will have from equation 4.5:

$$Area\ (CBED) = \frac{1}{2} x_4 y_1 - \frac{1}{2} x_4 y_4 - \frac{1}{2} x_1 y_4 \tag{4.9}$$

The total area or area of CBED is the area calculated along the trajectories of BC, CD, DE, and EB.

4.2 Calculation The Regular 8 Polygon
To calculate the regular 8 polygon area, using equation (4.3) which is carried out on 8 tracks, that is, the trajectories of BC, CD, DE, EF, FG, GH, HI, and the IB path. Figure 2 is an example.
Furthermore, the process of calculating the area for other (n) regular polygon is done by a similar process.
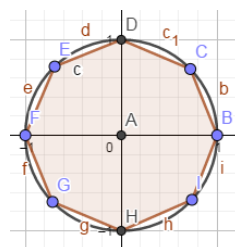

Figure 2. Regular 8 Polygon in a Unit Circle

The process of calculating the area of regular 4 polygon is done by counting on 4 tracks. For the greater number of passes, manually is no longer good enough. Therefore, the calculation process with the help of MS Excel or other software is needed to simplify and speed up the calculation process Following is the presentation of calculations using MS Excel.

4.2.1 Area of Regular 4 Polygon The following, table 1, is a table stating the area of regular 4 polygon.

Table 1. Area of Regular 4 Polygon

Points	X	y	Area_Regular_4
B	0	1	0.5
C	1	0	0.5
D	0	-1	0.5
E	-1	0	0.5

The area of a regular 4 polygon is 2, from the summation from column 4th from table 1.

4.2.2 Area of Regular 8 Polygon The following, table 2, is a table stating the area of regular 8 polygon

Table 2. Area of Regular 8 Polygon

Points	x	y	Area_Regular_4
B	1	0	0.353553
C	0.7071	0.70711	0.103553
D	3E-07	1	0.353553
E	-0.707	0.70711	0.603553
F	-1	6.5E-07	0.353554
G	-0.707	0.70711	0.103553
H	-1E-06	-1	0.353553
I	0.707	-0.70711	0.603554

The area of a regular 8 polygon is 2.828427. The following table, table 3, is resume for regular (n) polygon.

Table 3. Resume of Area (n) Polygon

| Regular (n) Polygon	Area
4	2
8	2.828427
16	3.061467132
32	3.122069058
64	3.13716

4.3 The Area Describe Outside the Unit Circle

The second part of this discussion about the area that could be describe outside the unit circle. We can choose the area that we could defined outside the unit circle by regular 4 polygon, regular 8 polygon, regular 16 polygon etc. Figure 3 is example for regular 4 polygon and figure 4 is example for regular 8 polygon.
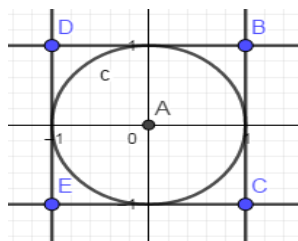
x	y	Integral along CBDE
1	1	0.5
-1	1	1.5
-1	-1	0.5
0.999999	-1	1.5

Total_area = 4

Figure 3. Area Describe Outside the Unit Circle (the figure was produced using GeoGebra [6])
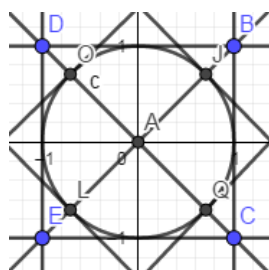
x	y	Integral along regular (8) polygon
1	0.414213	0.207107
0.414214	1	0.207107
-0.41421	1	0.62132
-1	0.414214	0.62132
-1	-0.41421	0.207107
-0.41421	-1	0.207106
0.414212	-1	0.62132
0.999999	-0.41421	0.621321

Total_area = 3.313708

Figure 4. Area Describe Outside The Unit Circle

If we use regular 8 polygon we will have figure 4. Figure 4 describe the calculation of the area for regular 8 polygon on the right side using MS Excel, and the left side is the ilustration was produced by GeoGebra. The process of arranging coordinates so as to ensure that the trajectories made do come from two consecutive points, as in figure 3 and figure 4 is done by an identical process with defining trigonometry using a unit circle.

