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Validation of ANUGA hydraulic model using exact solutions to shallow water wave problems

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Abstract. ANUGA is an open source and free software developed by the Australian National University (ANU) and Geoscience Australia (GA). This software is a hydraulic numerical model used to solve the two-dimensional shallow water equations. The numerical method underlying it is a finite volume method. This paper presents some validation results of ANUGA with respect to exact solutions to shallow water flow problems. We identify the strengths of ANUGA and comment on future work that may be taken into account for ANUGA development.

1. Introduction

Shallow water flows have been studied mathematically since the ninteenth century. One available mathematical model is the shallow water equations derived by de Saint-Venant [5]. Based on the shallow water equations, a well-known solution to a dam break problem was derived by Ritter [16]. More comprehensive study of shallow water flows has been conducted by other authors, such as Stoker [19, 20].

Nowadays, the shallow water equations are widely applied to model river flows, tsunami inundations, and floods. However, the exact analytical solutions to the shallow water equations are available only for some specific cases. Numerical solutions are then desired as approximations of the exact solutions. Due to the applications of the shallow water equations, numerical packages (softwares) to solve these equations have been made available. Some of the softwares are commercial and some are free. One available free software is ANUGA. ANUGA is an open source software developed by the Australian National University (ANU) and Geoscience Australia (GA).

This paper validates ANUGA with respect to some exact analytical solutions to the shallow water equations. We identify the strengths of ANUGA when used to simulate shallow water flow problems. Comments on ANUGA shall also be provided for consideration on ANUGA development.

This paper is organised as follows. We provide a brief review of ANUGA software in Section 2. Some exact analytical solutions to the shallow water equations are collected in Section 3. In Section 4, we present numerical results of ANUGA experiments in comparison to the exact analytical solutions. Section 5 concludes this paper with some remarks.

2. ANUGA software

ANUGA is written in Python programming language for the User Interface together with C programming language for the expensive computation parts. Python gives a flexible interface, while C leads to fast computations. The combination of these two languages make ANUGA a robust software. This shall be demonstrated in the numerical experiments we present in Section 4.

The mathematical background underlying ANUGA is a finite volume method used to solve the two-dimensional shallow water equations. A description of it can be found in some documents, such as the ANUGA User Manual [18] and our previous work [11].

ANUGA has advantages for simulation of shallow water flows or waves. First, it can be used to simulate tsunami, dam break, flood inundations and other types of shallow flows. Second, it can handle wetting and drying processes reasonably. Third, it is able to accurately resolve discontinuous water surface and velocity profiles, such as shock waves or hydraulic jumps. See previous publications [1, 3, 4, 6, 7, 8, 15, 17, 22, 23] about ANUGA for more detailed explanations on these advantages.

However, ANUGA has some limitations due to the implemented mathematical model [6]. First, it is not able to simulate turbulence and breaking waves. (To simulate turbulence and breaking waves, the Navier-Stokes equations or the three-dimensional shallow water equations should be used.) Second, ANUGA does not cover sedimentations, erosions and debris-type flows. Third, for simulating tsunamis, a sea floor displacement is not handled by ANUGA. (A tsunami generation should be taken into account for another model, such as Method of Splitting Tsunamis (MOST) and the URS Corporation's Probabilistic Tsunami Hazard Analysis [15].)

3. Some exact analytical solutions to the shallow water equations

In this section, we collect four analytical solutions to shallow water flow (wave) problems. The first three are analytical solutions to problems in one dimension. The fourth is an analytical solution to a problem in two dimensions. All of the analytical solutions can be implemented to test the performance of numerical methods used to solve two-dimensional problems.

In this paper,we use the following notations. For two-dimensional problems:

- x, y represent the two-dimensional spatial variables,
- t represents the time variable,
- $u = u(x, y, t)$ denotes the water velocity in the x-direction, that is, the x-velocity,
- $v = v(x, y, t)$ denotes the water velocity in the y-direction, that is, the y-velocity,
- $h = h(x, y, t)$ denotes the water height (depth),
- $z = z(x, y)$ denotes the topography elevation (bed),
- $w = h + z$ is called the stage, and
- g is the acceleration due to gravity.

