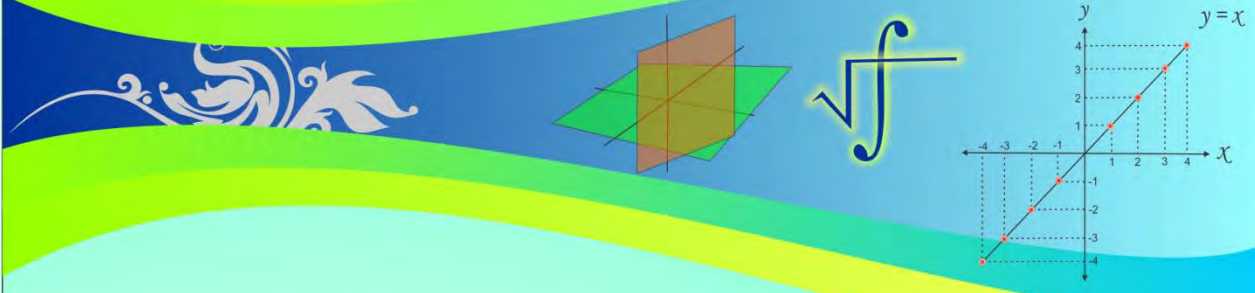




Proceeding

Dept of Mathematics Education
Dept of Math Education Graduate School
Yogyakarta State University



International Seminar on Innovation in Mathematics and Mathematics Education

1st ISIM-MED 2014

“ Innovation and Technology for Mathematics and Mathematics Education”

Joint Conference on Innovation and Technology
For Mathematics and Mathematics Education :

19th ATCM 1st ISIM-MED 3rd ISMEI 2014 2nd SeNdiMat

NOVEMBER, 26 - 30, 2014
YOGYAKARTA STATE UNIVERSITY

Organized by :

Dept of Mathematics Education
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Asian Technology Conference in Mathematics

P4TK Matematika, Yogyakarta



SEAMEO QITEP in Mathematics

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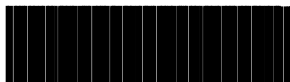
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Department of Mathematics Education
Faculty of Sciences
Yogyakarta State University
2014

Preface

Assalaamu'alaikum Warahmatullaahi Wabarakaatuh.

First of all, we would like to praise *alhamdulillah*, praise is only to Allah SWT, the most gracious and the most merciful, for blessing and giving us energy in conducting a Joint Conference on *Innovation and Technology for Mathematics and Mathematics Education* from 26 to 30 of November 2014, under coordination of Asian Technology Conference in Mathematics (ATCM), Department of Mathematics of Yogyakarta State University, SEAMEO QITEP in Mathematics, and PPPPTK Matematika.

The conference is a joint conference between the 19th ATCM, 1st ISIMMED of Department of Mathematics of Yogyakarta State University, 3rd ISMEI of SEAMEO QITEP in Mathematics, and 2nd SeNdiMat of PPPPTK Matematika. During the joint conference, there are 170 papers are presented – 7 papers presented on plenary sessions, 10 papers presented on invited speech sessions, 33 ATCM papers presented on parallel sessions, 120 papers of ISIMMED and ISMEI presented on parallel sessions. There are also 20 workshop papers and 3 posters presented in the joint conference. In addition, more than 50 SeNdiMat papers are presented separately at PPPPTK Matematika. Each conference has its own proceeding. We are honored to present the proceeding of 1st ISIMMED 2014. This proceeding contains 110 papers consist of 77 papers on mathematics education and 33 papers on mathematics and/or computer.

Presenters in this conference come from more than 60 universities and institutions from 20 countries around the world: Australia, Belgium, Brunei Darussalam, China, France, Hong Kong, India, Indonesia, Japan, Luxembourg, Malaysia, Netherlands, Oman, Papua New Guinea, Philippines, Singapore, Taiwan, Thailand, United Kingdom, and United State of America. We are fortunate to have presenters from a wide spectrum of scientists and educators, whose presentations and workshops demonstrate the most current innovation and technology for mathematics and mathematics education. Papers and presentations address a very wide spectrum of topics and ideas. We can find papers concentrating on using computer software in teaching mathematics, papers on using Internet, multimedia, and other tools for interactive and online mathematics courses delivery, as well as research papers from pure mathematics where technology was used to produce some new results.

Thanks to evolving technological tools, we are able to explore more interdisciplinary areas such as science, technology, and engineering with Mathematics which we could not before. Therefore, integrating technology into mathematics teaching, learning and research will definitely allow us to expand our knowledge horizon in mathematics.

We would like to express our appreciation to all members of the local organizers from Yogyakarta State University, SEAMEO QITEP in Mathematics, and PPPTK Matematika for the enormous task of planning and preparation of this joint conference. We thank also the International Program Committee of ATCM, especially Prof. Wei-Chi YANG for giving opportunity to host the 19th ATCM 2014, one of the most enjoyable and instructive conference in the world. We are very grateful to all plenary and invited speakers for their inspiring papers as well as all reviewers for their great contribution in reviewing papers.

We hope that the papers in this proceeding become very useful references to enrich the creative and innovative ideas that can support the advancement of mathematics education, especially in Indonesia.

Wassalaamu'alaikum Warahmatullaahi Wabarakaatuh.

Yogyakarta, November 26, 2014
Chairman of the YSU Local Committee
Sahid

This Event is presented by :



**Departement of Mathematics Education
Faculty of Mathematics and Natural Science
Yogyakarta State University**

**Graduate Program of Mathematics Education
Yogyakarta State University**



SEAMEO QITEP in Mathematics



P4TK Matematika, Yogyakarta

CONTENTS

Cover			i
Publishing Statement			ii
Paper Reviewers			iii
Preface			iv
Organizer			v
Contents			vii
Invited and Plenary Papers			
<i>(Invited and Plenary Papers can be found in Proceedings of 19th Asian Technology Conference in Mathematics)</i>			
1	Bill Blyth	Australian Scientific & Engineering Solutions (ASES), School of Mathematical and Geospatial Sciences RMIT University Australia bill.blyth@rmit.edu.au	Teaching Experimental Mathematics: Digital Discovery using Maple
2	Wei-Chi YANG	Department of Mathematics and Statistics, Radford University Radford, VA 24142 USA wyang@radford.edu	Technological tools have enhanced our Teaching, learning and doing mathematics, what is next?
3	Keng-Cheng Ang, Liang-Soon Tan,	National Institute of Education Nanyang Technological University 1, Nanyang Walk, Singapore 637616 kengcheng.ang@nie.edu.sg liangsoon.tan@stdmail.nie.edu.sg	Professional Development for Teachers in Mathematical Modelling
4	Antonio R. Quesada	Department of Mathematics The University of Akron aquesada@uakron.edu	A Capstone Course to Improve the Preparation of Mathematics Teachers on the Integration of Technology
5	Colette Laborde	Cabrilog, Grenoble, FRANCE Colette.Laborde@cabri.com	Interactivity and flexibility exemplified with Cabri
6	Paulina Pannen	Directorate General of Higher Education, Ministry of Education and Culture Indonesia ppanen@gmail.com ppannen@dikti.go.id	Integrating Technology in Teaching and Learning Mathematics
7	Marsigit	Department of Mathematics Education, Faculty of Mathematics and Science, Yogyakarta State University, Yogyakarta, Indonesia marsigitina@yahoo.com	Re-conceptualizing Good Practice of Mathematics Teaching Through Lesson Study in Indonesia
8	Leong Chee Kin	Training Programme Division SEAMEO RECSAM Malaysia ckleong@recsam.edu.my	Educating the Educators: Technology-Enhanced Mathematics Teaching and Learning
9	Hongguang Fu, Xiuqin Zhong,	University of Electronic Science and Technology of China, 611731	Mathematics Intelligent Learning Environment

