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**Abstract.** We study the quark sector of the universal seesaw model with $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times U(1)$. The model incorporates the seesaw mechanism with the vector-like quarks (VLQs). The purpose of this work is to study the model with the effective theory. After integrating the heavy five VLQs, we derive the effective theory with four up-type quark and three down type quark. In this work, the FCNC of Z boson for top quark and top′ quark is derived.

## 1. Introduction

Though the standard model is a very successful theory, the origin of the flavor and mass hierarchy of quarks can be explained only by tuning the Yukawa coupling [1]. For instance up quark mass and top quark mass are respectively given as,

$$\begin{aligned} m_u = y_u \frac{v}{\sqrt{2}} < m_t = y_t \frac{v}{\sqrt{2}} \\ y_u \simeq 1.25 \times 10^{-5} y_t, \quad y_t \simeq 0.99, \end{aligned} \tag{1}$$

where we use the top quark mass from the direct measurement $m_t = 172.69(\mathrm{GeV})$ and the up quark mass from the $\overline{MS}$ scheme $m_u(2\mathrm{GeV}) = 2.16(\mathrm{MeV})$ [2]. We also use $v = 246.22(\mathrm{GeV})$ to derive Yukawa coupling of the top quark. The universal seesaw model explains the smallness of the mass of the up quark with a tiny ratio of $\mathrm{SU}(2)_R$ breaking scale and a SU(2) singlet vector-like quark (VLQ) mass $M_U$. The standard model Yukawa coupling $y_u$ is given by the seesaw like formula,

$$\begin{aligned} y_u &= y_{uL}\left(\frac{v_R}{\sqrt{2}M_U}\right)y_{uR} \simeq \left(\frac{v_R}{\sqrt{2}M_U}\right) \simeq 10^{-5}, \\ y_{uL} &\simeq y_{uR} \simeq O(1), \end{aligned} \tag{2}$$

where $y_{uL}$ and $y_{uR}$ are mixing type Yukawa couplings between the ordinary quark and VLQ. We study the quark sector of the universal seesaw model with $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)$ [3, 4, 5, 6]. Our aim is to construct the effective theory obtained after integrating VLQs with their masses larger than $\mathrm{SU}(2)_R$ breaking scale. Then, we study the flavor structure of the effective theory. The same model was investigated with the full theory [7, 8, 9].

## 2. The Lagrangian of the model

We first present the quark and gauge sector. The two SU(2) doublet Higgs fields are introduced. $\phi_L$ stands for $\mathrm{SU}(2)_L$ doublet and $\phi_R$ for $\mathrm{SU}(2)_R$. Their vacuum expectation values (vevs) $v_L$ and $v_R$ repectively break $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$. From the Higgs potential study, one can show that the vevs satisfy $v_R \gg v_L$. The ordinary quarks $\psi_L(\psi_R)$ are also $\mathrm{SU}(2)_L$ ($\mathrm{SU}(2)_R$) doublets. The six VLQs $(U_1 \sim U_3, D_1 \sim D_3)$ are also introduced and the Lagrangian is,

$$\begin{aligned} \mathcal{L}_{Doublets} &= \sum_{i=1}^{3}\overline{\psi_{Li}}\left(i\partial\!\!\!/ - g_L W\!\!\!/_L - g_1\frac{1}{6}B\!\!\!/_1\right)\psi_{Li} + (L \to R), \\ \mathcal{L}_{VLQ} &= \sum_{I=1}^{3}\overline{U_I}\left(i\partial\!\!\!/ - g_1 B\!\!\!/_1\frac{2}{3} - M_{U_I}\right)U_I + \sum_{I=1}^{3}\overline{D_I}\left(i\partial\!\!\!/ + g_1 B\!\!\!/_1\frac{1}{3} - M_{D_I}\right)D_I, \\ \mathcal{L}_{VLQ-Doublets} &= -y^u_{LiJ}\overline{\psi_{iL}}\tilde{\phi}_L U_{JR} - y^u_{RiJ}\overline{\psi_{iR}}\tilde{\phi}_R U_{JL} - h.c. \\ &\quad -y^d_{LiJ}\overline{\psi_{iL}}\phi_L D_{JR} - y^d_{RiJ}\overline{\psi_{iR}}\phi_R D_{JL} - h.c., \end{aligned} \tag{3}$$

where $g_L$, $g_R$ and $g_1$ denote the gauge couplings for $\mathrm{SU}(2)_L$, $\mathrm{SU}(2)_R$ and $U(1)$, respectively.

