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ISSN 00001984, 00002005, 00001983

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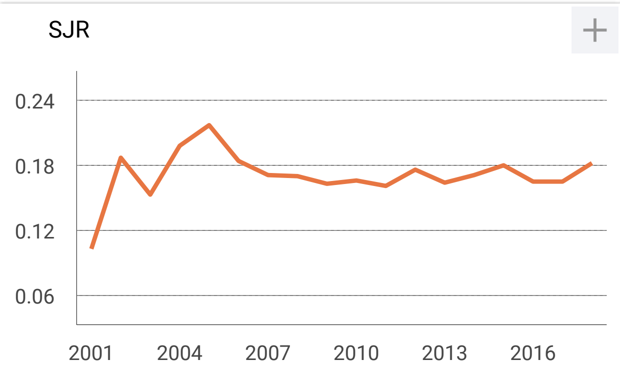

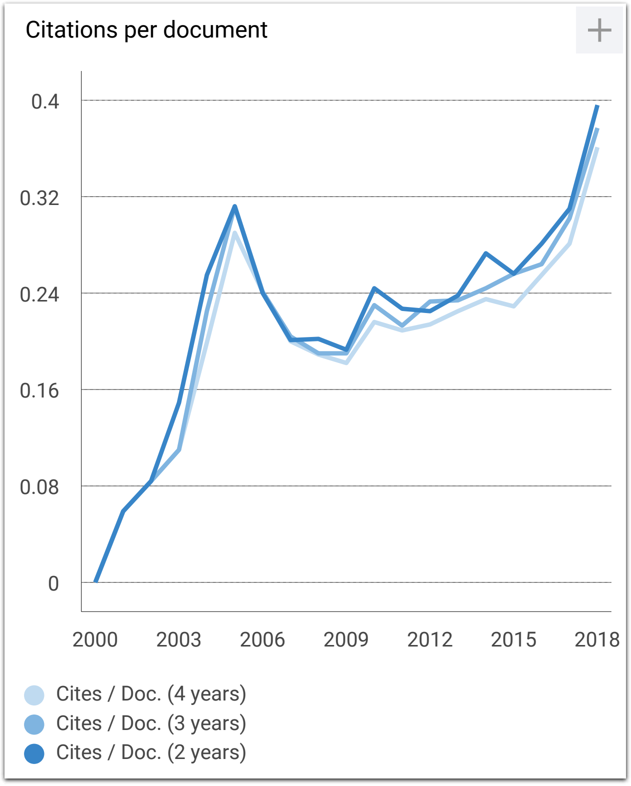

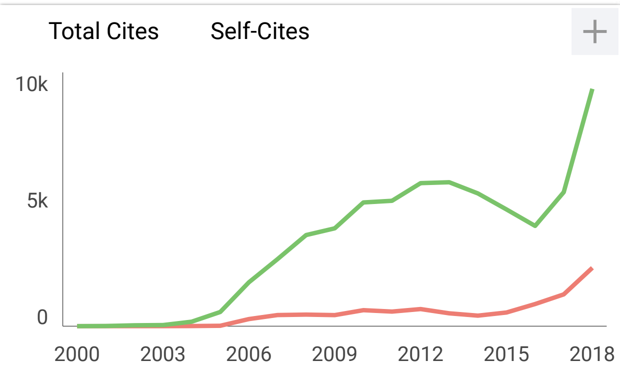

Performance of the Runge-Kutta methods in solving a mathematical model for the spread of dengue fever disease

Cite as: AIP Conference Proceedings **2202**, 020044 (2019); https://doi.org/10.1063/1.5141657
Published Online: 27 December 2019

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Performance of the Runge-Kutta Methods in Solving a Mathematical Model for the Spread of Dengue Fever Disease

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Abstract. Dengue fever disease has been a big problem in Indonesia. In this paper, we solve a mathematical model for the spread of dengue fever disease using the Runge-Kutta methods. We provide some cases to show the performance of the Runge-Kutta methods in solving the model. In particular, the Runge-Kutta methods to be applied are the Euler and the Heun methods. We focus on researching the performance of these methods in solving the model.

