*Journal of Science and Science Education*
Vol. 3 No. 1 May 2019 13–18
ISSN: 2598-3830 (Online)

# Modeling of three crossroad traffic flow queuing systems using max-plus algebra

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**ABSTRACT**

This study aims to determine the model of the traffic flow queuing system of three crossroads, that is the Monjali, Kentungan, and Gejayan crossroad in Yogyakarta using max-plus algebra. This research is based on data on traffic flow density, especially cars and traffic light duration data on the threecrossroads. Then, an image is drawn describing the condition of the crossroad and representing the direction of the movement of the car especially in the direction of Monjali to Gejayan. The model is then compiled which includes the waiting time of the car when the red lights are on, the time the car leaves the crossroad during the green light, and the travel time of the vehicle crossroad with max-plus algebra, so that the total length of time from Monjali to the Gejayan crossroad can be determined. The results of the study show that the queue system can be modeled using max-plus algebra. Furthermore, it can be determined the waiting time, the time to leave the intersection, and the total time needed to cross from the Monjali to exit at Gejayan.

**Keywords**: max-plus algebra, queuing system, traffic-light

**Received:** April 30, 2019; **Accepted:** June 12, 2019; **Published online:** August 13, 2019
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**Citation:** Janu, M. N. P., & Rudhito, A. (2019). Modeling of three crossroad traffic flow queuing systems using max-plus algebra. *Journal of Science and Science Education*, *3*(1), 13–18.

## 1. Introduction

Moderation often triggers complex problems in modernitation of human life, so a deeper analysis is needed to deal with them. The traffic flow system is one of them. Traffic flow systems are one example of a Discrete Event System (SED) problem. Class of SED mainly contains a man-made system that is able to control and help overcome these problems. Of course this requires a number of supporting components. The picture that will be achieved from the characteristics of SED is the dynamics of Event-Driven in contrast to Time-Driven (Subiono, 2015). In Yogyakarta, one of the steps taken to reduce congestion is by building underpasses at several points, one of which is an underpass located in the Monjali area.

Transportation problems become the dominant topic in Operations Research. This is certainly an indication that these problems are important to overcome in a city. Solutions to overcome transportation problems are several ways, one of which is by improving and improving the quality of public transportation service facilities and managing traffic flow systems that are integrated with each other. Congestion occurs because there is a conflict of movement at the crossroad. To reduce this problem, many arrangements have been made to optimize the effectiveness of the intersection by using traffic lights. Traffic lights indicate the time the vehicle must walk and stop alternately from various directions. There are many traffic lights with a long duration of red lights and a short duration of green light. This creates a queue that accumulates so that congestion occurs. Pradanti

& Sari (2016), proposed timing of traffic lights can be made using max-plus algebra. Max-plus algebraic models of the time the traffic lights can be arranged so that it can be determined the time of the lightsand obtained a traffic light timing which can overcome the problem of long vehicle queues. Therefore we need an optimization and integration of traffic light settings, especially in crossroads that have high vehicle densities and distances between adjacent traffic lights. Based on Wibowo *et al*. (2018) starting in the 90s until now the study of Max-Plus Algebra theory to model, analyze and control the transportation network and others continues to grow. Based on the characteristics of these problems, this study was examined using a max-plus algebra model to design traffic light settings.

Based on Rudhito *et al.* (2010), the general form of max-plus algebra with a system of linear equations max-plus $A \otimes x = b$ can be a reference in completing applied algebra by considering the properties of $A \otimes x = b$ in the matrix and its vectors. In addition, the theory of graphs is also a tool in describing traffic light conditions into a directed graph. The directed graph then becomes a precedent graph on the matrix that represents the traffic light condition. In this study, a traffic light setting design was created that was integrated with other traffic lights using max-plus algebra. Traffic light network structure includes network categories, road functions, classes of roads and status of the road.

## 2. Materials and Methods

### *2.1 Materials*

Based on Rudhito (2016) and Nurhidayah & Farkhatu (2016), Max-plus algebra is an algebraic structure in a set of real numbers consisting of the sum of max-plus algebra and maxplus algebra multiplication.

Definition 1:
Operation $\oplus$ which is called max-plus algebra addition and operation $\otimes$ which called max plus algebra multiplication is defined as:

$$a \oplus b = max(a, b)$$
$$a \otimes b = a + b$$

On the set $R_{max} = R \cup \{-\infty\}$, $\forall a, b \in R_{max}$. The natural element is the sum of max-plus algebra which is $-\infty$ and denoted with $\varepsilon$ , while the multiplication natural element is 0 and denoted by $e$.

