Re: Inquiry about Manuscript IDAQP-D-21-00003

Bhat IDAQP <bhatidaqp@gmail.com>
Thu 3/17/2022 11:23 AM
To: Herry P Suryawan <herrypribs@usd.ac.id>
Cc: uncigji@chungbuk.ac.kr <uncigji@chungbuk.ac.kr>

Dear Professor Suryawan
I have recently reminded the handling editor. He has promised to take action though it may not get reflected on the Editorial Manager website.
Best regards,
R. Bhat.

On Thu, Mar 17, 2022 at 9:31 AM Herry P Suryawan <herrypribs@usd.ac.id> wrote:

> Dear Prof. B V Rajarama Bhat
> Editor of IDAQP,
>
> It has been more than 6 months since we have submitted our manuscript entitled "A white noise approach to stochastic currents of Brownian motion" (ID: IDAQP-D-21-00003) to your journal via the online submission system on August 26, 2021 and the status has remained "with editor".
>
> I would be grateful if you could let me know whether there has been any further progress on our submission.
>
> Sincerely,
> Herry P. Suryawan
> Department of Mathematics, Sanata Dharma University
> Indonesia
>
> ---
>
> **From:** Bhat IDAQP <bhatidaqp@gmail.com>
> **Sent:** Friday, January 7, 2022 4:33 PM
> **To:** Herry P Suryawan <herrypribs@usd.ac.id>
> **Cc:** uncigji@chungbuk.ac.kr <uncigji@chungbuk.ac.kr>
> **Subject:** Re: Inquiry about Manuscript IDAQP-D-21-00003
>
> Dear Professor Suryawan
> your paper is with a handling editor and I am following it up. I am reminding him.
> Best regards,
> R. Bhat.
>
> On Wed, Jan 5, 2022 at 7:55 PM Herry P Suryawan <herrypribs@usd.ac.id> wrote:
>
>> Dear Prof. B V Rajarama Bhat
>> Editor of IDAQP,
>>
>> I have submitted my manuscript entitled "A white noise approach to stochastic currents of Brownian motion" (ID: IDAQP-D-21-00003) to your journal via the online submission system on August 26, 2021. Some days after, the status changed to "With Editor". However, the status has remained unchanged ever since.

I would be grateful if you could let me know whether there has been any further progress on my submission, i.e. whether the status means that the manuscript is still under review or already passed the review.

Sincerely,
Herry P. Suryawan
Department of Mathematics, Sanata Dharma University
Indonesia

Re: Your Submission: IDAQP-D-21-00003

José Luís da Silva <joses@staff.uma.pt>
Thu 4/7/2022 2:59 PM
To: Herry P Suryawan <herrypribs@usd.ac.id>

Thanks a lot for the work.
José Luís

> On 7 Apr 2022, at 05:38, Herry P Suryawan <herrypribs@usd.ac.id> wrote:
>
> Dear Martin, Dear José Luís,
>
> revised manuscript has been sent to IDAQP (receipt in the attachment).
>
> Best regards,
> Herry
>
> ---
>
> **From:** Martin Grothaus <grothaus@mathematik.uni-kl.de>
> **Sent:** Tuesday, April 5, 2022 8:40 AM
> **To:** Herry P Suryawan <herrypribs@usd.ac.id>; José Luís da Silva <joses@staff.uma.pt>
> **Subject:** Re: Your Submission: IDAQP-D-21-00003
>
> Dear Herry,
>
> both are ok. You can send them to IDAQP.
>
> Thanks for taking care!
>
> Best regards,
> Martin
>
> > Dear Martin, Dear José Luís,
> >
> > I'm sorry for my late response on the revision process due to busy week here. Please find in the attachment the revised manuscript and the rebuttal letter. As soon as it is ok, I will send them to IDAQP.
> >
> > Best regards,
> > Herry
> >
> > ---
> >
> > **From:** Martin Grothaus <grothaus@mathematik.uni-kl.de>
> > **Sent:** Thursday, March 31, 2022 9:27 PM
> > **To:** Herry P Suryawan <herrypribs@usd.ac.id>; José Luís da Silva <joses@staff.uma.pt>
> > **Subject:** Re: Your Submission: IDAQP-D-21-00003
> >
> > Dear Herry,
> >
> > are there any updates concerning the revision?
> >
> > Thanks!
> >
> > Greetings from a snowy Iowa,
> > Martin
> >
> > > Dear José Luís, Dear Martin,
> > >
> > > I hope both of you are fine! Finally, today we got news from IDAQP. Our paper will be accepted after minor revisions. It seems that the main concern of the reviewers is about the usefulness and probabilistic interpretation of our white noise definition of the stochastic current. Detailed comments from reviewers can be seen in the email below and also in the docs file in the attachment.
> > >
> > > Best regards,
> > > Herry
> > >
> > > ---
> > >
> > > **From:** em.idaqp.0.7a2321.c7ff7f4e@editorialmanager.com <em.idaqp.0.7a2321.c7ff7f4e@editorialmanager.com> on behalf of Infinite Dimensional Analysis, Quantum Probability and Related Topics (IDAQP) <em@editorialmanager.com>
> > > **Sent:** Tuesday, March 22, 2022 10:54 AM
> > > **To:** Herry P Suryawan <herrypribs@usd.ac.id>
> > > **Subject:** Your Submission: IDAQP-D-21-00003
> > >
> > > Ref.: Ms. No. IDAQP-D-21-00003
> > > A white noise approach to stochastic currents of Brownian motion
> > > Infinite Dimensional Analysis, Quantum Probability and Related Topics
> > >
> > > Dear Dr. Herry Suryawan,

We are sorry for the delay in handling your paper. It was due to some miscommunication. Reviewers have now commented on your paper. You can see that they are in favor of publication but have asked for minor revision to your manuscript.

For your guidance, their comments are appended below.