INTRODUCTION

Mathematics is a field of science that is very useful for calculating or predicting various aspects of life. Mathematics is not only applied in the life of a professional mathematician but also often used in various fields of medicine, computer engineering, mechanical engineering, economics, accounting, biology, physics, chemistry and other sciences.

One of mathematical topics is differential equations. Differential equation is an equation that states the relationship between derivatives of a dependent variable to one or more independent variables. In other words, differential equations are equations for the function of one or more variables, which have a relationship between the value of functions and their derivatives in all orders [1].

The differential equation models are available in the literature. An example is a system for the spread of dengue models in the form of Susceptible-Infected-Recovered (SIR) system [2]. This system relates to the Dengue Hemorrhagic Fever (DHF), which is an infectious disease that can be spread or transmitted by Aedes aegypti mosquitoes through viruses dengue from sufferers to others through their bites [3].

Euler method and Heun method are methods commonly used to solve differential equations with a certain order having results are numerical [4-6]. The Runge-Kutta methods, and to be specific, the Euler method and the Heun method, have been widely used to solve nonlinear equations by some researchers such as [7-10].

The purpose of this study was to determine the solution of the system of equations of dengue fever using the Runge-Kutta methods (the Euler method and the Heun method). The results of this solution are in the form of an approach or an approximation.

The next sections provide as follows: the mathematical model, the Euler and Heun methods used to solve the mathematical model, mathematical results of computations, and conclusions.

MATHEMATICAL MODEL AND METHODS

In this section, we provide the mathematical model that we want to solve and the numerical methods that we want to implement.

International Conference on Science and Applied Science (ICSAS) 2019
AIP Conf. Proc. 2202, 020044-1–020044-6; https://doi.org/10.1063/1.5141657
Published by AIP Publishing. 978-0-7354-1953-7/$30.00

SIR Model

The SIR model of dengue fever [2] is:

$$\frac{dx}{dt} = \mu_h(1 - x(t)) - \alpha x(t)z(t) \tag{1}$$

$$\frac{dy}{dt} = \alpha x(t)z(t) - \beta y(t) \tag{2}$$

$$\frac{dz}{dt} = \gamma\big(1 - z(t)\big)y(t) - \delta_1 z(t) \tag{3}$$

where t is time variable. The coefficients are constants. Furthermore, $x(t)$ is the proportion of the *susceptible* host (human) population with respect to the total number of the host population, $y(t)$ is the proportion of the *infectious* (infected) host population with respect to the total number of the host population, and $z(t)$ is the proportion of the infectious vector (Aedes aegypti mosquitoes) with respect to the total number of the vector population. Note that the proportion of the *recovered* population $r(t)$ with respect to the total number of the host population can be obtained easily by the formula $r(t) = 1 - x(t) - y(t)$. It can be calculated if $x(t)$ and $y(t)$ are obtained beforehand.

Euler Method

The Euler method is used to solve a system of nonlinear equations:

$$\frac{dx}{dt} = f(t, x, y, z), \tag{4}$$

$$\frac{dy}{dt} = g(t, x, y, z), \tag{5}$$

$$\frac{dz}{dt} = j(t, x, y, z), \tag{6}$$

with value $x(0) = x_0$, $y(0) = y_0$, $z(0) = z_0$ and choose a fixed h value that is $h = t_{n+1} - t_n$ is:

$$x_{n+1} = x_n + h\, f(t_n, x_n, y_n, z_n), \tag{7}$$

$$y_{n+1} = y_n + h\, g(t_n, x_n, y_n, z_n), \tag{8}$$

$$z_{n+1} = z_n + h\, j(t_n, x_n, y_n, z_n), \tag{9}$$

for $n = 0, 1, 2, \dots$.