	Zhong Liu	Chengdu, China fu_hongguang@hotmail.com		
10	Widodo¹, Muh.Tamimuddin H²	¹ Professor of Mathematics Gadjah Mada University Yogyakarta Indonesia, and Head of PPPPTK in Mathematics BPSDMPK PMP Ministry of Education and Culture Indonesia widodo_mathugm@yahoo.com ² PPPPTK in Mathematics Indonesia tamim@p4tkmatematika.org	Three Training Strategies for Improving Mathematics Teacher Competences in Indonesia 2015-2019 based on Teacher Competency Test (TCT) 2012-2014	
Code	Name	Institution	Title	Page
Papers in fields of Mathematics Education				
E - 1	Abdurahman Askois Jailani	Mahasiswa PPS UNY, rahmanaskois@gmail.com FMIPA UNY jailani@uny.ac.id	Effectiveness Worksheet with Problem Solving Approach	EP – 1
E- 2	AkhsanulIn'am	Mathematics Department University of Muhammadiyah Malang ahsanul_in@yahoo.com	Students' Problem in Writing the Article of Mathematics Learning Resources Development	EP – 11
E – 3	Ariyadi Wijaya^{a,c,*}, Marja van den Heuvel- Panhuizen^{a,b}, Michiel Doorman^a, Alexander Robitzsch^d *	^a Freudenthal Institute for Science and Mathematics Education, Utrecht University, the Netherlands ^b Faculty of Social and Behavioural Sciences, Utrecht University, the Netherlands ^c Mathematics Education Department, Yogyakarta State University, Indonesia ^d Federal Institute for Education Research, Innovation and Development of the Austrian School System, Austria a.wijaya@uu.nl / a.wijaya@staff.uny.ac.id ; m.vandenheuvel-panhuizen@uu.nl ; m.doorman@uu.nl	Identifying (Indonesian) Students' Difficulties in Solving Context-Based (PISA) Mathematics Tasks	EP – 15
E – 4	Hj. Epon Nur'aeni L, Tiara Penta Yurlita	S1 PGSD Fakultas Ilmu Pendidikan UPI Kampus Tasikmalaya	The Enhancement Ability Of Mathematical Connection In Parallelogram Material Through Learning Based On Van Hiele Theory	EP – 25
E – 5	Nurcholif Diah Sri Lestari Titik Sugiarti	Faculty of Teacher and Training Education, Jember University, Indonesia nurcholifdsl@yahoo.com titiksugiarti.fkip@unej.ac.id	Designing Mathematics Model of Teaching: The syntax of "Problem-Solving Performance Modelling" Model of Teaching	EP – 33
E – 6	Baiduri	Mathematics Education Department University of Muhammadiyah	Thinking Process of Elementary School Students in Word Problem	EP – 41

		Malang baiduriumm@gmail.com	Solving	
E – 7	Supandi ¹ , W. Kusumaningsih ² , L. Ariyanto ³	1,2,3 Department of Mathematics Education, University of PGRI Semarang, Indonesia hspandi@gmail.com	Blended Learning Design for Mathematics in Schools	EP – 49
E – 8	Desi Rahmatina	Universitas Maritim Raja Ali Haji. Tanjungpinang desirahmatina@gmail.com	The Effect Of Students Attitude Toward Mathematics For Mathematics Achievement In Indonesia	EP – 55
E – 9	Dasa Ismaimuza	Tadulako University	Associations Between Students’ Prior Knowledge with Critical and Creative Thinking Ability on Mathematics Junior High School Students Through Problem Based Learning and Cognitive Conflict Strategy	EP - 63
E - 10	Diar Veni Rahayu Ekasatya Aldila Afriansyah	Program Studi Pendidikan Matematika STKIP Garut Indonesia diar_math@yahoo.com e_satya@yahoo.com	Enhancing the Ability of Mathematics Student Problem Solving through Pembelajaran Pelangi Matematika	EP – 69
E – 11	Edy Bambang Irawan	Mathematics Department , State University of Malang ib_ide@yahoo.co.id_	Investigation Of Mathematical Concepts In Order To Increase Tacit Knowledge Of Mathematics Novice Teachers	EP – 77
E – 12	Eka Zuliana	Department of Primary School Teacher Education, Muria Kudus University, Central Java, Indonesia zulianaeka@yahoo.co.id	Caping Kalo As Kudus Cultural Heritage To Construct Circle Concept Of Primary School Students	EP – 83
E – 13	Fina Nurmita	Pascasarjana (S2) Pendidikan Matematika Universitas Bengkulu finanurmita91@gmail.com	Application Of Problem Posing Study Model At Group Study To Increase Result Of Learning Mathematics	EP – 89
E – 14	Jackson Pasini Mairing	Mathematics Education Department of Palangka Raya University jacksonmairing@yahoo.co.id	Students’ Concept Maps in Abstract Algebra	EP – 97
E – 15	Tri Dyah Prastiti, Jackson Pasini, Mairing	UPBJJ UT Yogyakarta; Email: Mathematics Education Department of Palangka Raya University; tridyahprastiti@ut.ac.id jacksonmairing@yahoo.co.id	Tutorial Based on Problems and Role Playing to Increase Yogyakarta Open University Students’ Understanding of Class Action Research	EP – 107
E – 16	Dewi Rahimah, Syafdi Maizora	The Study Program of Mathematics Education, The Department of Mathematics and Science Education The Faculty of Teacher Training and Education, The University of Bengkulu, Indonesia rahimah_dewi@yahoo.com	The Implementation of Cooperative Learning Course Review Horay Type Aided Macromedia Flash Media in Integral Calculus Course	EP – 115

