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A PIECEWISE ANALYTICAL ITERATIVE METHOD FOR A WIRE-MASS MODEL(Article)

Mungkasi, S.

Department of Mathematics, Faculty of Science and Technology, Sanata Dharma University, Yogyakarta, Indonesia

Abstract

A wire-mass model is considered, where the wire is elastic and the mass is attached to the wire. We propose a piecewise analytical iterative method for simulating the motion of the mass attached to the wire. Our proposed method combines Picard's successive approximation and Taylor series expansion methods. Picard's successive approximation method is simple to construct, but difficult to compute, as it involves a nonlinear term in the integrand. Taylor series expansion method is accurate, but only for small intervals. We combine these two methods to take advantage of the strengths and avoid the weaknesses of both methods. Therefore, we implement this combination piecewisely. Numerical tests show that our proposed method is simple to implement but produces accurate solutions © Işık University, Department of Mathematics, 2023; all rights reserved.

Author keywords

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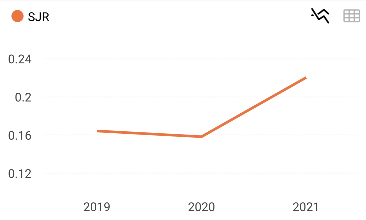

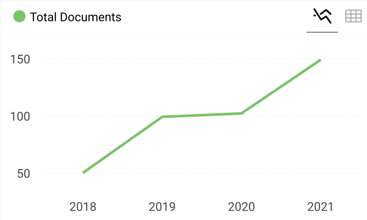

