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Mathematical model of referral marketing strategy

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Abstract. A marketing strategy is an attempt to campaign for a product so that information about the product becomes viral. Advertising strategy began to be combined by involving customers to campaign for the company's products. This marketing strategy is known as the referral marketing strategy. Viral information in a community can be analogous to the pandemic condition of a disease. This study aims to examine the relationship between the epidemiological model and the marketing model with a referral strategy. This research is a literature study with the results of a marketing model design with a referral strategy. Based on the research, there are 4 groups in marketing modeling with a referral strategy, namely Unaware, Potential Broadcaster, Broadcaster, and Inert. The epidemiological model that is suitable for this condition is the SEIR model. Adjustments to the SEIR Model need to be made because the dynamics of the Unaware can go directly to the Inert because of the trust factor in marketing information. The simulation results show that the parameters that have an important role in making information viral are the customer network and the incubation time of the information becoming viral.

Keywords: referral, viral, epidemiological model, compartment

1. Introduction

The development of information technology influences the dynamics of the development of marketing strategies. Conventional marketing methods have begun to be abandoned by companies. Very easy access to information provides an opportunity for a customer to be able to take a role in marketing the products of a company. One person's review of a product can be a consideration for others to decide whether or not to buy the product. This marketing method is known as Word of Mouth (WOM). As one of the strategies in marketing, WOM is very possible to spread information more widely. This viral information opens up opportunities for companies to get more and more customers. WOM can be given a stimulus to accelerate the viral information. Companies can provide incentives/rewards to customers who successfully disseminate information and bring in new customers for the company. In [1] it is recommended to further investigate the effect of incentives, customer judgment, and influence of brand attitude on the generation of WOM.

A person's trust in an advertisement is more significant if the person's friends or family provide recommendations for the product in the advertisement. Based on a survey of Thrust Advertising [2], it was found that 92% of people would trust advertisements more if their friends or family directly gave recommendations. Companies take advantage of the survey results by involving their customers in campaigning for their products. Based on a survey conducted by Credit Marketing Officers (CMO), companies have allocated marketing budgets of around 10% to 20% of the total budget. The budget is used to retain current customers by 28% and to acquire new customers by 21% [3]. Part of this marketing budget can be allocated to incentivize customers who can acquire new customers. The



marketing strategy by providing incentives to customers who succeed in getting new customers for the company is known as the referral strategy. Community social networks that are getting easier in the digital era are increasingly supporting this strategy to be implemented. The referral strategy can be done by direct contact or through digital contacts. Direct contact is carried out by conveying information directly to prospective customers so that they want to purchase a product. Meanwhile, what is currently being done is that customers who have made transactions will receive a code (referral code) which will later be distributed through digital social networks. Incentives will be given if there are new customers who transact using a certain referral code.

The referral strategy has the goal of spreading information about a company's products. The wider the information spread, the greater the opportunity for people to buy a product. These characteristics are similar to the spread of disease. Transmission of disease from individual to individual is caused by interactions between people who have been exposed to them and those around them. The more social interactions carried out by people who have been exposed to the disease with the local community, the faster the spread of the disease. The compartment model is one of the epidemiological models. The population is divided into several partitions according to their characteristics. In the Epidemiology model, efforts will be made to minimize the spread of the disease, while in the referral strategy, efforts will be made to maximize the spread of information. Compartment models for referral strategies have been developed by [4–7]. Each model is a system of differential equations that depends on customer dynamics based on marketing policies. Compartment models that have been developed have not been fully linked to the epidemiological model. In this study, a model will be designed that connects the referral strategy in marketing with one of the disease spread models.

