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Modeling and Analysis of Hexapod Robot Motion with Two Motors Using Max-Plus Algebra

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Abstract. This research will model and analyze the six-legged robot foot movement using two motors. The robot used is using the Lego Mainstroms EV3 robot. Searching for eigenvalues and vectors will use the MATLAB and Scilab programs. From the value and eigenvectors obtained, analysis will be performed for the movement of the six-legged robot. Tripod gait means that 3 legs that are in the same group perform the same movements both raised and tread and alternately with other groups of legs. (1) The main components making up the six-legged robot are the robot body, EV3 Brick (robot controller), two motors (moving the robot's legs) and connectors, (2) modelling when all feet move in iteration is enough to model one foot in each group of feet and modelling footwork requires a matrix measuring 12×12 to find out the total time needed for the foot to make movements in one cycle, (3) the eigenvalues obtained for all groups of legs are the same so that it can be known when each foot starts to make periodic movements, that is, when the time needed for each leg to lift and touch the ground.

INTRODUCTION

The current technology of robots has progressed rapidly along with the advancement of existing technologies. The development of robots is not only in the sophistication of mechanical design, but also the control system using computerized system. Along with the development of robotic programming techniques, it is increasingly easier for humans to create robots that have intelligence that follows human wishes and abilities. The running Robot that has been created to date has two, four, six or even eight feet [1].

Research on current running robots is driven by two application areas: two-legged humanoid robots for service or regional operations in the surrounding environment and four-legged, six-and eight-legged robots that are expected to be a search engine or lifesaver in some very difficult locations faced by humans. In this scenario, six-legged robots are more stable than four-legged robots and are more complicated than eight-legged robots, and subsequently various types of six-legged robots have been developed [2].

One of the many types of robots that are also developed is the six-legged Robot (Hexapod). The Robot is based on the movement of the tripod gait, consisting of both the front and back legs and the middle leg on the other [3]. The six-legged Robot (Hexapod) has a cycle of leg step movements using a tripod of gait that will continue to move in accordance with the rules that three feet must tread on the floor surface. In this study, it will be discussed the mechanical construction of modeling motion, control, and optimization on the six-legged robot using the two drive motors using Max-Plus algebra.

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BASIC THEORY

Max-Plus Algebra

In [4] algebra Max-Plus is a algebraic structure where the set of all real numbers $\mathbb{R} \cup \{-\infty\}$ equipped with Max and plus operations [5]. The maximum operation is $\oplus$ and summation operations is $\otimes$, so that $(\mathbb{R} \cup \{-\infty\}, \oplus, \otimes)$ denoted by $\mathbb{R}_{max}$ dan $-\infty$ denoted by ε. The ε element is a neutral element of operation $\oplus$ and 0 is an identity element to the Operation $\otimes$.

Given $\mathbb{R}_{\varepsilon}$: = ℝ □{□□} where ℝ is the set of all real numbers and $\varepsilon := -\infty$. In ℝ the following operations are defined:

$$\forall_{a,b} \in \mathbb{R}_{\varepsilon}, a \oplus b := \max(a, b) \text{ dan } a \otimes b := a + b$$

In this study, we will use matrix calculations on max-plus algebra. Addition and product of the max-plus matrix $A, B \in \mathbb{R}_{max}^{n \times m}$ defined by [6]:

$$[A \oplus B]_{ij} = a_{ij} \oplus b_{ij} \equiv \max(a_{ij}, b_{ij}) \quad (1)$$

$$[A \otimes B]_{ik} = \bigoplus_{j=1}^{m} a_{ij} \otimes b_{jk} \equiv \max_{j=1,\ldots,m}(a_{ij} + b_{jk}) \quad (2)$$

$$D^{\otimes k} \equiv \underbrace{D \otimes D \otimes \ldots \otimes D}_{k\ kali}, \quad k \in \mathbb{N}\{0\} \quad (3)$$

