Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

The existence of irregular n-shape containing a unit circle based on MATLAB

To cite this article: Beni Utomo 2021 *J. Phys.: Conf. Ser.* **1778** 012008

The existence of irregular n-shape containing a unit circle based on MATLAB

Beni Utomo

Department of Mathematics Education, Faculty of Teacher Training and Education, Sanata Dharma University, Jl. Affandi, Sleman, Yogyakarta 55281 Indonesia

E-mail: unlinearid@usd.ac.id

Abstract. This research will provide an implementation of the study about the area in terms of numerical point of view. Regular n-shaped constructions that load a unit circle and load in unit circles can be made for at least 3 points on the unit circle. The irregular n-shaped construction contained in the unit circle can also be made without conditions with a minimum of 3 points on the unit circle. The problem arises in the irregular n-shaped construction that contains the unit circle, apparently it cannot be made by taking 3 points freely. The purpose of this research is to conduct an analysis related to what conditions are needed so that the irregular n terms can be made so that it contains a unit circle. The method used is a literature review, carried out by comparing and analyzing research that has been done relating to irregular and irregular aspects, the analysis process is carried out on irregular n-construction construction with the help of MATLAB for visualization. The results of the research show that one of the requirements for irregular n-terms can be made and contain unit circles found, and the theory test is carried out empirically with the help of MATLAB for visualization. Another result is a theory about the existence of the irregular n-aspect mathematically.

1. Introduction

The unit circle has several unique things as the beginning of the definition of mathematical theory. One of the theories is about area that can be constructed using unit circle by irregular and regular polygons. The polygons must have two conditions, simple and closed. If the polygon or simple closed curve define inside the unit circle, the area could be calculated by Green's Theorem. The design and construction of an n-shape made inside a unit circle and containing a unit circle have different processes. For a regular n-design that includes a unit circle and is contained in a unit circle can be done by taking at least 3 points on the circle and the n-shape can be made. For the n-irregular aspect, it turns out that it has a different complexity. For an irregular n-shape created in a unit circle, the design can be made by taking at least 3 points on the circle. Not so for irregular n terms that contain unit circles. This will be the subject of study in this study. Based on this, the purpose of this study is to observe the behavior of irregular n-squares and to design irregular n-squares containing unit circles.

The unit circle is a circle whose center (0.0) and its radius are 1 and are expressed by the equation [1–3]

$$sin^2 t + cos^2 t = 1 \quad ; \quad 0 \le t \le 2\pi \tag{1}$$

and the area is π. Green Theorem [4–6], the calculation of the area of an area bounded by a simple curve and closed with the requirement that the curve must be smooth. An area S which is bounded by a trajectory C with the path condition C piecewise smooth, a simple closed curve, and the direction of its integration to keep the area at the left of the trajectory, the S area can be expressed as

$$Area(S) = \frac{1}{2}\oint_S (-ydx + xdy) \tag{2}$$

The design and construction of an n-shape made inside a unit circle and containing a unit circle have different processes. For a regular n-design that includes a unit circle and is contained in a unit circle can be done by taking at least 3 points on the circle and n-squares can be made. For the n-irregular aspect, it turns out that it has a different complexity. For an irregular n-shape created in a unit circle, the design can be made by taking at least 3 points on the circle. Not so for irregular n terms that contain unit circles. This will be the subject of study in this study. Based on this, the purpose of this research is to observe the design and construction behavior of the irregular n-plane containing the unit circle.

2. Method

The research was carried out by examining previous studies, those carried out by Utomo [7] concerning n-square designs made on unit circles with the help of GeoGebra [8] and MS Excel [9, 10] and MATLAB [11]. Regarding n-square designs made on unit circles and containing unit circles with help of MATLAB [10]. Bobby [11] has shown a n-shaped design that contains a unit circle and is contained in a unit circle, also has shown an irregular n-design design that contains a unit circle but has not been able to show an irregular n-design design that contains a unit circle.