The nonlinear equation in our SIR model is a system of differential equations (1)-(3) which is a system of simultaneous nonlinear differential equations. They can be written as:

$$\frac{dx}{dt} = f(t, x, y, z) = \mu_h\big(1 - x(t)\big) - \alpha x(t)z(t), \tag{10}$$

$$\frac{dy}{dt} = g(t, x, y, z) = \alpha x(t)z(t) - \beta y(t), \tag{11}$$

$$\frac{dz}{dt} = j(t, x, y, z) = \gamma\big(1 - z(t)\big)y(t) - \delta_1 z(t). \tag{12}$$

Using the Euler method, we solve equations (10)-(12) as follows:

$$x_{n+1} = x_n + h\,[\mu_h(1 - x_n) - \alpha x_n z_n], \tag{13}$$

$$y_{n+1} = y_n + h\,[\alpha x_n z_n - \beta y_n], \tag{14}$$

$$z_{n+1} = z_n + h\,[\gamma(1 - z_n)y_n - \delta_1 z_n]. \tag{15}$$

Heun Method

A system of first-order differential equations is given with three dependent variables namely equation (10)-(12) with $h = t_{n+1} - t_n$, then the Heun method scheme for the equation system is in the form of:

Predictor :

$$\tilde{x}_{n+1} = x_n + hf(t_n, x_n, y_n, z_n), \tag{16}$$

$$\tilde{y}_{n+1} = y_n + hg(t_n, x_n, y_n, z_n), \tag{17}$$

$$\tilde{z}_{n+1} = z_n + hj(t_n, x_n, y_n, z_n), \tag{18}$$

Corrector :

$$x_{n+1} = x_n + \frac{h}{2}[f(t_{n,}x_n, y_n, z_n) + f(t_{n+1,}\tilde{x}_{n+1}, \tilde{y}_{n+1}, \tilde{z}_{n+1})], \tag{19}$$

$$y_{n+1} = y_n + \frac{h}{2}[g(t_n, x_n, y_n, z_n) + g(t_{n+1,}\tilde{x}_{n+1}, \tilde{y}_{n+1}, \tilde{z}_{n+1})], \tag{20}$$

$$z_{n+1} = z_n + \frac{h}{2}[j(t_n, x_n, y_n, z_n) + j(t_{n+1,}\tilde{x}_{n+1}, \tilde{y}_{n+1}, \tilde{z}_{n+1})], \tag{21}$$

or we can rewrite them as:

Predictor :

$$\tilde{x}_{n+1} = x_n + h\,[\mu_h(1 - x_n) - \alpha x_n z_n], \tag{22}$$

$$\tilde{y}_{n+1} = y_n + h\,[\alpha x_n z_n - \beta y_n], \tag{23}$$

$$\tilde{z}_{n+1} = z_n + h\,[\gamma(1 - z_n)y_n - \delta_1 z_n], \tag{24}$$

Corrector :

$$x_{n+1} = x_n + \frac{h}{2}[(\mu_h(1 - x_n) - \alpha x_n z_n) + (\mu_h(1 - \tilde{x}_{n+1}) - \alpha\tilde{x}_{n+1}\tilde{z}_{n+1})], \tag{25}$$

$$y_{n+1} = y_n + \frac{h}{2}[(\alpha x_n z_n - \beta y_n) + (\alpha\tilde{x}_{n+1}\tilde{z}_{n+1} - \beta\tilde{y}_{n+1})], \tag{26}$$

$$z_{n+1} = z_n + \frac{h}{2}[(\gamma(1 - z_n)y_n - \delta_1 z_n) + (\gamma(1 - \tilde{z}_{n+1})\tilde{y}_{n+1} - \delta_1\tilde{z}_{n+1})]. \tag{27}$$

RESULTS

In this section, we provide our research results. As we have mentioned that we use the Euler method and the Heun method, this section will discuss about their results in solving the SIR model.