		syafdiichiemaizora@yahoo.com		
E – 17	Susilahudin Putrawangsa ¹ , Monica Wijers ² Agung Lukito ³ , Siti M Amin ⁴ ,	¹ STKIP Hamzanwadi Selong, Indonesia, ² Utrecht University, the Netherlands , ³ Universitas Negeri Surabaya, Indonesia , ⁴ Universitas Negeri Surabaya, Indonesia ¹ sis.putrawangsa@yahoo.com , ² m.wijers@uu.nl , ³ gung_lukito@yahoo.co.id ⁴ amin3105@yahoo.com	Educational Design Research: Developing Students' Understanding Of Measurement Units Of Area	EP – 125
E – 18	Adi Asmara	Pendidikan Matematika FKIP UMB Email: asmaraadi@gmail.com	Mathematical Representation Ability And Self Confidence Students Through Realistic Mathematics Approach	EP - 137
E – 19	Risnanosanti	Pendidikan Matematika FKIP UMB rnanosanti@yahoo.com	Development Pisa Problems With Cultural Context Of Bengkulu	EP – 145
E – 20	Sudirman	Supervisor of Shcool in National Education and Culture Departement Province West Sulawesi sudirmanlise@gmail.com	Improving Ability of Teachers in Action Learning of Mathematic Through Using Environment of School as Source of Learning	EP – 155
E – 21	Hedi Budiman	Mathematics Education Universitas Suryakencana Cianjur hbudiman2011@gmail.com	Developing Mathematical Discovery Ability Using Geometry Expression Software	EP – 163
E – 22	Gusniarti	Student MPd.mat candidate in the Mathematics Department of Education, The School of Teaching and Training (FKIP), University of Bengkulu gusniarti89@gmail.com	Applying Geogebra Software To Improve Students Of Learning Outcomes And Activities	
E – 23	Hamidah	STKIP Siliwangi Bandung Jl. Terusan Jenderal Sudirman Cimahi shiroimida@gmail.com	Relationships Between Retention Of Mathematical Critical Thinking And Self Regulated Learning Through Contextual Approach	EP – 185
E – 24	Harkirat S Dhindsa, P. K. Veloo, Parmjit Singh	MESH, Learning and Teaching Unit, University of Western Sydney, Australia SEGi University, Kota Damansara, Kuala Lumpur, Malaysia University of Technology MARA, Selangor, Malaysia hdhindsa11@gmail.com	The Distribution of Addition and Subtraction Word Problems in Bruneian Elementary Mathematics Texts	EP – 193
E – 25	Hasratuddin ¹	¹ Department of Mathematics Education, Medan State University, Indonesia siregarhasratuddin@yahoo.com	Mathematics Learning Now And Will Come	EP – 209

E – 26	Hayatun Nufus	Dosen Pendidikan Matematika Universitas Islam Riau ya2tunnufus@yahoo.com	Student Attitudes Towards Learning Mathematics, Mathematics Learning Problems And Reasoning Communication And Mathematical	EP – 219
E – 27	Heni Pujiastuti	Department of Mathematics Education, Universitas Sultan Ageng Tirtayasa, Banten hpharyadi@gmail.com	Developing Teaching Material of Inquiry Co-operation Model for Enhancing Student' Mathematical Communication Ability	EP – 227
E – 28	Herfa MD Soewardini	Department of Mathematics Education, Wijaya Kusuma Surabaya University herfa.soewardini@gmail.com	Assimilation and Accomodation Speed Detection of The Seventh Grade Students in Learning Special Triangle	EP – 237
E – 29	Heri Retnawati	Mathematics Department, Mathematics and Science Faculty, Yogyakarta State University retnawati_heriuny@yahoo.co.id	Assembling the Mathematics Test Using the Value of Information Functions	EP – 245
E – 30	Hongki Julie, St. Suwarsono, Dwi Juniati	Mathematics Education Department, Sanata Dharma University, Yogyakarta, and Surabaya State University, Surabaya, Indonesia hongkijulie@yahoo.co.id , stsuwarsono@gmail.com , dwi_juniati@yahoo.com	The Understanding Profiles Of The Subject 1 About The Philosophy, Principles, And Characteristics Of RME Before Subject 1 Learns From The Learning Resource	EP – 255
E – 31	¹ In Abdullah, ² Hery Suharna	^{1,2} Study Program of Mathematics Education ^{1,2} Faculty of Teacher Training and Education. Khairun University of Ternate ¹ Email: inabdullaha@yahoo.com . ² Email: hsuharna@yahoo.co.id .	The Innovative Reflective Thinking Process In Solving Calculus Problems	EP – 265
E – 32	Indah Widiati	Mathematics Education Islamics Riau University Email: indahwidiati@yahoo.com	Developing Mathematical Problem Solving Skills Of Students Junior High School Through Contextual Learning	EP – 273
E – 33	Ekasatya Aldila Afriansyah	Program Studi Pendidikan Matematika, STKIP Garut Indonesia e_satya@yahoo.com	What Students' Thinking about Contextual Problems is	EP – 279
E – 34	Janet Trineke Manoy	State University of Surabaya (Unesa) janet_manoy@yahoo.com	Creative Problem Solving with Higher Order Thinking Problem in Learning Mathematics	EP – 289
E – 35	Kadir, La Masi	Department of Mathematics Education, Universitas Halu Oleo, Indonesia kadirraea@yahoo.co.id lamasimbahido1966@yahoo.co.id	Mathematical Creative Thinking Skills Of Students Junior High School In Kendari City	EP – 295
E – 36	Kartini Hutagaol	Prodi Pendidikan Matematika	Integrating Faith And Learning	EP – 307

