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


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
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ANALYSIS OF STUDENTS' ERRORS IN WORKING ON NUMBER THEORY QUESTIONS (21ST CENTURY SKILLS)

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Abstract

This study aims to describe students' mistakes in solving number theory's problem. In this study, error analysis, especially on the topic of Diophantine equations and Chinese remainder theorem. This type of research is qualitative research. The subjects of this study were 46 students of mathematics education at Sanata Dharma University. The data collection technique used a test. The data were analyzed to determine errors in doing number theory problems using the type of error according to Newman. According to Newman, five types of errors are reading, comprehension, transformation, process skill, and encoding errors. The conclusion of this study showed that the most errors made were related to process skill errors and transformation errors, while only a few experienced comprehension and encoding errors. None of the students made reading error. This shows that students already understand the meaning of the problem, but when changing and looking for solutions to problems an error occurs. The factors that may cause this error are a lack of understanding of the concept of the Diophantine equations and Chinese remainder theorem, as well as a lack of problem practice.

Keywords: Error analysis, number theory, Newman error analysis, mathematics

INTRODUCTION

Problem solving skills are one of the 21st century skills. According to Zubaidah (2016), important skills in the 21st century are critical thinking skills, problem solving, metacognition, communication skills, collaboration, innovation and creation, information literacy, and various other skills. These skills are important to strive for in learning activities. According to Widjajanti (2009) a question is said to be a problem if the problem is challenging to solve and the procedure to solve it cannot be done routinely. Problem solving ability is the ability to determine the number of steps that must be taken, find the relationship between past experiences (schemes) and problems that are now faced and then act to resolve it. Prospective teacher students should have problem solving skills because later when they become teachers one of their duties is to guide students in solving mathematics problems.

One of the courses in mathematics education study program is number theory. In number theory lectures, there are many topics. These topics are mathematical induction, binomial theorem, congruence and its applications, Diophantine equations, Chinese remainder theorem, and cryptology (Burton, 1980; Rosen, 2011, Sukirman, 2013). Students should understand the concept of number theory and be able to solve the problems on all topics. However, so far it has been found that in solving problems, students often make mistakes.

There are several theories to identify students' mistakes, one of which is according to Newman's theory. According to Newman (Sign, Arba, Teoh, 2010), five types of errors are reading errors, comprehension errors, transformation errors, process skill errors, and encoding errors. Reading errors are errors in writing words or symbols that cause students to find the wrong solution. Comprehension error is when students can read questions but misunderstand what is known and asked in the questions, it cause the students make mistakes in finding solutions. Transformation error is when students

understand the meaning of the problem but are wrong in determining the appropriate mathematical operation or formula to solve the problem and find a solution. Process skill errors are when students are able to determine the appropriate mathematical formula or operation but are mistaken in calculating or carrying out the completion procedure correctly. Encoding error is when students have found a problem solution but incorrect writing down the final result or conclusion.

According to Prakitipong & Nakamura (Sign, Arba, Teoh, 2010), reading and comprehension are two significant initial steps in which students interpret the problem into a mathematical context correctly. The other three steps, namely transformation, process skill, and encoding, show that the student has successfully executed the mathematical processes required in order to solve the task. Thus identifying students' mistakes using Newman's theory can also determine the root of the problem originating from language understanding or student knowledge content. In addition, according to Chusnul, Mariyana, and Dewi (2017) by using Newman's error analysis we can see the relationship between literacy and numeracy because Newman analyzes student work from beginning to end. There were several questions that were asked to help analyze student errors according to Newman, namely whether students could read the questions? Do students understand the meaning of the questions? Can students choose the right operation or procedure to solve the problem? Can students apply calculations or work procedures correctly? And can students represent answers or conclude correctly?

The mistakes made by students are not only on difficult material but also on easy material. If this is allowed, the learning objectives cannot be carried out properly. Based on research conducted by Fatahilah, Yuli, and Susanto (2017), the errors that often appear are comprehension, transformation, and skill error processes. Another research conducted by Nurahman and Karim (2018) states that the student has not been able to optimize all capabilities, especially the proof of mathematics in working on the number theory that tends to give up in working on problems when experiencing difficulties. In line with that, Meilanawati and Pudjiastuti (2020) state that errors in working on number theory problems are conceptual errors, prosuderal errors and technical errors. The contributing factors include: lack of understanding of the problem, not being careful in calculations, and not memorizing theorems. According to Hartati (2020), students' difficulties in working on number theory problems include the difficulty of understanding and using congruent concepts and congruent applications, applying definitions, properties and theorems in mathematical proof, and converting base numbers and base number arithmetic operations. According to Karim and Nurrahman (2018) students have not been able to optimize all their abilities, especially their mathematical comprehension skills in working on number theory questions, so they tend to give up working on problems when experiencing difficulties. Based on this description, this study aims to identify students' errors in working on Number Theory questions using Newman's analysis on the topic of Diophantine equations and Chinese remainder theorem.