Results of Euler Method

We assume to have the following values for the parameters [11]:

$$c_1 = \frac{7675406}{7675893},\ \ c_2 = \frac{487}{7675893},\ \ c_3 = 0.056,\ \ \alpha = 0.232198,$$
$$\beta = 0.328879,\ \mu_h = 0.0000460,\ \gamma = 0.375\ \text{and}\ \ \delta_1 = 0.0323. \tag{28}$$

and take value $h = 0.1$. Then the solution of the system of equations (1)-(3) using the Euler scheme are as in equations (13)-(15). Solutions to equations (1)-(3) with the Euler method for the value of $0 \leq t \leq 6$ with the aid of the MATLAB software are recorded in Table 1.

Solutions to equations (1)-(3) with the Euler method for the value $0 \leq t \leq 6$ can be solved with the MATLAB software, as shown in Fig. 1. In this figure, the graph of x falls from the value of 0.999937 to the value of 0.909459102, the graph y rises from the value 0.0000634 to the value of 0.04279 and the z graph rises from the value of 0.056 to the value 0.093229.

TABLE 1. The values of $X(t)$, $Y(t)$ and $Z(t)$ of equations (1)-(3) using the Euler method. Here, $X(t)$, $Y(t)$ and $Z(t)$ are the Euler solutions to the variables $x(t)$, $y(t)$ and $z(t)$.

t_n	$X(t_n)$	$Y(t_n)$	$Z(t_n)$
0	0.99994	0.000063	0.05600
4.5	0.93668	0.034476	0.07759
4.6	0.93499	0.035030	0.07853
4.7	0.93329	0.035582	0.07948
4.8	0.93157	0.036135	0.08046
4.9	0.92983	0.036687	0.08144
5.0	0.92807	0.037239	0.08245
5.1	0.92629	0.037791	0.08346
5.2	0.92450	0.038343	0.08449
5.3	0.92268	0.038896	0.08553
5.4	0.92085	0.039449	0.08659
5.5	0.91900	0.040003	0.08766
5.6	0.91713	0.040558	0.08875
5.7	0.91524	0.041114	0.08985
5.8	0.91333	0.041672	0.09096
5.9	0.91140	0.042230	0.09209
6.0	0.90945	0.042790	0.09323

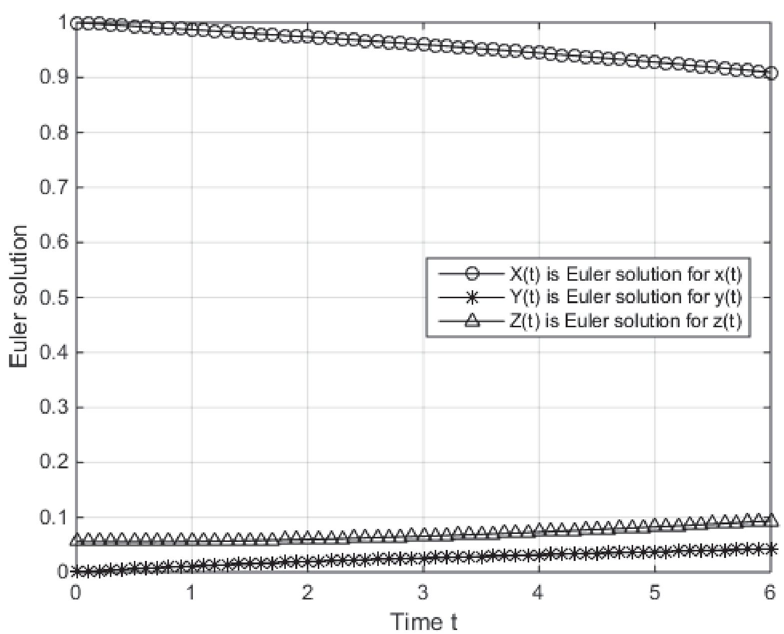

FIGURE 1. Solutions to the SIR model of equations (1)-(3) using the Euler method.

Results of Heun Method

We take the same data in equation (28) and take the value $h = 0.1$. Then the solution to the system of equations (1)-(3) with the Heun method are as in equations (22)-(27). Solutions to equations (1)-(3) with the Heun method for the value of $0 \leq t \leq 6$ with the MATLAB software are recorded in Table 2.