		Universitas Advent Indonesia Bandung-Indonesia kartinih_smant@yahoo.com		
E – 37	Khoirul Qudsiyah ¹ , Tika Dedy Prastyo ²	^{1,2} STKIP PGRI Pacitan ¹ azril.dito@gmail.com , ² kuliah.didiet@gmail.com	The Effect Of REOG Learning For Mathematical Analogic Ability From Junior State High School In Pacitan Regency	EP – 317
E – 38	Ariyani Muljo	STAIN Zwaiyah Cot Mala Langsa	The Development Of Teaching Macromedia Flash For Plane Material In Class X SMKN 1 Langsa	EP – 325
E – 39	Kodirun	Halu Oleo University	Effectiveness of Progressive Learning Approach toward Enhancement of Students' Competency on Mathematics Journal Writing	EP – 335
E – 40	La Misu	Mathematics Education University Of Halu Oleo Kendari lamisuhamid@yahoo.co.id	Mathematical Problem Solving Of Student Approach Behavior Learning Theory	EP – 341
E – 41	Nurina Happy	Universitas PGRI Semarang	The Effectiveness Of PBL On Mathematical Creative Thinking Skills And Self-Esteem Of Junior High School Students	EP – 349
E- 42	Heris Hendriana, Euis Eti Rohaeti	Siliwangi STKIP Bandung herishen@yahoo.com	Values And Characters-Nuanced Innovative Teaching To Develop Hard Skills And Soft Skills Of Junior And Senior High Students' Math	EP – 359
E – 43	Euis Eti Rohaeti	Siliwangi STKIP Bandung e2rht@yahoo.com	Developing A Balanced Hard Skill And Soft Skill Of Students' Math Through The Character-Oriented Scientific Approach	EP – 369
E – 44	Mimih Aminah, Yaya S. Kusumah	STKIP Sebelas April Sumedang mimih.aminah@yahoo.co.id Universitas Pendidikan Indonesia yayaskusumah@yahoo.com	The Implementation of Metacognitive Learning Approach in Developing Students' Mathematical Communication Ability	EP – 377
E – 45	Mujiyem Sapti	Mathematics Education Program Muhammadiyah University of Purworejo, Indonesia saptimoedji@yahoo.com	Teacher's Informal Learning Trajectory and Student's Actual Learning Trajectory On Learning Cube And Cuboid Nets	EP – 389
E – 46	Mustamin Idris [*] , Jusman Mansyur [*] , Darmawan [*] dan Sarintan N. Kaharu ^{**}	[*] FKIP of Tadulako University ^{**} STMIK Bina Mulia Palu	The Readiness and the Ability of Elementary School Teachers in Integrating Mathematics into Other Subjects on the Implementation of 2013 Curriculum	EP – 397
E – 47	Mustamin Anggo, Latief Sahidin	FKIP Universitas Halu Oleo Kendari	Development Of Learning Mathematics To Train Students' Metacognitive Ability	EP – 403

E – 48	Pham Sy NAM, Ha Xuan THANH, Max STEPHENS	Phan Boi Chau Gifted High School, Nghe An Province, Vietnam Ministry of Education and Training, Vietnam Graduate School of Education, The University of Melbourne, Australia	Teaching experiments in constructing mathematical problems that relate to real life	EP – 411
E – 49	Pham Sy NAM	Phan Boi Chau Gifted High School, Nghe An Province, Vietnam phamsynampbc@gmail.com	Teaching Experiments in exploring convex and concave functions	EP – 421
E – 50	Agustina Sri Purnami	Majoring in Mathematics Education Sarjanawiyata Tamansiswa University Email: purnami_mat@yahoo.com	Coaching Model To Certified Junior High School Math Teachers In Yogyakarta	EP – 429
E – 51	Neneng Tita Rosita ¹ , Mimih Aminah ² , Agus Jaenudin ³	^{1,2,3} STKIP Sebelas April Sumedang ¹ pyusepa@yahoo.com , ² mimih.aminah@yahoo.co.id , ³ ghoesjen@yahoo.com	An Analysis of Mathematical Problem Solving Ability of High Capability Students of the Islamic Elementary Schools at Sumedang	EP – 437
E – 52	Nila Kesumawati	Dosen Universitas PGRI Palembang nilakesumawati@yahoo.com	Realistic Mathematics Education Of Indonesia, Mathematically Disposition, and Mathematically Creative Thinking of Junior High School	EP – 445
E – 53	Hepsi Nindiasari, Novaliyosi, Nurul Anriani	Department of Mathematics Education, University Sultan Ageng Tirtayasa, Banten	Stages of Reflective Thinking Mathematically	EP – 453
E – 54	Erni Nuraeni, Rahayu Kariadinata, Iyon Maryono	State Islamic University- Bandung , Jl. A.H. Nasution No.105 Cibiru - Bandung	Implementation Of Project Based Learning As An Effort To Improve Student’s Strategic Competence And Productive Disposition	EP – 461
E – 55	Raoda Ismail, Okky Riswandha Imawan	Graduate Student in The Graduate Program of Yogyakarta State University raodaismail26@gmail.com , okkyriswandhaimawan@gmail.com	Project-Based Learning on Learning Mathematics	EP – 473
E – 56	Rezi Ariawan	Lecture of Mathematics Education, Islamic University of Riau Jl. Kaharuddin Nasution No. 113 Pekanbaru 28284 Email: ariawanrezi@rocketmail.com	The Implementation of Visual Thinking Approach in Learning Activity with a Quick on the Draw to Improve the Problem Solving Ability of Junior High School Students	EP – 483
E – 57	S. Uttunggadewa, E. Soewono, R. Hadianti, N. Nuraini, K. A. Sidarto, N. Sumarti	Institut Teknologi Bandung	The role of the Center for Mathematical Modeling and Simulation, Institut Teknologi Bandung, at Mathematical Modeling Course at Department of Mathematics, Institut Teknologi Bandung	EP – 493
E – 58	Salman Sakif,	State University of Malang	Defragmenting Of Thinking	EP – 505

	Subanji, Sisworo	salman_arudam@yahoo.com ¹ , subanji_mat@yahoo.co.id ² , sisworo_um@yahoo.com ³	Process Through Cognitive Mapping To Fix Student's Error In Solving The Problem Of Algebra	
E – 59	Sari Herlina	University Islam of Riau	Effectiveness Of React Strategy For Improve Of Problem Solving Ability On Mathematics In Junior High School	EP – 521
E – 60	Nahor Murani Hutapea	Mathematics Education Teacher Training and Education University of Riau, Pekanbaru e-mail: nahor_hutapea@yahoo.com	The Enhancement Of Mathematical Reasoning Ability Of Senior High School Students' Through Generative Learning	EP – 529
E – 61	Sindi Amelia	Lecturer of Mathematics Education Islamic University of Riau sindiamelia@gmail.com	The Influence Of Accelerated Learning Cycle On Junior High School Students' Mathematics Connection Abilities	EP – 539
E – 62	Siti Khomariyah and Janet Trineke Manoy	Department of Mathematics, State University of Surabaya	Application Of Model Problem Based Learning (PBL) With Creative Problem Solving (CPS) In Arithmetic Sequence And Series	EP – 535
E – 63	Somakim	Program Pascasarjana Universitas Sriwijaya	Improving Competence Mathematical Self-Efficacy of Junior Secondary School Students by Applying Realistic Mathematics Approach	EP – 545
E – 64	Susda Heleni	Mathematics Education Study Program Guidance and Counseling University of Riau	Application Model Learning Creative Problem Solving (CPS) Math Learning to Improve Results Class VIII SMPN 3 Pekanbaru	EP – 553
E – 65	Syaiful	Mathematic Education and Natural Sciences Department, FKIP University of Jambi	Student Comprehension About Line And Row From Apos Theory Point Of View	EP – 563
E – 66	Theresia Laurens, Christina Laamena, Christi Matitaputty	Faculty of Teaching and Training Pattimura University	Development a Set of Instructional Learning Based Realistic Mathematics Education and Local Wisdom	EP – 571
E – 67	Rasiman	Universitas PGRI Semarang	Development Of Mathematics Learning Equipment Based On Critical Thinking Using Savi Approach Assisted By Interactive CD	EP – 577
E – 68	Wardani Rahayu, Meyta Dwi Kurniasih	Universitas Negeri Jakarta, Universitas Prof. DR. UHAMKA	The Influence of React Strategy Towards Mathematical Belief	EP – 587
E – 69	Warman	State Junior High School 1 Gandusari, Blitar	Mathematics Learning through The Problems of Environment	EP – 595
E – 70	Winita Sulandari, Hartatik, Nughthoh Arfawi Kurdhi, Yudho Yudhanto,	Faculty of Mathematics and Natural Sciences Sebelas Maret University	Build an Interactive Application "Matica" for Teaching and Learning Mathematics	EP – 601