METHOD

This type of research is descriptive research. In this study, the types of students' errors in working on number theory questions were described on the topic of the Diophantine equation and Chinese remainder theorem based on Newman's error analysis. The subjects in this study were 46 students of second semester mathematics education at Sanata Dharma University Yogyakarta. The object of this research is the students' mistakes in working on number theory questions. Data collection was carried out in June 2021.

The data collection technique used test questions in the form of a description of the topic Diophantine equation and Chinese remainder theorem. The data analysis technique used was to determine the types of errors made by students based on the following table.

Table 1. Error analysis criteria according to Newman

Type of error	Criteria	Code
Reading error	Students cannot write down what is known in the problem	RE
Comprehension error	Students cannot write down what to ask or what to look for in the questions	CE
Transformation error	Students cannot determine the procedure for solving the problem	TE
Process Skill error	Students cannot count correctly and carry out work procedures correctly	PS
Encoding error	Students cannot write conclusions or final answers correctly	EE

After identifying the types of errors made by students, then the percentage of the number of each type of error was calculated on the topic Diophantine equation and Chinese remainder theorem. In addition, the average percentage of each type of error that students made was calculated.

RESULT AND DISCUSSION

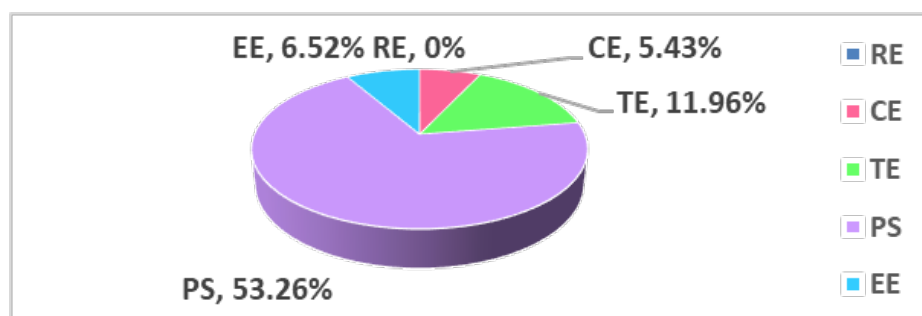
The results of the analysis of students' errors in working on number theory questions on the topic Diophantine equation and Chinese remainder theorem based on Newman's error theory are as follows.

Table 2. Percentage of types of errors made by students

	Diophantine Equation	Chinese Remainder Theorem	Average percentage of errors
Reading error (RE)	0%	0%	0%
Comprehension error (CE)	10.87%	0%	5.43%
Transformation error (TE)	0%	23.91%	11.96%
Process Skill error (PS)	56.52%	50%	53.26%
Encoding error (EE)	13.04%	0%	6.52%

From table 2, it can be seen that the most mistakes made by students on working Diophantine equation is the Process skill error. Likewise, in questions related to Chinese remainder theorem, the most mistakes made by students were also in the process skill error. Students did not make mistakes in doing Diophantine Equation questions on reading errors and transformation errors, while in questions about Chinese remainder theorem on reading, comprehension, and encoding errors. In the topic Diophantine equation, students still make errors in encoding and comprehension. On the topic Chinese remainder theorem, mistakes that are often made by students regarding transformation errors. The average percentage of student error is presented in the following diagram.

Figure 1. Diagram of the average percentage of student error



Based on the diagram of the average percentage of student errors, the most students' mistake is process errors and the second is transformation errors, while few make errors for encoding and comprehension errors, and for reading errors none of the students who make mistakes.

In addition, the following will identify the mistakes made by students when working on questions related to Diophantine equation and Chinese remainder theorem.

Diophantine equation topic

The five types of errors identified by Newman are as follows.

Reading error. Reading error is an error where students cannot write down what is known in the problem. Regarding the Diophantine equation, all students can read the questions correctly and write down what is known in the questions correctly.

Comprehension error. Comprehension error is a student's error in understanding the problem. This can be seen when students cannot write down or write wrong what is being asked in the questions. In the following, two example comprehension errors are presented.

Figure 2. Example comprehension error 1

$x = \text{banyak ice cream coklat yang dibeli}$
 $y = \text{banyak ice cream vanilla yang dibeli}$
 $\begin{cases} 5.000x + 7.000y = 80.000 - 1.000 \\ 5.000x + 7.000y = 79.000 \end{cases}$
 $5x + 7y = 79$

From Figure 2, it can be seen that students have been able to read the questions but did not write down what to look for in the questions. Students immediately write in the form of an equation.