2. Literature Review

In this section, we are focus on Referral Program and Epidemiological Model

2.1. Referral Program

Various ways can be done in marketing communications. Word of Mouth (WOM) is one of the most widely used marketing strategies in the last decade [8]. WOM can occur naturally through direct customer experience after making a purchase. Companies develop marketing strategies through WOM with intervention. The intervention carried out was providing incentives/gifts/discounts to customers who succeeded in bringing in new customers for the company. This kind of marketing strategy is known as the referral strategy. Customers who are active in carrying out this strategy are known as referred customers. Referral strategy has three main characteristics, which are intentionally initiated, actively managed, and continuously controlled by the company [9]. Referral strategy is different from multilevel marketing. One will only be incentivized based on the new customers that one manages to acquire. This referral strategy, of course, costs more than a naturally occurring WOM bad. Companies need to calculate the upper limit of incentives provided to their customers to remain profitable for the company.

2.2. Epidemiological Model

The compartment model is a model of the spread of disease outbreaks by dividing the population into several sub-populations that are mutually exclusive [10]. There are two simplest compartment models, namely the Susceptible-Infected-Susceptible (SIS) and Susceptible-Infected-Recovered (SIR) models. The SIS model is used for the spread of disease that allows individuals to be re-infected after being infected (no immunity is formed). Meanwhile, the SIR model is used for the spread of diseases that can form immunity after infecting a person. As development progressed, the compartment model underwent many modifications according to the characteristics of the ongoing pandemic. For example, in the Covid-19 modeling conducted by Loli and Zama [11], the Exposed (E) and Death (D) subpopulations were added. An exposed subpopulation is a subpopulation with individuals who have been infected but have not been able to transmit it because the virus is still in the incubation period. Meanwhile, the Death subpopulation is the subpopulation that died due to Covid-19. The system of differential equations is used to model the spread of disease under the above conditions.

3. Method

This research is literature research with the results in the form of a conceptual model of referral strategy in marketing. The stages of this research include 1) examining the characteristics of referral strategies in the world of marketing, 2) analyzing the relationship between the spread of information and disease outbreaks, 2) constructing a mathematical model with appropriate parameters, and 4) model simulation with variations in parameter values.

4. Result and Discussion

4.1. Conceptual Framework of Referral Marketing Strategy

The spread of information in marketing so that it becomes viral is one of the strategies undertaken by a company. With the motive to get incentives/rewards/discounts, a customer can influence other individuals to become new customers. These customers interact through social contact with other individuals so that they have the potential to become new customers for the company. In social interactions with referred customers, one needs time to decide whether to take part in the program. Someone tends to see trends in the products offered before deciding to participate. After considering various conditions, there will be two possibilities, namely participating in disseminating information or not participating in disseminating information. In [12] the factors that cause someone to want to be a referred customer are high incentives, easy program terms, and attractive products offered. Meanwhile, a person may decide not to participate in disseminating information due to the perception of the program being irrelevant, security level, or privacy concerns. Furthermore, the referred customer has the possibility not to redistribute the information. The basic things that affect him include feeling bored, having more demands from the company, or even forgetting to spread the information. A person who is already informed and decides not to share the information is also likely to become a preferred customer. This can happen when they are reminded again by their friends or motivated because they see the success of their friends getting rewards directly.

The Hierarchical Clustering Technique was carried out by [13] to cluster a community in a marketing strategy. There are three main groups in this system. The terminology used is the Unware, Broadcaster, and Inert groups. Unware cluster is a target market group which is individuals who have the potential to receive information from referred customers through persuasive shares. While the Broadcaster group is a group that actively disseminates information through social networks. The last cluster is Inert, which is a group that no longer takes part in disseminating information. This kind of community clustering was also carried out by [13–15]. In the marketing strategy, it is necessary to maximize the spread of information. This characteristic is of course closely related to the characteristics of the epidemiological model. The conceptual model of referral marketing strategy produced by [12–15] is the standard epidemiological model, namely SIR.

