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Strong edge antimagic total labeling on multistar

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Abstract. Strong edge antimagic total labelling of a simple graph $G(V,E)$ is graph labelling which the vertex labels are consecutive integers from 1 to $|V|$ such that, the weight of edges, i.e. the total label of the vertices incident to the edge, will form an ascending arithmetic sequence. This article discusses this kind of vertex labelling on multistar graphs. A multistar graph is an unconnected combination of star graphs. The study is a literature study with a mathematical proof. The results of the study is that multistar graphs have a strong edge antimagic total labelling. Furthermore, the labelling can be done on an unconnected combination of m star graphs and the difference of the weight of the edges are 1 and 2 with initial term $a=\dfrac{5mn+3m+4}{2}$ and $a=\dfrac{4mn+3m+5}{2}$, respectively.

1. Introduction

Graph labeling is defined as a function that maps the set of elements of a simple graph $G(V,E)$ to a set of positive integers. In 1970, Kotzig and Rosa [1] introduced magic labeling as one-to-one mapping from graph elements to a set of positive integers and the weights of the graph elements were same. The weight of the graph element depends on how the labeling is evaluated. Hartsfield and Ringel [2] introduced an antimagic labeling as a way to label the edges of a graph with consecutive integers from 1 to $|E|$, so that the weights for each element that evaluated are different. Furthermore, Bodendiek and Walther [3] define an antimagic labeling (a, d) as an edge labeling with the weights of every vertices form an ascending arithmetic sequence with the initial terms is a and the different is d.

Several studies about antimagic total labeling (a, d) have been done on several graphs. Baca, et al. [4] has shown the applicability of vertex antimagic total labelling on paths, Petersen's graphs, odd cycles and some other graphs.

Prasetyo [5,6] has shown vertex antimagic total labeling on multicycle and multicomplete bipartite graphs. Sugeng and Bong (in Gallian [7]) have shown vertex antimagic total labeling on circulant graph C_n (1,2,3). The vertex antimagic total labeling on union of suns was found by Parestu, Silaban and Sugeng [8]. In a study conducted by Abdussakir [9], it was shown that there was a strong edge magic total labeling on a multistar graph.

Muttaqien [10] has researched that every double star graph has an edge antimagic total labeling. Riza [11] has shown that the star graph has a strong edge antimagic total labeling (a, d). Sanjaya [12] has shown (a, d) edge magic total labeling on multicycle graph. Whereas Irawati [13] and Chusna [14] have researched the edge magic total labelling on star graph and super edge magic labelling on star graphs that its center vertex are connected by single hook vertex, respectively. And Shiu et al. [15] have shown supermagic labeling of an s-duplicate of $K_{n,n}$. Rusmansyah [16] and Nurainun, et al [17] have shown an (*a*, *d*)-super edge antimagic total labelling on subdivision of star graph and super edge antimagic total labelling on union of double star graphs and path, respectively.

This article will research strong edge antimagic labeling on multistar graph. The multistar graph in this reseacrh is a combination of unconnected star graphs. The discussion begins with a literature study and detemine a basic calculation of labeling on multistar graphs to find initial term *a* and different *d*.

2. Labeling of Graph

Simple and undirected graph G is defined as a set of ordered pairs $G(V,E)$ where V is a non-empty finite set of vertices and E is a set of all edges that connect vertices in V. The number of vertices and edges of G are $|V|$ and $|E|$, respectively. Labelling a graph is an one-to-one mapping that maps elements from G to a set of positive integers. There are some definitions that will be used in this research [18].

2.1 Definition 1
An one-to-one mapping $f: V(G) \cup E(G) \rightarrow \{1,2,\cdots,|V|+|E|\}$ is called an edge total labelling of $G(V,E)$ if the weight of edge is $w_f(uv) = f(u) + f(v) + f(uv)$, for every $u,v \in V(G)$ and $uv \in E(G)$.

2.2 Definition 2
An one-to-one mapping $f: V(G) \cup E(G) \rightarrow \{1,2,\cdots,|V|+|E|\}$ is called an (*a*, *d*) edge antimagic total labelling of $G(V,E)$ if the weight of edges will form an ascending arithmetic sequence with initial term *a* and different *d*, i.e $W = \{w_f(uv) : uv \in E\} = \{a, a+d, a+2d, \cdots, a+(|V|-1)d\}$.