Please submit a list of changes or a rebuttal against each point raised when you submit the revised manuscript.

Your revision is due by .

To submit a revision, go to https://www.editorialmanager.com/idaqp/ and log in as an Author. From the main menu, select the item "Submissions Needing Revision" and you will find your submission in that folder.

Yours sincerely

B V Rajarama Bhat
Managing Editor
Infinite Dimensional Analysis, Quantum Probability and Related Topics

Reviewers' comments:

Several heuristic objects of stochastic
analysis may be defined as generalized
functions in WNA framework.
The Main question is if we can use such
definition. In most cases the answer is
negative. I know few exceptions and do
not sure that the case considered in the
paper is such an example. My request is the following:

the authors shall show any reason for their
results from stochastic point of view.
Attached.

____

<IDAQP-D-21-00003_R1.pdf>

Re: Your Submission: IDAQP-D-21-00003R1

Bhat IDAQP <bhatidaqp@gmail.com>
Mon 10/3/2022 1:06 PM
To: Herry P Suryawan <herrypribs@usd.ac.id>;Pan Suqi <sqpan@wspc.com>
Cc: Infinite Dimensional Analysis, Quantum Probability and Related Topics (IDAQP) <idaqp@wspc.com>

Dear Prof. Suryanwan
the Publisher should be contacting you for source files and to have the final proofreading.
Your article will appear in one of the forthcoming issues. Probably in the first or second issue of 2023.
Best regards,
R. Bhat.

On Fri, Sep 30, 2022 at 12:29 PM Herry P Suryawan <herrypribs@usd.ac.id> wrote:

> Dear Prof. Rajarama Bhat,
>
> we would like to know the progress of our accepted paper, what is the next step and when will it be published? Thank you.
>
> Sincerely,
> Herry Pribawanto Suryawan
>
> ---
>
> **From:** em.idaqp.0.7c44c8.00f85587@editorialmanager.com <em.idaqp.0.7c44c8.00f85587@editorialmanager.com> on behalf of Infinite Dimensional Analysis, Quantum Probability and Related Topics (IDAQP) <em@editorialmanager.com>
> **Sent:** Monday, June 27, 2022 11:02 AM
> **To:** Herry P Suryawan <herrypribs@usd.ac.id>
> **Subject:** Your Submission: IDAQP-D-21-00003R1
>
> Ref.: Ms. No. IDAQP-D-21-00003R1
> A white noise approach to stochastic currents of Brownian motion
> Infinite Dimensional Analysis, Quantum Probability and Related Topics
>
> Dear Dr. Herry Suryawan,
>
> I am pleased to inform you that your work has now been accepted for publication in Infinite Dimensional Analysis, Quantum Probability and Related Topics.
>
> It was accepted on Jun 27, 2022.
>
> Should you wish to publish your paper on an Open Access basis, please refer to our OA options for IDAQP at https://www.worldscientific.com/IDAQP/openaccess and inform us of your choice at idaqp@wspc.com. The page includes information regarding special arrangements and subsidies World Scientific has with select institutions and organisations for publishing in OA. For enquiries on possible Open Access publication funding from your institution, please contact us at openaccess@wspc.com.

Comments from the Editor and Reviewers can be found below.

Thank you for submitting your work to this journal.

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With kind regards

B V Rajarama Bhat
Managing Editor
Infinite Dimensional Analysis, Quantum Probability and Related Topics

Comments from the Editors and Reviewers:

Re: Your Submission: IDAQP-D-21-00003R1

Nithin ACES <nithin@acesworldwide.net>
Thu 10/6/2022 1:27 PM
To: Herry P Suryawan <herrypribs@usd.ac.id>
Cc: Pan Suqi <sqpan@wspc.com>;Bhat IDAQP <bhatidaqp@gmail.com>

Dear Dr. Herry Suryawan

Thanks for the prompt reply.
I will move the paper for production and send you the 1st proof for checking once it is ready.

Regards
Nithin

On Thu, Oct 6, 2022 at 11:50 AM Herry P Suryawan <herrypribs@usd.ac.id> wrote:

> Dear Nithin,
>
> please find in the attachment the tex file of the paper. Thank you.
>
> Best regards
> Herry
>
> ---
>
> **From:** Nithin ACES <nithin@acesworldwide.net>
> **Sent:** Thursday, October 6, 2022 11:54 AM
> **To:** Herry P Suryawan <herrypribs@usd.ac.id>
> **Cc:** Pan Suqi <sqpan@wspc.com>; Bhat IDAQP <bhatidaqp@gmail.com>
> **Subject:** FW: Your Submission: IDAQP-D-21-00003R1
>
> Dear Dr. Herry Suryawan
>
> We are currently working on your paper "A white noise approach to stochastic currents of Brownian motion".
>
> For typesetting, we need doc or tex format of the paper.
> Kindly provide the same at your earliest convenience.
>
> Regards
> Nithin
> On Fri, Sep 30, 2022 at 12:29 PM Herry P Suryawan <herrypribs@usd.ac.id> wrote:
>
> Dear Prof. Rajarama Bhat,
>
> we would like to know the progress of our accepted paper, what is the next step and when will it be published? Thank you.

Sincerely,

Herry Pribawanto Suryawan

--

Nithin Jayan
Journal Production Administrator
Academic Consulting and Editorial Services (ACES) Pvt Ltd
(World Scientific Group Company)
First Floor 22/40, Arshiya Chambers Thanikachalam Road,
T. Nagar, Chennai 600 017
India
Mobile: 8056250778

Websites:
ACES: www.acesworldwide.net
World Scientific: www.wspc.com

--

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T. Nagar, Chennai 600 017
India
Mobile: 8056250778

Websites:
ACES: www.acesworldwide.net
World Scientific: www.wspc.com

Re: IDAQP Vol: 26 Iss: 01 pp(2250025) A white noise approach to stochastic currents of Brownian motion

José Luís da Silva <joses@staff.uma.pt>
Tue 3/28/2023 2:29 PM
To: Herry P Suryawan <herrypribs@usd.ac.id>

Thank you very much.
José Luís
PS: this week we have a small workshop in Madeira with Bock, Tyll and others about Covid-19. Unfortunately I have 10h/week lectures and can't join all the time.