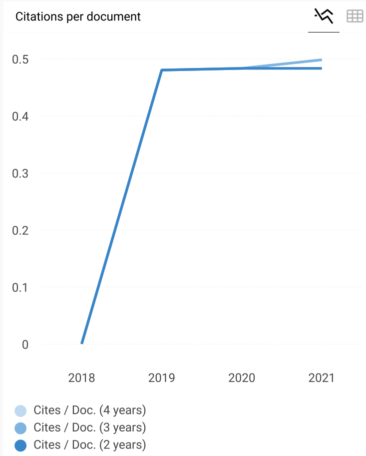

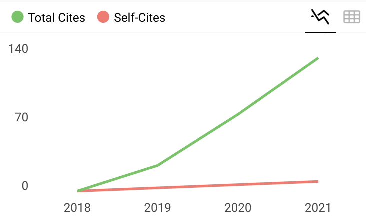

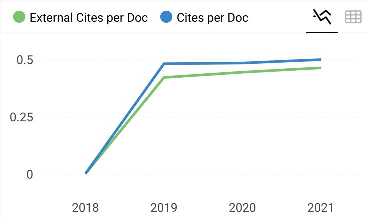

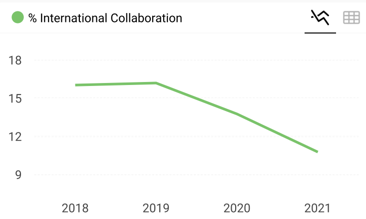

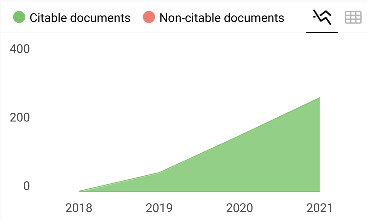

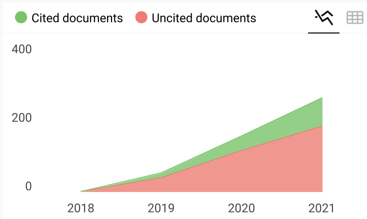

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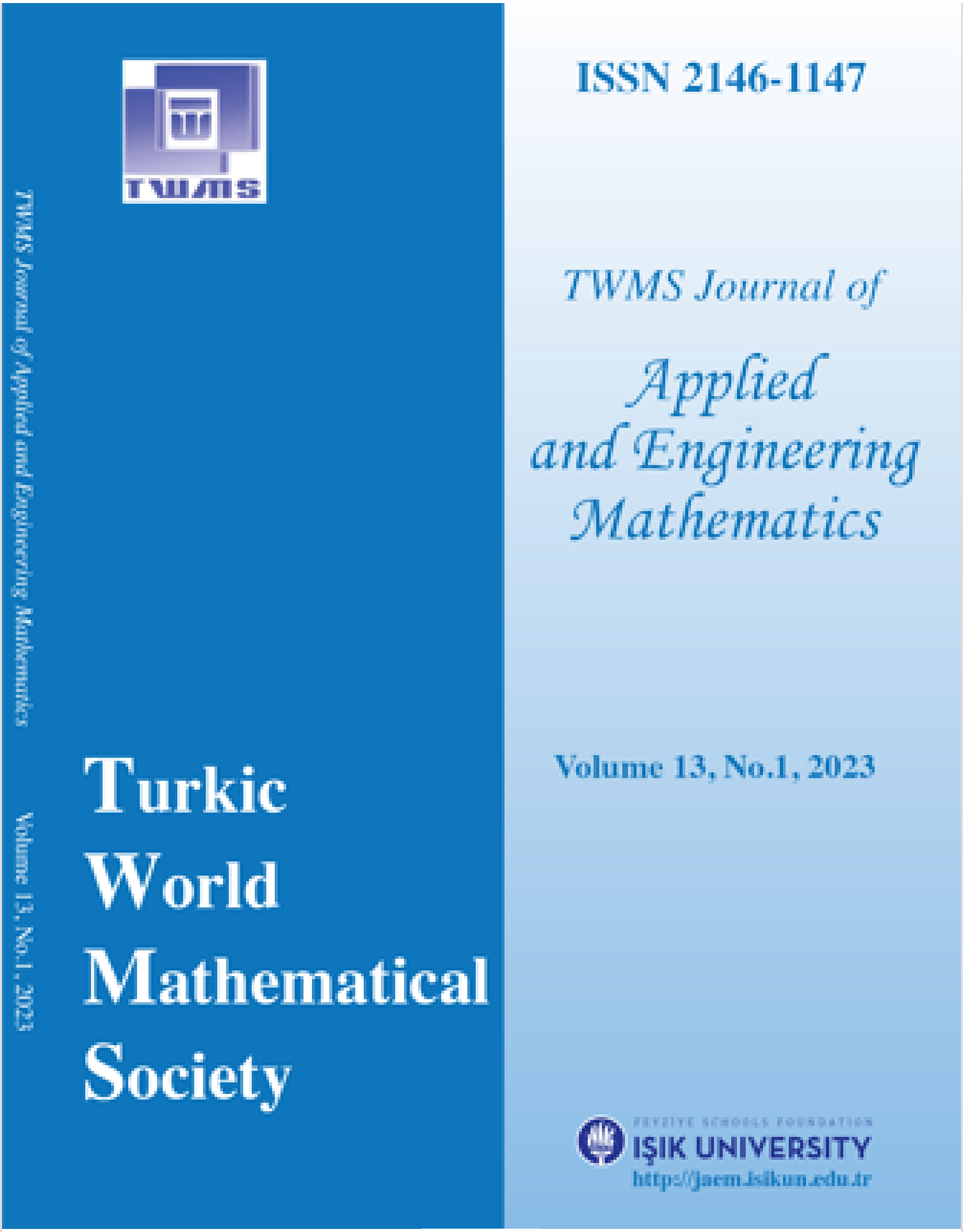
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A PIECEWISE ANALYTICAL ITERATIVE METHOD FOR A WIRE-MASS MODEL

SUDI MUNGKASI[1], §

Abstract. A wire-mass model is considered, where the wire is elastic and the mass is attached to the wire. We propose a piecewise analytical iterative method for simulating the motion of the mass attached to the wire. Our proposed method combines Picard's successive approximation and Taylor series expansion methods. Picard's successive approximation method is simple to construct, but difficult to compute, as it involves a nonlinear term in the integrand. Taylor series expansion method is accurate, but only for small intervals. We combine these two methods to take advantage of the strengths and avoid the weaknesses of both methods. Therefore, we implement this combination piecewisely. Numerical tests show that our proposed method is simple to implement but produces accurate solutions.