This study will examine the phenomenon of irregular n-aspect design that includes unit circles with numerical and visual approaches with the help of MATLAB. The study was conducted by observing triangles, rectangulars, pentagons, and hexagons containing unit circles to make conclusions about the nature of the irregular n-shape with the help of MATLAB. Based on these observations, will be analyzed whether there are mathematical theories related to it.

3. Result and discussion

The activity begins by looking at the construction of the regular n-section containing the unit circle and the irregular n-section contained in the unit circle. Previously it was known that a regular n-shaped design could take any point and an n-aspect construction could be made. Next, we pay attention to the design in terms of the n-plane containing the unit circle. Both designs have differences.

Design to make both regular and irregular n-elements can be made without conditions if they are contained in a unit circle. Points taken either randomly or not randomly will become vertices on the n-aspect. To make these n-points, known points must be sorted, the ordering process an important part of this design. Design guarantees can be made and area of n count can be guaranteed by the effect of the green theorem to calculate the area of a simple and closed area. When points have been chosen either randomly or not, n-terms can be created by connecting each of the two closest points.

The design for n-squares containing unit circles has differences. Each selected point will either be random or will not be the point of tangency on the unit circle. From the tangent point will be made tangent line with the tangent also through the tangent point. The design of the irregular n-shape is apparently not as easy as the design for n-ordered irregularity.

The following study is carried out in order to calculate and create an irregular n-unit containing a unit circle with sufficient input of a certain number of points. Following are the theories developed.

3.1 Design of i*rregular n-shape* c*ontaining unit* c*ircle*

For every two points $A\left(x_1, y_1\right)$ and $B\left(x_2, y_2\right)$ could be connected by line equation:

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \tag{3}$$

or

$$y = y_1 + \left(\frac{x - x_1}{x_2 - x_1}\right)(y_2 - y_1) \tag{4}$$

equation (4) has differential

$$dy = \frac{y_2 - y_1}{x_2 - x_1} dx \tag{5}$$

In the study of calculating the area using an integral line and as a result of the Green theorem it is found that for any area that is closed and simple S, the area is expressed by the equation:

$$Area = \frac{1}{2} \oint_S (-ydx + xdy) \tag{6}$$

For example, take any rectangular. For any S in the form of a rectangular, for example the curve S is rectangular *ABCD*, the area can be obtained by adding up the integration along all four sides, namely integration along *AB*, along *BC*, along *CD*, and along *DA*. For path *AB* with $A\left(x_1, y_1\right)$ and $B\left(x_2, y_2\right)$ the integration value is calculated by substituting equations (4) and (5) to equation (6) and obtained:

$$Area(ABCD) = \frac{1}{2} \int_{x_1}^{x_2} \left(x_1 \frac{y_2 - y_1}{x_2 - x_1} - y_1\right) dx \tag{7}$$

Solution for equation (7) given by:

$$Area(ABCD) = \frac{1}{2} x_1 y_2 - \frac{1}{2} x_1 y_1 - \frac{1}{2} x_2 y_1 \tag{8}$$

that depends only to point A and B.

The process continues for integration along the other path and is obtained:

3.1.1. On the BC track. Suppose that $B(x_2, y_2)$, and $C(x_3, y_3)$ then we have from equation (8):

$$Area(ABCD) = \frac{1}{2} x_2 y_3 - \frac{1}{2} x_2 y_2 - \frac{1}{2} x_3 y_2 \tag{9}$$

3.1.2. On the CD track. Suppose that $C(x_3, y_3)$, and $D(x_4, y_4)$ then we will have from equation (7):

$$Area(ABCD) = \frac{1}{2} x_3 y_4 - \frac{1}{2} x_3 y_3 - \frac{1}{2} x_4 y_3 \tag{10}$$

3.1.3. On the DA track. Suppose that $D(x_4, y_4)$, and $A(x_1, y_1)$ then we will have from equation (7):

$$Area\ (CBED) = \frac{1}{2} x_4 y_1 - \frac{1}{2} x_4 y_4 - \frac{1}{2} x_1 y_4 \tag{11}$$

The total area or area of *ABCD* is the area calculated along the trajectories of *AB*, *BC*, *CD*, and *DA*.