> On 28 Mar 2023, at 06:21, Herry P Suryawan <herrypribs@usd.ac.id> wrote:
>
> Dear Martin, Dear José Luís,
>
> our paper is finally published (IDAQP Vol 26 No 01, 2023).
>
> Best regards
> Herry
>
> ---
>
> **From:** sales@wspc.com.sg <sales@wspc.com.sg>
> **Sent:** Monday, March 27, 2023 8:50 AM
> **To:** Herry P Suryawan <herrypribs@usd.ac.id>
> **Subject:** IDAQP Vol: 26 Iss: 01 pp(2250025) A white noise approach to stochastic currents of Brownian motion
>
> Dear Herry Pribawanto Suryawan,
> Greetings from World Scientific Publishing and thank you for your contribution to Infinite Dimensional Analysis, Quantum Probability and Related Topics (IDAQP). You may like to know that your article has been published in IDAQP's Volume No. 26, Issue No. 01, Article No. 2250025, Year 2023.
> The DOI for your article is: 10.1142/S0219025722500254. You could use https://dx.doi.org/10.1142/S0219025722500254 to connect readers of your personal webpage (if you have one) to the article's abstract page, or forward it to your peers.
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>
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>
> 1. do a Search (keywords that is representative of your interest),
> 2. name the search,
> 3. set the frequency for the alert,
> 4. and 'Save'.

Thank you for your attention and best regards,
Cindy Lee (Ms)
World Scientific Publishing
<S0219025722500254.pdf>

Infinite Dimensional Analysis, Quantum Probability and Related Topics

A white noise approach to stochastic currents of Brownian motion

--Manuscript Draft--

Manuscript Number:	IDAQP-D-21-00003R1
Full Title:	A white noise approach to stochastic currents of Brownian motion
Article Type:	Research Paper
Keywords:	stochastic currents; extended Skorokhod integral; White Noise Analysis
Corresponding Author:	Herry Pribawanto Suryawan Sanata Dharma University: Universitas Sanata Dharma INDONESIA
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Corresponding Author's Secondary Institution:	
First Author:	Martin Grothaus, Prof. Dr.
First Author Secondary Information:	
Order of Authors:	Martin Grothaus, Prof. Dr.
	Herry Pribawanto Suryawan
	Jose Luis da Silva, Prof. Dr.
Order of Authors Secondary Information:	
Abstract:	In this paper we study stochastic currents of Brownian motion [[EQUATION]] , [[EQUATION]] , by using white noise analysis. For [[EQUATION]] and for [[EQUATION]] we prove that the stochastic current [[EQUATION]] is a Hida distribution. Moreover for [[EQUATION]] with [[EQUATION]] we show that the stochastic current is not a Hida distribution.
Response to Reviewers:	Our response to the reviewers' comments led to changes in the paper as follow: (1). In the Section 4 (Conclusion and Outlook) we replace the last sentence (In a future paper we plan to extend these results to a larger class of stochastic processes, e.g., fractional Brownian motion and grey Brownian motion.) by the following: There have been some other approaches to study stochastic current, such as Malliavin calculus and stochastic integrals via regularization, see [3-5,7], among others. In [4] ξ was constructed in a negative Sobolev space, i.e., in a generalized function space in the variable $x \in \mathbb{R}^d$. Then the constructed distribution was applied to a model of random vortex filaments in turbulent fluids. Using the improved characterization of regular generalized functions from $\mathcal{G}^\prime$, see [8], we plan to show more regularity of $\xi(x)$, $x \in \mathbb{R}^d \setminus \{ 0 \}$. For general $d \in \mathbb{N}$ we are analysing, whether $\xi(x) \in \mathcal{G}^\prime$. The improved characterization provided in [8] also enables to check, whether a given Hida distributions is even a square integrable function. We already obtained very promising estimates for the S-transform of ξ. These lead us to the following conjecture: For $d=1$ and all $x \in \mathbb{R}$ is $\xi(x) \in L^2(\mu)$. As far as we know, this would be the first pointwise construction of stochastic current. (2). In References we add the following paper: M. Grothaus, J. Müller, and A. Nonnenmacher, An improved characterisation of regular generalised functions of white noise and an application to singular SPDEs, \textit{Stochastics and Partial Differential Equations: Analysis and Computations}, (2021) https://doi.org/10.1007/s40072-021-00200-2

	(3). The numbers of the references are taken as in the revised manuscript.

Infinite Dimensional Analysis, Quantum Probability and Related Topics

A white noise approach to stochastic currents of Brownian motion

Martin Grothaus

Department of Mathematics, University of Kaiserslautern
67653 Kaiserslautern, Germany
grothaus@mathematik.uni-kl.de

Herry Pribawanto Suryawan*

Department of Mathematics, Sanata Dharma University
55281 Yogyakarta, Indonesia
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José Luís da Silva

CIMA, University of Madeira, Campus da Penteada
9020-105 Funchal, Portugal
joses@staff.uma.pt

Received (Day Month Year)
Revised (Day Month Year)
Published (Day Month Year)
Communicated by (xxxxxxxxxx)

In this paper we study stochastic currents of Brownian motion $\xi(x)$, $x \in \mathbb{R}^d$, by using white noise analysis. For $x \in \mathbb{R}^d \backslash \{0\}$ and for $x = 0 \in \mathbb{R}$ we prove that the stochastic current $\xi(x)$ is a Hida distribution. Moreover for $x = 0 \in \mathbb{R}^d$ with $d > 1$ we show that the stochastic current is not a Hida distribution.