2.3 Definition 3
An (*a*, *d*) edge antimagic total labelling of $G(V,E)$ is called strong lebelling if vertices label are $\{1,2,\cdots,|V|\}$.

3. Discussion and Result

3.1 Multistar and Basic Counting

The multistar graph that developed in this study is a combination of as many unconnected star graphs. Figure 1 is a multistar graph with symbols mS_n.

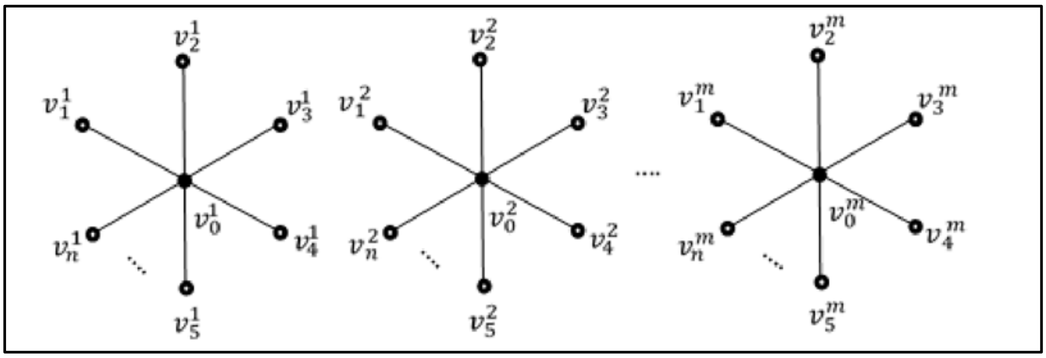

Figure 1. Multistar Graph mS_n

Multistar graph have a set of vertices $V(mS_n)=\{v_i^j : 0\le i\le n, 1\le j\le m\}$ and a set of edges $E(mS_n)=\{v_0^j v_i^j : 1\le i\le n, 1\le j\le m\}$. So, they have $(mn+m)$ vertices, mn edges and totally $(2mn+m)$.

Based on Definition 1 we get the sum of the edge weights on edge antimagic total labelling of multistar with adding the labels of all edges and all vertices and $(n-1)$ times the center label. The results of the sum is

$$S_w = 1+2+\cdots+m(2n+1)+(n-1)\sum_{j=1}^{m} c_j \tag{1}$$

where S_w is sum of the edge weights and c_j is center label of star j.

In other hand, based on Definition 2 the sum of edge weights is defined as the sum of mn terms in arithmetic sequence with initial a and difference d. so that

$$\begin{aligned} S_w &= a+(a+d)+\cdots+(a+(mn-1)d) \\ &= mna+\frac{mn(mn-1)}{2}d \end{aligned} \tag{2}$$

From equations (1) and (2) we get

$$mna+\frac{mn(mn-1)}{2}d = 1+2+\cdots+m(2n+1)+(n-1)\sum_{j=1}^{m} c_j \tag{3}$$

Since the center of the star is vertex that must be labeled, we will choose the labels for this center are $\{mn+1, mn+2, \cdots, mn+m\}$, so

$$\begin{aligned} (n-1)\sum_{j=1}^{m} c_j &= (n-1)(mn+1+mn+2+\cdots+mn+m) \\ &= \frac{(n-1)m(2mn+m+1)}{2} \end{aligned} \tag{4}$$