Keywords: High order method, Picard–Taylor method, piecewise method, successive approximation method, wire-mass model.

AMS Subject Classification: 34A34.

1. Introduction

Wire is often needed in engineering installations. When a mass is attached to a wire, the mass gets involved in the determination of the motion of the whole system. To solve the problem, it needs to be modelled and simulated. As exact analytical solutions are generally not available, numerical methods provide a way to deal with the problem.

Other than numerical methods, an analytical approximation can also be taken. An available analytical approximation is by using Picard's successive approximation method, which is a special case of variational iteration method [1–6] due to He [7–9]. The successive approximation method is simple to construct, but difficult to compute when we have nonlinear terms in the integrand. Another analytical approximation is by using truncated Taylor series expansions. Taylor series polynomials are easy to compute, but they are accurate only for small intervals.

In this paper, we propose a combination of methods to solve the problem of the motion of a mass attached to a stretched elastic wire. To overcome the difficulty in the integration process of the successive approximation method, we replace the original nonlinear term

[1] Department of Mathematics, Faculty of Science and Technology, Sanata Dharma University, Yogyakarta, Indonesia.
e-mail: sudi@usd.ac.id; ORCID: https://orcid.org/0000-0001-5319-9744.

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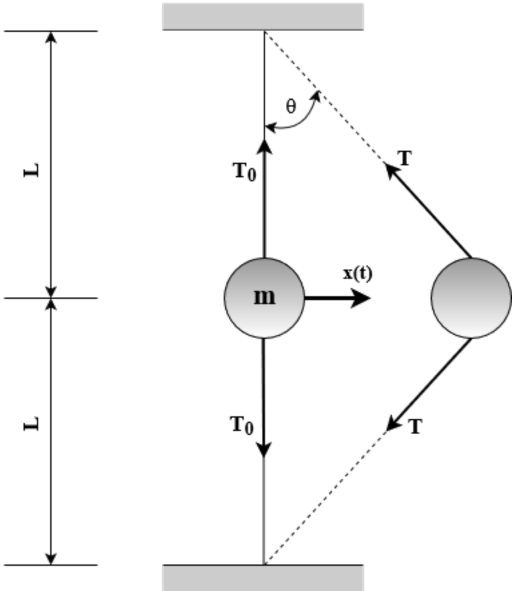


FIGURE 1. Illustration of the wire-mass model, where a mass attached to a stretched elastic wire [10].

with a truncated Taylor series expansion. To maintain the accuracy of the solution, we implement this combination of Picard and Taylor methods into small subdomains consecutively. Computational tests confirm that our proposed method can be made high accuracy simply by taking more number of Picard iterations and more Taylor terms in the piecewise evolution.

We formulate the problem of the motion of a mass attached to a stretched elastic wire into a mathematical equation called the wire-mass model following Durmaz et al. [10]. The wire-mass model is of the type of vibration problems. Its applications can be extended for the vibration of a bridge, the vibration of a solid structure, etc. Therefore, modelling and simulation of the wire-mass problem provide insights for solving these extended versions of the problem, which includes the wire-mass motion in the wire installation problem itself.

The rest of this paper is structured as follows. We first provide the mathematical model of the problem. Then we provide methods for solving the problem. After that, results and discussion are presented. Finally, some concluding remarks will close the paper.