Keywords: Stochastic currents; extended Skorokhod integral; white noise analysis.

AMS Subject Classification: 60H40, 60J65, 46F25

1. Introduction

The concept of current is fundamental in geometric measure theory. The simplest version of current is given by the functional

$$\varphi \mapsto \int_0^T (\varphi(\gamma(t)), \gamma'(t))_{\mathbb{R}^d} \, dt, \quad 0 < T < \infty,$$

*corresponding author

in a space of vector fields $\varphi : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ and γ is a rectifiable curve in $\mathbb{R}^d$. Informally, this functional may be represented via its integral kernel

$$\zeta(x) = \int_0^T \delta(x - \gamma(t))\gamma'(t)\,\mathrm{d}t,$$

where δ is the Dirac delta distribution on $\mathbb{R}^d$. The interested reader may find comprehensive account on the subject in the books.[2,16]

The stochastic analog of the current $\zeta(x)$ rises if we replace the deterministic curve $\gamma(t)$, $t \in [0,T]$, by the trajectory of a stochastic process $X(t)$, $t \in [0,T]$, in $\mathbb{R}^d$. In this way, we obtain the following functional

$$\xi(x) := \int_0^T \delta(x - X(t))\,\mathrm{d}X(t). \tag{1.1}$$

The stochastic integral (1.1) has to be properly defined. Now we consider a d-dimensional Brownian motion $B(t)$, $t \in [0,T]$, and the main object of our study is

$$\xi(x) = \int_0^T \delta(x - B(t))\,\mathrm{d}B(t). \tag{1.2}$$

In this work the stochastic integral (1.2) is interpreted as an extension of the Skorokhod integral developed in.[10] It coincides with the extension given by the adjoint of the Malliavin gradient. There have been some other approaches to study stochastic current, such as Malliavin calculus and stochastic integrals via regularization, see,[3–5,7] among others.

An initial study of the stochastic current (1.2) using white noise theory was done in.[6] The authors showed that $\xi(x)$ in (1.2) is well defined as a Hida distribution for all $x \in \mathbb{R}^d$ and all dimensions $d \in \mathbb{N}$. However the proof of Theorem 3.3 in[6] is not carefully written which lead the authors to an inaccurate conclusion. In fact, for $x = 0 \in \mathbb{R}^d$, $d > 1$, we show that $\xi(0)$ is not a Hida distribution. This is confirmed by first orders of the chaos expansion we obtained. Moreover, we got the impression that the authors were not checking integrability of the integrand in (1.1). Hence, they cannot apply Corollary 2.1 below. We in turn could check the assumptions of Corollary 2.1, below, for all nonzero $x \in \mathbb{R}^d$, $d \in \mathbb{N}$, and for $x = 0 \in \mathbb{R}$. The aim of this paper is to fill this gap and obtain kernels of first orders of the chaos expansion of $\xi(x)$.

The organization of the paper is as follows. Section 2 provides some background of white noise analysis. In Section 3 we prove the main results of this paper on the existence of the Brownian currents.

2. Gaussian White Noise Analysis

In this section we summarize pertinent results from white noise analysis used throughout this work, and refer to[10,12,15] and references therein for a detailed presentation.

2.1. *White Noise Space*

We start with the Gel'fand triple

$$S_d \subset L^2_d \subset S'_d,$$

where $S_d := S(\mathbb{R}, \mathbb{R}^d)$, $d \in \mathbb{N}$, is the space of vector valued Schwartz test functions, S'_d is its topological dual and the central Hilbert space $L^2_d := L^2(\mathbb{R}, \mathbb{R}^d)$ of square integrable vector valued measurable functions. For any $f \in L^2_d$ given by $f = (f_1, \dots, f_d)$ its norm is

$$|f|^2 = \sum_{i=1}^{d} \int_{\mathbb{R}} |f_i(x)|^2 \,\mathrm{d}x.$$

Let $\mathcal{B}$ be the σ-algebra of cylinder sets on S'_d. Since S_d equipped with its standard topology is a nuclear space, by Minlos' theorem there is a unique probability measure μ_d on $(S'_d, \mathcal{B})$ with the characteristic function given by

$$C(\varphi) := \int_{S'_d} e^{i\langle w, \varphi\rangle} \,\mathrm{d}\mu_d(w) = \exp\left(-\frac{1}{2}|\varphi|^2\right), \quad \varphi \in S_d.$$

Hence, we have constructed the white noise probability space $(S'_d, \mathcal{B}, \mu_d)$. In the complex Hilbert space $L^2(\mu_d) := L^2(S'_d, \mathcal{B}, \mu_d; \mathbb{C})$ a d-dimensional Brownian motion is given by

$$B(t, w) = (\langle w_1, \eta_t\rangle, \dots, \langle w_d, \eta_t\rangle),\ w = (w_1, \dots, w_d) \in S'_d, \quad \eta_t := 1\!\!1_{[0,t)},\ t \geq 0.$$

In other words, $(B(t))_{t\geq 0}$ consists of d independent copies of 1-dimensional Brownian motions. For all $F \in L^2(\mu_d)$ one has the Wiener-Itô-Segal chaos decomposition

$$F(w) = \sum_{n=0}^{\infty} \langle : w^{\otimes n} :, F_n\rangle, \quad F_n \in (L^2_{d,\mathbb{C}})^{\hat{\otimes} n},$$

where $: w^{\otimes n} :\in (S'_{d,\mathbb{C}})^{\hat{\otimes} n}$ denotes the n-th order Wick power of $w \in S'_{d,\mathbb{C}}$ and $\langle \cdot, \cdot\rangle$ denotes the dual pairing on $(S'_{d,\mathbb{C}})^{\otimes n} \times (S_{d,\mathbb{C}})^{\otimes n}$ which is a bilinear extension of $(\cdot, \bar{\cdot})$, where $(\cdot, \cdot)$ is the inner product on $(L^2_{d,\mathbb{C}})^{\otimes n}$ in the sense of a Gel'fand triple. Here $V_{\mathbb{C}}$ denotes the complexification of the real vector space V and $\hat{\otimes} n$ denotes the n-th power symmetric tensor product. Note that $\langle : \cdot^{\otimes n} :, \cdot\rangle$, $n \in \mathbb{N}_0$, in the second variable extends to $(L^2_{d,\mathbb{C}})^{\hat{\otimes} n}$ in the sense of an $L^2(\mu_d)$ limit.

