Investigating of sine curve in polar coordinates using GeoGebra

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Investigating of Sine Curve in Polar Coordinates using GeoGebra

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Abstract. In daily life, some functions are circular, such as the orbit of the planets, the shape of a flower that blooms. Curves in the form of Cartesian coordinates can be mapped into curves in polar coordinates. The Cartesian coordinate system is used more often than polar coordinates, so students are less familiar with mathematical equations using polar coordinates. This paper discusses the visualization of the sine wave curve at polar coordinates in the form of the effect of wave frequency (integer and fractional frequency), the number of waves (single and multi-wave), the effect of offset using GeoGebra. The results were obtained in the form of a curve that resembles a blooming flower. Variation of sine with fractional frequency produces a more complex pattern than sine with integer frequency. The effect of the offset on the curve affects the density of the center point of the curve.

INTRODUCTION

Sine curves are found in many lives around us, such as the projection of rotating wheels that will form sine functions or cosine functions. The sine curve is formed from the $\sin(t)$ equation, where t is time while the cosine curve is formed from the $\cos(t)$ equation. Sine and cosine curves are often described in time domains and square coordinates (Cartesian). In daily life, polar coordinates are found in the form of circular natural objects such as flowers. The application of the polar form is also found in recording in spirals, antenna radiation patterns, planetary orbitals, etc. The exploration of polar coordinates using GeoGebra was carried out by [1–3]. The reason for using GeoGebra in this research is that GeoGebra is a web-based tool for teaching and learning Mathematics. GeoGebra is a free digital tool for class activities, graphing, geometry, collaborative whiteboard, and more.

The novelty of this research is reviewed from the GeoGebra community, including Nick Benallo Polar Graphics 2021, Tim Brzezinski Polar Coordinate (intro) 2016, AB Corn Polar Grapher with Transformations 2014. This study discussed the representation of sine curves in polar coordinates using GeoGebra. In this study is the influence of the number of sine waves on the polar graph, the influence of sine amplitude on the polar graph, the construction protocol on GeoGebra.

METHODS

A polar coordinate graph with a grid of circles that shows the radius and angular line representing angles from 0 to 2π radians is shown in Fig. 1. The angle is repeating when the angle is more than 2π. Waves in the time function are mapped in polar coordinates with $0 < t < 2\pi$, as seen in Fig. 2, where the function $y = \sin(3t)$ is mapped into polar form $r = \sin(3t)$, where $\theta = t$.
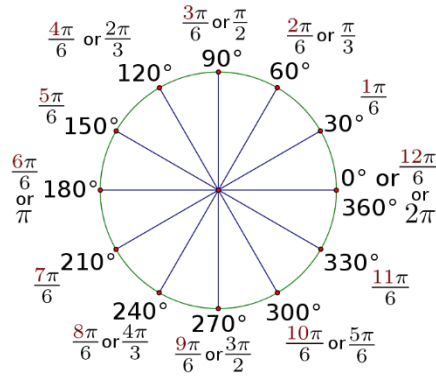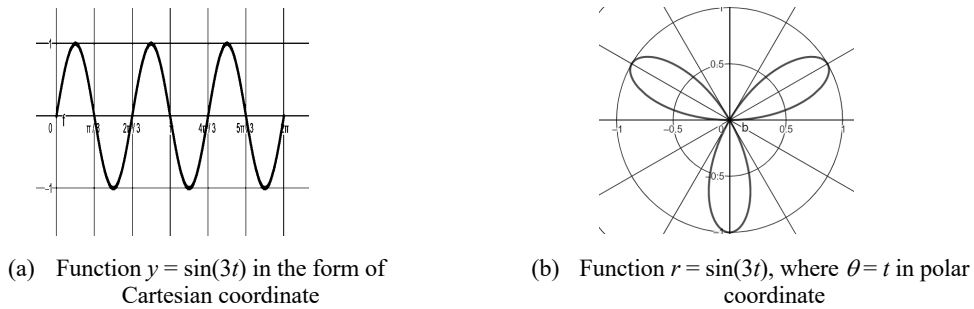
FIGURE 1. 2D polar coordinate system.