Figure 3. Example comprehension error 2

Harga ice cream coklat = Rp5.000,00
 Harga ice cream vanilla = Rp7.000,00
 Jika membayar Rp80.000,00 sisa Rp1.000,00
 $5000x + 7000y \equiv 1000 \pmod{80.000}$

In Figure 3, it can be seen that students do not understand the meaning of the questions. What is meant in the question is to buy chocolate and vanilla ice cream by paying 80,000 and getting a 1,000 in return means the purchase price is 79,000. Students mistakenly write it to be congruent.

Transformation error. Transformation error is a student error where students cannot determine the procedure for solving the problem. When working on problems related to diophantine equations, all students can perform transformations, none of the students carry out transformation errors.

Process skill error. Process skill is the student's ability to carry out the completion procedure and do calculations correctly. The following are some of the mistakes students made.

Figure 4. Example process skill error 1

→ kongruensi linear
 $5x \equiv 79 \pmod{7}$
 $5x \equiv 100 \pmod{7}$
 $x \equiv 20 \pmod{7}$
 $x = 20 + 7t, t \in \mathbb{Z}$
 → substitusi ke pers
 $5(20 + 7t) + 7y = 79$
 $100 + 35t + 7y = 79$
 $7y = -21 - 35t$
 $y = -3 - 5t$

From Figure 4 it can be seen that the students were correct in carrying out the procedure, it was just wrong in determining the t value so that a negative value was obtained even though the number of items could not be negative.

Figure 5. Example process skill error 2

\rightarrow Kongruensi Linear
 $5x \equiv 79 \pmod{7}$
 $5x \equiv 100 \pmod{7}$
 $x \equiv 20 \pmod{7}$
 $x = 20 + 7t, t \in \mathbb{Z}$
 \rightarrow Sub. nilai x ke persamaan
 $5x + 7y = 79$
 $5(20 + 7t) + 7y = 79$
 $100 + 7t + 7y = 79$
 $7y = 79 - 100 - 7t$
 $7y = -21 - 7t$
 $y = -3 - t$
 Untuk $t=0$, maka
 $x = 20 + 7t$
 $= 20 + 7(0)$
 $= 20$
 $y = -3 - t$
 $= -3 - 0$
 $= -3$
 Untuk $t=1$, maka
 $x = 20 + 7t$
 $= 20 + 7(1)$
 $= 27$
 $y = -3 - t$
 $= -3 - 1$
 $= -4$

From Figure 5, it can be seen that students made a calculation error in the process and incorrectly determined the t value, causing the final answer to be wrong.

Figure 6. Example process skill error 3

$5x + 7y = 79$
 $7x \equiv 79 \pmod{5}$
 $7x \equiv 49 \pmod{5}$
 $x \equiv 12 \pmod{5}$
 $x \equiv 2 \pmod{5}$
 $x = 2 + 5t$
 \rightarrow Substitusikan ke persamaan awal
 $5x + 7y = 79$
 $5(2 + 5t) + 7x = 79$
 $10 + 25t + 7x = 79$
 $7x = 69 - 25t$
 $x = 69 - 25t$

From Figure 6 it can be seen that students are not careful in writing the variable, it should be $7y$ but written as $7x$, causing the answer to be wrong in the next substitution process.

Encoding error Encoding error is a student's error in writing the final answer or conclusion. The following are examples of encoding errors made by students.

Figure 7. Example encoding error 1

penyelesaian persamaan adalah
 $x = 6 \pmod{7}$ dan $y = 7 \pmod{7}$

Figure 8. Example encoding error 2

$\therefore x = 6 + 7t$
 $y = 7 - 5t$

From Figures 7 and 8, it can be seen that students have been able to determine the x and y values, but they writing the conclusion in mathematical form, it should be a sentence of the number of chocolate and vanilla ice cream.

Figure 9. Example encoding error 3

Figure 9. Example encoding error 3

8. banyak es krim yang dibeli maudy ayunda adalah 20 es krim coklat dan -3 es krim vanilla.

From Figure 9 it can be seen that the students were wrong in determining the amount of vanilla ice cream, namely -3 should be positive.

Chinese remainder theorem topic

The identification of five mistakes when working on questions related to Chinese remainder theorem according to Newman's error analysis is as follows.

Reading error In reading questions related to Chinese remainder theorem topic, all students can read the questions correctly and write down what is known in the questions.

Comprehension error In understanding the questions related to Chinese remainder theorem, all students can understand and write down what is being asked in the questions correctly.

Transformation error At the transformation stage, some students were still wrong in transforming questions related to Chinese remainder theorem as follows.

