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The efficiency and effectiveness of fins made from two different materials in unsteady-state

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The efficiency and effectiveness of fins made from two different materials in unsteady-state

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Abstract. This paper describes the calculation of the temperature distribution on the fins, the rate of heat flow released by the fins, the efficiency and effectiveness of the straight fins in the unsteady-state. Calculations are performed using numerical computational methods. The cross section of the fins has a capsule-shape. Fins are made of two different metal materials. The heat -transfer that occurs in fins only occurs in one dimension, namely in the x direction, or perpendicular to the base of the fin. The method used is an explicit finite different method. This paper also explains the influence of fin material on the value of temperature distribution, heat flow rate, fin efficiency and effectiveness. Temperature distribution, heat flow rate, efficiency and efficiency of fins on fins that have two materials with different materials, have different values. This difference is due to material properties of fins such as: material density, specific heat and thermal conductivity. Research can be continued for various fins and cross sections, which are composed of several different materials, according to the needs of the era. Research can also be developed by using boundary conditions which have a fin base temperature that changes with time or receives heat flux.

1. Introduction

In fin design, knowledge of the temperature distribution of the fins, the rate of heat flow released by the fins, the efficiency of the fins and the effectiveness of the fins in steady-state and unsteady-state are very much needed. Related research on fins in unsteady-state has been carried out by several researchers [3, 4, 5,..14]. Various examples of fin shapes that can be used to release heat can be seen in reference-books. For certain fin shapes, fin efficiency in steady-state can be found in these reference books [1,2]. So far, research is usually only on fins made from one fin material.

A mathematical model for obtaining a temperature distribution on a straight fin that has a fixed cross-section, with two different materials, in a non-steady state, can be determined by Equation (1). If the fins at the initial conditions have a uniform temperature, and are the same as the temperature at the base of the fins, then the initial conditions on the fins can be expressed by Equation (1a). If the condition at the base of the fin (x = 0) always has a constant temperature of Tb, and the cross-section surface at the tip of the fin, it always processes heat transfer by convection with the surrounding fluid, then the boundary conditions at the base of the fin and at the tip of the fin in this mathematical model, can be stated by Equations (1b), and (1c). For conditions on the boundaries of the two ingredients can be expressed by Equation (1d). Fig. 1 presents the fin image that is the object of the problem.

$$\frac{\partial^2 T(x,t)}{\partial x^2} + \frac{hP}{kAp}\left(T(x,t) - T_\infty\right) = \frac{\rho c}{k}\frac{\partial T(x,t)}{\partial t} \qquad 0<x<L,\ x \neq L_1,\ t>0 \qquad (1)$$

Initial Conditions :

$$T(x,0) = T_i = T_b \qquad 0 \leq x \leq L,\ t=0 \tag{1a}$$

Boundary Conditions :

$$T(0,t) = T_b \qquad x=0,\ t>0 \tag{1b}$$

$$k_2 \frac{\partial T(x,t)}{\partial x} = h(T(x,t) - T_\infty) \qquad x=L,\ t>0 \tag{1c}$$

$$k_1 \frac{\partial T(x,t)}{\partial x} = k_2 \frac{\partial T(x,t)}{\partial x} \qquad x=L_1,\ t>0 \tag{1d}$$

In Equations (1), (1a), (1b), (1c), and (1d):

T(x,t)	:	The temperature at the x position, at the t time, °C
x	:	Stating the position on fin, m
t	:	Stating the time, seconds
h	:	Convection heat transfer coefficient, W/(m^{2}°C)
k_1	:	Conduction heat transfer coefficient of fin material 1, W/(m°C)
k_2	:	Conduction heat transfer coefficient of fin material 2, W/(m°C)
P	:	Around the cross section, m
A_p	:	Cross-sectional of the fin, m^2
T_∞	:	Fluid temperature around the fin, °C.
T_i	:	Initial temperature of the fin, °C
T_b	:	Temperature at the base of the fin , °C
L	:	The length of the fin , m
L_1=L_2	:	The length of the fin with material 1or the length of the fin with material 2, m