2. The Wire-Mass Model

We consider the motion problem of a mass attached in a stretched elastic wire, as shown in Figure 1. Wire installation may involve a mass in the joint of two pieces of wire. To determine the position of the mass (the joint) at anytime, a mathematical model has been proposed in the literature [10–22], but accurately solving the model remains to be an open problem. To make our paper to be self-contained, we shall rederive the mathematical model of a mass attached in a stretched elastic wire following the work of Durmaz et al. [10].

Let us assume that the motion of the mass in the wire-mass system is one dimensional, that is, in the x-direction. Based on Newton's second law, the mathematical model governing the motion of the mass attached to a stretched elastic wire, as shown in Figure 1, is [10]:

$$m\ddot{x} = -2T\sin\theta, \tag{1}$$

with initial conditions

$$x(0) = x_0, \qquad \dot{x}(0) = 0. \tag{2}$$

Here, the free variable is time t; m is the mass; $x = x(t)$ is the position of the mass dependent on time variable t; $\dot{x}$ means the first derivative of x with respect to time t; $\ddot{x}$ means the second derivative of x with respect to time t; constant x_0 is the initial amplitude of the mass. Observing Figure 1, we note that

$$\sin\theta = \frac{x}{\sqrt{L^2 + x^2}}, \tag{3}$$

where L is the distance from the mass to the walls in the still condition.

The tension T of the wire is given by

$$T = T_0 + R\frac{\sqrt{L^2 + x^2} - L}{L}. \tag{4}$$

Here, T_0 and R are known constants, where T_0 is the tension of the wire when there is no motion, and R is the axial rigidity of the wire such that $0 \le T_0 \le R$. (Note that there were typographical errors in the formulation of tension T in the work of Durmaz et al. [10].)

Substituting $\sin\theta$ given by equation (3) and tension T given by equation (4) into equation (1), we eliminate θ from the mathematical model, so the mathematical model for the motion of a mass attached in a stretched elastic wire becomes

$$m\ddot{x}(t) + 2R\frac{x(t)}{L} + \frac{2(T_0 - R)\frac{x(t)}{L}}{\sqrt{1 + \left(\frac{x(t)}{L}\right)^2}} = 0, \tag{5}$$

with initial conditions

$$x(0) = x_0, \qquad \dot{x}(0) = 0. \tag{6}$$

Taking dimensionless variables

$$u = \frac{x}{L}, \qquad \tau = \frac{t}{\sqrt{\frac{mL}{2R}}}, \tag{7}$$

and introducing new parameters

$$\mu = 1 - \frac{T_0}{R}, \qquad u_0 = \frac{x_0}{L}, \tag{8}$$

we obtain that the dimensionless model for the motion of the mass attached to the stretched elastic wire is

$$u''(\tau) + u(\tau) - \frac{\mu u(\tau)}{\sqrt{1 + u(\tau)^2}} = 0, \tag{9}$$

with initial conditions

$$u(0) = u_0, \qquad u'(0) = 0. \tag{10}$$

In dimensionless model (9), τ is the free variable, u is the dependent variable, and the constant μ is on interval $0 \le \mu \le 1$. Here, u' means the first derivative of u with respect to the dimensionless time τ; and u'' means the second derivative of u with respect to the dimensionless time τ. Solving model (9) with initial conditions (10) leads to solving the problem.

3. Methods for Solving the Wire-Mass Model

In this section, we recall an existing iterative method, which is Picard's successive approximation method, for solving dimensionless model (9) with initial conditions (10). As the existing iterative method is not accurate for a long period of time and it has difficult task in the integration process, we need a better solving method. Therefore, in this paper, we propose a combination of Picard's successive approximation method for initial value problems and Taylor's series method for function approximations in a piecewise domain. We call the method that we propose the piecewise Picard–Taylor iterative method.

3.1. **Existing iterative method and its limitations.** An available (existing) iterative method is Picard's successive approximation method. It works as follows. We consider a general initial value problem

$$y'(x) = \varphi(x, y), \qquad y(x_0) = y_0, \tag{11}$$

where $\varphi(x, y)$ is assumed to be continuous on a domain containing the point (x_0, y_0). Then, we have the following theorem.