The momentum in the x-direction called the x-momentum is given by uh, and the momentum in the y-direction called the y-momentum is given by vh. The notations for one-dimensional problems are similar to those for two-dimensional problems, but note that variables y and v do not arise in one-dimensional problems.

3.1. Dam break involving a dry area in one dimension

In this subsection, we recall the analytical solution to a dam break problem in one dimension.

The initial condition is described as follows. A dam wall is located at points $x = x_0$. The topography is horizontal. Water height on the left of the dam wall is $h = h_1$, and on the right of the dam wall is $h = h_0$.

For the case with $x_0 = 0$, $h_1 > 0$, and $h_0 = 0$, the analytical solution to the dam break is as follows. After the dam wall is removed $(t > 0)$, the water height is

$$h(x,t) = \begin{cases} h_1 & \text{if } x \leq -t\sqrt{gh_1} \\ \frac{4}{9g}(\sqrt{gh_1} - \frac{x}{2t})^2 & \text{if } -t\sqrt{gh_1} < x \leq 2t\sqrt{gh_1} \\ 0 & \text{if } x \geq 2t\sqrt{gh_1}\,, \end{cases} \tag{1}$$

and the velocity is

$$u(x,t) = \begin{cases} 0 & \text{if } x \leq -t\sqrt{gh_1} \\ \frac{2}{3}(\sqrt{gh_1} + \frac{x}{t}) & \text{if } -t\sqrt{gh_1} < x \leq 2t\sqrt{gh_1} \\ 0 & \text{if } x \geq 2t\sqrt{gh_1}\,. \end{cases} \tag{2}$$

The derivation of this analytical solution can be found in some references, such as Ritter [16], Stoker [19, 20], Mangeney, Heinrich, and Roche [10], as well as Mungkasi and Roberts [12, 13].

3.2. Dam break involving a shock wave in one dimension

Recall the dam break problem stated in the previous subsection. A dam wall is located at points $x = x_0$. The topography is horizontal. Water height on the left of the dam wall is $h = h_1$, and on the right of the dam wall is $h = h_0$.

For $h_1 \geq h_0 > 0$, the solution to the dam break problem is

$$h(x) = \begin{cases} h_1 & \text{if } x \leq -t\sqrt{gh_1} \\ \frac{4}{9g}(\sqrt{gh_1} - \frac{x}{2t})^2 & \text{if } -t\sqrt{gh_1} < x \leq t(u_2 - \sqrt{gh_2}) \\ \frac{h_0}{2}\left(\sqrt{1 + \frac{8\dot{\xi}^2}{gh_0}} - 1\right) & \text{if } t(u_2 - \sqrt{gh_2}) < x < t\dot{\xi} \\ h_0 & \text{if } x \geq t\dot{\xi} \end{cases} \tag{3}$$

and

$$u(x) = \begin{cases} 0 & \text{if } x \leq -t\sqrt{gh_1} \\ \frac{2}{3}(\sqrt{gh_1} + \frac{x}{t}) & \text{if } -t\sqrt{gh_1} < x \leq t(u_2 - \sqrt{gh_2}) \\ \dot{\xi} - \frac{gh_0}{4\dot{\xi}}\left(1 + \sqrt{1 + \frac{8\dot{\xi}^2}{gh_0}}\right) & \text{if } t(u_2 - \sqrt{gh_2}) < x < t\dot{\xi} \\ 0 & \text{if } x \geq t\dot{\xi} \end{cases} \tag{4}$$

at any time $t > 0$, where $\dot{\xi}$ is the shock speed constant

$$\dot{\xi} = 2\sqrt{gh_1} + \frac{gh_0}{4\dot{\xi}}\left(1 + \sqrt{1 + \frac{8\dot{\xi}^2}{gh_0}}\right) - \left(2gh_0\sqrt{1 + \frac{8\dot{\xi}^2}{gh_0}} - 2gh_0\right)^{\frac{1}{2}}. \tag{5}$$

The derivation of this analytical solution can be found in some references, for example, Stoker [19, 20] and Mungkasi and Roberts [13].