Where $A, B \in \mathbb{R}_{max}^{n \times m}$, $D \in \mathbb{R}_{max}^{n \times n}$. In this case the zero matrix ε and the identity matrix E are defined as follows:

$$[\varepsilon]_{ij} := \varepsilon$$

$$[E]_{ij} := \begin{cases} e & if\ i = j \\ \varepsilon & others \end{cases}$$

In this paper we will use Max Plus's iterative (SPLI) linear equation system. SPLI max plus has a general form:

$$x = A \otimes x \oplus b \quad (4)$$

With $A \in \mathbb{R}_{max}^{n \times n}$ and $x, b \in \mathbb{R}_{max}^{n \times 1}$. Futhermore given [6]:

$$A^* \equiv \bigoplus_{k=0}^{\infty} A^{\otimes k}$$

If A^* has $\mathbb{R}_{max}^{n \times n}$ then the solution of system (4) is

$$x = A^* \otimes b \quad (5)$$

In this discussion, eigenvalues (λ) and eigenvectors (v) will be used for the analysis process and define:

$$A \otimes v = \lambda \otimes v, \quad v \neq \varepsilon$$

For the linear max system plus the eigenvalue of the matrix system represents the total time cycle of the robot leg. Meanwhile, the eigenvector signifies that the leg starts periodic movements in an iteration.

Basic Legged Robot Motion

In this section we will model robot-legged movements. From the gait $t_i(k)$ = time of leg stepping, $l_i(k)$ = time of leg lifted off the ground. The footing time $t_i(k)$ is the time the leg starts to lift from the ground $l_i(k)$ plus the time when the leg floats τ_f or can be written in the following equation:

$$t_i(k) = l_i(k) + \tau_f \quad (6)$$

Using same analogy, can be obtained for the time the feet are lifted from the ground as follows:

$$l_i(k) = t_i(k-1) + \tau_g \quad (7)$$

With $t_i(k-1)$ is the time to tread on the previous event and τ_g is the time of standing feet. From equations (6) and (7), we can use to synchronize or synchronize two different legs, i.e. for example i and j. When we start stepping, a process occurs where i can only lift τ_Δ after j leg touches the ground. τ_Δ is the time when i and j feet both touch the ground. From the explanation above we can get a relationship that is:

$$l_i(k) = \max(t_i(k-1) + \tau_g, t_j(k-1) + \tau_\Delta) = [\tau_g \tau_\Delta] \otimes \begin{bmatrix} t_i(k-1) \\ t_j(k-1) \end{bmatrix} \quad (8)$$

From Equation (8) we know legs i remain a little τ_g seconds in an upright position and will only be raised less than τ_Δ seconds after leg j lands. When the two situations above are fulfilled, the leg lifting event is in progress. For this reason, a person can be efficient to demonstrate synchronous movements at a certain time.

For n-legged robots, the position vector when the leg is tread and hovering is discrete is defined as follows:

$$x(k) = [\underbrace{t_1(k) \ldots t_n(k)}_{\boldsymbol{t(k)}} \quad \underbrace{l_1(k) \ldots l_n(k)}_{\boldsymbol{l(k)}}]^T$$

Equations (6), (7) can be written in the form of a two-dimensional equation system as follows:

$$\begin{bmatrix} t(k) \\ l(k) \end{bmatrix} = \begin{bmatrix} \varepsilon & \tau_f \otimes E \\ \varepsilon & \varepsilon \end{bmatrix} \otimes \begin{bmatrix} t(k) \\ l(k) \end{bmatrix} \oplus \begin{bmatrix} \varepsilon & \varepsilon \\ \tau_g \otimes E & \varepsilon \end{bmatrix} \otimes \begin{bmatrix} t(k-1) \\ l(k-1) \end{bmatrix} \quad (9)$$

Based on the system of equation (9) all feet follow the same rhythm i.e moving with the same period of at least $\tau_f + \tau_g$ seconds. It is assumed that all feet are synchronized when the relationship between the time when the leg is lifted and the time when the other leg touches the ground is reached. This assumption is expressed by adding the P and Q matrices to equation (9), resulting in a synchronized system as follows:

$$\begin{bmatrix} t(k) \\ l(k) \end{bmatrix} = \begin{bmatrix} \varepsilon & \tau_f \otimes E \\ P & \varepsilon \end{bmatrix} \otimes \begin{bmatrix} t(k) \\ l(k) \end{bmatrix} \oplus \begin{bmatrix} \varepsilon & \varepsilon \\ \tau_g \otimes E \oplus Q & \varepsilon \end{bmatrix} \otimes \begin{bmatrix} t(k-1) \\ l(k-1) \end{bmatrix} \quad (10)$$