Equation (4) irregular n-shape design that contains unit circles. To design an irregular n-shaped sketch design, take any two points on the circle, for example $A\left(x_1, y_1\right)$ and $B\left(x_2, y_2\right)$. The unit circle in equation (1) has a gradient or slope for any point (x, y) is:

$$\frac{dy}{dx} = -\frac{x}{y} \tag{12}$$

Because of tanget gradient at $A\left(x_1, y_1\right)$ is $\frac{dy}{dx} = -\frac{x_1}{y_1}$ and also tanget gradient at $B\left(x_2, y_2\right)$ is $\frac{dy}{dx} = -\frac{x_2}{y_2}$ then the equation with gradient $m = \frac{dy}{dx}$ and pass through point $A\left(x_1, y_1\right)$ given by equation: $y - y_1 = m\left(x - x_1\right)$and then $y - y_1 = -\frac{x_1}{y_1}\left(x - x_1\right)$ or

$$y+\frac{x_1}{y_1}x=\frac{{x_1}^2+{y_1}^2}{y_1} \tag{13}$$

With similar process, for every point $B\left(x_2,y_2\right)$ will have equation:

$$y+\frac{x_2}{y_2}x=\frac{{x_2}^2+{y_2}^2}{y_2} \tag{14}$$

The next process is to find the intersection points for each of the two adjacent tangents. Searching for this intersection point as input in MATLAB so that the irregular n-shaped side in the form of tangents (line segments) can be made. Based on equations (13) and (14) with the substitution method the following intersection points are obtained:

$$x=\frac{{x_1}^2y_2+{y_1}^2y_2-{x_2}^2y_1-{y_2}^2y_1}{x_1y_2-x_2y_1} \tag{15}$$

and the value of y given by:

$$y=\frac{{x_1}^2+{y_1}^2-x_1x}{y_1} \tag{16}$$

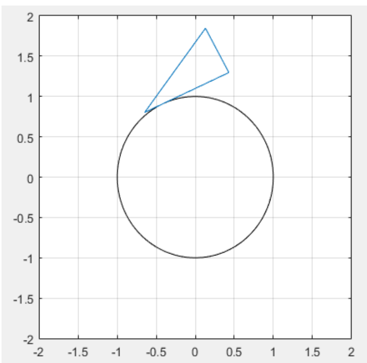
or

$$y=\frac{{x_2}^2+{y_2}^2-x_2x}{y_2} \tag{17}$$

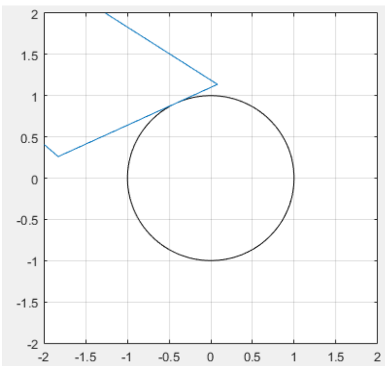
This process is carried out for all adjacent tangents. At this stage a sequence of points (x,y) has been obtained and if these points are connected, an irregular n-shaped side is formed and the area can be calculated. To connect the closest intersection points and calculate the area of the irregular n-plane using visualization with MATLAB. The MATLAB program can already be made and then used to observe the properties that appear to take random points to get the possibility of theory related to the existence of an irregular n-plane containing a unit circle.

3.2 Irregular triangle contains unit circle

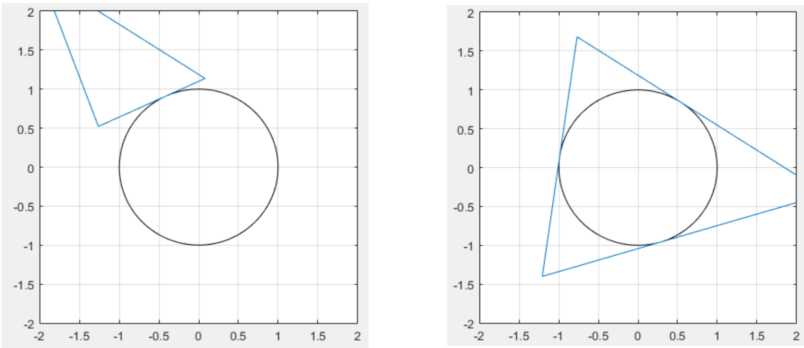
Based on previous studies, with the help of MATLAB the irregular n-shaped design process can be made and then an observation is made to obtain the existence conditions of any triangle containing unit circles with the sides of the triangle constituting tangents to the unit circle.