2.2. *Hida Distributions and Characterization*

By the standard construction with the Hilbert space $L^2(\mu_d)$ as central space, we obtain the Gel'fand triple of Hida test functions and Hida distributions.

$$(S_d) \subset L^2(\mu_d) \subset (S_d)'.$$

We denote the dual pairing between elements of $(S_d)'$ and (S_d) by $\langle\!\langle \cdot, \cdot \rangle\!\rangle$. For $F \in L^2(\mu_d)$ and $\varphi \in (S_d)$, with kernel functions F_n and φ_n, respectively, the dual pairing yields

$$\langle\!\langle F, \varphi \rangle\!\rangle = \sum_{n=0}^{\infty} n! \langle F_n, \varphi_n \rangle.$$

This relation extends the chaos expansion to $\Phi \in (S_d)'$ with distribution valued kernels $\Phi_n \in (S'_{d,\mathbb{C}})^{\hat{\otimes} n}$ such that

$$\langle\!\langle \Phi, \varphi \rangle\!\rangle = \sum_{n=0}^{\infty} n! \langle \Phi_n, \varphi_n \rangle,$$

for every generalized test function $\varphi \in (S_d)$ with kernels $\varphi_n \in (S_{d,\mathbb{C}})^{\hat{\otimes} n}$, $n \in \mathbb{N}_0$.

Instead of repeating the detailed construction of these spaces we present a characterization in terms of the S-transform.

Definition 2.1. Let $\varphi \in S_d$ be given. We define the Wick exponential by

$$e_{\mu_d}(\cdot, \varphi) := \frac{e^{\langle \cdot, \varphi \rangle}}{\mathbb{E}(e^{\langle \cdot, \varphi \rangle})} = C(\varphi) e^{\langle \cdot, \varphi \rangle} = \sum_{n=0}^{\infty} \frac{1}{n!} \langle :\cdot^{\otimes n}:, \varphi^{\otimes n} \rangle \in (S_d)$$

and the S-transform of $\Phi \in (S_d)'$ by

$$S\Phi(\varphi) := \langle\!\langle \Phi, e_{\mu_d}(\cdot, \varphi) \rangle\!\rangle.$$

Example 2.1. For $d \in \mathbb{N}$ the S-transform of d-dimensional white noise $(W(t))_{t \geq 0}$ is given by $SW(t)(\varphi) = \varphi(t)$, for all $\varphi \in S_d$, $t \geq 0$, see.[10] Here $(W(t))_{t \geq 0}$ is the derivative of $(B(t))_{t \geq 0}$ as a Hida space valued process. That is, each of its components takes values in $(S_d)'$.

Definition 2.2 (U-functional). A function $F : S_d \longrightarrow \mathbb{C}$ is called a U-functional if:

(1) For every $\varphi_1, \varphi_2 \in S_d$ the mapping $\mathbb{R} \ni \lambda \mapsto F(\lambda \varphi_1 + \varphi_2) \in \mathbb{C}$ has an entire extension to $z \in \mathbb{C}$.
(2) There are constants $0 < C_1, C_2 < \infty$ such that

$$|F(z\varphi)| \leq C_1 \exp\left(C_2 |z|^2 \|\varphi\|^2\right), \quad \forall z \in \mathbb{C}, \varphi \in S_d$$

for some continuous norm $\|\cdot\|$ on S_d.

We are now ready to state the aforementioned characterization result.

Theorem 2.1 (cf.,[12,18]). *The S-transform defines a bijection between the space $(S_d)'$ and the space of U-functionals. In other words, $\Phi \in (S_d)'$ if and only if $S\Phi : S_d \to \mathbb{C}$ is a U-functional.*

Based on Theorem 2.1 a deeper analysis of the space $(S_d)'$ can be developed. The following corollary concerns the Bochner integration of functions with values in $(S_d)'$ (for more details and proofs see e.g.,[10],[12,18] for the case $d = 1$).

Corollary 2.1. *Let $(\Omega, \mathcal{F}, m)$ be a measure space and $\lambda \mapsto \Phi_\lambda$ be a mapping from Ω to $(S_d)'$. We assume that the S-transform of Φ_λ fulfills the following two properties:*

(1) The mapping $\lambda \mapsto S\Phi_\lambda(\varphi)$ is measurable for every $\varphi \in S_d$.
(2) The U-functional $S\Phi_\lambda$ satisfies

$$|S\Phi_\lambda(z\varphi)| \leq C_1(\lambda) \exp\left(C_2(\lambda)|z|^2\|\varphi\|^2\right), \quad z \in \mathbb{C}, \varphi \in S_d,$$

for some continuous norm $\|\cdot\|$ on S_d and for some $C_1 \in L^1(\Omega, m)$, $C_2 \in L^\infty(\Omega, m)$.

Then

$$\int_\Omega \Phi_\lambda \, \mathrm{d}m(\lambda) \in (S_d)'$$

and

$$S\left(\int_\Omega \Phi_\lambda \, \mathrm{d}m(\lambda)\right)(\varphi) = \int_\Omega S\Phi_\lambda(\varphi) \, \mathrm{d}m(\lambda), \quad \varphi \in S_d.$$

Moreover, the integral exists as a Bochner integral in some Hilbert subspace of $(S_d)'$.