By example $A_0 = \begin{bmatrix} \varepsilon & \tau_f \otimes E \\ P & \varepsilon \end{bmatrix}$ *dan* $A_1 = \begin{bmatrix} \varepsilon & \varepsilon \\ \tau_g \otimes E \oplus Q & \varepsilon \end{bmatrix}$

Then the equation (10) can be written as:

$$x(k) = A_0 \otimes x(k) \oplus A_1 \otimes x(k-1) \quad (11)$$

Based on (4), (5) then the solution for equation (11) is

$$x(k) = A_0^* \otimes A_1 \otimes x(k-1) \quad (12)$$

$$x(k) = A \otimes x(k-1) \quad (13)$$

The P and Q matrices are used to encode all the feet in each set of feet that make the same movements. The set of legs that performs the same movement is notated as follows:

$$\{L_1\} \prec \{L_2\} \prec \cdots \prec \{L_m\} \quad (14)$$

Notation (14) represents that the set of L_i legs will be lifted and landed simultaneously preceding the appointment of the set of L_{i+1}. legs. The L_{i+1} leg set will be lifted after the L_i leg set has touched and has been on the ground for at least τ_Δ. For example, one of the tripod gait patterns on a hexapod robot if written using notation (14) is {1,4,5} ≺ {2,3,6}. This means that the robot's legs numbered 2,3 and 6 are allowed to be lifted off the ground after feet 1, 4 and 5 land and have been on the ground for at least τ_Δ. By using notation (14) the elements in the P and Q matrices in (10) can be defined. Given the pattern of leg movements defined in (14) with $\forall_j \in \{1, \ldots, m-1\}, \forall_p \in L_{j+1}, \forall_q \in L_j$.

$$[P]_{p,q} = \tau_\Delta \quad (15)$$

And $\forall_p \in L_1, \forall_q \in L_m$

$$[Q]_{p,q} = \tau_\Delta \quad (16)$$

Where for the other elements of the P and Q matrices are ε.

RESULT AND DISCUSSION

Six-Legged Motor Robot Construction

In this study, using the Lego Mainstroms EV3 robot. The hexapod robot used for the simulation of leg movements is the robot in Figure 1.

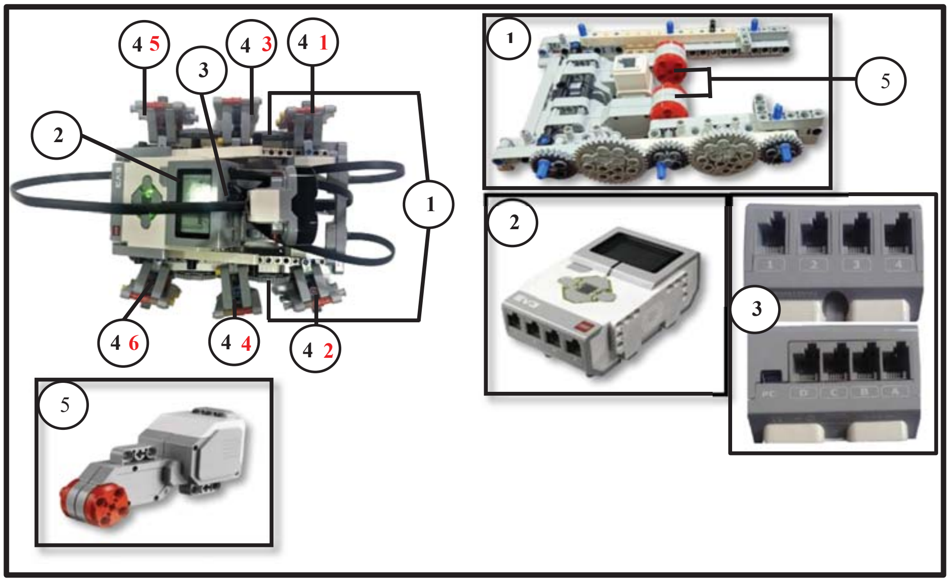

FIGURE 1. Hexapod Robot with two motors and their components

Caption of Figure 1:

1. Robot body construction. In order to move the Hexapod robot's legs, all robot legs are connected using gear and other components.
2. Brick EV3. Functioning as a controller (brain and power source for EV3 robots). Programs that have been created can be uploaded to EV3 Brick to be compiled or run.*Brick EV3*. Berfungsi sebagai pengendali (otak dan sumber tenaga untuk robot EV3). Program yang sudah dibuat dapat di upload ke EV3 Brick untuk di *compile* atau dijalankan.
3. Port Brick. The picture above serves as the output port used to connect the motor to the EV3 Brick. Whereas the bottom picture functions as an input port that is used to connect the sensor with Brick.
4. Hexapod robot leg. Hexapod robot legs are constructed using a tripod gait movement pattern that is alternately a group of legs totaling three to make the same movement (tread or float). The feet are labeled with numbers 1-6 (red) to make it easier to mention the tripod gait formation.
5. Motor. In the Lego Mindstroms EV3 robot, the motor used is a DC servo motor equipped with an encoder that functions as feedback, so that the control center can provide a current that matches the load on the motor. The maximum angular speed of the motor is one rotation per second. The motor functions to move the robot parts, such as turning a wheel or becoming a joint. The motors used are of two motors. Where each motor moves the 3 legs of the Hexapod robot.

Modelling Six-Legged Two-Robot Robot Motion

In previous studies, there were 3 types of ways for six-legged robots to move, namely Quintaple gait, Quadruped gait and Tripod gait [6]. Quintaple gait is alternately from leg number 1 to leg number 6 raised and tread. Quadruped gait is alternately a pair of legs that are adjusted up and down. Tripod gait that is alternately 3 legs that are in the same group do the same movements both raised and tread and alternately with other groups of legs. Examples of Quintaple Gait, Quadruped Gait and Tripod Gait in succession are $(\{1\} \prec \{2\} \prec \{3\} \prec \{4\} \prec \{5\} \prec \{6\}), (\{1,4\} \prec \{3,6\} \prec \{2,5\})$, and $(\{1,4,5\} \prec \{2,3,6\})$. In this paper the six-legged robot uses two motors. So that each motor moves three legs and each leg is connected using a gear. Therefore, the type of stepping robot hexapod that fits the two motor drives is a tripod gait. All tripod gait formations can be seen in the table below:

Table 1. Tripod Gait Formation on the Hexapod robot

$\{1,2,3\} \prec \{4,5,6\}$	$\{1,3,5\} \prec \{2,4,6\}$
$\{1,2,4\} \prec \{3,5,6\}$	$\{1,3,6\} \prec \{2,4,5\}$
$\{1,2,5\} \prec \{3,4,6\}$	$\{1,4,5\} \prec \{2,3,6\}$
$\{1,2,6\} \prec \{3,4,5\}$	$\{1,4,6\} \prec \{2,3,5\}$
$\{1,3,4\} \prec \{2,5,6\}$	$\{1,5,6\} \prec \{2,3,4\}$

There are 10 formations or movement patterns of the tripod gait type on the hexapod robot. In the first part we will model the time that the movement of each leg when it starts and starts to float on each iteration (k). Using equation (6), (8) we will use one of the tripod gait patterns namely $\{1,4,5\} \prec \{2,3,6\}$ to model the time for each leg. Tripod gait with the pattern $\{1,4,5\} \prec \{2,3,6\}$ means that alternately 3 legs namely 1, 4 and 5 and feet 2, 3 and 6 make the same movements both raised and tread, so based on equation (6), (8) obtained:

Table 2. Modeling the time of leg movement with the pattern $\{1,4,5\} \prec \{2,3,6\}$ in iteration k