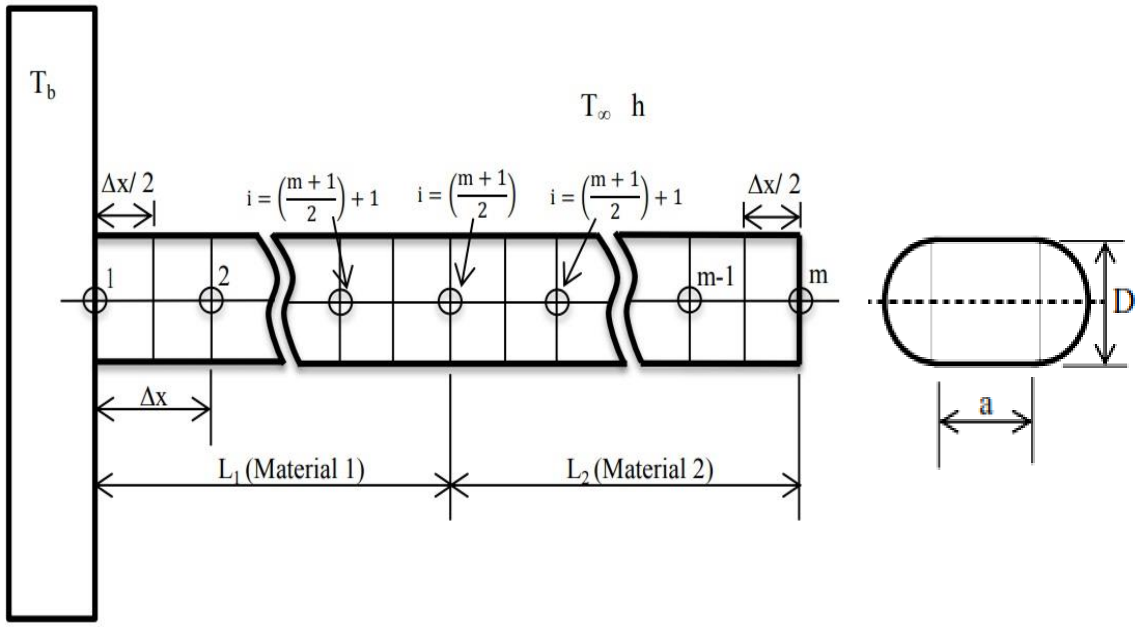

Figure 1. Straight fins with the capsule-shaped cross section

The sequence of calculation steps is carried out as follows: (a) searching the distribution on the fins (b) calculating the rate of heat flow released by the fins to the fluid (c) calculating the efficiency

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of the fins and (d) calculating the effectiveness of the fins. The calculation is done by an explicit finite-difference method. Fins are divided by many small elements. This small element is called the control volume. Total the control volume is m. Each the volume control is numbered, starting with numbers 1, 2, 3, ... m. The control volume i = 1 is at the base of the fin. The control volume to i = m is at the tip of the fin. The control volume (m + 1) / 2 is at the center of the fin. The control volume at the base of the fin i = 1 and at the tip of the same fin i = m has a thickness (Δx / 2). The control volume from i=2 to i = m-1 has a thickness Δx. The distance between the control volume is Δx. Each the control volume has a uniform temperature.

1.1. Temperature distribution

The equation to calculate the temperature in each control volume on the fins is derived by using the principle of energy balance in each control volume. In this calculation, the initial condition of the fins (when t = 0) has a uniform temperature, the value of which is the same as the temperature at the base of the fin. Equations are numerically presented in Equation (2). To calculate the temperature at the volume control located at: (a) at the base of the fin, (b) from the bottom of the fin to the meeting of the fin material (c) at the meeting of the fin material (d) from the meeting place of the fin material to the tip of the fin, and (e) at the end of the fin, use Equations (3), (4), (5), (6) and (7).

$$T_i^n = T_b \qquad i=1, 2, 3...m \;;\; n=0 \tag{2}$$

$$T_i^{n+1} = T_b \qquad i=1, \tag{3}$$

$$T_i^{n+1} = \frac{k_1 \Delta t}{\rho_1 c_1 \Delta x^2}\left(T_{i-1}^n - 2T_i^n + T_{i+1}^n + \frac{h\Delta x}{k_1}\frac{A_{s,i}}{A_P}(T_\infty - T_i^n)\right) + T_i^n \qquad i=2, 3, 4,....[(m+1)/2]-1. \tag{4}$$

$$T_i^{n+1} \frac{\Delta t}{(\rho_1\ c_1\ V_1 + \rho_2\ c_2\ V_2)}\left(k_1\ A_p \frac{T_{i-1}^n - T_i^n}{\Delta x} + k_2\ A_p \frac{T_{i+1}^n - T_i^n}{\Delta x} + h\ A_s(T_\infty - T_i^n)\right) + T_i^n \qquad i= (m+1)/2 \tag{5}$$