(a) Function $y = \sin(3t)$ in the form of Cartesian coordinate

(b) Function $r = \sin(3t)$, where $\theta = t$ in polar coordinate

FIGURE 2. A function in Cartesian and polar coordinates.

The construction protocol using GeoGebra for drawing the function $y = \sin(3t)$ in the form of Cartesian coordinates and polar coordinates is shown in Fig 3. Writing points in Cartesian coordinates in GeoGebra uses the format $A = (x, y)$ while writing points in polar coordinates use the format $A = (radius, angle)$. Angles in GeoGebra have units of radians, if it uses degrees, add a degree symbol by pressing [alt][o]. To get the symbol pi by pressing [alt][p], the keyword pi is also known by GeoGebra as the constant pi. The curve command in GeoGebra is used to create curves with parameters such as equations, limits, and increments. Command format: curve(<expression>,<var>, <start>,<end>,<increment>).

$$f(t) = \sin(3\,t), \quad (0 \le t \le 2\,\pi)$$

$$b = \text{Curve}((f(t); t), t, 0, 2\,\pi)$$

$$\rightarrow \quad (f(t); t), \quad (0 \le t \le 6.28)$$

FIGURE 3. Construction protocol in GeoGebra to draw $y = \sin(3t)$.

This research will examine the effect of the number of frequencies of the sine function (single frequency and multi-frequency) in polar form, the effect of frequency (fractional and integer), the effect of wave count, the effect of the offset value on the polar form. To convert from Cartesian coordinates to polar coordinates, use equation 3.

$$y(t) = A \sin(f(t)) + \text{offset} \tag{1}$$
$$y(t) = A_1 \sin(f_1(t)) + A_2 \sin(f_2(t)) + \text{offset} \tag{2}$$
$$\text{in Cartesian coordinate: } A = (t, y(t)) \tag{3}$$

RESULTS AND DISCUSSION

The effect of the number of waves in the polar form is shown in Fig. 4, while the sine waves as a time-domain function are shown in Fig. 5. shows that the period wave of $N = 1$ in the polar form is a petal form. This is due to the time the amplitude is positive, the angle $0 < \theta < \pi$ occurs, when the amplitude is negative, the angle $\pi < \theta < 2\pi$ occurs. A negative amplitude with an angle greater than π would be a positive amplitude. For waves with $N = 2$ periods, each part of the wave is depicted in polar form so that it draws with 4 petals.
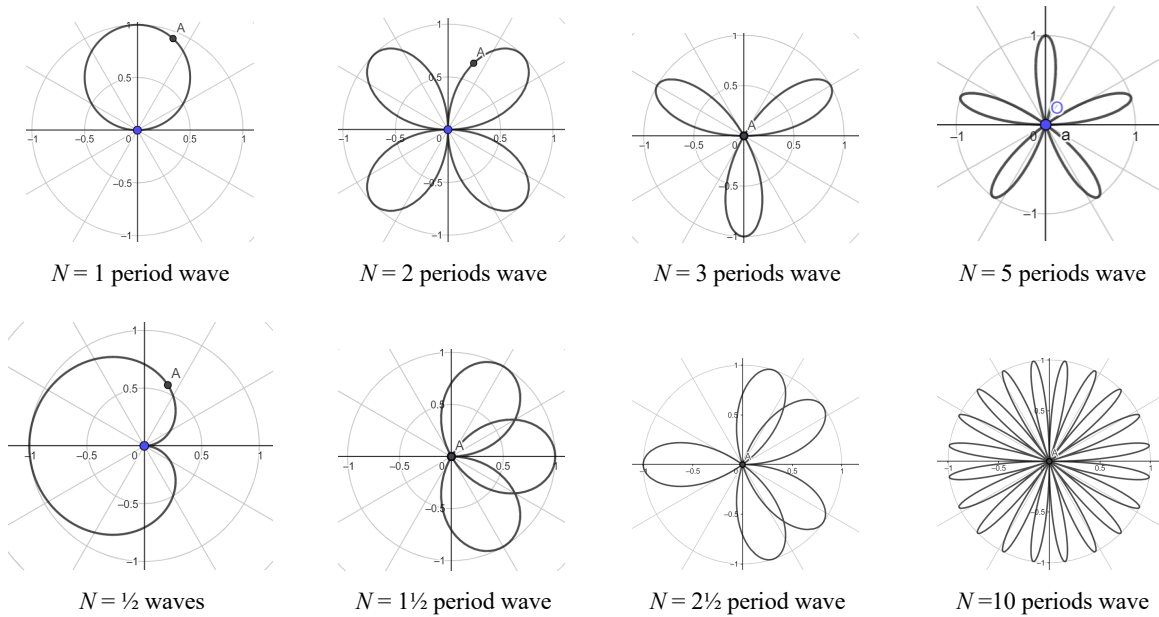