Example 2.2 (Donsker's delta function). As a classical example of a Hida distribution we have the Donsker delta function. More precisely, the following Bochner integral is a well defined element in $(S_d)'$

$$\delta(x - B(t)) = \frac{1}{(2\pi)^d} \int_{\mathbb{R}^d} e^{i(\lambda, x - B(t))_{\mathbb{R}^d}} \, \mathrm{d}\lambda, \quad x \in \mathbb{R}^d.$$

The S-transform of $\delta(x - B(t))$ for any $z \in \mathbb{C}$ and $\varphi \in S_d$ is given by

$$S\delta(x - B(t))(z\varphi) = \frac{1}{(2\pi t)^{d/2}} \exp\left(-\frac{1}{2t}\sum_{j=1}^{d}(x_j - \langle z\varphi_j, \eta_t\rangle)^2\right). \tag{2.1}$$

It is well known that the Wick product is a well defined operation in Gaussian analysis, see for example,[13,11] and.[14]

Definition 2.3. For any $\Phi, \Psi \in (S_d)'$ the Wick product $\Phi \Diamond \Psi$ is defined by

$$S(\Phi \Diamond \Psi) = S\Phi \cdot S\Psi. \tag{2.2}$$

Since the space of U-functional s is an algebra, by Theorem 2.1 there exists an element $\Phi \Diamond \Psi \in (S_d)'$ such that (2.2) holds.

3. Stochastic Currents of Brownian Motion

In this section we investigate in the framework of white noise analysis the following functional

$$\varphi \mapsto \int_0^T (\varphi(B(t)), \mathrm{d}B(t))_{\mathbb{R}^d}, \tag{3.1}$$

on a given space of vector fields $\varphi : \mathbb{R}^d \longrightarrow \mathbb{R}^d$. The functional (3.1) can be represented via its integral kernel

$$\xi(x) := \int_0^T \delta(x - B(t))\,\mathrm{d}B(t), \quad x \in \mathbb{R}^d.$$

We interpret the stochastic integral as an extended Skorokhod integral

$$\begin{aligned}&\int_0^T \delta(x - B(t))\,\mathrm{d}B(t)\\ &:= \left(\int_0^T \delta(x - B(t))\Diamond W_1(t)\,\mathrm{d}t, \dots, \int_0^T \delta(x - B(t))\Diamond W_d(t)\,\mathrm{d}t\right)\\ &=: (\xi_1(x), \dots, \xi_d(x)),\end{aligned}$$

where $W = (W_1, \dots, W_d)$ is the white noise process as in Example 2.1. If the integrand is a square integrable function then this stochastic integral coincides with the Skorokhod integral. In this interpretation, we call $\xi(x)$ stochastic currents of Brownian motion.

Below we show that $\xi(x)$, $x \in \mathbb{R}^d \backslash \{0\}$ is a well defined functional in $(S_d)'$. From now on, C is a real constant whose value is immaterial and may change from line to line.

Theorem 3.1. *For $x \in \mathbb{R}^d \backslash \{0\}$, $0 < T < \infty$, the Bochner integral*

$$\xi_i(x) = \int_0^T \delta(x - B(t))\Diamond W_i(t)\,\mathrm{d}t \tag{3.2}$$

is a Hida distribution and its S-transform at $\varphi \in S_d$ is given by

$$S\left(\int_0^T \delta(x - B(t))\Diamond W_i(t)\,\mathrm{d}t\right)(\varphi) = \frac{1}{(2\pi)^{d/2}} \int_0^T \frac{1}{t^{d/2}} e^{-\frac{|x - \langle \eta_t, \varphi\rangle|^2_{\mathbb{R}^d}}{2t}} \varphi_i(t)\,\mathrm{d}t. \tag{3.3}$$

Proof. First we compute the S-transform of the integrand

$$(0, T] \ni t \mapsto \Phi_i(t) := \delta(x - B(t))\Diamond W_i(t).$$

Using Definition 2.3, Example 2.2, and Example 2.1 for any $\varphi \in S_d$ we have

$$\begin{aligned}t \mapsto S\Phi_i(t)(\varphi) &= S\big(\delta(x - B(t))\big)(\varphi) SW_i(t)(\varphi)\\ &= \frac{1}{(2\pi t)^{d/2}} \exp\left(-\frac{1}{2t}|x - \langle \eta_t, \varphi\rangle|^2_{\mathbb{R}^d}\right) \varphi_i(t),\end{aligned}$$

which is Borel measurable on $(0, T]$. Furthermore, for any $z \in \mathbb{C}$, $t \in (0, T]$ and all

$\varphi \in S_d$ we obtain

$$\begin{aligned}
&\left|S\Phi_i(t)(z\varphi)\right| \\
&\leq \left|\frac{1}{(2\pi t)^{d/2}}\exp\left(-\frac{1}{2t}\left|x-\langle\eta_t, z\varphi\rangle\right|^2_{\mathbb{R}^d}\right) z\varphi_i(t)\right| \\
&\leq \frac{1}{(2\pi t)^{d/2}}\exp\left(-\frac{1}{2t}|x|^2\right)\exp\left(\frac{1}{t}|x|\left|\langle\eta_t, z\varphi\rangle\right|\right)\exp\left(\frac{1}{2t}|z|^2\left|\langle\eta_t,\varphi\rangle\right|^2\right)\left|z\varphi_i(t)\right| \\
&\leq \frac{1}{(2\pi t)^{d/2}}\exp\left(-\frac{1}{2t}|x|^2\right)\exp\left(|x||z||\varphi|_\infty\right)\exp\left(\frac{1}{2}|z|^2|\varphi|^2\right)\exp\left(|z||\varphi|_\infty\right) \\
&\leq \frac{C}{(2\pi t)^{d/2}}\exp\left(-\frac{1}{2t}|x|^2\right)\exp\left(\frac{1}{2}|x|^2\right)\exp\left(\frac{1}{2}|z|^2|\varphi|^2\right)\exp\left(\frac{1}{2}|z|^2|\varphi|^2_\infty\right) \\
&\leq \frac{C}{(2\pi t)^{d/2}}\exp\left(-\frac{1}{2t}|x|^2\right)\exp\left(\frac{1}{2}|x|^2\right)\exp\left(\frac{1}{2}|z|^2\|\varphi\|^2\right),
\end{aligned}$$