3.3. Periodic wave on a sloping beach in one dimension

Consider a sloping beach with a positive constant slope. When the water is unperturbed, we have the dimensional setting: (i). the origin of the spatial domain is on the water surface, (ii). the horizontal distance of the water surface between the origin and the shoreline is L , (iii). the vertical distance (water depth or height) between the origin and the topography is h_0 .

One available analytical solution is as follows. The stage is

$$w = -\frac{1}{2}u^2 + \mathcal{A}J_0\left(\frac{4\pi\sqrt{w+1-x}}{T}\right)\cos\left(\frac{2\pi\,(u+t)}{T}\right), \tag{6}$$

and the velocity is

$$u = -\frac{\mathcal{A}J_1\left(\frac{4\pi\sqrt{w+1-x}}{T}\right)}{\sqrt{w+1-x}}\sin\left(\frac{2\pi\,(u+t)}{T}\right). \tag{7}$$

Note that this analytical solution is dimensionless, where the original dimensional quantities have been scaled [2, 9, 14]. The scalings are as follow:

- horizontal distances are scaled by L,
- vertical distances by h_0,
- time by $L/\sqrt{gh_0}$, and
- velocity by $\sqrt{gh_0}$.

3.4. Periodic wave on a paraboloid channel in two dimensions
Now we consider a two-dimensional domain. We define parameters $D_0 = 1000$, $L = 2500$, $R_0 = 2000$ such that

$$\begin{aligned} A &= \frac{L^4 - R_0^4}{L^4 + R_0^4}, && (8)\\ \omega &= \frac{2}{L\sqrt{2gD_0}}, && (9)\\ T &= \frac{2\pi}{\omega} && (10)\end{aligned}$$

are the amplitude of oscillation at the origin, the angular frequency, and the period of oscillation. Furthermore, the topography is given by

$$z(x,y) = -D_0\left(1 - \frac{r^2}{L^2}\right), \tag{11}$$

where

$$r = \sqrt{x^2+y^2}. \tag{12}$$

One available analytical solution is as follows. The water height is

$$h(x,y) = D_0\left[\frac{\sqrt{1-A^2}}{1-A\cos(\omega t)} - 1 - \frac{r^2}{L^2}\left(\frac{1-A^2}{[1-A\cos(\omega t)]^2} - 1\right)\right]. \tag{13}$$

In polar coordinates (r,θ), the velocity u_r in r-direction and the velocity u_θ in θ-direction are given by

$$u_r = \frac{\omega r A\sin(\omega\theta)}{2\left[1-A\cos(\omega\theta)\right]}, \tag{14}$$

$$u_\theta = 0. \tag{15}$$

This analytical solution was derived by Thacker [21] and implemented by Yoon and Cho [24].

4. Computational experiments on problems in two-dimensional settings

In this section, we present our computational results. The results regard validations of ANUGA using exact solutions to shallow water flow (wave) problems. Note that we use the four analytical solutions presented in the previous section as the benchmarks. Some of those analytical solutions are for one-dimensional problems originally, but they can also be used to test two-dimensional numerical methods.

The numerical setting is as follows. The second order method is used. The spatial domain is discretised using the `rectangular_cross` function available in ANUGA. Quantities are measured in SI units. The acceleration due to the gravity is taken as $g = 9.81$. The friction term is set to be zero.

Four computational experiments are performed. Results for a planar dam break involving a dry area are presented in Subsection 4.1. In Subsection 4.2, results for a planar dam break involving a shock wave are provided. Subsection 4.3 presents results for a periodic wave on a sloping beach. Finally, results for a periodic wave on a paraboloid channel are summarised in Subsection 4.4.