$t_1(k+1)$	$= t_4(k+1) = t_5(k+1)$	$= \max\left(t_1(k) + \tau_g + \tau_f, t_2(k) + \tau_\Delta + \tau_f\right)$ $= \max\left(t_1(k) + \tau_g + \tau_f, t_3(k) + \tau_\Delta + \tau_f\right)$ $= \max\left(t_1(k) + \tau_g + \tau_f, t_6(k) + \tau_\Delta + \tau_f\right)$
$l_1(k+1)$	$= l_4(k+1) = l_5(k+1)$	$= \max\left(t_1(k) + \tau_g, t_2(k) + \tau_\Delta\right)$ $= \max\left(t_1(k) + \tau_g, t_3(k) + \tau_\Delta\right)$ $= \max\left(t_1(k) + \tau_g, t_6(k) + \tau_\Delta\right)$
$t_2(k+1)$	$= t_3(k+1) = t_6(k+1)$	$= \max\left(t_2(k) + \tau_g + \tau_f, t_1(k+1) + \tau_\Delta + \tau_f\right)$ $= \max\left(t_2(k) + \tau_g + \tau_f, t_4(k+1) + \tau_\Delta + \tau_f\right)$ $= \max\left(t_2(k) + \tau_g + \tau_f, t_5(k+1) + \tau_\Delta + \tau_f\right)$
$l_2(k+1)$	$= l_3(k+1) = l_6(k+1)$	$= \max\left(t_2(k) + \tau_g, t_1(k+1) + \tau_\Delta\right)$ $= \max\left(t_2(k) + \tau_g, t_4(k+1) + \tau_\Delta\right) =$ $\max\left(t_2(k) + \tau_g, t_5(k+1) + \tau_\Delta\right)$

In the next section we will describe the process of making matrix A in equation (13). There are ten possible patterns or tripod gait formations. But to make the A matrix we only show it using the tripod gait pattern $\{1,4,5\} \prec \{2,3,6\}$, because the same method also applies to other patterns. The parameters used to construct the matrix A are $\tau_f = 0{,}5\ s, \tau_g = 0{,}7\ s,$ and $\tau_\Delta = 0.1\ s$

We begin by making a matrix P, $\tau_f \otimes E, \tau_g \otimes E$ and Q in equation (10).

$$P = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 0.1 & \varepsilon & \varepsilon & 0.1 & 0.1 & \varepsilon \\ 0.1 & \varepsilon & \varepsilon & 0.1 & 0.1 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 0.1 & \varepsilon & \varepsilon & 0.1 & 0.1 & \varepsilon \end{bmatrix}, \tau_f \otimes E = \begin{bmatrix} 0.5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 0.5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 0.5 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 0.5 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0.5 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0.5 \end{bmatrix}$$

$$t_g \otimes E = \begin{bmatrix} 0.7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 0.7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 0.7 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 0.7 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0.7 & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0.7 \end{bmatrix}, P = \begin{bmatrix} \varepsilon & 0.1 & 0.1 & \varepsilon & \varepsilon & 0.1 \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 0.1 & 0.1 & \varepsilon & \varepsilon & 0.1 \\ \varepsilon & 0.1 & 0.1 & \varepsilon & \varepsilon & 0.1 \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$

Next we construct the matrix A_0 and A_1 in equation (11), and use them to construct matrix ${A_0}^*$ and A. Using the *MATLAB* [5] matrix program ${A_0}^*$ in equation (12) can be obtained as follows:

$$
{A_0}^* = \begin{bmatrix}
0 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0.5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
0.6 & 0 & \varepsilon & 0.6 & 0.6 & \varepsilon & 1.1 & 0.5 & \varepsilon & 1.1 & 1.1 & \varepsilon \\
0.6 & \varepsilon & 0 & 0.6 & 0.6 & \varepsilon & 1.1 & \varepsilon & 0.5 & 1.1 & 1.1 & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & 0 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0.5 & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & 0 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0.5 & \varepsilon \\
0.6 & \varepsilon & \varepsilon & 0.6 & 0.6 & 0 & 1.1 & \varepsilon & \varepsilon & 1.1 & 1.1 & 0.5 \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
0.1 & \varepsilon & \varepsilon & 0.1 & 0.1 & \varepsilon & 0.6 & 0 & \varepsilon & 0.6 & 0.6 & \varepsilon \\
0.1 & \varepsilon & \varepsilon & 0.1 & 0.1 & \varepsilon & 0.6 & \varepsilon & 0 & 0.6 & 0.6 & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0 & \varepsilon & \varepsilon \\
\varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0 & \varepsilon \\
0.1 & \varepsilon & \varepsilon & 0.1 & 0.1 & \varepsilon & 0.6 & \varepsilon & \varepsilon & 0.6 & 0.6 & 0
\end{bmatrix}
$$