$$T_i^{n+1} = \frac{k_2 \Delta t}{\rho_2 c_2 \Delta x^2}\left(T_{i-1}^n - 2T_i^n + T_{i+1}^n + \frac{h\Delta x}{k_2}\frac{A_{s,i}}{A_P}(T_\infty - T_i^n)\right) + T_i^n \qquad i= [(m+1)/2]+1,\dots ..m-1. \tag{6}$$

$$T_i^{n+1} = \frac{k\Delta t}{0{,}5\rho_2\, c_2 \Delta x^2}\left((T_{i-1}^n - T_i^n) + \left(\frac{h\Delta x}{k_2}(T_\infty - T_i^n)\right) + \left(\frac{h\Delta x}{k_2}\frac{A_{s,i}}{A_P}(T_\infty - T_i^n)\right)\right) + T_i^n \qquad i= m. \tag{7}$$

The stability requirements for Equations (4), (5), (6) and (7) are respectively expressed by Equations (8), (9), (10) and (11).

$$\Delta t \le \frac{\rho_1 c_1 \Delta x^2}{k_1\left(2 + \frac{h\Delta x A_{s,i}}{k_1\ A_p}\right)} \tag{8}$$

$$\Delta t \le \frac{(\rho_1\, c_1\, V_1 + \rho_2\, c_2\, V_2)}{\left(\frac{k_1 A_p}{\Delta x} + \frac{k_2 A_p}{\Delta x} + h\ A_s\right)} \tag{9}$$

$$\Delta t \le \frac{\rho_2 c_2 \Delta x^2}{k_2\left(2 + \frac{h\Delta x A_{s,i}}{k_2\ A_p}\right)} \tag{10}$$

$$\Delta t \le \frac{(0{,}5\ \rho_2 c_2 \Delta x^2)}{k_2\left(1 + \frac{h\Delta x}{k_2} + \frac{h\Delta x A_{s,i}}{k_2\ A_p}\right)} \tag{11}$$

In Equation (2) to Equation (11) :

T_{i-1}^{n}	:	Temperature of the control volume in position i at time n-1, ^oC
T_{i+1}^{n}	:	Temperature of the control volume at the i position i at the n time, ^oC
T_{i}^{n+1}	:	Temperature of the control volume at the i position at the (n+1) time, ^oC
T_∞	:	Fluid temperature around the fin, ^oC
$A_{s,i}$	:	The fin surface area of the control volume is in contact with the fluid at the i position, m^2
A_p	:	Area of the cross-sectional of the fin, m^2.
Δx	:	Distance between the control volumes, m
Δt	:	Time step, from the n to the n+1, seconds
k_1	:	Conduction heat transfer coefficient of material 1, W/m^oC
k_2	:	Conduction heat transfer coefficient of material 2, W/m^oC.
ρ_1	:	Density of the fin material 1, kg / m^3
c_1	:	Specific heat of the fin material 1, $J/(kg^oC)$
ρ_2	:	Density of the fin material 2, kg / m^3
c_2	:	Specific heat of the fin material 2, $J/(kg^oC)$
h	:	Convection heat transfer coefficient, $W/(m^{2o}C)$
m	:	The number of the control volumes

1.2. Heat flow rate

The actual heat flow rate released by the fins ($q_{fin,\ actual}$) at time t, the maximum heat flow rate released by the fins ($q_{fin,\ ideal}$) and the heat flow rate if the object is not finned (q_{nofin}) in the steady-state are successively calculated using Equation (12), Equation (13) and Equation (14). The maximum heat flow rate released by the fins is the rate of heat flow released, if all fins have the same uniform temperature as the base temperature of the fins. The rate of heat flow if the object is not finned is the rate of heat flow released if the length of the fin is zero, and the heat released is from the base of the fin.

$$q_{fin,actual}^{n} = h\sum_{i=1}^{m} A_{s,i}(T_i^n - T_\infty) \tag{12}$$

$$q_{fin,ideal} = hA_s(T_b - T_\infty) \tag{13}$$

$$q_{no\,fin} = hA_b(T_b - T_\infty) \tag{14}$$

In Equations (12), (13) and (14) :