Theorem 3.1. *Any solution to the initial value problem (11) is also a solution to the integral equation*

$$y(x) = y_0 + \int_{x_0}^{x} \varphi(z, y(z))dz \tag{12}$$

and conversely.

The proof of this theorem can be found in the literature, such as, Agarwal and O'Regan [23]. Based on Theorem 3.1, Picard's successive approximations are constructed as

$$y_n(x) = y_0 + \int_{x_0}^{x} \varphi(z, y_{n-1}(z))dz, \tag{13}$$

where $n = 1, 2, 3, ...$ If the sequence $\{y_n(x)\}$ converges uniformly to a function $y(x)$, then the solution to the initial value problem (11) is

$$y(x) = \lim_{n\to\infty} y_n(x), \tag{14}$$

where $y(x)$ is continuous in an interval containing x_0.

Model (9) can be written equivalently into

$$u' = v, \tag{15}$$

$$v' = -u + \frac{\mu u}{\sqrt{1+u^2}}. \tag{16}$$

Solving model (9) is equivalent to solving the system of equations (15) and (16). Picard's successive approximation method for solving the system of equations (15) and (16) with initial conditions (10) is

$$u_n(\tau) = u_0 + \int_0^{\tau} v_{n-1}(\xi)d\xi, \tag{17}$$

$$v_n(\tau) = v_0 + \int_0^{\tau} \left[-u_{n-1}(\xi) + \frac{\mu u_{n-1}(\xi)}{\sqrt{1+u_{n-1}(\xi)^2}}\right] d\xi, \tag{18}$$

where $n = 1, 2, 3, ...$ and $v_0 = u'(0) = 0$.

There are at least two limitations of the existing Picard's successive approximation method. First, in order to obtain accurate solutions, we need to iterate formulas (17) and (18) many times, but doing so is impractical. Second, the integration of equation (18) is difficult to do when n is large, because of the presence of the nonlinear term $\mu u/\sqrt{1+u^2}$

in the integrand. This motivates that a simple but highly accurate method for solving the model is desired.

3.2. **Proposed method to overcome the limitations of the existing method.** In this section, we propose a new strategy for solving the system of equations (15) and (16). Two actions are taken to overcome two limitations of Picard's successive approximation method. The first action is to subdivide the time domain into a finite number of subdomains consecutively. This will make the iterative method produce accurate solutions for a long period of time. The second action is to replace factor $u/\sqrt{1+u^2}$ using a truncated Taylor series expansion about the initial point of each of subdomains. This will make the integration in Picard's successive approximation method easy to do. The resulting proposed method is called the piecewise Picard–Taylor iterative method (PPTIM).

Suppose that we are given the time domain $[\tau_0, \tau_f]$ for determining the position of the mass. We subdivide the domain into K subdomains consecutively, that is, $[\tau_{k-1}, \tau_k]$ for $k = 1, 2, 3, ..., K$. We implement Picard's successive approximation method up to N iteration(s) for each of subdomains, where N is a positive integer. We denote $u_{n,k}(\tau)$ the solution of PPTIM at the nth iteration of Picard's successive approximation method on the kth subdomain.

Our proposed PPTIM works as follows. For each $k = 1, 2, 3, ..., K$ and for each $n = 1, 2, 3, ..., N$:

$$u_{n,k}(\tau) = u_{0,k} + \int_{\tau_{k-1}}^{\tau} v_{n-1,k}(\xi) d\xi, \tag{19}$$

$$v_{n,k}(\tau) = v_{0,k} + \int_{\tau_{k-1}}^{\tau} \left[-u_{n-1,k}(\xi) + \mu P_{N-1,k}(\xi)\right] d\xi, \tag{20}$$