Figure 10. Example transformation error 1

$$M = m_1 \cdot m_2 \cdot m_3 = 4 \cdot 5 \cdot 6 = 120$$

$$M_1 = \frac{120}{4} = 30$$

$$M_2 = \frac{120}{5} = 24$$

$$M_3 = \frac{120}{6} = 20$$

$$20y_2 = 2 \pmod{6}$$

$$20y_2 = 40 \pmod{6}$$

$$y_2 = 2 \pmod{6}$$

$$X = a_1 \cdot m$$

Figure 11. Example transformation error 2

$$30y_1 \equiv 1 \pmod{4}$$

$$24y_2 \equiv 1 \pmod{5}$$

$$20y_3 \equiv 1 \pmod{6}$$

$$y_1 \equiv 1 \pmod{4}$$

$$y_2 \equiv 1 \pmod{5}$$

$$y_3 \equiv 1 \pmod{6}$$

$$X \equiv 1 \pmod{4}$$

$$X \equiv 2 \pmod{5}$$

$$X \equiv 3 \pmod{6}$$

From Figures 10 and 11, it can be seen that students directly use the theorem to work on problems related to Chinese remainder theorem, even though the requirement to be able to use the theorem is that each pair of modulo must be relatively prime.

Figure 12. Example transformation error 3

$$X \equiv 1 \pmod{4}$$

$$X \equiv 2 \pmod{5}$$

$$X \equiv 3 \pmod{6}$$

$$a_1 = 1, a_2 = 2, a_3 = 3$$

$$m_1 = 4, m_2 = 5, m_3 = 6$$

$$(m_1, m_2) = 1, (m_2, m_3) = 1, (m_1, m_3) = 2$$

In Figure 12 students realize that modulo pairs are not relatively prime, but there are still other ways to solve problems related to Chinese remainder theory, namely by substitution.

Process skill error The process skill error that students do when working on questions related to Chinese remainder theory is as follows.

Figure 13. Example process skill error 1

$$\begin{aligned}
 1. \quad 17 + 20u &\equiv 3 \pmod{6} & x &= 17 + 20(3 + 6z) \\
 20u &\equiv -14 \pmod{6} & & \\
 20u &\equiv 6 \pmod{6} & & \\
 20u &\equiv 60 \pmod{6} & & \\
 u &\equiv 3 \pmod{6} & & \\
 u &\equiv 3 + 6z & & \\
 \therefore x &= 77 + 120z \\
 x &\equiv 77 \pmod{120} \\
 \therefore \text{minimal coklat yang dimiliki ditan adalah } 77.
 \end{aligned}$$

In Figure 13, it can be seen that students made a mistake in determining the value of u it should have obtained $u = 2 + 6z$.

Figure 14. Example process skill error 2

$$\begin{aligned}
 \text{Rongmani 1 dapat dibagi } n &= 4t + 1 \\
 4t + 1 &\equiv 2 \pmod{5} \\
 4t &\equiv 1 \pmod{5} \\
 t &\equiv 2 \pmod{5} \\
 t &= 5u + 2 \quad \text{Substitusi}
 \end{aligned}$$

In Figure 14, students made a mistake in determining t , the value of $t = 5x + 4$.

Encoding error Students can write down the conclusion of the final result correctly, none of the students do an encoding error.

From the description above, it can be seen that the most mistakes made by students occurred in the process skill error, as much as 53.26%. Next, the errors that are often made related to transformation errors are 11.96%. Only a few students did comprehension and encoding errors, namely 5.43% and 6.52%, respectively. And there are no students who read errors. This indicates that some students still have difficulty understanding the concept of the Diophantine equation and Chinese remainder theorem as well as a lack of practice questions related to these topics. Therefore, students should do more practice questions on the topic of Diophantine and Chinese remainder theorem.

This is in line with research conducted by Macromah and Mega (2017) that there are still many students who make mistakes when working on questions on calculus material. As many as 80.87% of students made mistakes when working on integral calculus problems. Some of the things that cause students to make mistakes are: students are not careful in reading questions and writing mathematical symbols and errors in calculations, low initial student ability or student mastery of prerequisite material, students do not understand the concept being studied, and students cannot choose the method right to solve the problem.

CONCLUSIONS AND SUGGESTIONS

The conclusion of this study showed that the most errors made were related to process skill errors and transformation errors, while only a few experienced comprehension and encoding errors. None of the students made reading error. This shows that students already understand the meaning of the problem, but when changing and looking for solutions to problems an error occurs. The factors that may cause this error are a lack of understanding of the concept of the Diophantine equations and Chinese remainder theorem, as well as a lack of problem practice.

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