4.1. Planar dam break involving a dry area

We consider the spatial domain $\{(x, y)|x \in [-60, 60], y \in [-10, 10]\}$. It is descretised into 100 by 20 rectangular-crosses, in which each rectangular cross has four uniform triangles. Therefore, we have 8000 triangles as the discretisation of the spatial domain. The dam wall is at $x = 0$. Still water of height 10 is on the left of the dam wall and dry area is on the right of the dam wall.

The numerical solutions of the stage are shown in Figures 1, 3, and 5 for time $t = 0, 1, 2$ respectively. The corresponding x-momenta are depicted in Figures 2, 4, and 6 respectively. From these figures, we see that the numerical solutions match well with the analytical solution. However, a small inaccuracy around wet/dry interface occurs.

4.2. Planar dam break involving a shock wave

Recall the spatial domain and its discretisation used in the previous subsection, but now with a different initial condition. Taking the initial condition as still water of height 10 on the left of the dam wall and of height 1 on the right of the dam wall, we have a shock wave which involves in the solution after dam wall is removed.

The numerical solutions of the stage are shown in Figures 7, 9, and 11 for time $t = 0, 1, 2$ respectively. The corresponding x-velocities are depicted in Figures 8, 10, and 12 respectively. From these figures, the numerical solutions match well with the analytical solution. In addition, the shock wave is resolved accurately. It is interesting to see that diffusion is more visible around the rarefaction than around the shock. It suggests that the limiter implemented in ANUGA works very well for the second order method that we test in resolving the shock.

4.3. Periodic wave on a sloping beach

We consider the spatial domain $\{(x, y)|x \in [0, 55000], y \in [0, 500]\}$. It is descretised into 550 by 5 rectangular-crosses, in which each rectangular cross has four uniform triangles. Therefore, we have 11000 triangles as the discretisation of the spatial domain.

We set some required parameters. The dimensional parameters are as follows:

- the reference length is $L = 5 \cdot 10^4$,
- the reference depth is $h_0 = 5 \cdot 10^2$,
- the period of oscillation is $T_p = 9 \cdot 10^2$, and
- the amplitude at origin is $a = 1.0$.

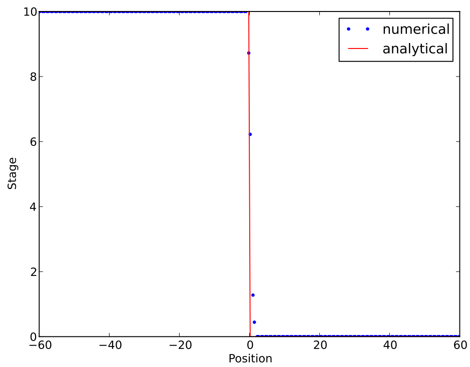

Figure 1. Stage of a cross section of the planar dam break involving a dry area at $t = 0$.

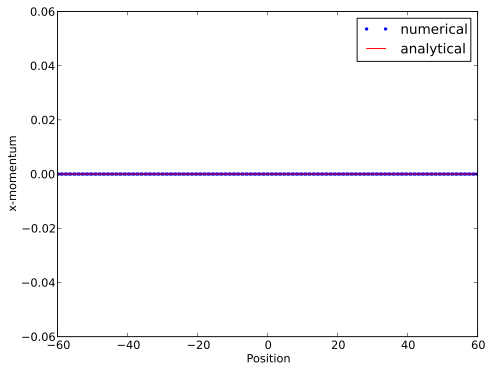

Figure 2. x-momentum of a cross section of the planar dam break involving a dry area at $t = 0$.

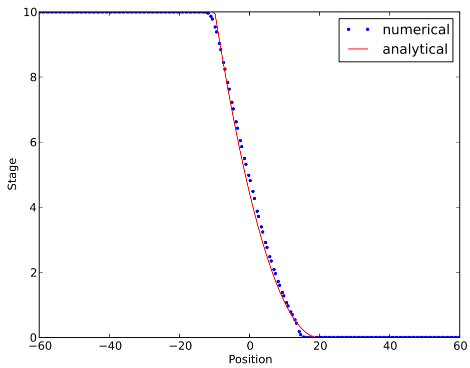

Figure 3. Stage of a cross section of the planar dam break involving a dry area at $t = 1$.

