Finite Volume Simulations of Fast Transient Flows in a Pipe System

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Abstract

The use of a finite volume method is proposed to solve a water hammer flow problem in a pipe. This flow is of the type of fast transient pipe flow. The relating mathematical model is hyperbolic partial differential equations. Therefore, our proposed choice of finite volume method is appropriate. In particular, we consider water flows through a pipe from a pressurized water tank at one end to a valve at the other end. We want to know the pressure and velocity values in the pipe when the valve closes as a function of time. Our research results may be used to prepare valves for pipe systems, so that valves have enough strength to stop pipe flows periodically.

Keywords: finite volume, pipe flow, water hammer

Introduction

Fast transient flow often occurs in pipe systems. This flow is usually caused by a valve fast closing the pipe, when there is a steady flow. This kind of flow is also known as the water hammer problem. Studies on the water hammer problem are available in the literature, for example those listed in the References [1-5].

This poster is a promotion of our collaborative work [3] and very much inline with the work of Markendahl [1]. Among the literature, the work of Markendahl [1] is simpler but provides enough knowledge to solve the water hammer problem. Markendahl [1] tested various values of closing times.

In this poster, instead of varying the closing time, we vary the initial velocity of water in the pipe. The closing time is fixed. We show that different initial velocities of water lead to different values of maximum pressure hitting the pipe. Furthermore we research the relation between the varying initial velocities and the values of maximum pressure, exactly at the instant time when the valve is completely closed. This work extends the research of Markendahl [1]. A finite volume method with the standard Lax-Friedrichs formulation is applied in this work.

The Considered Problem

We want to solve the fast transient pipe flow problem with the model shown in Figure 1. If units of quantities are omitted, they should be understood to have SI units. The dimensional space is [0,170], where the value 170 m is the total of three segments 60, 10, 90 and 10 m.

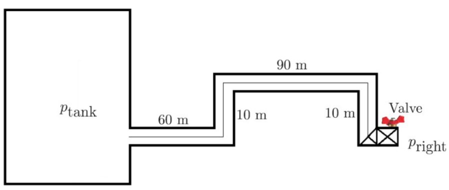

Figure 1. The considered pipe system to be solved.

Some notes should be taken into account. In the mathematical model, the effect of pipe bendings are assumed to be negligible, so that the model is one-dimensional. Frictions to the pipe wall is also neglected, so the pipe is assumed to be ideal. Turbulent flow is out of the scope of this research.

Mathematical Model & Method

The mathematical model for the one-dimensional water hammer problem is

$$\frac{\partial p(x,t)}{\partial t} + \rho c^2 \frac{\partial u(x,t)}{\partial x} = 0,$$

$$\frac{\partial u(x,t)}{\partial t} + \frac{1}{\rho}\frac{\partial p(x,t)}{\partial x} = 0.$$

Here,

- x is the one-dimensional space variable,
- t is the time variable,
- p represents the pressure,
- u represents the velocity,
- ρ is the density, and
- c is the propagation speed of pressure wave.

We discretize the mathematical model using the standard finite volume method with uniform space width and uniform time step. The space is discretized into a finite number of cells.

The fully discrete finite volume method is

$$\mathbf{Q}_i^{n+1} = \mathbf{Q}_i^n - \frac{\Delta t}{\Delta x}\left(\mathbf{F}_{i+1/2}^n - \mathbf{F}_{i-1/2}^n\right).$$

Here,

- $\mathbf{Q}_i^n$ represents the vector of averaged conserved quantities over the i-th cell at the n-th time step,
- $\mathbf{F}_{i-1/2}^n$ represents the vector of averaged fluxes flowing through the interface at the corresponding subscript during one time step,
- Δt is the time step, and
- Δx is the cell width.

In this research, we use the Lax-Friedrichs formulation to compute the numerical flux. We refer to the work of LeVeque [1] for the Lax-Friedrichs flux formulation.

Result: Varied Initial Velocity

Our numerical test is set as follows. We take the tank pressure to be 100,000 Pa and the pressure at the right side is zero. The propagation speed of pressure wave is 1,500 m/s. The water density is taken 1,000 kg/m^3. The initial pressure in the pipe is considered to be the same as the pressure in the tank. The initial velocity of water flowing in the pipe is varied and we investigate the effect of such variation to the system.

Table 1. The pressure and velocity hitting the valve.

Initial velocity	Results for p: p_N	Results for p: p order	Results for u: u_N	Results for u: u order
0.5	8.2488×10^5		0.0167	
2	3.0508×10^6	0.9435	0.0328	0.4869
8	1.2001×10^7	0.9879	0.0657	0.5011
32	4.7902×10^7	0.9985	0.1319	0.5027

We present the results for various values of initial velocity in Table 1. Here we have four scenarios, where the initial velocity is varied by four times a previous value. The first scenario takes the initial velocity to be 0.5 m/s. The second, third and fourth take the initial velocity to be 1, 8, 32 m/s, respectively. In all simulations the number of cells is N = 10,000.

As written in Table 1, when we vary the initial velocity, the pressure order is approximately 1. This means that the varying initial velocity is in linear relationship with the maximum pressure hitting the closing valve. In addition, the velocity order is about 0.5. This means that the varying initial velocity is in square root relationship with the velocity hitting the closing valve. We infer that the initial velocity influences the pressure hitting the closing valve significantly.

Result: Pressure & Velocity

Numerical results for the first scenario are shown in Figures 2 and 3. Here Figure 2 depicts the pressure versus the position exactly at the time when the valve is completely closed, that is, at time 0.1 s. Figure 3 shows the velocity versus the position exactly at the same time as that in Figure 2. These two figures are typical results for the pressure and velocity distributions at the time when the valve is completely closed.

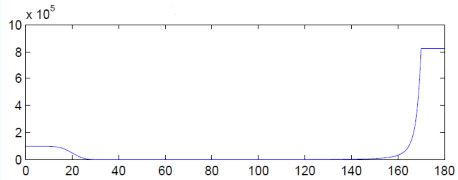

Figure 2. The pressure (vertical axis) with respect to the position (horizontal axis) exactly at the time when the valve is completely closed at time 0.1 second.

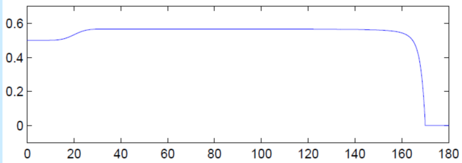

Figure 3. The velocity (vertical axis) with respect to the position (horizontal axis) exactly at the time when the valve is completely closed at time 0.1 second.

At the time when the valve is fully closed, the computed value of the pressure to the valve is 8.2488×10^5 Pa and the cell velocity next to the valve is 0.0167. The plotting interval is [0,180] having a ten meter additional extension of the given space [0,170]. This spatial extension is taken into account to clearly see the pressure and velocity values hitting the closing valve.

The largest pressure is found at the position where the fluid hits the valve, which is correct physically.

Conclusion

We obtain the followings:

- The higher the initial velocity of the pipe flow leads to the higher the pressure hitting the closing valve.
- The relation between the initial velocity and the pressure hitting the closing valve is linear.
- As an example, if we increase the initial velocity to be four times a benchmark initial velocity of a problem, then the maximum pressure hitting the closing valve will be four times the pressure of the resulting benchmark problem.

References

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