(a). Three points in [0, Pi]



(b). Three points with two points in [0,Pi]



(c). Three Points with each point on first-second-third quadrant

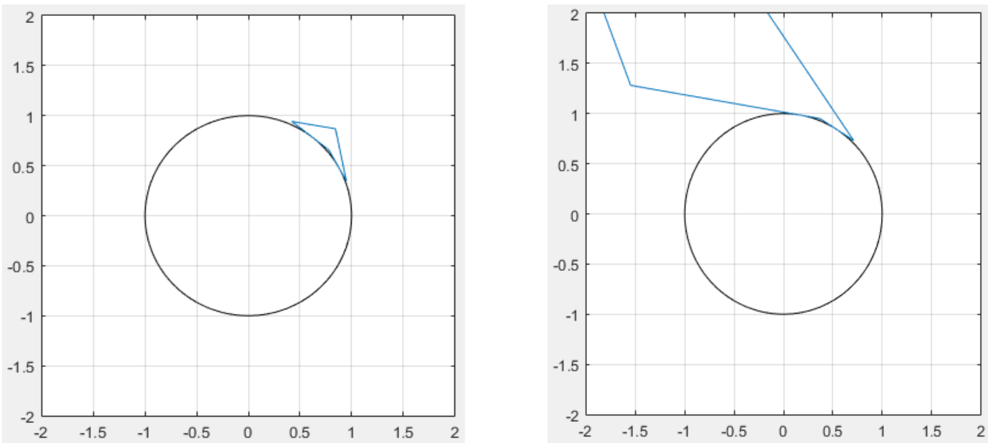
(d). Three points with each point on first-second-fourth quadrant

Figure 1. Irregular triangle containing unit circle

Based on observations on several points that were tried to make irregular triangles, it turns out that not all points can produce triangles that contain unit circles. The example in Figure 1 shows that a triangle that is *outside* the unit circle is a triangle that cannot contain a unit circle or cannot be made in the desired triangle. This can be seen in Figures 1a, 1b, and 1c. Based on observations, the existence of a triangle containing a circle is related to the distance of one point from another point.

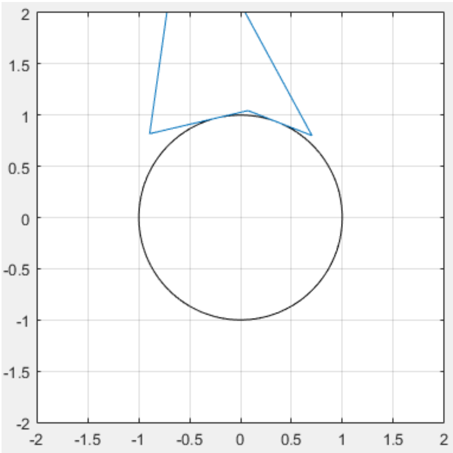
3.3. Irregular rectangular contains unit circle

Taking a random point on a regular rectangular made with certain restrictions to obtain properties based on the hypothesis in the observation of irregular triangle design. Here are some results obtained from many experiments.