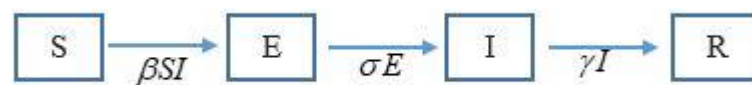
The analogy of the SEIR model can be used to model the referral marketing strategy. The Susceptible (S) compartment corresponds to the Unware/Target Market (T) cluster. The Exposed (E) compartment (epidemiological model) can represent people who have received information from the referred customer but have not yet made a decision. The Infectious (I) compartment corresponds to the Broadcaster/Active Sharing (A) cluster. Finally, the Recovered (R) compartment corresponds to the Inert/Dormant (D) cluster. In addition to determining the appropriate clusters between the epidemiological model and the referral marketing strategy, it is also necessary to adjust the parameters in the model. The following table describes the parameters of the model of referral marketing strategy.

Table 1. Description of Parameters in Referral Strategy Model

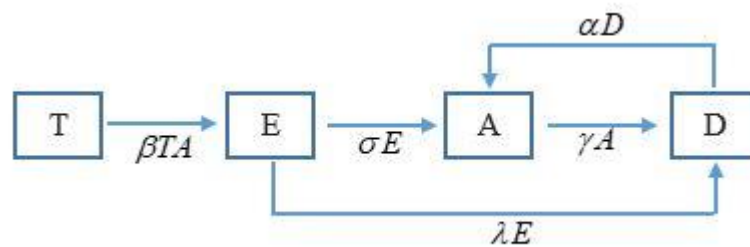
Parameter	Notation	Description
Transmission Rate of Information	β	Social Network Persuasive Share
Incubation rate	σ	The time needed to decide Adequacy of Information
Recovered rate	γ	Bored Forgetfulness

Not affected to spread	λ	uncompleted information not relevant not secured
Re-infected rate (to be Active)	α	remain by friend friend get reward

The rate of population flow rate from group T to A is determined by the value of the parameter β . Suppose the success rate (rate of success to persuade) a person in group A to bring a new referred customer is p then the population flow rate from group E to D is one of them determined by the value of $1 - p$. In addition, the level of social interaction also has a role in this flow rate. Suppose the level of social interaction is δ then it will be obtained $\beta = p\delta$ and $\lambda = (1 - p)\delta$. The flow chart of the standard SEIR model and the model of referral strategy can be seen in the following figure.



(a) SEIR Standard Model



(b) Conceptual Model of Referral Strategy

Figure 1. Flow Chart of SEIR and Referral Strategy Models

Based on the diagram in Figure 1 above, a system of differential equations is obtained for the Model of Referral Strategy.

$$\begin{cases} \frac{dT}{dt} = -\beta TA \\ \frac{dE}{dt} = \beta TA - (\sigma + \lambda) E \\ \frac{dA}{dt} = \sigma E - \gamma A + \alpha D \\ \frac{dD}{dt} = \gamma A + \lambda E - \alpha D \end{cases} \quad (1)$$

4.2. Simulation

Model simulations need to be carried out to analyze the dynamics of each compartment at various parameter values. Simulate this model using simple software, namely Microsoft Excel. The system of differential equations (1) will first be converted into a system of differential equations. This is done to simplify iterations to be performed by Microsoft Excel. Based on [16], the system of differential equations is obtained as follows.

$$\begin{cases} \Delta T = (-\beta TA) \Delta t \\ \Delta E = (\beta TA - \sigma E - \lambda E) \Delta t \\ \Delta A = (\sigma E - \gamma A + \alpha D) \Delta t \\ \Delta D = (\gamma A + \lambda E - \alpha D) \Delta t \end{cases} \quad (2)$$

The rate of population change in each compartment was measured every one-time unit. In this simulation, initial values for each compartment are required. For example, $T_0 = 900$, $E_0 = 0$, $A_0 = 100$ and $D_0 = 0$. In the first part of this simulation will be analyzed the influence of parameters β . The parameter value of $\sigma = 500$, $\gamma = 0.08$, and $\alpha = 0.01$. Variations values of β are 0.25, 0.5, and 0.75. The simulation results can be seen in Figure 2 below.