	Berliana Kusuma Riasti, Titin Sri Martini			
E – 71	Yaya S. Kusumah and Vara Nina Yulian	Study Program of Mathematics Education, School of Postgraduate Studies, UPI	Enhancing Students' Mathematical Reasoning by Algebrator-Assisted Inquiry Method	EP – 607
E – 72	Yenita Roza, Sakur, Syarifah Nur Siregar, Suci Nitia Edwar	Universitas Riau, Pekanbaru	Developing Mathematics Teaching Material for 4 th Grade of Elementary School Based on Traditional Folk Games of Riau Province	EP – 613
E – 73	Yenita Roza, Titi Solfitri, Eka Puspita Sari.	Universitas Riau, Pekanbaru	Developing Mathematics Learning Material for 5 th Grade of Elementary School Based on Traditional Games of Riau Province	EP – 621
E – 74	Zulkarnain	Mathematics Education Majors FKIP UR	The Influence of Realistic Mathematics Approach to Mathematics Problem Solving Capability of Students Class VII State Junior High School 3 Mandau	EP – 631
E – 75	Sutarto Hadi, Karim, Kamaliyah, Rizki Amalia	Lambung Mangkurat University, Banjarmasin, Indonesia e-mail: shadiunlam@gmail.com	Using the Ornaments of Historical Mosque to Learn Two-Dimensional Shapes	EP – 637
E – 76	Nur Hadi Waryanto ¹ Wahyu Setyaningrum ²	Department of Mathematics Education Yogyakarta State University ¹ nurhadiw@gmail.com , ² setyaningrum.w@gmail.com	E-Learning Readiness In Indonesia: A Case Study In Junior High School Yogyakarta	EP – 645
E – 77	Himmawati Puji Lestari ¹ , Kuswari Hernawati ²	^{1,2} Department of Mathematics Education Yogyakarta State University ¹ himmawati@uny.ac.id , ² kuswari@uny.ac.id	The Student's Response To Solid Geometry Learning Using Information Communication Technology (ICT)	EP – 655
Papers in fields of Statistic				
S - 1	Edy Widodo ¹ , Suryo Guritno ² , Sri Haryatmi ²	¹ Doctoral Student of Mathematics UGM, ¹ Department of Statistics UII, ² Department of Mathematics UGM	M-estimation of Multivariate Response Surface Models with Data Outliers	SP – 1
S - 2	Yuliana Susanti, Sri Sulistijowati H. and Hasih Pratiwi	Mathematics Department, Sebelas Maret University	Analysis of Rice Availability in Indonesia Using Multi-Dimensional Scaling	SP – 11
S - 3	Mohamad Fatekurohman, Subanar Danardono	Department of Mathematics Jember University, Department of Mathematics Gadjah Mada University	An Algorithm of Nonparametric Maximum Likelihood Estimation for Bivariate Censored Data	SP – 21
S - 4	Nita Delima	Department of Mathematics	Martingale and The Efficient	SP – 27

		Education, Subang University	Market Hypothesis (EMH)	
S - 5	Septianusa, Maulina Supriyaningsih, Ayu Septiani, RB Fajriya Hakim	Department of Statistics, Faculty of Mathematics and Sciences, Universitas Islam Indonesia	Mining the Traffic Conditions via Twitter based on Rough Set Theory	SP – 31
S - 6	SitiNurashikenBinti Md. Sabudin ¹ , Dr. Zaleha Ismail ²	¹ Institut Pendidikan Guru KampusTengkuAmpuanAfzan, Kuala Lipis, Malaysia ² Universiti Teknologi Malaysia, Johor	Statistical Reasoning Learning Enviroment (SRLE) in Teaching Video Improved Statistical Reasoning Skills	SP – 43
S - 7	Sukono, Akik Hidayat, Wulan Marselly Firdaus	Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Indonesia	Risk Analysis of Credit Default on Rural Bank by Using Back Propagation Neural Networks Approach	SP – 53
S - 8	Zaky Musyarof, Dwi Yono Sutarto, Dwima Rindy Atika, RB Fajriya Hakim	Department of Statistics, Faculty of Mathematics and Sciences, Universitas Islam Indonesia	To Select Evacuation Route	SP – 63
S – 9	Dhoriva Urwatul Wutsqa ¹), Rosita Kusumawati ²), Retno Subekti ³)	Department of Mathematics, Yogyakarta State University, Indonesia dhoriva@yahoo.com ¹), rosita.kusumawati@gmail.com ²), safina.rere@gmail.com ³)	Forecasting Consumer Price Index of Education, Recreation, and Sport, using Feedforward Neural Network Model	SP – 73
Papers in fields of Analysis and Algebra				
A – 1	M. Andy Rudhito	Department of Mathematics Education, Sanata Dharma University, Indonesia rudhito@usd.ac.id	Fuzzy Number Min-Plus Algebra and Matrix	AP – 1
A – 2	Denik Agustito	Jurusan Pendidikan Matematika, Fakultas Keguruan dan Ilmu Pendidika, Universitas Sarjanawiyata Tamansiswa Email : denikagustito@yahoo.co.id	Constructon Of Finite Limit And Their Properties	AP – 9
A – 3	Muzamil Huda	Teacher of Mathematics from MAN Babat, Lamongan, Email: muzamilhudam.pd90@yahoo.co.id	Finding New Numbers From Square Matrix	AP – 15
A – 4	Musthofa	Math Education Department Yogyakarta State University	An application of Max Plus Algebra in Cryptography	AP – 23
A – 5	Yeni Susanti	Mathematics Departments, Gajah Mada University, Indonesia	On Regular Elements of Semigroups of Finitary Operations	AP – 29
A – 6	L. H. Wiryanto, W. Djohan	Department of Mathematics, ITB, Indonesia Email: leo@math.itb.ac.id , warsoma@math.itb.ac.id	Numerical solution of KdV equation model of interfacial wave	AP – 37
A – 7	Herry Suryawan	Sanata Dharma University	Donsker’s Delta Function of the Generalized Mixed Fractional	AP – 45