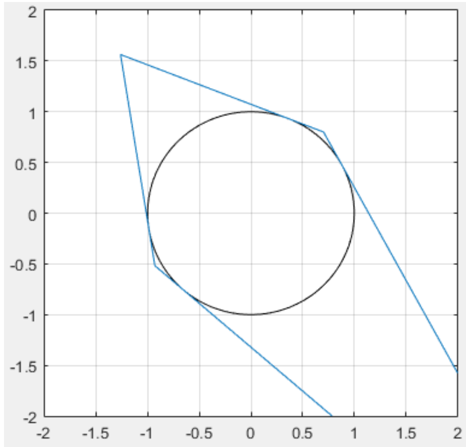


(a). Four points in one quadrant

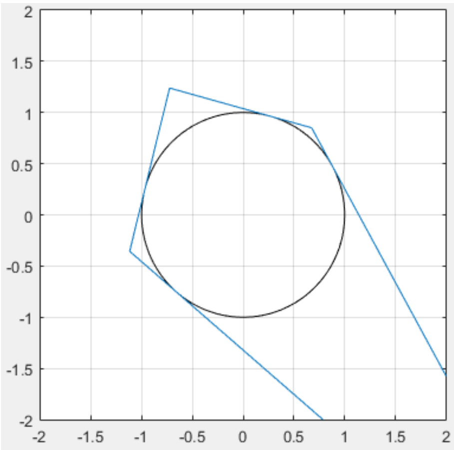
(b). Three points in one quadrant



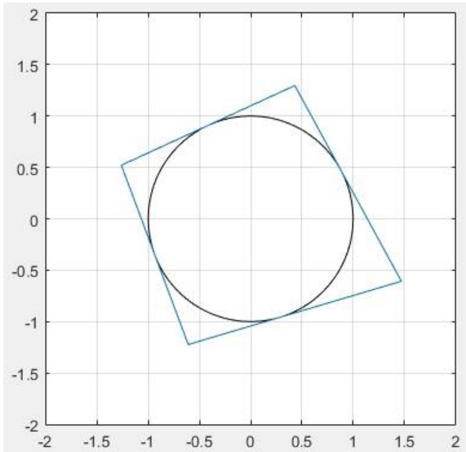
(c). Two points in one quadrant



(d). Two points in one quadrant



(e). Two points in one quadrant, one point in another quadrant



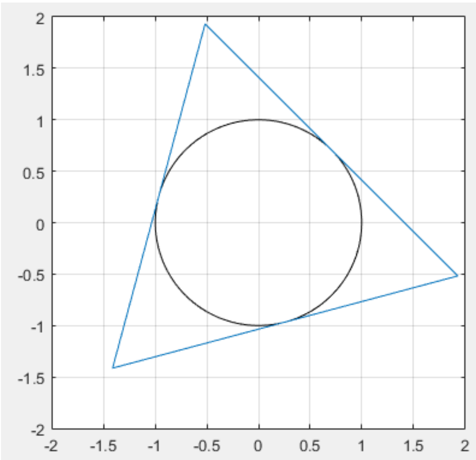
(f). One point on each quadrant

Figure 2. Irregular rectangular contain unit circle

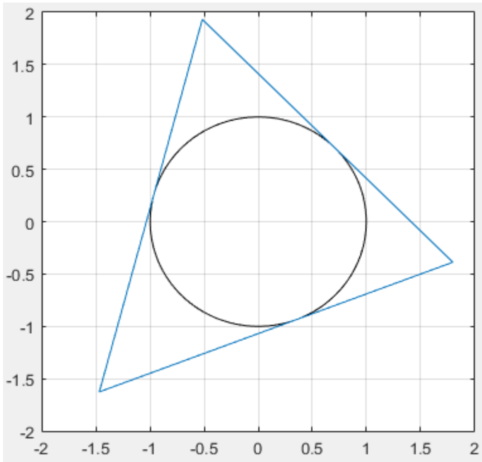
Based on the rectangular obtained from Figure 2, the hypothesis in the irregular triangle observations is increasingly strengthened. That the existence of a rectangular is irregular so that it can contain a unit circle related to the distance between points.

3.4 Regular and irregular triangle contain unit circle

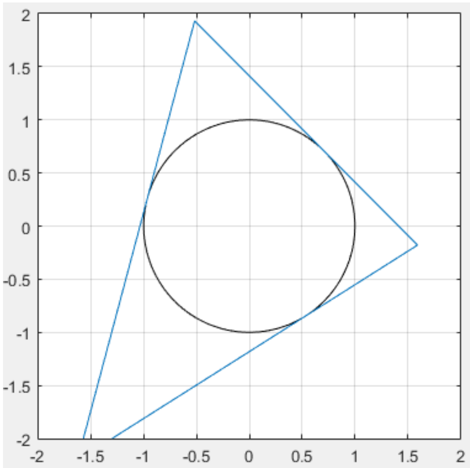
The design of an irregular triangle containing a unit circle will be used as a reference in observing an irregular triangle to strengthen the hypothesis of the existence of an irregular triangle containing a unit circle.