The results below, in the figure 5, we can get the calculation of regular 16 polygon and regular 32 polygon. Detail calculation is shown in the last column and in the last line are the area.
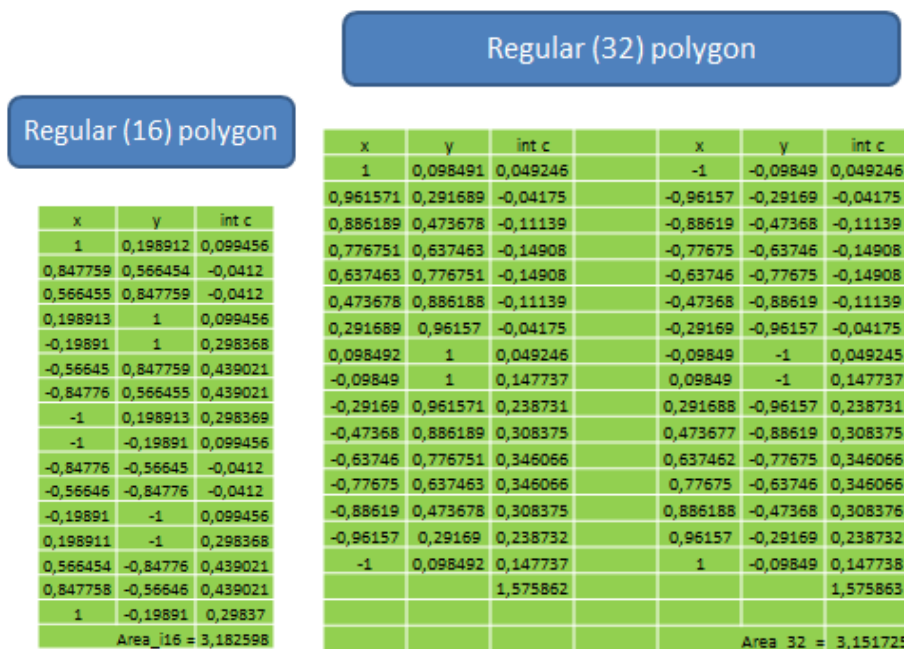
Figure 5. Area Describe Outside The Unit Circle

5. Resume

Based on the results of the discussion, it can be concluded that :

a. Area construction in terms of regular (n) terms defined inside and outside MS Excel-assisted unit circles can be obtained using the Green Theorem.

b. The concept of limits as presented by Varberg et al. [5] can be clarified by an empirical example in
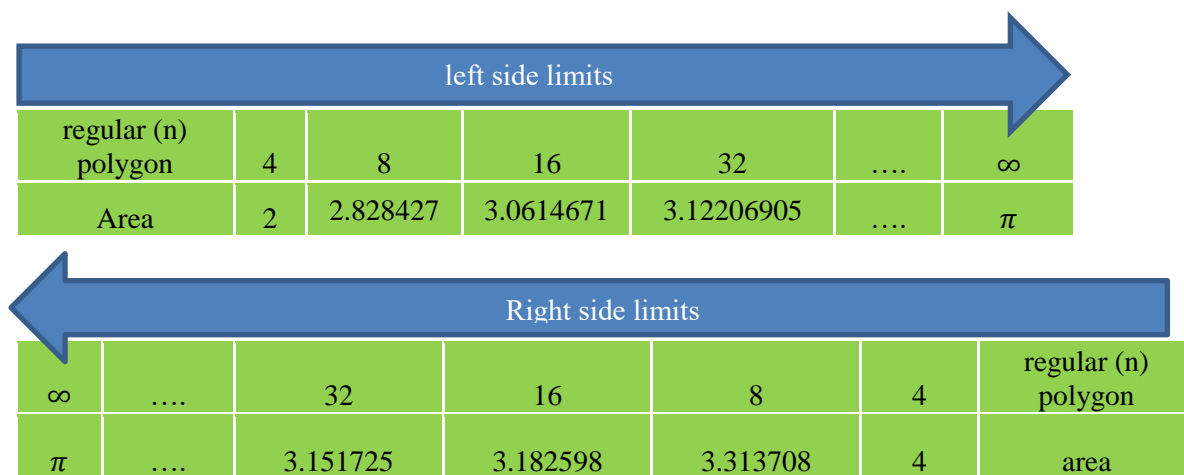   figure 6.

left side limits						
regular (n) polygon	4	8	16	32	….	∞
Area	2	2.828427	3.0614671	3.12206905	….	π

Right side limits						
∞	….	32	16	8	4	regular (n) polygon
π	….	3.151725	3.182598	3.313708	4	area

Figure 6. An Empirical Example

References

[1] Joseph Billo E 2007 *Excel for Scientists and Engineers: Numerical Methods* (Canada: John Wiley and Sons)

[2] Stewart J 2010 *Calculus: Concepts and Contexts, Fourth Edition* (Canada: Brooks/Cole)

[3] Spiegel M R 1974 *Theory and Problems of Vector Analysis. Schaum's Outline Series* (New York: McGraw-Hill)

[4] Sullivan M 2012 *Trigonometry: A Unit Circle Approach,* 9th Ed (Boston: Pearson).

[5] Varberg D, Purcell E J and Rigdon S E 2000 *Calculus: Eight Edition* (New Jersey: Prentice-Hall)

[6] https://geogebra.org