where

$$u_{0,1} = u_0, \qquad v_{0,1} = v_0, \tag{21}$$

and

$$u_{0,k} = u_{N,k-1}(\tau_{k-1}), \quad v_{0,k} = v_{N,k-1}(\tau_{k-1}), \tag{22}$$

when $k = 2, 3, 4, ..., K$. Here

$$P_{N-1,k}(\xi) \approx \frac{u(\xi)}{\sqrt{1+u(\xi)^2}}, \tag{23}$$

where $P_{N-1,k}(\xi)$ denotes the Taylor polynomial of degree $N-1$, which approximates $u(\xi)/\sqrt{1+u(\xi)^2}$ about point $\xi = \tau_{k-1}$.

We remark that the upper bound of the integrals in formulas (19) and (20) is τ, because these formulas are defined for all τ in the whole subdomain $[\tau_{k-1}, \tau_k]$. When the formulas (19) and (20) are used to obtain the solution at τ_k, the upper bound τ is replaced by τ_k. In addition, to show the simplicity of our proposed PPTIM, we provide a pseudocode of the PPTIM written as Algorithm 1.

4. Results and Discussion

In this section, we report our research results and discussion. The exact analytical solution to the model is not known, but we can take an available highly accurate solution as the reference. In our computational tests, we take the following parameters: $\mu = 0.5$, $u_0 = 1$, $\tau_0 = 0$, $\tau_f = 8$, $\Delta\tau = \tau_k - \tau_{k-1}$ for all k. We use the notation PPTIMN for the PPTIM method using N successive iterations. Specially when we take $N = 1$, we observe from formulas (19) and (20) that PPTIM1 evaluated at τ_k is the same as

Algorithm 1 Pseudocode for the piecewise Picard–Taylor iterative method

procedure PPTIM($\tau_0, \tau_f, u_0, v_0, \mu, K, N$) ▷ Defined inputs for the PPTIM
  $\Delta\tau \leftarrow (\tau_f - \tau_0)/K$ ▷ Compute the time-step
  $T \leftarrow \tau_0 : \Delta\tau : \tau_f$ ▷ Define the discrete time
  $U \leftarrow 0 * T$ ▷ Define the storage for values of u
  $V \leftarrow 0 * T$ ▷ Define the storage for values of v
  $U(1) \leftarrow u_0$ ▷ Initial value of u
  $V(1) \leftarrow v_0$ ▷ Initial value of v
  for $k \leftarrow 1 : K$ **do** ▷ Loops through each subdomain
    $Y(1) \leftarrow U(k)$ ▷ Initial Picard's approximation of u at the kth subdomain
    $Z(1) \leftarrow V(k)$ ▷ Initial Picard's approximation of v at the kth subdomain
    TaylorSeries $\leftarrow$ The Nth order Taylor series of $u/\sqrt{1+u^2}$ about τ_k
    for $n \leftarrow 1 : N$ **do** ▷ Loops of Picard's successive approximation method
      $Y(n+1) \leftarrow Y(1) + \int_{\tau_k}^{\tau} Z(n)\, d\xi$
      $Z(n+1) \leftarrow Z(1) + \int_{\tau_k}^{\tau} (-Y(n) + \mu{*}\text{TaylorSeries})\, d\xi$
    $U(k+1) \leftarrow$ Value of $Y(n+1)$ at τ_{k+1}
    $V(k+1) \leftarrow$ Value of $Z(n+1)$ at τ_{k+1}
  return U ▷ Discrete solution values of u are returned in vector U