			Brownian Motion	
Papers in fields of Geometry				
G - 1	Dwi Juniati I Ketut Budayasa	Department of Mathematics, State University of Surabaya, Indonesia dwi_juniati@yahoo.com , ketutbudayasa@yahoo.com	The Dimension of Fractal Geometry and Its Applications	
Papers in fields of Applied Mathematics and Computer				
C - 1	Bethany Elvira ¹ , Yudi Satria ² , dan Rahmi Rusin ³	¹ Student in Department of Mathematics, University of Indonesia, Depok, Indonesia e-mail : bethany.elvira@sci.ui.ac.id ² Lecturer in Department of Mathematics, University of Indonesia, Depok, Indonesia e-mail : y-satria@ui.ac.id ³ Lecturer in Department of Mathematics, University of Indonesia, Depok, Indonesia e-mail : rahmirus@gmail.com	Modified Genetic Algorithm to Solve Time-varying Lot Sizes Economic Lot Scheduling Problem	CP - 1
C - 2	¹ Dewi Nurmalitasari, ² Basuki Widodo	¹ Jurusan Matematika, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia, ² Jurusan Matematika, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia ¹ b_widodo@matematika.its.ac.id	Pollutant And Sedimentation Dispersion Pattern In The Confluence Of Two Rivers	CP - 13
C - 3	Achmad Buchori ¹ , Sutrisno ² , Noviana Dini Rahmawati ³	Study Program of Mathematics Education PGRI University of Semarang buccherypgri@gmail.com , sutrisno100jt@gmail.com , fadiniz@gmail.com	Development of Matiklopedia Based Character Building in Junior High School	CP - 23
C - 4	Budi Lazarusli ¹ , Rosalina Ginting ² and Achmad Buchori ³	Faculty of Social Education , PGRI Semarang University , Semarang, Indonesia. e-mail: blazarusli@yahoo.com	Development of Model Learning Character Building Based e-comic in The Elementary School	CP - 31
- 5	Hisyam Hidayatullah	SMAN 8 Batam, Bengkong Sadai-Bengkong; Kota Batam email : maysihwahidiyah@gmail.com	Exploiting Domino Games "Cassava H154M" in order to improve students' understanding about the value of trigonometry in various quadrants	CP - 41
C - 6	¹ Rani Kurnia Putri dan ² Basuki Widodo	¹ Post Graduate Student, Department of Mathematics ² Lecture of Mathematic's Department, Institut Teknologi Sepuluh Nopember (ITS) Email: ¹ rani.kurnia@ymail.com ² b_widodo@matematika.its.ac.id	The Influence Of Hydrodynamics On The Spread Of Pollutants And Sedimentation In The Confluence Of Two Rivers	CP - 53
C - 7	Lusi Rachmiazasi	Unit Program Belajar Jarak Jauh -	The Benefits of Compact Disc	CP - 63

	Masduki	UniversitasTerbuka Semarang lusi@ut.ac.id	Interactive Mathematic as a Builder Democratic Mindset Sub Tema: (Using Technology in Mathematics Education)	
C – 8	Miranda Eliyan ^{1*} , Basuki Widodo ^{2*}	Institut Teknologi Sepuluh Nopember,Surabaya, Indonesia ^{1*} eliyanmiranda@gmail.com Institut Teknologi Sepuluh Nopember,Surabaya, Indonesia ^{2*}	The Implementation of Meshless Local Petrov Galerkin (MLPG) Method for Determine Pollutant Sources in Brantas River	CP – 69
C – 9	Rifky Fauzi ^{1*} , Mashuri ¹ , Idha Sihwaningrum ¹	¹ Universityof Jenderal Soedirman, Indonesia , *rifkyfauzi9@gmail.com	On the Pseudo-spectral Methods for Solving MKdV Equation	CP – 77
C – 10	Rosita Kusumawati ^{1*} , Aran Puntosadewo ²	¹ Mathematics Education Department, Yogyakarta State University , ² Mathematics Education Department, Yogyakarta State University *rosita.kusumawati@gmail.com	The Application Of Goal Programing For Portfolio Selection Problem In Indonesia	CP – 85
C – 11	Kuswari Hernawati, Nur Insani, Nur Hadi Waryanto, Bambang Sumarno	Department of Mathematics Education, Yogyakarta State University kuswari@uny.ac.id, nurinsani@uny.ac.id, nurhw@uny.ac.id, bambang@uny.ac.id	Application Of Association Rules With Apriori Algorithm To Determine The Pattern Of The Relationship Between SBMPTN Database And Student's Grade Point Average	CP – 91
C – 12	Sri Andayani	Department of Mathematics Education, Yogyakarta State University Indonesia andayani_uny@yahoo.com	The 2-Tuple Linguistic Representation Approach for Learning Competence Evaluation	CP – 101
C – 13	Imam Solekhudin	Depertment of Mathematics, Gadjah Mada University, Indonesia Email: imams@ugm.ac.id	A Numerical Method For Infiltration Problems	CP – 111
C – 14	¹ Kistosil Fahim, ² Lukman Hanafi, ³ Subiono, ⁴ Fatma Ayu	^{1,2,3,4} Department of Mathematics, Institut Teknologi Sepuluh Nopember, Indonesia Email: ¹ kfahimt@gmail.com	Monorail and Tram Scheduling which Integrated Surabaya using Max-Plus Algebra	CP – 119
C – 15	Fitriana Yuli Saptaningtyas	Department of Mathematics Educational, Yogyakarta State University anamathuny@gmail.com	Picard Iteration To Solve Linier And Nonlinier IVP Problem	CP – 127
C – 16	Nur Insani	Department of Mathematics Education, Yogyakarta State University, INDONESIA nurinsani.utomo@gmail.com	A Comparison of Heuristics Algorithms to Solve Vehicle Routing Problem with Multiple Trips and Intermediate Facility	CP - 131

Fuzzy Number Min-Plus Algebra and Matrix

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Abstract

This paper discusses the algebra of the set of all fuzzy numbers with the operations minimum and addition (plus) and matrix over its algebra. This algebra is an extension of the min-plus algebra through interval min-plus algebra and Decomposition Theorem in fuzzy sets. It can be shown that the minimum and addition operation defined by a closed interval alpha-cut in the set of all fuzzy numbers. The set of all fuzzy numbers with minimum and addition operations is a commutative idempotent semiring. Furthermore, the set of all matrices over this algebra is a semimodule. Given also an example calculation using MATLAB program.

Keywords: semiring, min-plus algebra, fuzzy number, matrix, semimodule.

1. Introduction

Min-plus algebra, which is the set of all real numbers \mathbf{R} equipped with operation min (minimum) and plus (addition), has been used well to model and analyze the shortest path problem [1] and [2]. In the problem of modeling and analysis of a network sometimes travel time unknown, for example because it is still at the design stage. Travel time can be estimated based on the experience and opinion of the experts and the network operator. In this case the travel time of the network can be modeled by a fuzzy number, and its travel time is called fuzzy travel time.