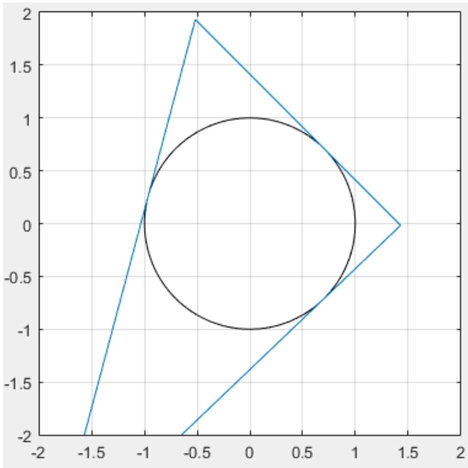
(a). Regular triangle



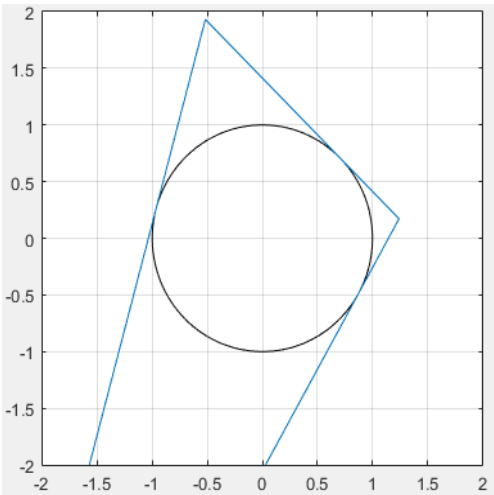
(b). Irregular triangle



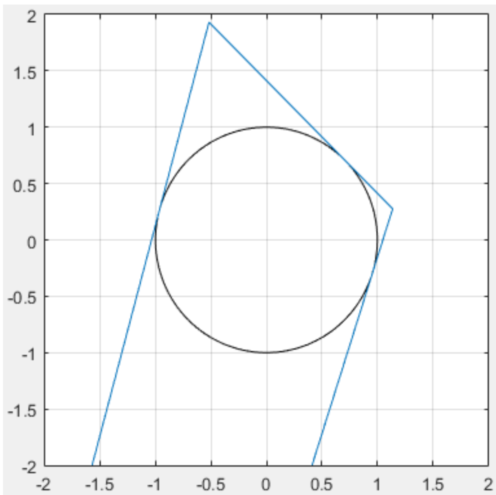
(c). Irregular triangle



(d). Irregular triangle



(e). Irregular triangle



(f). Irregular triangle

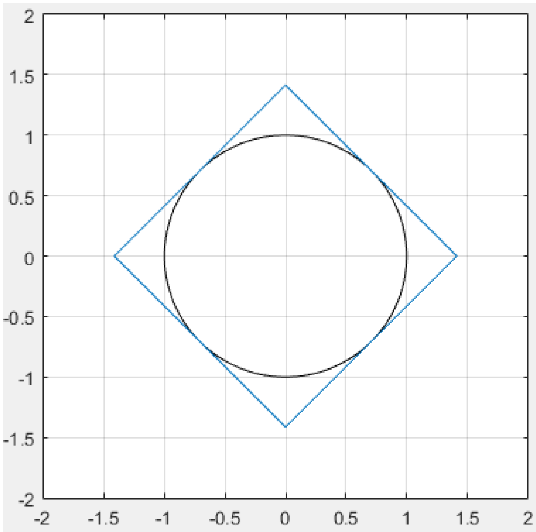
Figure 3. Regular and irregular triangle

Journal of Physics: Conference Series **1778** (2021) 012008 doi:10.1088/1742-6596/1778/1/012008