Solutions to equations (1)-(3) with the Heun method for the value $0 \leq t \leq 6$ can be described with the MATLAB software, as shown in Fig. 2. In the figure, the x graph falls from the value 0.999937 to the value 0.908791, the y graph rises from the value 0.0000634 to the value of 0.04288 and the z graph rises from the value 0.056 to the value 0.093689836.

TABEL 2. The values of $X(t)$, $Y(t)$ and $Z(t)$ of equations (1)-(3) using the Heun method. Here, $X(t)$, $Y(t)$ and $Z(t)$ are the Heun solutions to the variables $x(t)$, $y(t)$ and $z(t)$.

t_n	$X(t_n)$	$Y(t_n)$	$Z(t_n)$
0	0.999937	0.000063	0.056000000
4.5	0.936273	0.034469	0.077951839
4.6	0.934570	0.035029	0.078899432
4.7	0.932849	0.035589	0.079862031
4.8	0.931111	0.036148	0.080839501
4.9	0.929354	0.036707	0.081831718
5.0	0.927579	0.037265	0.082838564
5.1	0.925786	0.037824	0.083859927
5.2	0.923975	0.038383	0.084895703
5.3	0.922144	0.038942	0.085945792
5.4	0.920295	0.039502	0.087010100
5.5	0.918427	0.040062	0.088088537
5.6	0.916539	0.040624	0.089181019
5.7	0.914631	0.041186	0.090287465
5.8	0.912705	0.041750	0.091407799
5.9	0.910758	0.042314	0.092541946
6.0	0.908791	0.042880	0.093689836

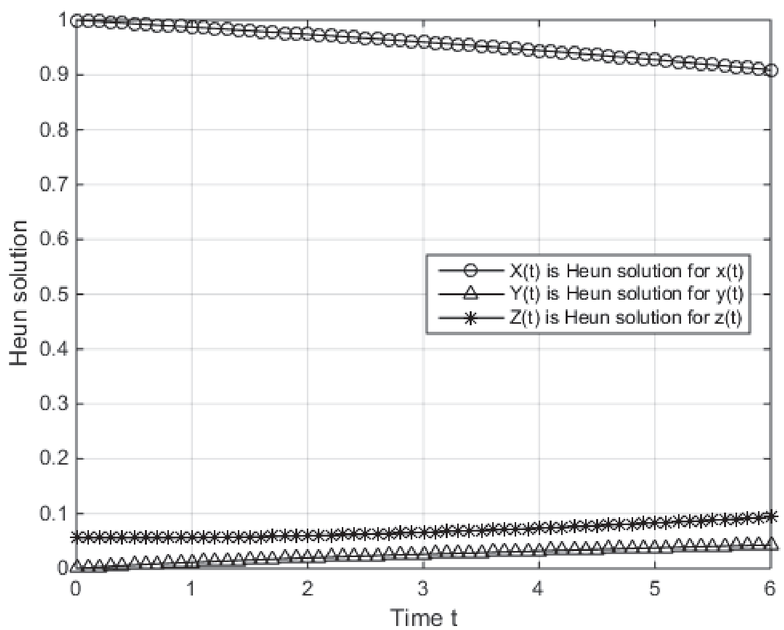

FIGURE 2. Solutions to the SIR model of equations (1)-(3) using the Heun method.

Before we conclude this paper, it is worthwhile to note that our results of the SIR numerical solutions in this paper can be used to test emerging numerical methods called "nonstandard finite difference methods" for solving differential equations. These nonstandard finite difference methods are due to Mickens [12-15]. This could be a future research direction.

CONCLUSION

We can solve the system of equations for the dengue fever SIR model using the Euler method and the Heun method. The resulting solutions are approximations to the exact ones. We observe that the differences between the Euler's solution and the Heun's solution at small and large time values remain small.

ACKNOWLEDGMENTS

Some of this work was a part of the first author's thesis [16] written under the supervision of the second author. This research was supported financially by Sanata Dharma University. The authors are very grateful to Sanata Dharma University for the financial support.

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