Modeling and analysis of the shortest path problem with fuzzy travel time, as far as we know, no one has discussed using min-plus algebra approach such as has been done for deterministic and probabilistic models. As has been known to approach network solution using min-plus algebra can provide analytical results and more ease in computation, compared to other approaches which tend to be heuristic.

Min-plus algebra approach to solve the shortest path problem is also using the basic concepts in the min-plus algebra, such as matrix on min-plus algebra and system of min-plus linear equations, as has been discussed in [1] and [2]. Thus, to resolve the fuzzy shortest path problem, the min-plus algebra approach, first min-plus algebra needs to be generalized into fuzzy numbers min-plus algebra, where the elements of the set are discussed in the form of fuzzy numbers and concepts related thereto, such as matrix over fuzzy numbers min-plus algebra. Therefore in this article will discuss the definition and basic concepts in min-plus algebra and matrix fuzzy numbers on min-plus algebra fuzzy numbers. We will first review some basic concepts and results in the interval min-plus algebra and matrix. A more complete discussion can be found in [7] and [8].

2. Interval Min-Plus Algebra and Matrix

Let $\mathbf{R}_\varepsilon := \mathbf{R} \cup \{\varepsilon\}$ with \mathbf{R} the set of all real numbers and $\varepsilon := \infty$. In \mathbf{R}_ε defined two operations : $\forall a, b \in \mathbf{R}_\varepsilon$, $a \oplus b := \min(a, b)$ and $a \otimes b := a + b$. We can show

that $(\mathbf{R}_\varepsilon, \oplus, \otimes)$ is a commutative idempotent semiring [1] with neutral element $\varepsilon = \infty$ and unity element $e = 0$. Moreover, $(\mathbf{R}_\varepsilon, \oplus, \otimes)$ is a semifield, that is $(\mathbf{R}_\varepsilon, \oplus, \otimes)$ is a commutative semiring [1], where for every $a \in \mathbf{R}$ there exist $-a$ such that $a \otimes (-a) = 0$. Thus, $(\mathbf{R}_\varepsilon, \oplus, \otimes)$ is a *min-plus algebra*, and is written as \mathbf{R}_{\min} [2]. One can define $x^{\otimes 0} := 0$, $x^{\otimes k} := x \otimes x^{\otimes k-1}$, $\varepsilon^{\otimes 0} := 0$ and $\varepsilon^{\otimes k} := \varepsilon$, for $k = 1, 2, \dots$.

The operations \oplus and \otimes in \mathbf{R}_{\min} can be extend to the matrices operations in $\mathbf{R}_{\min}^{m \times n}$, with $\mathbf{R}_{\min}^{m \times n} := \{A = (A_{ij}) \mid A_{ij} \in \mathbf{R}_{\min}, \text{ for } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$, the set of all matrices over max-plus algebra. Specifically, for $A, B \in \mathbf{R}_{\min}^{n \times n}$ we define $(A \oplus B)_{ij} = A_{ij} \oplus B_{ij}$ and $(A \otimes B)_{ij} = \bigoplus_{k=1}^n A_{ik} \otimes B_{kj}$. We also define matrix $E \in \mathbf{R}_{\min}^{n \times n}$, $(E)_{ij} := \begin{cases} 0, & \text{if } i = j \\ \varepsilon, & \text{if } i \neq j \end{cases}$ and $\mathcal{E} \in \mathbf{R}_{\min}^{m \times n}$, $(\mathcal{E})_{ij} := \varepsilon$ for every i and j . For any matrices $A \in \mathbf{R}_{\min}^{m \times n}$, one can define $A^{\otimes 0} = E_n$ and $A^{\otimes k} = A \otimes A^{\otimes k-1}$ for $k = 1, 2, \dots$. We can show that $(\mathbf{R}_{\min}^{m \times n}, \oplus)$ is an idempotent commutative semigroup, $(\mathbf{R}_{\min}^{n \times n}, \oplus, \otimes)$ is an idempotent semiring with matrix \mathcal{E} as neutral element and matrix E as unity element, and $\mathbf{R}_{\min}^{m \times n}$ is a semimodule over \mathbf{R}_{\min} [1] and [6].

Definitions and concepts in the interval min-plus algebra are analogous to the concepts in the interval max-plus algebra which can be seen in [6]. The (closed) interval x in \mathbf{R}_{\min} is a subset of \mathbf{R}_{\min} of the form $x = [\underline{x}, \bar{x}] = \{x \in \mathbf{R}_{\min} \mid \underline{x} \preceq_m x \preceq_m \bar{x}\}$. The interval x in \mathbf{R}_{\min} is called *min-plus interval*, which is in short is called *interval*. Define

$$\mathbf{I}(\mathbf{R})_\varepsilon := \{x = [\underline{x}, \bar{x}] \mid \underline{x}, \bar{x} \in \mathbf{R}, \varepsilon \prec_m \underline{x} \preceq_m \bar{x}\} \cup \{\varepsilon\}, \text{ where } \varepsilon := [\varepsilon, \varepsilon].$$

In the $\mathbf{I}(\mathbf{R})_\varepsilon$, define operation $\bar{\oplus}$ and $\bar{\otimes}$ as

$$x \bar{\oplus} y = [\underline{x} \oplus \underline{y}, \bar{x} \oplus \bar{y}] \text{ and } x \bar{\otimes} y = [\underline{x} \otimes \underline{y}, \bar{x} \otimes \bar{y}], \forall x, y \in \mathbf{I}(\mathbf{R})_\varepsilon.$$

Since $(\mathbf{R}_\varepsilon, \oplus, \otimes)$ is an idempotent semiring and it has no zero divisors, with neutral element ε , we can show that $\mathbf{I}(\mathbf{R})_\varepsilon$ is closed with respect to the operation $\bar{\oplus}$ and $\bar{\otimes}$. Moreover, $(\mathbf{I}(\mathbf{R})_\varepsilon, \bar{\oplus}, \bar{\otimes})$ is a commutative idempotent semiring with neutral element $\varepsilon = [\varepsilon, \varepsilon]$ and unity element $0 = [0, 0]$. This commutative idempotent semiring $(\mathbf{I}(\mathbf{R})_\varepsilon, \bar{\oplus}, \bar{\otimes})$ is called *interval min-plus algebra* which is written as $\mathbf{I}(\mathbf{R})_{\min}$.