Definition 2:
Summing up max-plus algebra for matrix $A, B \in R_{max}^{n \times m}$ is defined $[A \oplus B]_{ij} = [A]_{ij} \oplus [B]_{ij}$

Definition 3:
Max-plus algebra multiplication for matrix $A \in R_{max}^{n \times l}$and $B \in R_{max}^{l \times m}$ defined by

$$[A \otimes B]_{ij} = \bigotimes_{k=1}^{l} \left([A]_{ij} \otimes [B]_{kl}\right) = \max_{k=1,\dots,l}\{[A]_{jk} + [B]_{kl}\}$$

In modeling using max-plus algebra, there is an eigenvalue and eigenvectors related to matrices that can be used for analyze the periodicity.

Definition 4:
For matrices $A \in R_{max}^{n \times m}$, scalar$\lambda \in R_{max}$ is the max-plus eigen value of matrix $A$ if there is a vector $v \in R_{max}^{n \times m}$ with $v \neq \varepsilon_{nx1}$, so $A \otimes v = \lambda \otimes v$ with vectors $v$ are max-plus matrix $A$ eigen vectors which correspond to $\lambda$.

### *2.2 Research methods*

Based on Khisty & Lall ( 2002) this research method is based on the study of literature and field studies which include theoretical studies, field observations for real problems, mathematical modeling and analysis by doing simulations. In this process the researcher directly observes the place of research and looks for information needed in the research process until the details will be obtained so that the research process runs smoothly and gets the expected results [3]. The steps taken are conducting field observations on the object of research, namely at Monjali, Kentungan, and Gejayan crossroads. The types of data collected include (1) Data about the length of time the traffic lights are applied now (2) Data on many motorized vehicles and passing cars.Steps taken are conducting field observations of research objects namely at the Kentungan crossroad, Monjali crossroad and Gejayan crossroad using traffic light which exists. The chosen place is east of Monjali towards the west of Gejayan. The choice of place aims to find the harmony of data used in research and to ensure that the data can be examined using the Max-Plus Algebra tool (Fahim, 2013). The types of data collected include data on the length of time of traffic lights from the west and east, Data of many cars passing through, data on car density, distance data between cars, car length and length of car trajectories. The method used in collecting this research data is direct observation and recording to take primary data from the study site, namely the duration of red, yellow, and green lights on each intersection. From the observation process, researchers can analyze using the help of max-plus algebra to obtain a queuing system model to find out the total time required for a queuing unit to pass from Monjali to Gejayan on a sufficiently crowded queue.

## 3. Results and Discussion

The model of three crossroads can be seen in Figure 1. The results of field observations show the time data of the red, green and yellow lights (in seconds) at the three crossroads queue length data and the number of vechile can be seen in Tables 1–4.

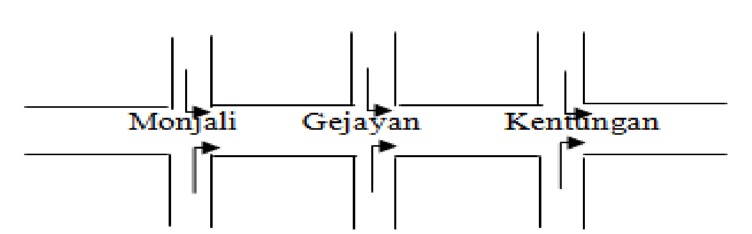


**Figure 1**. Three crossroad traffic flow.

**Table 1.** Traffic ligth duration in Monjali.

| Monjali | Red | Green | Yellow |
|---|---|---|---|
| South | 110 | 33 | 3 |
| East | 112 | 41 | 3 |
| North | 120 | 26 | 2 |
| West | 145 | 38 | 3 |

**Table 2.** Traffic ligth duration in Kentungan.

| Monjali | Red | Green | Yellow |
|---|---|---|---|
| South | 175 | 20 | 3 |
| East | 137 | 60 | 3 |
| North | 180 | 31 | 4 |
| West | 140 | 63 | 3 |

**Table 3.** Traffic ligth duration in Gejayan.

| Monjali | Red | Green | Yellow |
|---|---|---|---|
| South | 133 | 24 | 4 |
| East | 133 | 45 | 3 |
| North | 131 | 53 | 4 |
| West | 133 | 53 | 3 |

**Table 4.** Queue length data, and the number of vehicles.

| Red light | Queue length (m) | The number of vehicles |
|---|---|---|
| 1st | 50 | 12 |
| 2nd | 45 | 10 |
| 3rd | 45 | 10 |
| 4th | 47 | 10 |
| 5th | 60 | 13 |
| 6th | 58 | 13 |
| 7th | 47 | 10 |
| 8th | 58 | 13 |
| 9th | 53 | 12 |
| 10th | 51 | 10 |

In addition to knowing the duration of the red, green and yellow lights in the three intersections in the model arrangement, another assumption considered in this study is that the first red light is assumed to emerge at the same time, the vehicle volume is fixed by looking at the road going to the road modeled when someone comes out of the first straight area in Monjali, Kentungan, or Gejayan.