The simulation results show the referred customer does not increase significantly from the initial value given at $\beta = 0.25$. The increase in the group that decided not to disseminate information was very rapid. This indicates that loyal customer marketing communications are not going well. There are about 40% of the population who will be referred customers at $\beta = 0.75$. Such a high level of information dissemination meant that the information was already known to most of the population by the 15th time. With the possibility of group D re-entering A , referred customers remained stable from 50th time, which is about 11%. This indicates that the referred customer remains loyal to the company, it just needs to expand its social network.

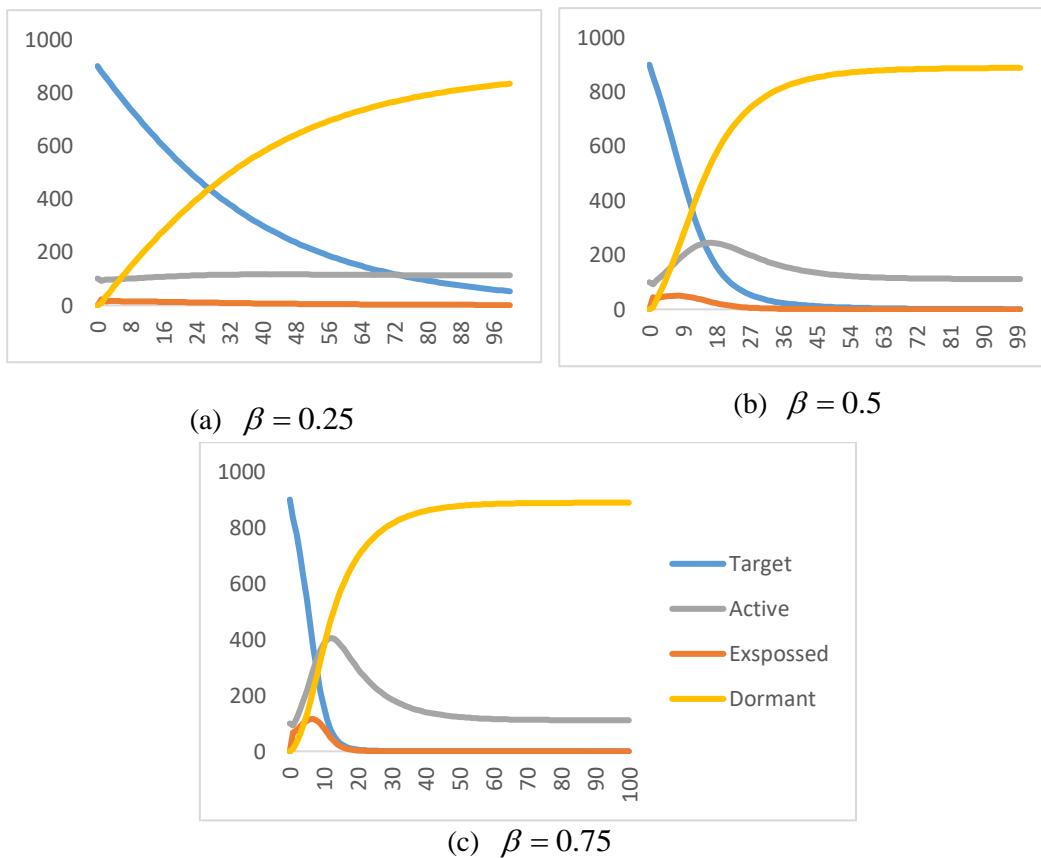


Figure 2. Simulation with various of the transmission rate of information

The next simulation is done by selecting the reinvented rate variation. The parameters σ , γ , and β are set to 500, 0.08, and 0.5, respectively. The simulation results can be seen in Figure 3 below. Variations in reinfection rate determine the latency of compartment A . The higher value of α , the faster group A becomes latent. At the $\alpha = 0.01$, about 60% of the population can be referred customers. However, the number of individuals who will not disseminate information has remained 40%. These companies and referred customers need to expand their campaigns to other population groups so that this company's customer dynamics can be expanded.