The final step will be made A matrix as in equation (13), where $A = {A_0}^* \otimes A_1$

$$
A = {A_0}^* \otimes A_1 = \begin{bmatrix}
1.2 & 0.6 & 0.6 & \varepsilon & \varepsilon & 0.6 & 0.5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
1.8 & 1.2 & 1.2 & 1.8 & 1.8 & 1.2 & 1.1 & 0.5 & \varepsilon & 1.1 & 1.1 & \varepsilon \\
1.8 & 1.2 & 1.2 & 1.8 & 1.8 & 1.2 & 1.1 & \varepsilon & 0.5 & 1.1 & 1.1 & \varepsilon \\
\varepsilon & 0.6 & 0.6 & 1.2 & \varepsilon & 0.6 & \varepsilon & \varepsilon & \varepsilon & 0.5 & \varepsilon & \varepsilon \\
\varepsilon & 0.6 & 0.6 & \varepsilon & 1.2 & 0.6 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0.5 & \varepsilon \\
1.8 & 1.2 & 1.2 & 1.8 & 1.8 & 1.2 & 1.1 & \varepsilon & \varepsilon & 1.1 & 1.1 & 0.5 \\
0.7 & 0.1 & 0.1 & \varepsilon & \varepsilon & 0.1 & 0 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
1.3 & 0.7 & 0.7 & 1.3 & 1.3 & 0.7 & 0.6 & 0 & \varepsilon & 0.6 & 0.6 & \varepsilon \\
1.3 & 0.7 & 0.7 & 1.3 & 1.3 & 0.7 & 0.6 & \varepsilon & 0 & 0.6 & 0.6 & \varepsilon \\
\varepsilon & 0.1 & 0.1 & 0.7 & \varepsilon & 0.1 & \varepsilon & \varepsilon & \varepsilon & 0 & \varepsilon & \varepsilon \\
\varepsilon & 0.1 & 0.1 & \varepsilon & 0.7 & 0.1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 0 & \varepsilon \\
1.3 & 0.7 & 0.7 & 1.3 & 1.3 & 0.7 & 0.6 & \varepsilon & \varepsilon & 0.6 & 0.6 & 0
\end{bmatrix}
$$

The resulting matrix A is used to determine the eigenvalues and eigenvectors to be analyzed in section 3.3.

Analysis of Six-Legged Robot Motion with Two Motors

In section 3.2 we have determined the matrix A which is represented by the tripod gait pattern $\{1,4,5\} \prec \{2,3,6\}$ for all possible patterns of leg movement. In this section we will determine the eigenvalues and eigenvectors for each of the tripod gait formations or patterns in Table 1. Determination of eigenvalues and eigenvectors for all matrix A using the *Scilab* program [7].

Based on calculations using the *Scilab* program, the eigenvalues of the A matrix for all tripod gait formations are the same, namely $\lambda = 1.2$. This means that the time needed for a leg (on all tripod gait formations) to carry out one cycle (from being lifted up from the ground to returning to the ground) is 1.2 seconds.

Based on the results of calculations using the eigenvector *Scilab* program generated from each matrix A, it can be seen in Table 3.