## 3. The mixing of the Neutral Gauge Bosons

The symmetry breaking of $\mathrm{SU}(2)_L\times$ $\mathrm{SU}(2)_R\times$ U(1) into $\mathrm{U}(1)_{em}$ leads to the following relation between weak eigenstates $(W^3_L, B_1.W^3_R)$ and mass eigenstates $(Z, A, Z')$ of the three neutral gauge bosons,

$$\begin{pmatrix} W^3_{L\mu} \\ B_{1\mu} \\ W^3_{R\mu} \end{pmatrix} = O_{23}(\theta_{W_R})O_{12}(-\theta_W)O_{13}(\theta_{13})\begin{pmatrix} Z_\mu \\ A_\mu \\ Z'_\mu \end{pmatrix}, \tag{4}$$

where the rotation matrices are $O_{23}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$, $O_{12}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $O_{13}(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$ and the mixing angles are given as $\tan\theta_{W_R} = \frac{g_1}{g_R}, \tan\theta_W = \frac{g'}{g_L}, \tan 2\theta_{13} = \frac{\sin^2\theta_{W_R}\sin 2\theta_{W_R}}{\sin\theta_W}\frac{v_L^2}{v_R^2}$. We note that $\theta_{13}$ is suppressed by the ratio of vevs $\frac{v_L^2}{v_R^2}$. With the mixing angles, the standard model-like $\mathrm{U}(1)_Y$ hypercharge and the electromagnetic charge are given by $g' = g_1 \cos\theta_{W_R}, e = g' \cos\theta_W$. The isospin current of Z boson is,

$$\begin{aligned}\mathcal{L}_{ZI_3} &= -\left\{\frac{1}{2\cos\theta_W}\left(\overline{u^i_L}\gamma_\mu u^i_L - \overline{d^i_L}\gamma_\mu d^i_L\right)(g_L\cos\theta_{13} + e\tan\theta_{W_R}\sin\theta_{13})\right.\\ &\left.+\frac{g_R}{2\cos\theta_{W_R}}\left(\overline{u^i_R}\gamma_\mu u^i_R - \overline{d^i_R}\gamma_\mu d^i_R\right)\sin\theta_{13}\right\}Z^\mu. \end{aligned} \tag{5}$$

## 4. Effective Lagrangian

We integrate the five VLQs except an up-type VLQ denoted by $U_3$ which mass parameter $M_{U3}$ is smaller than $v_R$. The following effective Lagrangian is obtained,

$$\begin{aligned}\mathcal{L}_{\rm eff} &= \sum_{i=1}^{3}\overline{\psi_{Li}}\left(i\partial\!\!\!/ - g_L W\!\!\!\!/_L - g_1\frac{1}{6}B\!\!\!/_1\right)\psi_{Li} + (L\to R)\\ &- \sum_{ij}\frac{v_L}{\sqrt{2}}\overline{u_{Li}}u_{Rj}y^u_{ij} - h.c. - \sum_{ij}\frac{v_L}{\sqrt{2}}\overline{d_{Li}}d_{Rj}y^d_{ij} - h.c.\\ &- \sum_i \frac{y^u_{Li3}v_L}{\sqrt{2}}\overline{u_{Li}}U_{R3} - \sum_i\frac{y^u_{Ri3}v_R}{\sqrt{2}}\overline{u_{Ri}}U_{L3} - h.c. - \overline{U}_3 M_{U_3}U_3,\end{aligned} \tag{6}$$

where we have ignored the terms suppressed by a factor $\frac{v_L^2}{M_X^2} \ll \frac{v_R^2}{M_X^2} \ll 1$ and standard model like Yukawa couplings for light quarks $u, d, c, s, b$ are given by,

$$y^u_{ij} = -\sum_{\alpha=1}^{2} y^u_{Li\alpha}y^{u*}_{Rj\alpha}\frac{v_R}{\sqrt{2}M_{U\alpha}}, \quad y^d_{ij} = -\sum_{\alpha=1}^{3} y^d_{Li\alpha}y^{d*}_{Rj\alpha}\frac{v_R}{\sqrt{2}M_{D\alpha}}, \tag{7}$$

where we have substituted the vevs to two Higgs doublets. $\mathcal{L}_{\rm eff}$ includes four up-type quarks and their $4\times 4$ mass matrix is given by,

$$\mathcal{L}_{\rm eff\ up-type\ mass} = -\begin{pmatrix}\overline{u_{Li}} & \overline{U_{L3}}\end{pmatrix} M_{\mathcal{U}}\begin{pmatrix}u_{jR}\\ U_{R3}\end{pmatrix}, M_{\mathcal{U}} = \begin{pmatrix}-\sum_{\alpha=1}^2\frac{\mathbf{y}^u_{L\alpha}\mathbf{y}^{u\dagger}_{R\alpha}v_Lv_R}{2M_{U\alpha}} & \frac{\mathbf{y}^u_{L3}v_L}{\sqrt{2}}\\ \frac{\mathbf{y}^{u\dagger}_{R3}v_R}{\sqrt{2}} & M_{U_3}\end{pmatrix}, \tag{8}$$

where $\frac{\mathbf{y}^{uT}_{L(R)3}v_L}{\sqrt{2}} = \begin{pmatrix}\frac{y^u_{L(R)13}v_L}{\sqrt{2}} & \frac{y^u_{L(R)23}v_L}{\sqrt{2}} & \frac{y^u_{L(R)33}v_L}{\sqrt{2}}\end{pmatrix}$. We apply the following bi-unitary transformation on $M_{\mathcal{U}}$ with two $3\times 3$ unitary matrices $V^u$ and $W^u$,