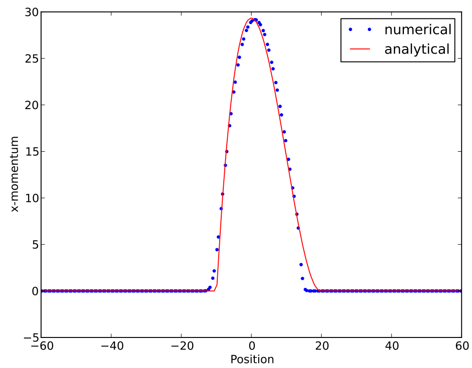

Figure 4. x-momentum of a cross section of the planar dam break involving a dry area at $t = 1$.

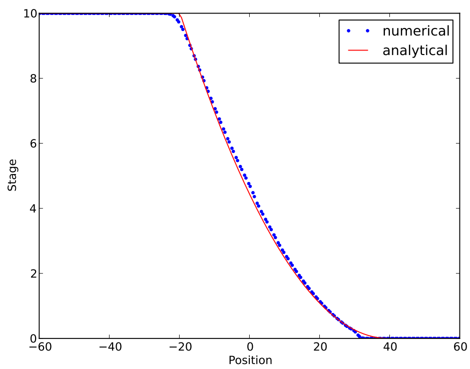

Figure 5. Stage of a cross section of the planar dam break involving a dry area at $t = 2$.

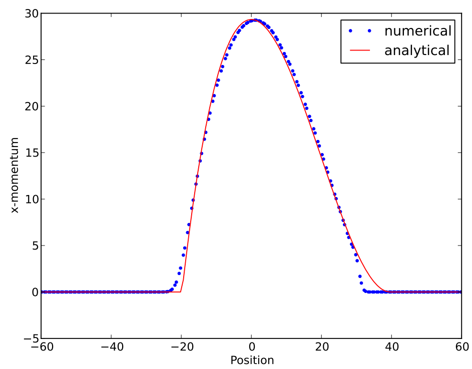

Figure 6. x-momentum of a cross section of the planar dam break involving a dry area at $t = 2$.

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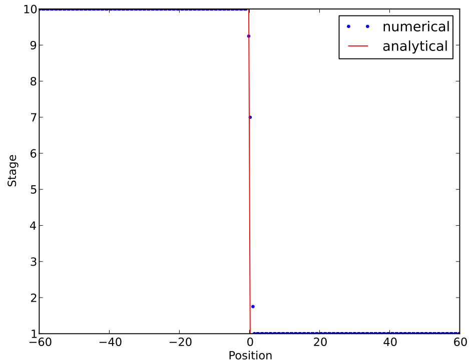

Figure 7. Stage of a cross section of the planar dam break involving a shock wave at $t = 0$.

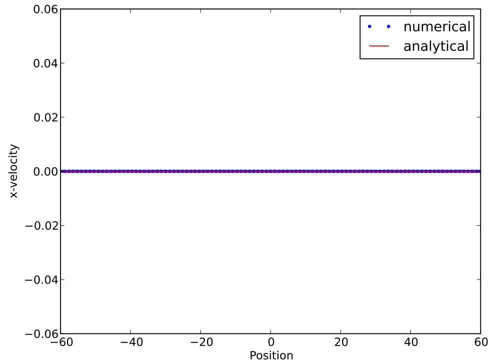

Figure 8. x-velocity of a cross section of the planar dam break involving a shock wave at $t = 0$.

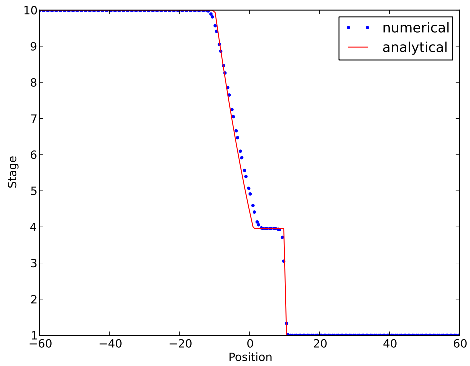

Figure 9. Stage of a cross section of the planar dam break involving a shock wave at $t = 1$.