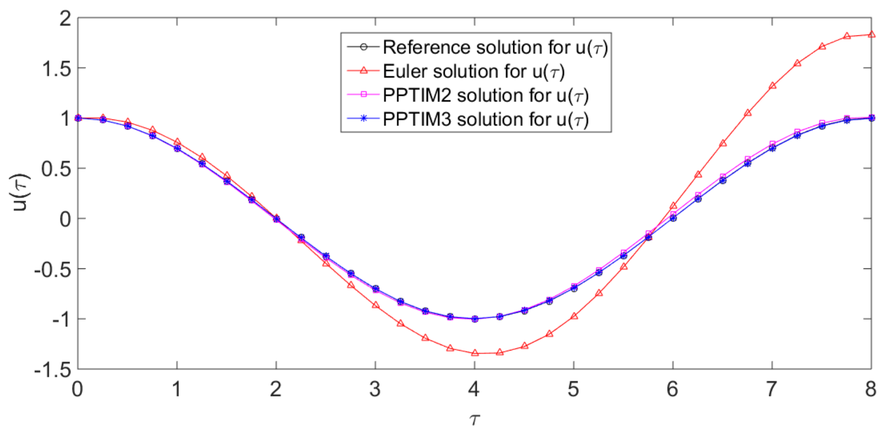


FIGURE 2. Euler's, PPTIM2 and PPTIM3 solutions produced using the dimensionless time step $\Delta\tau = 0.25$.

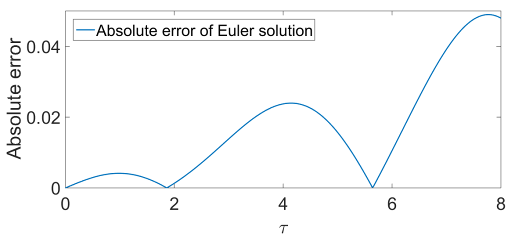
the standard Euler's method for solving equations (15) and (16). This standard Euler's method (PPTIM1 evaluated at τ_k) is

$$u_k = u_{k-1} + \Delta\tau v_{k-1}, \tag{24}$$

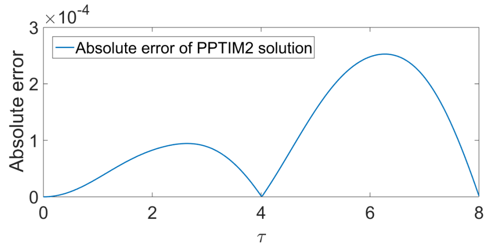
$$v_k = v_{k-1} + \Delta\tau \left[-u_{k-1} + \frac{\mu u_{k-1}}{\sqrt{1+u_{k-1}^2}} \right], \tag{25}$$

where $u_k = u_{1,k}(\tau_k)$, $v_k = v_{1,k}(\tau_k)$, $u_{k-1} = u_{1,k-1}(\tau_{k-1})$, and $v_{k-1} = v_{1,k-1}(\tau_{k-1})$.

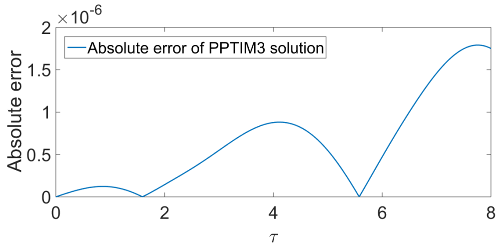
Euler's, PPTIM2 and PPTIM3 solutions together with the reference solution are shown in Figure 2 for $\Delta\tau = 0.25$, which is a relatively coarse subdivision of the time domain. In this figure, Euler's solution is not accurate; PPTIM2 solution is more accurate than



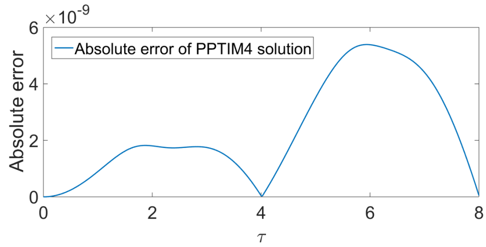
(a) Absolute error of Euler's solution.



(b) Absolute error of PPTIM2 solution.



(c) Absolute error of PPTIM3 solution.



(d) Absolute error of PPTIM4 solution.

FIGURE 3. Absolute errors of Euler's, PPTIM2, PPTIM3 and PPTIM4 solutions.