Table 3. Eigenvectors of all Tripod Gait Robot Hexapod formations

Formasi Tripod Gait	Vektor Eigen (V_o)
$\{1,2,3\} \prec \{4,5,6\}$	$[2.4\ \ 2.4\ \ 2.4\ \ 3.0\ \ 3.0\ \ 3.0\ \ 1.9\ \ 1.9\ \ 1.9\ \ 2.5\ \ 2.5\ \ 2.5]^T$
$\{1,2,4\} \prec \{3,5,6\}$	$[2.4\ \ 2.4\ \ 3.0\ \ 2.4\ \ 3.0\ \ 3.0\ \ 1.9\ \ 1.9\ \ 2.5\ \ 1.9\ \ 2.5\ \ 2.5]^T$
$\{1,2,5\} \prec \{3,4,6\}$	$[2.4\ \ 2.4\ \ 3.0\ \ 3.0\ \ 2.4\ \ 3.0\ \ 1.9\ \ 1.9\ \ 2.5\ \ 2.5\ \ 1.9\ \ 2.5]^T$
$\{1,2,6\} \prec \{3,4,5\}$	$[2.4\ \ 2.4\ \ 3.0\ \ 3.0\ \ 3.0\ \ 2.4\ \ 1.9\ \ 1.9\ \ 2.5\ \ 2.5\ \ 2.5\ \ 1.9]^T$
$\{1,3,4\} \prec \{2,5,6\}$	$[2.4\ \ 3.0\ \ 2.4\ \ 2.4\ \ 3.0\ \ 3.0\ \ 1.9\ \ 2.5\ \ 1.9\ \ 1.9\ \ 2.5\ \ 2.5]^T$
$\{1,3,5\} \prec \{2,4,6\}$	$[2.4\ \ 3.0\ \ 2.4\ \ 3.0\ \ 2.4\ \ 3.0\ \ 1.9\ \ 2.5\ \ 1.9\ \ 2.5\ \ 1.9\ \ 2.5]^T$
$\{1,3,6\} \prec \{2,4,5\}$	$[2.4\ \ 3.0\ \ 2.4\ \ 3.0\ \ 3.0\ \ 2.4\ \ 1.9\ \ 2.5\ \ 1.9\ \ 2.5\ \ 2.5\ \ 1.9]^T$
$\{1,4,5\} \prec \{2,3,6\}$	$[2.4\ \ 3.0\ \ 3.0\ \ 2.4\ \ 2.4\ \ 3.0\ \ 1.9\ \ 2.5\ \ 2.5\ \ 1.9\ \ 1.9\ \ 2.5]^T$
$\{1,4,6\} \prec \{2,3,5\}$	$[2.4\ \ 3.0\ \ 3.0\ \ 2.4\ \ 3.0\ \ 2.4\ \ 1.9\ \ 2.5\ \ 2.5\ \ 1.9\ \ 2.5\ \ 1.9]^T$
$\{1,5,6\} \prec \{2,3,4\}$	$[2.4\ \ 3.0\ \ 3.0\ \ 3.0\ \ 2.4\ \ 2.4\ \ 1.9\ \ 2.5\ \ 2.5\ \ 2.5\ \ 1.9\ \ 1.9]^T$

Based on Table 3, it is known that the eigenvectors for all tripod gait formations on hexapod robots are the same size ie 12 × 1. The first 6 rows represent the time i feet tread (t_i) in sequence from $i = 1,\dots,6$ and rows 7 to 12 represent the time i feet floated (l_i)in sequence from $i = 7,\dots,12$. Based on this table it is also known that the eigenvectors for all tripod gait formations on hexapod robots are different from one another. This can be seen in the different eigenvector matrix structures. This difference is due to the elements in the eigenvectors produced in accordance with the order of the leg movements in each tripod gait formation. For example eigenvectors $[2.4 \quad 3.0 \quad 3.0 \quad 2.4 \quad 2.4 \quad 3.0 \quad 1.9 \quad 2.5 \quad 2.5 \quad 1.9 \quad 1.9 \quad 2.5]^T$ represents the time of hovering and stepping on each leg in the formation $\{1,4,5\} \prec \{2,3,6\}$. Note that the first, fourth and fifth rows have the same value of 2.4, and the sixth, tenth and eleventh rows have the same value of 1.9. This shows that time the legs 1, 4 and 5 touches the ground on an iteration is 2,4 seconds and the time to float the other three feet on an iteration is 1,9 seconds. The same explanation also applies to feet 2, 3 and 6.

The eigenvector signifies that the leg starts periodic movements in an iteration. This can be seen in Table 4 and Table 5. The formation used in both tables is $\{1,4,5\} \prec \{2,3,6\}$.

TABLE 4. Periodic iteration when $x(0) = 0$

	k					
	0	**1**	**2**	**3**	**4**	**5**
t_1	0	1.2	2.4	3.6	4.8	6.0
t_2	0	1.8	3.0	4.2	5.4	6.6
t_3	0	1.8	3.0	4.2	5.4	6.6
t_4	0	1.2	2.4	3.6	4.8	6.0
t_5	0	1.2	2.4	3.6	4.8	6.0
t_6	0	1.8	3.0	4.2	5.4	6.6
l_1	0	0.7	1.9	3.1	4.3	5.5
l_2	0	1.3	2.5	3.7	4.9	6.1
l_3	0	1.3	2.5	3.7	4.9	6.1
l_4	0	0.7	1.9	3.1	4.3	5.5
l_5	0	0.7	1.9	3.1	4.3	5.5
l_6	0	1.3	2.5	3.7	4.9	6.1