3.2 Bound of a and d.

Now, we will determine the bound for the initial term *a*. In this labeling, based on Definition 3 the labels of the vertices are $\{1,2,\cdots,mn+m\}$ and the labels of the edges are $\{mn+m+1, mn+m+2, \cdots, 2mn+m\}$. So, the smallest value *a* (or the smallest edge weight) is the sum of the smallest vertex label, the smallest center label and the smallest edge label, i.e $1+(m+1)+(mn+m+1)$. So, we have

$$a \ge mn+2m+3 \tag{5}$$

In the other hand, we also have the biggest weight (or the last term of the line, $a+(mn-1)d$) of this labeling, that is the sum of the largest vertex label, the largest center label and the largest edge label, i.e $mn+(mn+m)+(2mn+m)$. So, we have

$$a+(mn-1)d \le 4mn+2m\,. \tag{6}$$

Based on equation (5) and (6), we get the bound of *d*, that is

$mn+2m+3 \le a \le 4mn+2m-(mn-1)d$ or

$mn+2m+3 \le 4mn+2m-(mn-1)d$ or

$(mn-1)d \le 4mn+2m-(mn+2m+3)$ or

$$d \le \frac{3mn-3}{mn-1} = 3 \tag{7}$$

Based on equation (7), there are three *d* values, i.e 1, 2, and 3. Futhermore, we will determine each of initial term *a* based on the *d* values.

3.2.1 Case d = 1

Based on equation (3) dan (4), we get $a = \frac{6mn+3m+3-mn+1}{2} = \frac{5mn+3m+4}{2}$.

This result doesn't contradict with equation (5) ang (6), i.e $mn+2m+3 \le a \le 3mn+2m+1$. So the labeling can be done in case *d* = 1. In order *a* is a positive integer, then *m* must be an even positive integer or *m* and *n* are both odd positive integers. So, the multistar graph (mS_n) can be labelled with $\left(\frac{5mn+3m+4}{2},1\right)$ - super edge antimagic total labeling (SEATL).

3.2.2 Case d = 2

Based on equation (3) dan (4), we get $a = \frac{6mn+3m+3-2mn+2}{2} = \frac{4mn+3m+5}{2}$.

This result doesn't contradict with equation (5) ang (6), i.e $mn+2m+3 \le a \le 2mn+2m+2$. So the labeling can be done in case *d* = 2. In order *a* is a positive integer, then *m* must be an odd positive integer. So, the multistar graph (mS_n) can be labelled with $\left(\frac{4mn+3m+5}{2},2\right)$- super edge antimagic total labeling (SEATL).

3.3.3 Case d = 3

Based on equation (3) dan (4), we get $a = \frac{6mn+3m+3-3mn+3}{2} = \frac{3mn+3m+6}{2}$.

This result contradicts with equation (5) dan (6), i.e $mn+2m+3 \le a \le mn+2m+3$. The initial term *a* exceed the given bound. So the labeling can't be done in case *d* = 3 or the multistar graph (mS_n) can't be labelled with this way.

3.3 $\left(\frac{4mn+3m+5}{2},2\right)$*-SEATL on multistar*

Now, we will show the $\left(\frac{4mn+3m+5}{2},2\right)$-SEATL on multistar. Figure 2, Figure 3, Figure 4, Figure 5 and Figure 6 shown that the multistar can be labelled by this labelling. They are respestively $(8,2)-$SEATL on S_2, $(19,2)-$SEATL on $3S_2$, $(30,2)-$SEATL on $5S_2$, $(10,2)-$SEATL on S_3 and $(25,2)-$SEATL on $3S_3$.