where $\|\cdot\|$ is the continuous norm on S_d defined by $\|\varphi\| := \sqrt{|\varphi|^2 + |\varphi|^2_\infty}$. The first factor $\frac{C}{(2\pi t)^{d/2}}\exp\left(-\frac{1}{2t}|x|^2\right)$ is integrable with respect to the Lebesgue measure $\mathrm{d}t$ on $[0,T]$. To be more precise, using the formula

$$\int_u^\infty y^{\nu-1}e^{-\mu y}\,\mathrm{d}y = \mu^{-\nu}\Gamma\left(\nu,\mu u\right), \quad u>0, \mathrm{Re}(\mu)>0,$$

where $\Gamma\left(\cdot,\cdot\right)$ is the complementary incomplete gamma function, one can show that

$$\int_0^T \frac{1}{t^{d/2}}\exp\left(-\frac{1}{2t}|x|^2\right)\mathrm{d}t = 2^{d/2-1}|x|^{2-d}\Gamma\left(\frac{d}{2}-1,\frac{|x|^2}{2T}\right).$$

As the second factor $\exp\left(\frac{1}{2}|x|^2\right)\exp\left(\frac{1}{2}|z|^2\|\varphi\|^2\right)$ is independent of $t\in(0,T]$, the result now follows from Corollary 2.1. □

Corollary 3.1. *For $x=0$ and $d=1$ the stochastic current $\xi(0)$ is a Hida distribution, that is, the Bochner integral*

$$\xi(0) = \int_0^T \delta(B(t))\Diamond W(t)\,\mathrm{d}t$$

is a Hida distribution. Moreover its S-transform at $\varphi \in S_1$ is given by

$$S\left(\int_0^T \delta(B(t))\Diamond W(t)\,\mathrm{d}t\right)(\varphi) = \frac{1}{\sqrt{2\pi}}\int_0^T \frac{1}{\sqrt{t}}e^{-\frac{\langle\eta_t,\varphi\rangle^2}{2t}}\varphi(t)\,\mathrm{d}t.$$

Proof. By adapting the proof of Theorem 3.1 we obtain for any $z\in\mathbb{C}$, $t\in(0,T]$ and all $\varphi\in S_1$

$$|S\Phi(t)(z\varphi)| \leq \frac{C}{(2\pi t)^{1/2}}\exp\left(\frac{1}{2}|z|^2\|\varphi\|^2\right).$$

Since the function $(0,T]\ni t\mapsto t^{-1/2}$ is integrable with respect to the Lebesgue measure, Corollary 2.1 implies the statement of the corollary. □

Remark 3.1. We would like to comment on the chaos expansion of the stochastic current of Brownian motion. To this end we identify the space L^2_d with the Hilbert space $L^2(m) := L^2(E, \mathcal{B}, m)$, where $E := \mathbb{R} \times \{1, \ldots, d\}$, $\mathcal{B}$ is the product σ-algebra on E of the Borel σ-algebra on $\mathbb{R}$ and the power set of $\{1, \ldots, d\}$ and $m = \mathrm{d}x \otimes \Sigma$ is the product measure of the Lebesgue measure on $\mathbb{R}$ and the counting measure on $\{1, \ldots, d\}$. That is, for all $f, g \in L^2(m)$ we have

$$(f, g)_{L^2(m)} = \int_E f(x,i) g(x,i) \,\mathrm{d}m(x,i) = \sum_{i=1}^{d} \int_{\mathbb{R}} f(x,i) g(x,i) \,\mathrm{d}x.$$

The n-th order chaos of a Hida distribution can be computed by the n-th order derivative of its S-transform at the origin. More precisely, for $\Psi \in (S_d)'$ and $\varphi \in (S_d)$ consider the function

$$\mathbb{R} \ni s \mapsto U(s) := S\Psi(s\varphi) \in \mathbb{C}.$$

Then the n-th order chaos $\Psi^{(n)}$ of Ψ applied to $\varphi^{\otimes n} \in S_{d,\mathbb{C}}^{\hat{\otimes} n}$ is given by

$$\langle \Psi^{(n)}, \varphi^{\otimes n} \rangle = \frac{1}{n!} \frac{\mathrm{d}^n}{\mathrm{d}s^n} U(s)\big|_{s=0},$$

see [17, Lemma 3.3.5]. In our situation we have

$$S\Phi_i(s\varphi) = \frac{1}{(2\pi t)^{d/2}} \exp\left(-\frac{1}{2t} |x - \langle \eta_t, s\varphi \rangle|^2_{\mathbb{R}^d} \right) s\varphi_i(t), \quad i \in \{1, \ldots, d\}.$$