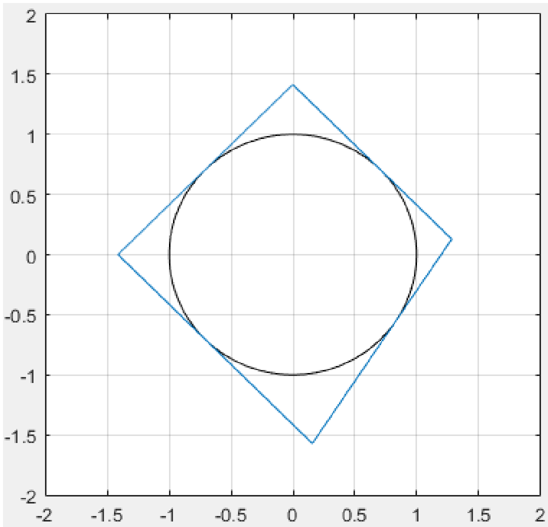
Based on Figure 3, we can obtain evidence that an irregular triangle will contain a unit circle if one of the points farther away from the other point is a maximum of pi. If the tangent point in the unit circle contains the distance of one point of contact with another adjacent point that is pi, then the irregular triangle does not contain the unit circle. In figure 3 it can also be noted that the area of the irregular triangle is getting bigger and bigger. The area of the triangle will become infinite if it contains adjacent tangents whose distance is at least pi.

3.5. Regular and irregular rectangular contain unit circle

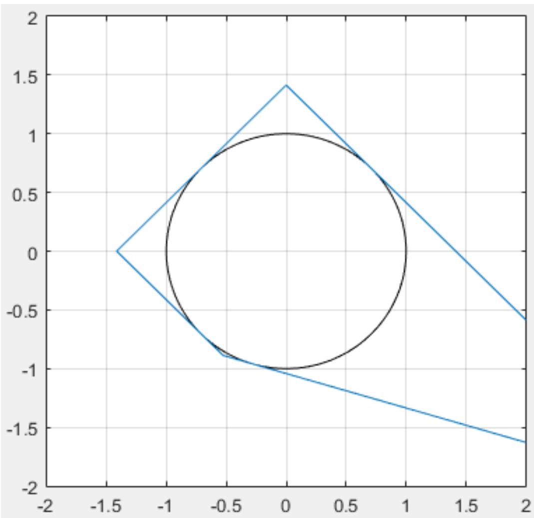
The proof of the hypothesis in Figure 3 will be strengthened by observing the regular rectangular and irregular rectangular with the following illustration.