T_i^n	:	Temperature of the control volume in position i at time n, ^oC
T_∞	:	Fluid temperatur, ^oC.
$A_{s,i}$	:	The blanket area of the control volume in position i, m^2
A_s	:	Surface area of the fin, m^2
A_b	:	Surface area of the base of fin, m^2
T_b	:	Temperature of the base of the fin, ^oC
T_∞	:	Fluid temperature around the fin, ^oC
m	:	The number of the control volumes

1.3. *Eficiency and effectiveness*

The efficiency of fins in an unsteady-state can be calculated by Equation (15) and the effectiveness of fins in an unsteady-state can be calculated by using Equation (16). The efficiency value of fins lies between 0% to 100%, while the value of the effectiveness of fins can be greater than 1. The greater the effectiveness of the fins, the greater the ability of fins to release heat.

$$\eta = \frac{q^{n}_{fin,actual}}{q_{fin,\ ideal}} = \frac{h\sum_{i=1}^{m} A_{s,i}(T_i^n - T_\infty)}{h\ A_s(T_b - T_\infty)} \tag{15}$$

$$\epsilon = \frac{q^{n}_{fin,actual}}{q_{nofin}} = \frac{h\sum_{i=1}^{m} A_s(aT_i^n - T_\infty)}{h\ A_b(T_b - T_\infty)} \tag{16}$$

2. Calculation of temperature distribution, heat flow rate, efficiency and effectiveness

Calculations of temperature distribution, heat flow rate, efficiency and effectiveness in an unsteady state are performed on a capsule-shaped cross section straight fin (Fig. 1). Fin material is composed of two fin materials. The calculation is done with use the explicit finite-difference method. The properties of fin material are stated in Table 1. In this calculation the total length of the fin is used = L= 0.05 m. The length of material fin 1 is the same as the length of material fin 2, $L_1 = L_2 = 0.5L = 0.025$ m. Fin height = D = 0.01 m. The width of the fin L_b = D + a = 0.015 m. The distance between the control volumes is $\Delta x = 0.002083$ m.The base temperature of the fins is constant at any time, $T_b = 100^oC$. The temperature of the fluid around the fin is fixed and uniform, $T_\infty = 30^oC$. Convection heat transfer coefficient h = 200 W/ (m^{2o} C). Time interval $\Delta t = 0.01$ second. Total volume control m = 25.

Table 1. Properties of the material (Y.A. Çengel, *Heat Transfer A Practical Approach*, pp. 868 - 870)

Material	Density (ρ) (kg/m^3)	Thermal Conductivity (k), ($W/m^2.^oC$)	Specific Heat (c) ($J/kg.^oC$)
Copper (Cu)	8933	401	385
Aluminum (Al)	2702	237	903
Zink (Zn)	7140	116	389
Iron (Fe)	7870	80.2	447
Steel (St)	7854	60.5	434

3. Results of the Calculation

The results of the calculation of temperature distribution, heat flow rate, efficiency, and effectiveness of straight fins in unsteady-state conditions, having the capsule-shaped cross section, with an explicit difference finite method are presented in Fig. 2, Fig. 3, Fig. 4 and Fig. 5. Fins with different materials give different results. From Equations (4), (5), (6) and (7), the temperature distribution of the fins in an unsteady-state is determined by the fin material. The fin material properties that affect the temperature distribution include: specific density, specific heat and thermal conductivity of the fin material. Because thermal diffusivity is a function of density, specific heat and thermal conductivity of the material, it can also be said, the value of the temperature distribution produced on the fins is also affected by the thermal conductivity of the material. This is different for the steady-state. For steady-state, the temperature distribution on the fins is only determined by the thermal conductivity of the material, so it can be said that under steady-state, the temperature distribution cannot be said to be a function of thermal diffusivity. Different temperature distributions on the fins cause the heat flow rate released by the fins, the efficiency of the fins and the effectiveness of the fins are also not the same. From Equation (15), the efficiency of the fin is determined by the temperature distribution that occurs in the fin. From Equation (16), the effectiveness of fins is also determined by the temperature distribution of fins.