Also defined that one queue unit is the number of vehicles calculated based on the queue length. The following Figure 2 shows a graph of the vehicle queue on all three crossroads.

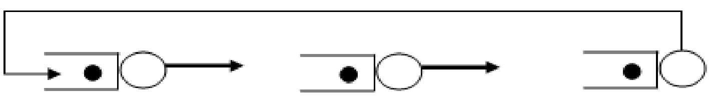

**Figure 2**. Vechile queque graph.

A mathematical model will be created using max-plus algebra to find out the total time needed for a queue unit to get out of Gejayan. Let:

$a_i^{(k)}$ : Time of arrival of the first car queue unit at the $i$-crossroad

$d_i^{(k)}$ : Departure time for the first car queue unit from the $i$-crossroad

$t_i$ : Long red lights at the *b*-crossroad when the car queue unit is at the front

$th_i$ : Long green lights at the *i*-crossroad when the car queue unit is at the front
$tj_i$ : Duration of travel on $i$-road
So, the mathematical model becomes:

$$d_1(k) = max(t_1 + a_1(k), t_1 + d_1(k-1))$$
$$d_2(k) = max(th_1 + tj_1 + +t_2 + a_2(k), th_1 + tj_1 + t_2 + d_2(k-1))$$
$$d_3(k) = max(th_3 + tj_2 + +t_3 + a_3(k), th_2 + tj_2 + t_3 + d_3(k-1))$$

If the max operation is denoted by $\oplus$, the sum operation is denoted by $\otimes$, then the form can be written as follows:

$$d_1(k) = (t_1 \otimes a_1(k)) \oplus (t_1 \otimes d_1(k-1))$$
$$d_2(k) = (th_1 \otimes tj_1 \otimes t_2 \otimes a_2(k)) \oplus (th_1 \otimes tj_1 \otimes t_2 \otimes d_2(k-1))$$
$$d_3(k) = (th_3 \otimes tj_2 \otimes t_3 \otimes a_3(k)) \oplus (th_2 \otimes tj_2 \otimes t_3 \otimes d_2(k-1))$$

where:

$$a_1(k) = d_3(k-1)$$
$$a_2(k) = d_1(k-1)$$
$$a_3(k) = d_2(k-1)$$

By using existing data we obtain the following forms:

$$d_1(k) = (145 \otimes d_3(k-1)) \oplus (145 \otimes d_1(k-1))$$
$$d_2(k) = (35 \otimes 180 \otimes 140 \otimes d_1(k-1)) \oplus (35 \otimes 180 \otimes 140 \otimes d_2(k-1))$$
$$d_3(k) = (70 \otimes 180 \otimes 180 \otimes d_2(k-1)) \oplus (70 \otimes 180 \otimes 180 \otimes d_3(k-1))$$

We can write in form:

$$d_1(k) = (145 \otimes d_3(k-1)) \oplus (145 \otimes d_1(k-1))$$
$$d_2(k) = (355 \otimes d_1(k-1)) \oplus (355 \otimes d_2(k-1))$$
$$d_3(k) = (430 \otimes d_2(k-1)) \oplus (430 \otimes d_3(k-1))$$

The mathematical model is converted into a matrix for updating the output of a mathematical model. Form a matrix of the model the math above is:

$$\begin{bmatrix} d_1(k) \\ d_2(k) \\ d_3(k) \end{bmatrix} = \begin{bmatrix} 145 & \varepsilon & 145 \\ 355 & 355 & \varepsilon \\ \varepsilon & 430 & 430 \end{bmatrix} \otimes \begin{bmatrix} d_1(k-1) \\ d_2(k-1) \\ d_3(k-1) \end{bmatrix}$$

Using MATLAB, the following results are obtained can be seen in Table 5. The result shows that the total travel time of one queue unit can exit Gejayan after 10 iterations.

**Table 5.** Total tavel time of one queue unit.

| Total travel time of one queue unit in second | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $d_1(k)$ | 145 | 290 | 720 | 1150 | 1580 | 2010 | 2440 | 2870 | 3300 | 3730 |
| $d_2(k)$ | 145 | 500 | 855 | 1210 | 1565 | 1935 | 2365 | 2795 | 3225 | 3655 |
| $d_3(k)$ | 145 | 575 | 1005 | 1435 | 1865 | 2295 | 2725 | 3155 | 3585 | 4015 |

## 4. Conclusion

The results of the study show that the queue system can be modeled using max-plus algebra. Furthermore, it can be determined the waiting time, the time to leave the crossroad, and the total time needed to cross from the Monjali crossroad to exit the Gejayan crossroad. With 10 iterations, the total time needed for one queue unit to get out of the Gejayan crossroad is 4015 seconds.

## Acknowledgment

The author thanks  to LPPM Sanata Dharma University for supporting all research processes so that this article can be completed also to Mr. Andy Rudhoto who has directed and assisted the writer in the process of making this article.

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