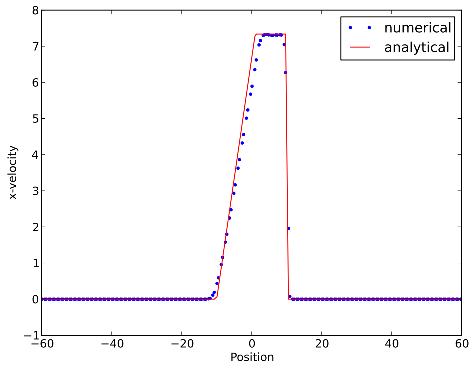

Figure 10. x-velocity of a cross section of the planar dam break involving a shock wave at $t = 1$.

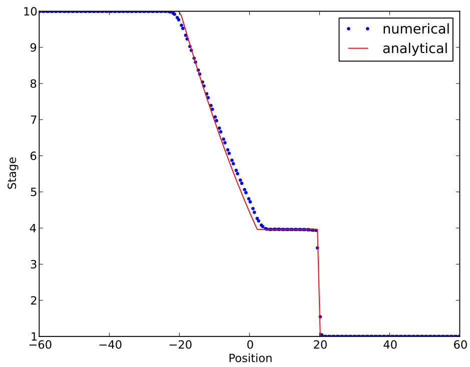

Figure 11. Stage of a cross section of the planar dam break involving a shock wave at $t = 2$.

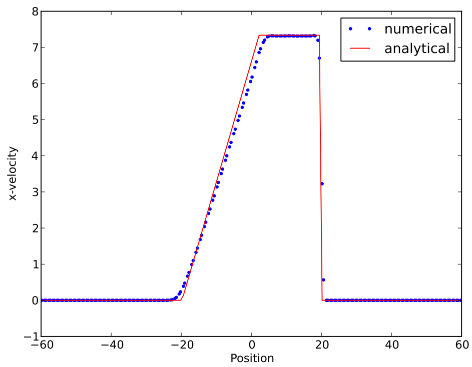

Figure 12. x-velocity of a cross section of the planar dam break involving a shock wave at $t = 2$.

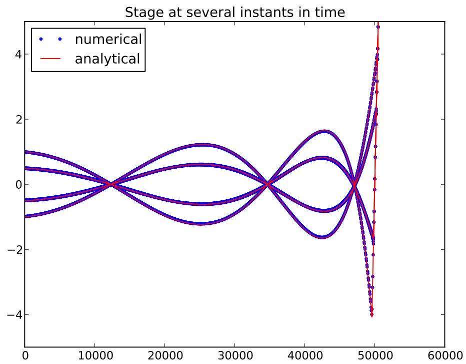

Figure 13. Stage of a cross section of the Carrier-Greenspan periodic wave at several instants in time ($t = kT_p/6$ where $k = 55, ..., 60$).

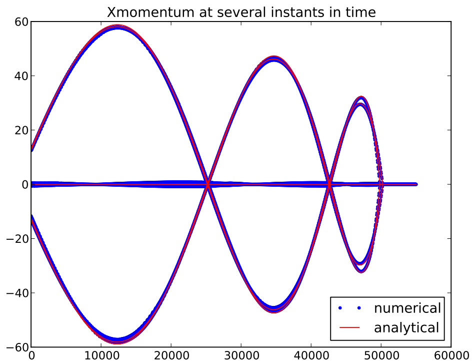

Figure 14. x-momentum of a cross section of the Carrier-Greenspan periodic wave at several instants in time ($t = kT_p/6$ where $k = 55, ..., 60$).