TABLE 1. List of error and Experimental Order of Convergence (EOC) of the proposed piecewise Picard–Taylor iterative method. We limit our simualtions for $N = 1, 2, 3, 4$.

Value of $\Delta\tau$	Error ($N = 1$)	EOC ($N = 1$)	Error ($N = 2$)	EOC ($N = 2$)	Error ($N = 3$)	EOC ($N = 3$)	Error ($N = 4$)	EOC ($N = 4$)
0.32	3.53855E-01	–	2.90628E-02	–	2.36565E-03	–	1.66754E-04	–
0.16	1.51637E-01	1.223	7.10398E-03	2.032	3.08423E-04	2.939	1.00044E-05	4.059
0.08	7.01945E-02	1.111	1.74974E-03	2.021	3.93000E-05	2.972	6.11111E-07	4.033
0.04	3.37961E-02	1.054	4.33791E-04	2.012	4.95870E-06	2.986	3.77328E-08	4.018
0.02	1.65839E-02	1.027	1.07966E-04	2.006	6.22748E-07	2.993	2.34359E-09	4.009

Euler's solution; PPTIM3 is most accurate. PPTIM3 is able to coincide graphically with the reference solution. These results indicate that more number of iterations in the PPTIM evolution leads to a more accurate solution. This means that if we take PPTIM4 for solving the problem, we shall obtain a more accurate solution.

Taking smaller $\Delta\tau$ shall also lead to smaller error of the approximate solution. This is confirmed in Figure 3 plotting the absolute errors of these solutions for $\Delta\tau = 0.02$. We observe from Subfigure 3a that the absolute error of the Euler's solution and the error is at the magnitude of 10^{-2}. In addition, Subfigure 3b shows the absolute error of the PPTIM2 solution and the error is at the magnitude of 10^{-4}. Furthermore, Subfigure 3c shows the absolute error of the PPTIM3 solution and the error is at the magnitude of 10^{-6}. Moreover, Subfigure 3d shows the absolute error of the PPTIM4 solution and the error is at the magnitude of 10^{-9}. Overall, higher order accurate method can be achieved simply by taking larger value of N, where N represents the number of Picard iterations and $N - 1$ is the polynomial degree of the truncated Taylor series expansion involved in the PPTIM formulation.

To investigate the Experimental Order of Convergence (EOC) of PPTIM, we take various values of $\Delta\tau$ and calculate the EOC. Table 1 records the average of absolute errors on the whole domain. We observe that, as $\Delta\tau$ tends to 0, EOC approaches the value of N. This confirms that if we take $N = 1$, then the order of accuracy of PPTIM is of the first order. If we take $N = 2$, then PPTIM is of the second order accurate method. Taking $N = 3$ in the PPTIM evolution, we obtain a third order accurate method. Taking $N = 4$ leads to that our PPTIM solution is of the fourth order accuracy.

5. Conclusion

We have proposed a new method that we call the piecewise Picard–Taylor iterative method for simulating the motion of a mass attached to a stretched elastic wire. The method is simple, but it is accurate. The order of accuracy can be made higher simply by taking more number of Picard successive iterations and more Taylor series terms in the piecewise evolution. The model that we solve is for one-dimensional problems. Future research direction could extend the proposed method to solve higher dimensional problems involving two and three dimensions.

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Sudi Mungkasi is an associate professor in the Department of Mathematics, Faculty of Science and Technology, Sanata Dharma University, Yogyakarta, Indonesia. He obtained his bachelor's degree in Mathematics from Gadjah Mada University, Yogyakarta, Indonesia in 2004. He received the degrees of Master of Mathematical Sciences and Doctor of Philosophy in Mathematical Sciences from The Australian National University, Canberra, Australia in 2008 and 2013, respectively. His research interests include applied and computational mathematics. Currently, he is the Dean of the Faculty of Science and Technology, Sanata Dharma University.