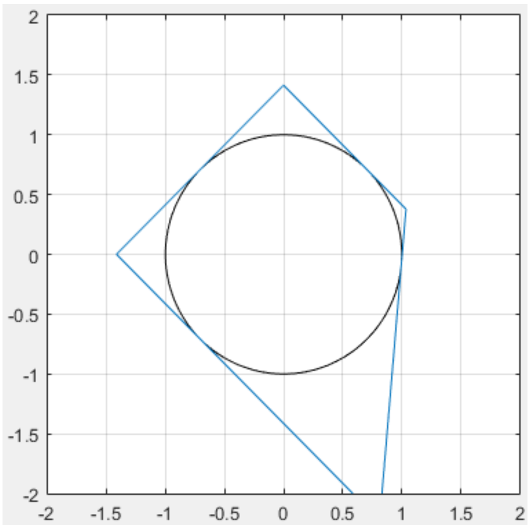
(a). Regular rectangular



(b). Irregular rectangular



(c). Irregular rectangular

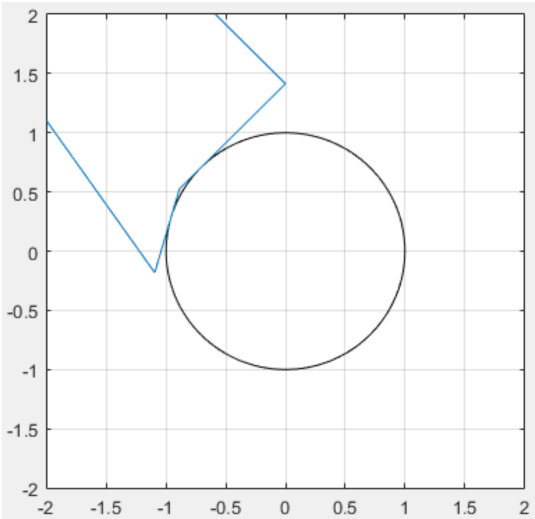


(d). Irregular rectangular

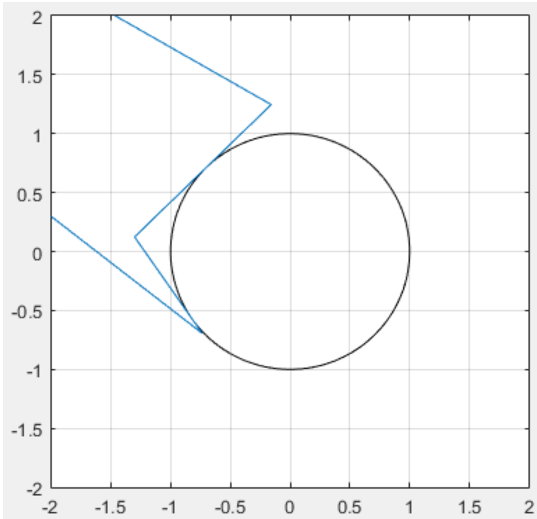
Figure 4. Regular and irregular rectangular

In an irregular rectangular if three fixed points are made like an irregular rectangular, then a rectangular containing a fixed unit circle exists. Based on the observation in Figure 3, this is because if the closest point from one of the tangent lines is pi, then what is obtained is approaching an irregular triangle like Figure 3f.

If a rectangular contains one of the tangents whose closest distance to another point is pi then the irregular rectangular will not be able to contain the unit circle because the rectangular formed is an open area.



(a) Irregular Rectangular Contain Maximum Distance Pi



(b) Irregular Rectangular Contain Maximum Distance Pi

Figure 5. Irregular rectangular with 3 points in [0,Pi]

Conclusion

Based on the discussion it can be concluded the following matters: (1) an irregular triangle and an irregular rectangular will contain a unit circle if the distance of the adjacent tangent is more or equal to Pi; (2) based on the area of an irregular triangle and an irregular rectangular based on figures 1, 2, 3, 4, and 5 then an irregular triangle and an irregular rectangular will exist containing a unit circle if the area is finite; and (3) the results of this study are still intuitive even though it has been proven, it needs to be supported by analytical mathematical evidence.

Acknowledgment

I thank the Mathematics Education Department, Faculty of Education and Teacher Training, and LPPM Universitas Sanata Dharma for giving support to this research.

References

[1] Maknun C L, Rosjanuardi and Jupri 2019 From ratios of right triangle to unit circle: An introduction to trigonometric functions *J. Phys.: Conf. Ser.* **1157** 022124
[2] Sullivan M 2012 *Trigonometry: A unit circle approach 9th ed* (Boston: Pearson)
[3] Wiryanto L H 2018 Line integral on engineering mathematics *IOP Conf. Ser.: Mater. Sci. Eng.* **296** 012046
[4] Stewart J 2010 *Calculus: Concepts and contexts fourth edition* (Canada: Brooks/Cole Publisher)
[5] Spiegel M R 1974 *Theory and problems of vector analysis Schaum's outline series* (New York: McGraw-Hill)
[6] Varberg D, Purcell E and Rigdon S 2000 *Calculus: Eight edition* (New Jersey: Prentice-Hall)

[7] Utomo B 2020 Keacakan konstruksi segi n tidak beraturan pada lingkaran satuan berbantu MATLAB *PRISMA Prosiding Seminar Nasional Matematika* vol 3 (Semarang: Universitas Negeri Semarang) pp 41–5
[8] Joseph B 2007 *Excel for scientists and engineers: Numerical methods* (Canada: John Wiley and Sons)
[9] Bobby M 2019 *Aplikasi teorema green pada bidang dalam menghitung luas segi-n dengan bantuan MATLAB* (Yogyakarta: Universitas Sanata Dharma)