$N = 1$ period wave	$N = 2$ periods wave	$N = 3$ periods wave	$N = 5$ periods wave
$N = \frac{1}{2}$ waves	$N = 1\frac{1}{2}$ period wave	$N = 2\frac{1}{2}$ period wave	$N = 10$ periods wave

FIGURE 4. Patterns that occur based on the number of periods wave.

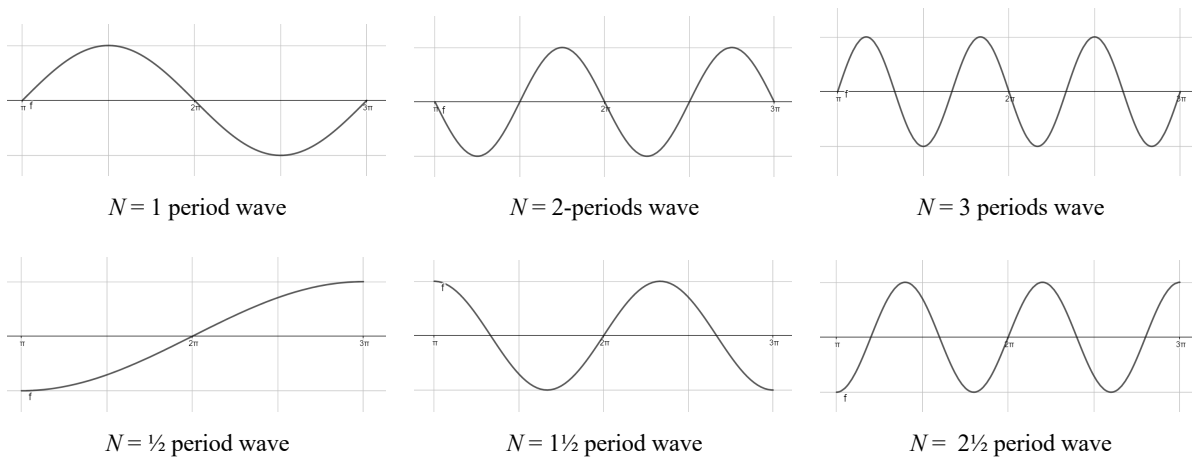|		
$N = 1$ period wave	$N = 2$-periods wave	$N = 3$ periods wave
$N = \frac{1}{2}$ period wave	$N = 1\frac{1}{2}$ period wave	$N = 2\frac{1}{2}$ period wave

FIGURE 5. Number of waves within 2π seconds.