TABLE 5. Periodic iteration when $x(0) = V_o$

	k					
	0	**1**	**2**	**3**	**4**	**5**
t_1	0.5	2.4	3.6	4.8	6.0	7.2
t_2	1.1	3.0	4.2	5.4	6.6	7.8
t_3	1.1	3.0	4.2	5.4	6.6	7.8
t_4	0.5	2.4	3.6	4.8	6.0	7.2
t_5	0.5	2.4	3.6	4.8	6.0	7.2
t_6	1.1	3.0	4.2	5.4	6.6	7.8
l_1	0	1.9	3.1	4.3	5.5	6.7
l_2	0.6	2.5	3.7	4.9	6.1	7.3
l_3	0.6	2.5	3.7	4.9	6.1	7.3
l_4	0	1.9	3.1	4.3	5.5	6.7
l_5	0	1.9	3.1	4.3	5.5	6.7
l_6	0.6	2.5	3.7	4.9	6.1	7.3

Based on Table 4, it is known that in the beginning all hexapod robot legs were in the same condition namely tread. 1,4 and 5 feet start to lift from the ground at $t = 0{,}7$ seconds and begin to tread again at $t = 1{,}2$. Furthermore, at 1,3 feet 2,3 seconds, and 6 just lifted off the ground and stepped back at $t = 1{,}8$. It is clear that it takes 0,1 second for both feet to tread together and this corresponds to the τ_Δ used. In the first iteration the time (hovering and treading) needed by each leg starting from a stationary position is different. While in the second iteration the time (hovering and retracing) required by each leg increases to the same magnitude of 1,2 seconds. This is in accordance with the magnitude of λ that is 1,2. Note that in the second iteration the time required for each leg to make a move (hover or tread) is the same as the eigenvector structure. After the second iteration the movement of all legs starts periodically, increasing by λ for the next iteration. When $k = 0$ the components in table 5 have been modified to make it more realistic. So that all components do not have a negative value and at least one component is zero. Based on table 5 it is known that in the first iteration the time used is the same as the eigenvector, which means that from the beginning the feet begin to move periodically.

CONCLUSION

There are 3 types of ways the six-legged robot moves, Quintaple gait, Quadruped gait and Tripod gait. The type of stepping robot hexapod that fits the two motors is a tripod gait. There are 10 tripod gait formations or patterns on the hexapod robot that are presented in Table 1. The movement of the tripod gait means that the 3 legs that fit in the same group perform the same movements both uplifted and tread and replace with another group of legs. So, to model the time whenever the leg moves in an iteration it is enough to model one of the legs in each group of leg as in Table 2. When modeling footwork, a 12×12 matrix is required. From the matrix you can know the total time legs are needed to move in one cycle (floating and then rising) by looking at the eigenvalue. The eigenvalues obtained for all formations are the same. In addition, from this matrix, we can know at any time that the periodic compilation necessary for each leg to do the floating and tread movements is equal to the elements in the eigenvector.

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REFERENCES

1. J. Andika and K.S. Salamah, J. Teknol. Elektro, Univ. Mercu Buana 9, **83** (2018).
2. P. Yang and F. Gao, Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci. 228, **2217** (2014).
3. A. Najmurrokhman, Kusnandar, B.H. Wibowo, and N.A. Rafanca, JUMANJI (Jurnal Masy. Inform. 1, **1** (2017).
4. Kasie Geralyn Farlow, Max-Plus-Algebra, Virginia Polytechnic Institute and State University in, 2009.
5. M.A. Rudhito, ALJABAR MAX-PLUS DAN PENERAPANNYA (Program Studi Pendidikan Matematika FKIP, Universitas Sanata Dharma, Yogyakarta, 2016).
6. G.A.D. Lopes, T.J.J. Van Den Boom, B. De Schutter, and R. Babuska, Proceed- Ings 10th Int. Work. Discret. Event Syst. (WODES 2010) **392** (2010).
7. Subiono, Aljabar Min-Max Plus Dan Terapannya, 3rd ed. (FMIPA, Institut Teknologi Sepuluh Nopember, Surabaya, 2015).