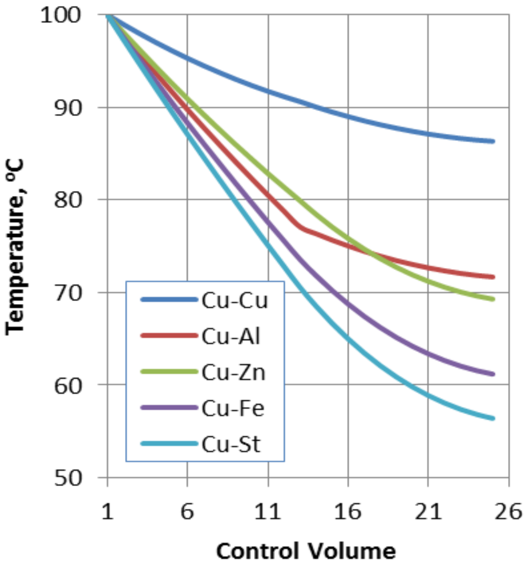

Figure 2. Temperature distribution time :180 seconds

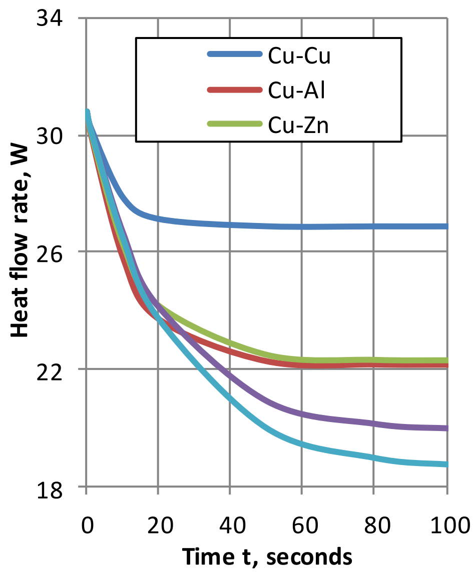

Figure 3. Heat flow rate with respect to time

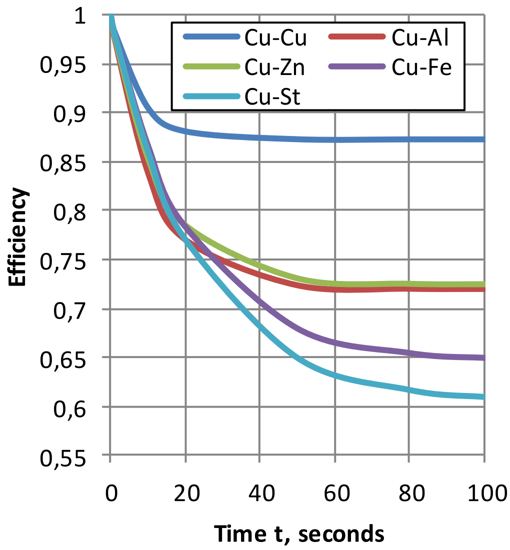

Figure 4. Efficiency of the fin with respect to time

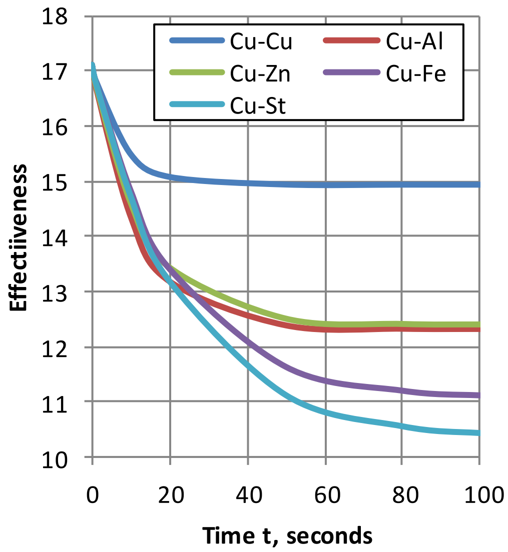

Figure 5. Effectiveness of fin with respect to time

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4. Conclusions

Finding the temperature distribution on the fins, the rate of heat flow released by the fins, the efficiency of the fins, and the effectiveness of the fins on the unsteady state can be accomplished well by using an explicit finite difference method. The efficiency and effectiveness of fins is determined by the temperature distribution produced on the fins. The temperature distribution of the fins is determined by the combination of the two fin materials selected. Material properties that affect the temperature distribution include: density, specific-heat and thermal-conductivity of the material. Calculation of efficiency and effectiveness of fins in unstable conditions can be developed on fins that have a cross-sectional area of fins which is a function of x. Research can be continued for various fins and cross sections, which are composed of several different materials, according to the needs of the era. Research can also be developed by using boundary conditions which have a fin base temperature that changes with time or receives heat flux.

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