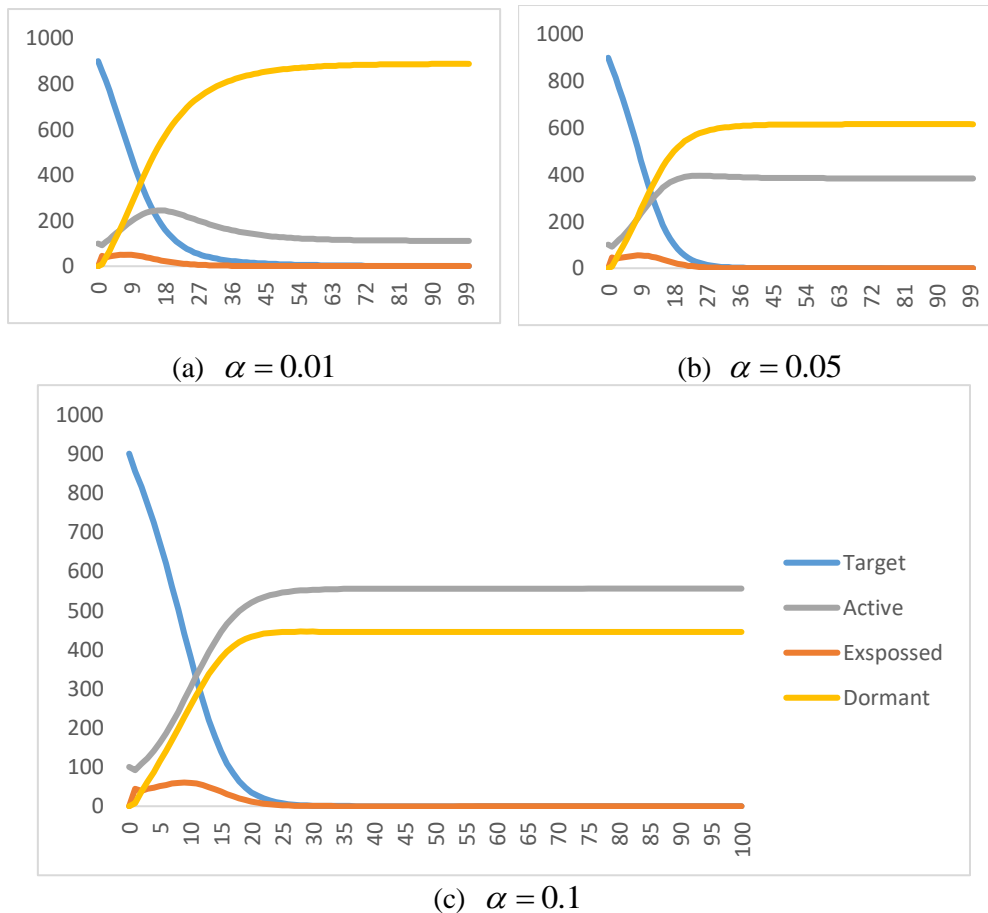


Figure 3. Simulation with various of re-infected rate

5. Conclusion

The referral strategy has characteristics in common with the spread of disease outbreaks. The mathematical model of referral strategy in marketing has been successfully built by analogy to the SEIR in the epidemiological model. The simulation results show that the higher the level of social interaction referred customers in disseminating information, the possibility of the company's customers tends to increase. The opposite impact is that the viral time of information in the community is getting shorter. This needs to be anticipated by the company by updating its marketing policy or increasing its product marketing reach.

6. Acknowledge

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