$$\begin{aligned}&\begin{pmatrix}V^{u\dagger} & 0_{3\times1}\\ 0_{1\times3} & 1\end{pmatrix} M_{\mathcal{U}}\begin{pmatrix}W^u & 0_{3\times1}\\ 0_{1\times3} & 1\end{pmatrix} = \begin{pmatrix}-V^{u\dagger}(\sum_{\alpha=1}^2\frac{\mathbf{y}^u_{L\alpha}\mathbf{y}^{u\dagger}_{R\alpha}v_Lv_R}{2M_{U\alpha}})W^u & V^{u\dagger}\frac{\mathbf{y}^u_{L3}v_L}{\sqrt{2}}\\ \frac{\mathbf{y}^{u\dagger}_{R3}v_R}{\sqrt{2}}W^u & M_{U_3}\end{pmatrix}\\ &= \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{|\mathbf{y}^u_{L3}|v_L}{\sqrt{2}}\\ 0 & 0 & \frac{|\mathbf{y}^u_{R3}|v_R}{\sqrt{2}} & M_{U_3}\end{pmatrix} + \begin{pmatrix}-V^{u\dagger}(\sum_{\alpha=1}^2\frac{\mathbf{y}^u_{L\alpha}\mathbf{y}^{u\dagger}_{R\alpha}v_Lv_R}{2M_{U\alpha}})W^u & 0_{3\times1}\\ 0_{1\times3} & 0\end{pmatrix}.\end{aligned} \tag{9}$$

Below we diagonalize the first term of Eq.(9) to obtain the spectrum of the heavier up-type quark. The second term is ignored since the contribution is suppressed by $\frac{1}{M_{U\alpha}}$ $(\alpha = 1, 2)$. With the bi-orthogonal transformation, it is diagonalized as,

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&\cos\theta_R&-\sin\theta_R\\0&0&\sin\theta_R&\cos\theta_R\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&\cos\theta_l&\sin\theta_l\\0&0&-\sin\theta_l&\cos\theta_l\end{pmatrix}$$
$$\times\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&\frac{|\mathbf{y}^u_{L3}|v_L}{\sqrt{2}}\\0&0&\frac{|\mathbf{y}^u_{R3}|v_R}{\sqrt{2}}&M_{U_3}\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&\cos\theta_R&\sin\theta_R\\0&0&-\sin\theta_R&\cos\theta_R\end{pmatrix}$$
$$=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&\cos\theta_L&-\sin\theta_L\\0&0&\sin\theta_L&\cos\theta_L\end{pmatrix}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&m_L&0\\0&0&M_{U_3}&m_R\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&\cos\theta_R&\sin\theta_R\\0&0&-\sin\theta_R&\cos\theta_R\end{pmatrix}=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&m_t&0\\0&0&0&m_{t'}\end{pmatrix},$$
$$m_{t(t')}=\frac{\sqrt{M_{U_3}^2+(m_R+m_L)^2}}{2}\mp\frac{\sqrt{M_{U_3}^2+(m_R-m_L)^2}}{2}, \tag{10}$$

where $m_L=\frac{|\mathbf{y}^u_{L3}|v_L}{\sqrt{2}}, m_R=\frac{|\mathbf{y}^u_{R3}|v_R}{\sqrt{2}}, \theta_L=\theta_R-\theta_l, \tan\theta_l=\frac{M_{U3}}{m_R+m_L}$ and $\tan 2\theta_R=\frac{2M_{U_3}m_R}{m_R^2-m_L^2-M_{U_3}^2}$.

## 5. Z FCNC for top quark and its partner

One can compute the Z FCNC by rewriting left and right up-type quarks in SU(2) doublet in terms of top quark $t$ and its partner $t'$,

$$u'_{3R}=-\sin\theta_R t_R+\cos\theta_R t'_R,\quad u'_{3L}=\cos\theta_L t_L+\sin\theta_L t'_L. \tag{11}$$

where $u'_3$ denote the basis obtained after unitary transformations $V$ and $W$. With Eq.(11) and Eq.(5), Z FCNC for top quark and top prime quark is,

$$\mathcal{L}^{tt'}_{\mathrm{Z\ I_3}}=-\frac{g_R\sin\theta_{13}Z^\mu}{2\cos\theta_{W_R}}\left\{\sin^2\theta_R\overline{t_R}\gamma_\mu t_R-\sin\theta_R\cos\theta_R\left(\overline{t_R}\gamma_\mu t'_R+h.c.\right)+\cos^2\theta_R\overline{t'_R}\gamma_\mu t'_R\right\}-$$
$$\frac{Z^\mu(g_L\cos\theta_{13}+e\tan\theta_{W_R}\sin\theta_{13})}{2\cos\theta_W}\left\{\cos^2\theta_L\overline{t_L}\gamma_\mu t_L+\sin\theta_L\cos\theta_L\left(\overline{t_L}\gamma_\mu t'_L+h.c.\right)+\sin^2\theta_L\overline{t'_L}\gamma_\mu t'_L\right\}. \tag{12}$$

When $M_{U_3}\ll m_R$, one can show $\theta_L\simeq 0\ll\theta_R\simeq\frac{M_{U_3}}{m_R}\ll 1$ .

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