The waveform of the following equation shown in Figure 6:
$$y(t) = \sin(t) + \frac{1}{3}\sin(9t).$$
The sum of waves on the graph of the time function produces a unique pattern, curves formed in the polar form produce a leaf-like pattern.
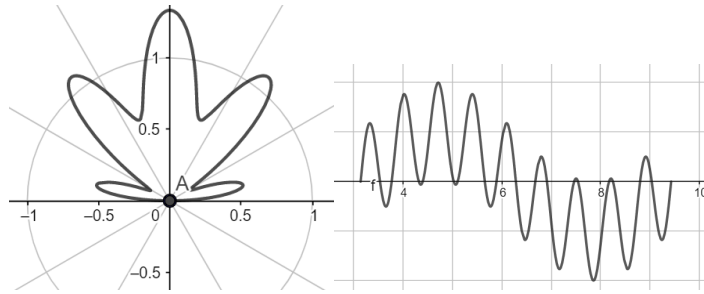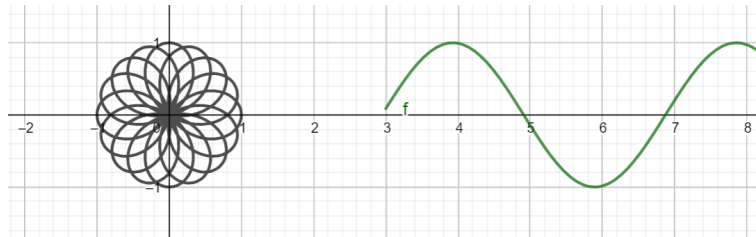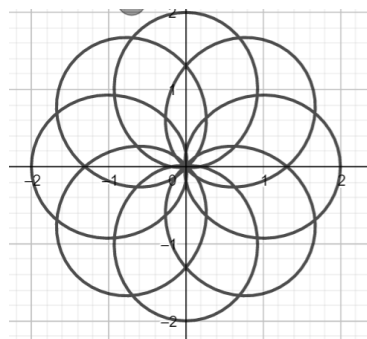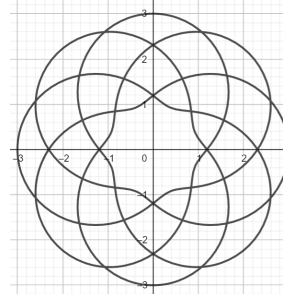
FIGURE 6. Polar shapes and waveforms of $y(t) = \sin(t) + \frac{1}{3}\sin(9t)$.

(a) $f(t) = \sin(1.6t)$

(b) $f(t) = \sin(1.6t) + 1$

(c) $f(t) = \sin(1.6t) + 1$

FIGURE 7. Effect of adding offset to waves and their effect on polar shape curves.

Figure 7(a) on the right shows the sine function $y(t) = \sin(1.6t)$ in the time domain, with a frequency of 1.6 radians per second or having a period of $2\pi/1.6$ or 3.92. In the polar form, it will turn into a flower with 16 petals as shown in Fig. 7(a) on the left. The effect of adding offsets to the waves causes the waves to shift up or down. A positive offset causes the wave to shift upwards as shown in Fig. 7(b) and Fig. 7(c). The larger the offset causes the formed curve to decompose (no concentrated of the middle curve) as shown in Fig. 7(c). The effect of offset on the curved petal is that the center of the flower will bloom more [4], as shown in Fig. 8.

The general sine function equation is

$$y(t) = A\sin(\omega t + \varphi) + \text{offset},$$

where A is the amplitude, ω is the frequency or number of waves per second, phase φ is the shift of the curve towards the time axis, while the offset will shift the curve to the y axis. The offset position is shown in Fig 9. The offset does not change the curve shape in Cartesian coordinates but rather shifts the curve up or down. The effect on the polar shape will be different, that is, the curve is getting further away from the center point.

Table 1 shows the effect of the sine frequency on the image formed in the polar form. The sine frequency starts from 1 Hz to 5 Hz in 0.1Hz increments. A sine with an integer frequency produces a simple pattern, while a sine with a fractional frequency produces a more complicated pattern. A sine curve with a fractional frequency will be depicted completely when the multiple of the fractional frequency becomes an integer. For example, with a frequency of 2.4 rad/s, the curve will return to its original position when it reaches 5 periods.
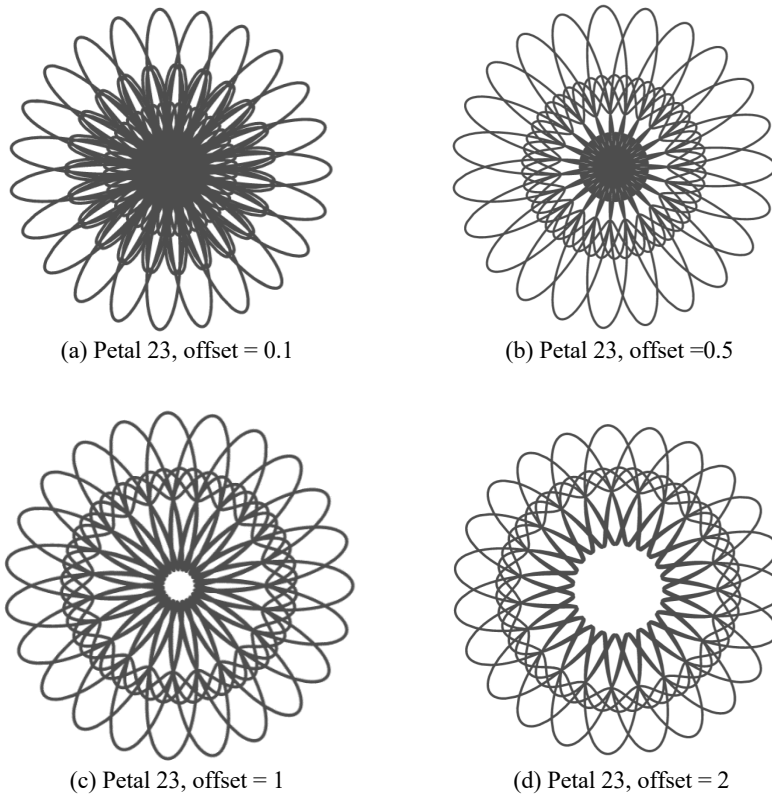
(a) Petal 23, offset = 0.1   (b) Petal 23, offset =0.5