Hence, for the stochastic currents of Brownian motion the first chaos are given by

$$\xi^{(0)}(x) = (0, \ldots, 0).$$

$$\xi_i^{(1)}(x) = \left(\underbrace{0, \ldots, 0}_{i-1}, \frac{1}{(2\pi)^{d/2}} \int_0^T \frac{1}{t^{d/2}} e^{-\frac{|x|^2}{2t}} \delta_t \,\mathrm{d}t, 0, \ldots, 0 \right).$$

$$\left(\xi_i^{(2)}(x) \right)_{j,k} = -\frac{1}{4(2\pi)^{d/2}} \int_0^T \frac{1}{t^{d/2+1}} e^{-\frac{|x|^2}{2t}} \left(\mathrm{Id}_{ki} x_j \eta_t \otimes \delta_t + \mathrm{Id}_{ji} x_k \delta_t \otimes \eta_t \right) \mathrm{d}t,$$

where Id denotes the identity matrix on $\mathbb{R}^d$ and δ_t denotes the Dirac distribution at $t > 0$. Note that for $x = 0$ and $d > 1$ the first chaos $\xi_i^{(1)}(0)$ is divergent, hence in this case $\xi(0)$ cannot be a Hida distribution. In all the other cases the integrals are well defined as Bochner integrals in a suitable Hilbert subspace of $(S_d')^{\hat{\otimes} n}$, $n = 0, 1, 2$. This follows from the estimates for integrability we derived in the proof of Theorem 3.1. Indeed the estimates derived to apply Corollary 2.1 imply that the integrands Φ_i, $1 \le i \le d$, are Bochner integrable in some Hilbert subspace H_- of (S_d') equipped with a norm $\|\cdot\|_-$, see proof of [12, Thm. 17]. More precisely, there one shows that $\|\Phi_i\|_-$, $1 \le i \le d$, is integrable. That implies Bochner integrability of the kernels of n-th order in the generalized chaos decomposition in a suitable Hilbert subspace of $S'(\mathbb{R}^n)$.

4. Conclusion and Outlook

In this paper we give a mathematical rigorously meaning to the stochastic current $\xi(x)$, $x \in \mathbb{R}^d \setminus \{0\}$ and $\xi(0)$, $0 \in \mathbb{R}$, of Brownian motion in the framework of white noise analysis. On the other hand, for $x = 0 \in \mathbb{R}^d$, $d > 1$, we showed that $\xi(0)$ is not a Hida distribution. The first orders of the chaos expansion leave open whether the $\xi(x)$, $x \in \mathbb{R}^d \backslash \{0\}$, are regular generalized functions or even square integrable. That is, it is not obvious whether $\xi^{(n)}(x) \in (L^2_{d,\mathbb{C}})^{\hat{\otimes} n}$ or not for $x \in \mathbb{R}^d \backslash \{0\}$ and $n \in \mathbb{N}$.

There have been some other approaches to study stochastic current, such as Malliavin calculus and stochastic integrals via regularization, see,[3–5,7] among others. In[4] ξ was constructed in a negative Sobolev space, i.e., in a generalized function space in the variable $x \in \mathbb{R}^d$. Then the constructed distribution was applied to a model of random vortex filaments in turbulent fluids.

Using the improved characterization of regular generalized functions from $\mathcal{G}'$, see,[8] we plan to show more regularity of $\xi(x)$, $x \in \mathbb{R}^d \setminus \{0\}$. For general $d \in \mathbb{N}$ we are analysing, whether $\xi(x) \in \mathcal{G}'$. The improved characterization provided in[8] also enables to check, whether a given Hida distributions is even a square integrable function. We already obtained very promising estimates for the S-transform of ξ. These lead us to the following conjecture: For $d = 1$ and all $x \in \mathbb{R}$ is $\xi(x) \in L^2(\mu)$. As far as we know, this would be the first pointwise construction of stochastic current.

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Rebuttal letter accompanying the revised version of manuscript IDAQP-D-21-00003

M. Grothaus, H. P. Suryawan, and J. L. da Silva

Below we address the reviewers' comments:

1. **Reviewer 1**: Several heuristic objects of stochastic analysis may be defined as generalized functions in WNA framework. The main question is if we can use such definition. In most cases the answer is negative. I know few exceptions and do not sure that the case considered in the paper is such an example.

   My request is the following: the authors shall show any reason for their results from stochastic point of view.

2. **Reviewer 2**: The paper is carefully written and includes important corrections to the previous studies in this problem. But I have a general remark concerning the topic of the paper.

   Many heuristic expressions in mathematical physics, quantum field theory and stochastics may be defined (in more or less complicated way) as objects of WNA. The latter is known from the beginning of this theory. Such attempts to "give a sense to nonsense" may have certain aesthetical meaning. But there appears a natural question: how we can apply such interpretations?

   It will nice if the authors may give at least certain comments in this direction.

   In general, I support the publication of this paper in IDAQP.

Our response to these comments led to changes in the paper as follow:

1. In the Section 4 (Conclusion and Outlook) we replace the last sentence (In a future paper we plan to extend these results to a larger class of stochastic processes, e.g., fractional Brownian motion and grey Brownian motion.) by the following: There have been some other approaches to study stochastic current, such as Malliavin calculus and stochastic integrals via regularization, see [3-5,7], among others. In [4] ξ was constructed in a negative Sobolev space, i.e., in a generalized function space in the variable $x \in \mathbb{R}^d$. Then the constructed distribution was applied to a model of random vortex filaments in turbulent fluids.

   Using the improved characterization of regular generalized functions from $\mathcal{G}'$, see [8], we plan to show more regularity of $\xi(x)$, $x \in \mathbb{R}^d \setminus \{0\}$. For general $d \in \mathbb{N}$ we are analysing, whether $\xi(x) \in \mathcal{G}'$. The improved characterization provided in [8] also enables to check, whether a given Hida distributions is even a square integrable function. We already obtained very promising estimates for the S-transform of ξ. These lead us to the following conjecture: For $d = 1$ and all $x \in \mathbb{R}$ is $\xi(x) \in L^2(\mu)$. As far as we know, this would be the first pointwise construction of stochastic current.

2. In References we add the following paper: M. Grothaus, J. Müller, and A. Nonnenmacher, An improved characterisation of regular generalised functions of white noise and an application to singular SPDEs, *Stochastics and Partial Differential Equations: Analysis and Computations*, (2021) https://doi.org/10.1007/s40072-021-00200-2

3. The numbers of the references are taken as in the revised manuscript.

Herry Suryawan | Logout

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