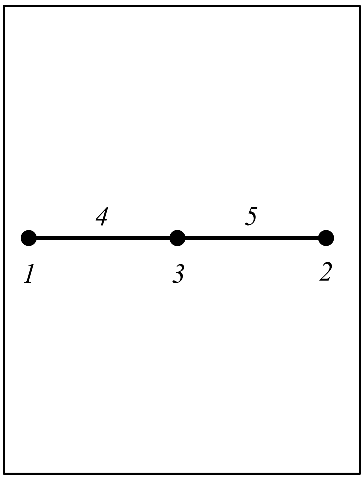

Figure 2.
$(8,2)-$SEATL on S_2

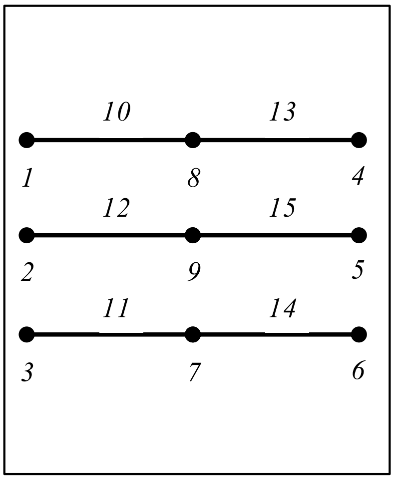

Figure 3.
$(19,2)-$SEATL on $3S_2$

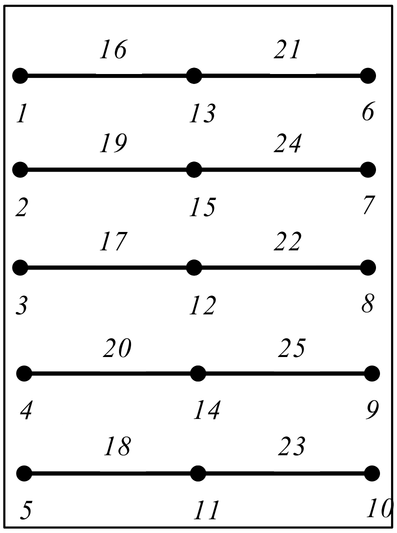

Figure 4.
$(30,2)-$SEATL on $5S_2$

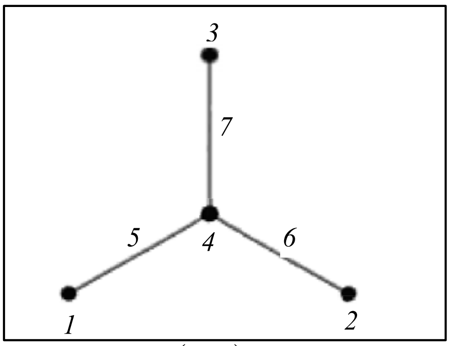

Figure 5. $(10,2)-$SEATL on S_3

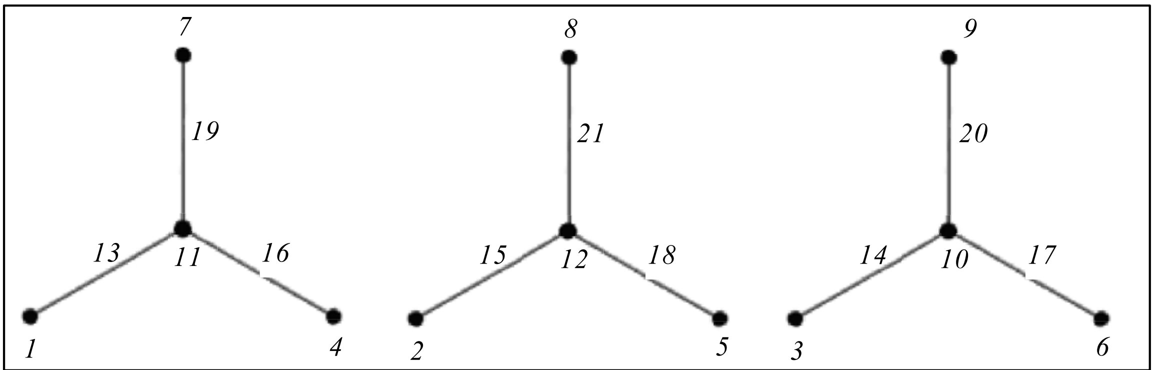

Figure 6. $(25,2)-$SEATL on $3S_3$

4. Conclusion and Suggestion

Based on result and discussion, we can conclude that several multistar graph can be labelled by (a,d) super edge antimagic total labelling (SEATL). Bound of the initial term a is $a \geq mn + 2m + 3$. Bound of the difference d is $d \leq 3$. In case $d = 1$ and $d = 2$, the multistar graph can be labelled by (a,d)-SEATL with $a = \frac{5mn + 3m + 4}{2}$ and $a = \frac{4mn + 3m + 5}{2}$, respectively. But, in case $d = 3$, the multistar can't be labelled by SEATL.

I suggest that for further research you can determine the formulas of $\left(\frac{5mn+3m+4}{2},1\right)$-SEATL and $\left(\frac{4mn+3m+5}{2},2\right)$-SEATL. You can also determine the applicability of this SEATL to other multigraphs.

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