(c) Petal 23, offset = 1   (d) Petal 23, offset = 2

FIGURE 8. Number of petals and effect of offset waveform.
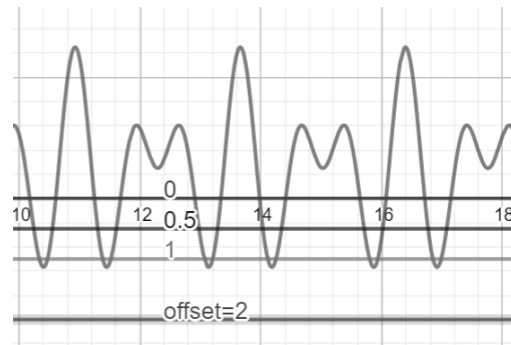

FIGURE 9. offset position on the wave.

Table 1. The formation of number petals at various sinus frequencies.

Number of Petals	Fraction Frequency (Hz)									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1 Hz	1	11	6	13	7	3	8	17	8	19
2 Hz	2	21	11	23	12	5	13	9	14	29
3 Hz	3	31	15	11	17	7	18	37	19	39
4 Hz	4	41	21	43	11	9	23	47	24	49

Notes: Shaded cell for the number of petals more than 10.

Patterns formed by non-sine functions such as spiral shapes (bifilar coil shapes) are shown in Fig. 10. Eq (4) and Eq. (5) show the GeoGebra construction protocol to create a spiral with a spacing of 1, with the center point at point O and the number of turns N.

$$y(t) = \frac{1}{2\pi} \tag{4}$$

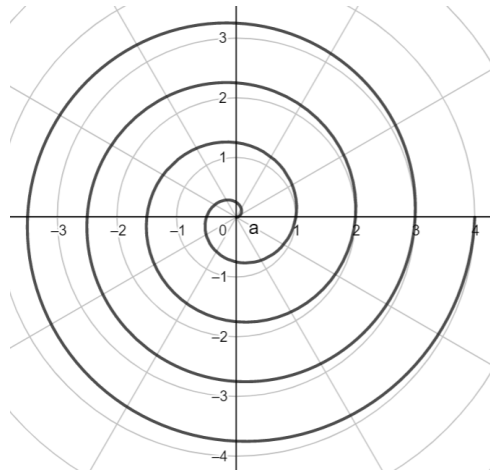$$\text{Curve}((y(t); t), t, 0, 2\pi N) \tag{5}$$

FIGURE 10. Bifilar coil as spiral.

CONCLUSION

The results of research and experiment can be concluded as follows:

1. Curves in Cartesian coordinates are converted into curves in polar coordinate by changing the time axis of the Cartesian coordinate to angles at the polar coordinate.
2. Through GeoGebra, visualization of sine waves in polar coordinate is easy. Changing sine wave parameters such as frequency, and adding offset is easy.
3. Sine waves form petals, the more waves in a 2π angle will be described as the number of petals. The number of petals is indicated by the frequency of the sinuses. The offset value of the sine is shown as the inner circle of the curve.

ACKNOWLEDGMENT

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