For the periodic wave on a sloping beach, we use the notation T_p for the dimensional period of oscillation and the notation T for the dimensionless one. The dimensionless parameters are found by some scalings as described in Subsection 3.3. We set the following dimensionless parameters:

- perturbation at origin
$$\epsilon = \frac{a}{h_0}\,, \tag{16}$$
- period of oscillation
$$T = \frac{T_p\sqrt{gh_0}}{L}\,, \tag{17}$$
- amplitude at shoreline
$$A = \frac{\epsilon}{J_0(4\pi/T)}\,, \tag{18}$$

where J_0 is the Bessel function of the first kind of order 0. See our previous work [14] for more detailed dimensional and dimensionless settings.

The numerical solutions for the stage and x-momentum are shown in Figures 13 and 14 respectively at time $t = kT_p/6$ where $k = 55, ..., 60$. These numerical solutions are found by considering the initial condition given by the analytical solution set up at time $t = 0$. Based on these results, ANUGA can handle long waves accurately. The errors for the stage and momentum are indeed seen to be negligible.

4.4. Periodic wave on a paraboloid channel

Now we consider the spatial domain $\{(x, y)|x \in [-4000, 4000]\,, y \in [-4000, 4000]\}$. It is descretised into 100 by 100 rectangular-crosses, in which each rectangular cross has four uniform triangles. Therefore, we have $4 \cdot 10^5$ triangles as the discretisation of the spatial domain.

The initial condition is given by the analytical solution set up at time $t = 0$. The numerical solution of the stage at time $t = 3.75T$ is shown in Figure 15. The stage corresponding to the origin of the spatial domain with respect to time is depicted in Figure 16. The exact solution in this test is much harder to resolve than the previous test. The difficulty occurs due to large vertical variations of the water. The vertical variations are indeed much larger than the previous test. Even though the results shown in Figure 15 lead to small error, as the time evolves, the error grows as illustrated in Figure 16. The oscillation is damped gradually. This means that

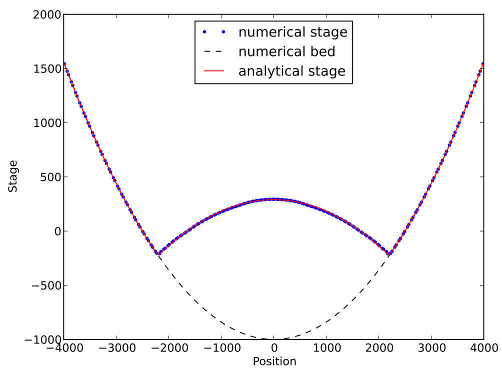

Figure 15. Stage of a cross section of the water oscillation on a paraboloid channel at time $t = 3.75T$.

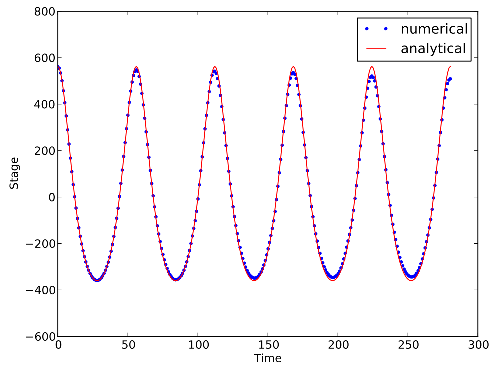

Figure 16. Stage corresponding to the origin of the spatial domain for time $t = 0, ..., 5T$.

the physical energy of this smooth flow decreases over time. One way to reduce the damping or the loss of the physical energy is by running the code with a large number of rectangular crosses (hence a large number of triangles for the spatial domain discretisation), but this leads to an expensive computation.

5. Conclusions

ANUGA has been found to be a stable and robust software to simulate shallow water flows or waves. It can handle wetting and drying process and solve problems with shock waves accurately. For a very long running time, however, it seems that there exists a loss of physical energy, even when the flow is smooth and frictionless. A possible future development of ANUGA is to minimise this loss of physical energy of a smooth and frictionless flow with a reasonably cheap computation.

Acknowledgments

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