Contents

Weak local residual as the refinement indicator for the shallow water equations

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Abstract

Weak local residual (WLR) detects the smoothness of numerical solutions to conservation laws. In fact, it can also be used for balanced laws with nonzero source term. Here we consider the shallow water equations (SWE). WLR is used as the refinement indicator in an adaptive finite volume method for solving SWE. We limit our presentation to one dimensional domain. Numerical simulations show the effectiveness of WLR as the refinement indicator. This is the first study in implementing WLR into an adaptive finite volume method used to solve the SWE.
Keywords: weak local residual, smoothness indicator, grid refinement, finite volume method

Weak local residuals (WLRs), also known as local truncation errors, as smoothness indicators for conservation laws were originally proposed by Kurganov et al. [2]. They have been successfully implemented as refinement indicators in adaptive methods for solving some conservation laws. A conservation law is homogeneous, so its source term is zero.

In this article we extend the implementation of WLR as the refinement indicator for balanced laws with a nonzero source term. In particular we consider the one-dimensional shallow water equations. These equations are

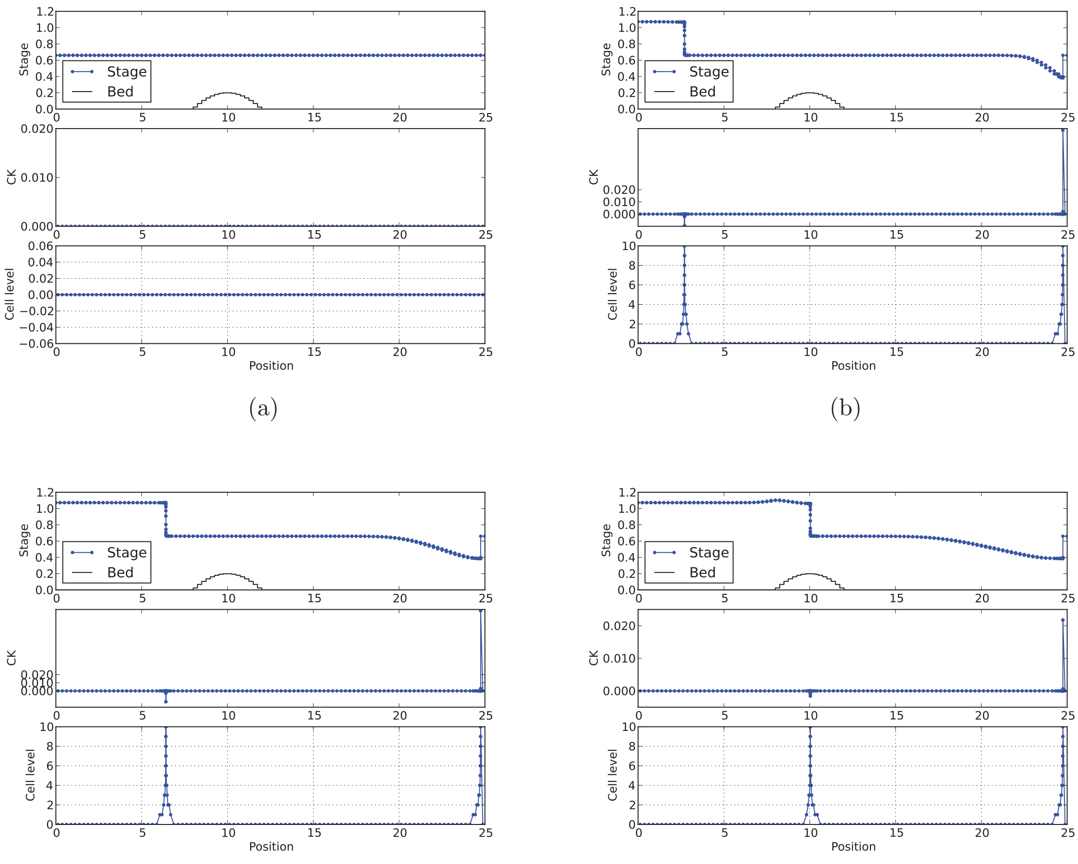
$$h_t + (hu)_x = 0\,, \tag{1}$$

$$(hu)_t + \left(hu^2 + \tfrac{1}{2}gh^2\right)_x = -ghz_x. \tag{2}$$

Here, x represents the coordinate in one-dimensional space, t represen[illegible]me variable, $u = u(x,t)$ denotes the water velocity, $h = h(x,t)$ denotes the water height, $z = z(x)$ is the bed topography, and g is the acceleration due to gravity. We define stage $w(x,t)$ as $w = h + z$.

The numerical method is summarised as follows. Quantities are measured in SI units. We use the finite volume method described in our previous work [3]. The WLR is computed using the scheme given by Constantin and Kurganov (CK) [1]. At each time step, the WLR of CK is computed. Rough areas are refined up to a given tolerance.

For a test case, we consider a flow over a bump. The initial condition is a river at rest with a parabolic bump at the bottom, as shown in Subfigure 1(a). For time $t > 0$ there is a constant inflow from the left-end and constant outflow at the right-end of the domain. We set that the maximum level of cell refinement is 10. The inflow results in shock waves coming into the domain. The numerical results for time $t > 0$ can be represented by Subfigures 1(b), 1(c) and 1(d), which show the results at time $t = 0.67$, 1.67 and 2.67 respectively. Shock waves are rough, so our adaptive method take action by refining the grids around the shocks. Therefore, we get accurate results. Note that if the standard uniform grid were used, the shocks could not be sharply resolved.



(a) (b) (c) (d)

Figure 1: Results with adaptive grids at: (a) $t = 0$, (b) $t = 0.67$, (c) $t = 1.67$, (d) $t = 2.67$.

References

[1] L. A. Constantin, A. Kurganov, *Adaptive central-upwind schemes for hyperbolic systems of conservation laws*, in Hyperbolic problems: Theory, numerics, and applications, Volume 1 (F. Asakura et al., editors), Yokohama Publishers, Yokohama, 2006, pages 95–103.

[2] S. Karni, A. Kurganov, G. Petrova, *A smoothness indicator for adaptive algorithms for hyperbolic systems*, Journal of Computational Physics **178**(2002), 323–341.

[3] S. Mungkasi, S. G. Roberts, *On the best quantity reconstructions for a well balanced finite volume method used to solve the shallow water wave equations with a wet/dry interface*, ANZIAM Journal **51**(2010), C48–C65.