Define $\mathbf{I}(\mathbf{R})_{\min}^{m \times n} := \{A = (A_{ij}) \mid A_{ij} \in \mathbf{I}(\mathbf{R})_{\min}, \text{ for } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n\}$. The element of $\mathbf{I}(\mathbf{R})_{\min}^{m \times n}$ are called *matrix over interval min-plus algebra*. Furthermore, this matrix is called *interval matrix*. The operations $\bar{\oplus}$ and $\bar{\otimes}$ in $\mathbf{I}(\mathbf{R})_{\min}$ can be extended to the matrices operations of in $\mathbf{I}(\mathbf{R})_{\min}^{m \times n}$. Specifically, for $A, B \in \mathbf{I}(\mathbf{R})_{\min}^{n \times n}$ and $\alpha \in \mathbf{I}(\mathbf{R})_{\min}$ we define

$$(\alpha \bar{\otimes} A)_{ij} = \alpha \bar{\otimes} A_{ij}, (A \bar{\oplus} B)_{ij} = A_{ij} \bar{\oplus} B_{ij} \text{ and } (A \bar{\otimes} B)_{ij} = \bigoplus_{k=1}^n A_{ik} \bar{\otimes} B_{kj}.$$

Matrices $A, B \in \mathbf{I}(\mathbf{R})_{\min}^{m \times n}$ are equal if $A_{ij} = B_{ij}$, that is if $\underline{A}_{ij} = \underline{B}_{ij}$ and $\overline{A}_{ij} = \overline{B}_{ij}$ for every i and j . We can show that $(\mathbf{I}(\mathbf{R})_{\min}^{n \times n}, \oplus, \otimes)$ is a idempotent semiring with neutral element is matrix ε , with $(\varepsilon)_{ij} := \varepsilon$ for every i and j , and unity element is matrix E , with $(E)_{ij} := \begin{cases} 0, & \text{if } i = j \\ \varepsilon, & \text{if } i \neq j \end{cases}$. We can also show that $\mathbf{I}(\mathbf{R})_{\min}^{m \times n}$ is a semi-module over $\mathbf{I}(\mathbf{R})_{\min}$.

For any matrix $A \in \mathbf{I}(\mathbf{R})_{\min}^{m \times n}$, define the matrices $\underline{A} = (\underline{A}_{ij}) \in \mathbf{R}_{\min}^{m \times n}$ and $\overline{A} = (\overline{A}_{ij}) \in \mathbf{R}_{\min}^{m \times n}$, which is called lower bound matrices and upper bound matrices of A , respectively. Define matrices interval of A , that is

$$[\underline{A}, \overline{A}] = \{ A \in \mathbf{R}_{\min}^{m \times n} \mid \underline{A} \preceq_m A \preceq_m \overline{A} \} \text{ and } \mathbf{I}(\mathbf{R}_{\min}^{m \times n})^* = \{ [\underline{A}, \overline{A}] \mid A \in \mathbf{I}(\mathbf{R})_{\min}^{m \times n} \}.$$

Specifically, for $[\underline{A}, \overline{A}], [\underline{B}, \overline{B}] \in \mathbf{I}(\mathbf{R}_{\min}^{m \times n})^*$ and $\alpha \in \mathbf{I}(\mathbf{R})_{\min}$ we define

$$\alpha \otimes [\underline{A}, \overline{A}] = [\alpha \otimes \underline{A}, \alpha \otimes \overline{A}], [\underline{A}, \overline{A}] \oplus [\underline{B}, \overline{B}] = [\underline{A} \oplus \underline{B}, \overline{A} \oplus \overline{B}]$$

$$\text{and } [\underline{A}, \overline{A}] \otimes [\underline{B}, \overline{B}] = [\underline{A} \otimes \underline{B}, \overline{A} \otimes \overline{B}].$$

The matrices interval $[\underline{A}, \overline{A}]$ and $[\underline{B}, \overline{B}] \in \mathbf{I}(\mathbf{R}_{\min}^{m \times n})^*$ are equal if $\underline{A} = \underline{B}$ and $\overline{A} = \overline{B}$. We can show that $(\mathbf{I}(\mathbf{R}_{\min}^{n \times n})^*, \oplus, \otimes)$ is an idempotent semiring with neutral element matrix interval $[\varepsilon, \varepsilon]$ and the unity element is matrix interval $[E, E]$. We can also show that $\mathbf{I}(\mathbf{R}_{\min}^{n \times n})^*$ is a semimodule over $\mathbf{I}(\mathbf{R})_{\min}$.

The semiring $(\mathbf{I}(\mathbf{R})_{\min}^{n \times n}, \oplus, \otimes)$ is isomorphic with semiring $(\mathbf{I}(\mathbf{R}_{\min}^{n \times n})^*, \oplus, \otimes)$. We can define a mapping f , where $f(A) = [\underline{A}, \overline{A}], \forall A \in \mathbf{I}(\mathbf{R})_{\min}^{n \times n}$. Also, the semimodule $\mathbf{I}(\mathbf{R})_{\min}^{n \times n}$ is isomorphic with semimodule $\mathbf{I}(\mathbf{R}_{\min}^{n \times n})^*$. So, for every matrices interval $A \in \mathbf{I}(\mathbf{R}_{\min}^{n \times n})^*$ we can determine matrices interval $[\underline{A}, \overline{A}] \in \mathbf{I}(\mathbf{R}_{\min}^{n \times n})^*$. Conversely, for every $[\underline{A}, \overline{A}] \in \mathbf{I}(\mathbf{R}_{\min}^{n \times n})^*$, then $\underline{A}, \overline{A} \in \mathbf{R}_{\min}^{n \times n}$, such that $[\underline{A}_{ij}, \overline{A}_{ij}] \in \mathbf{I}(\mathbf{R})_{\min}, \forall i$ and j . The matrix interval $[\underline{A}, \overline{A}]$ is called matrix interval associated with the interval matrix A and which is written $A \approx [\underline{A}, \overline{A}]$. So we have $\alpha \otimes A \approx [\alpha \otimes \underline{A}, \alpha \otimes \overline{A}], A \oplus B \approx [\underline{A} \oplus \underline{B}, \overline{A} \oplus \overline{B}]$ and $A \otimes B \approx [\underline{A} \otimes \underline{B}, \overline{A} \otimes \overline{B}]$.

3. Fuzzy Number Min-Plus Algebra

In this section we assume that readers have some knowledge of basic concepts of fuzzy set and fuzzy number. Further details can be found in [3] and Susilo [9]. In this article, fuzzy number operations defined by α -cuts. Firstly, we will review a theorem in the fuzzy set that we will use to following discussion.

Theorem 1. (Decomposition Theorem). If A^α is a α -cut of fuzzy set \tilde{A} in X and \tilde{A}^α is a fuzzy set in X with membership function $\mu_{\tilde{A}^\alpha}(x) = \alpha \chi_{A^\alpha}(x)$, where χ_{A^α} is the characteristic function of set A^α , then $\tilde{A} = \bigcup_{\alpha \in [0,1]} \tilde{A}^\alpha$.

Proof: [6].

Definition 1. Let \tilde{a} and \tilde{b} are fuzzy number with $a^\alpha = [\underline{a}^\alpha, \overline{a}^\alpha]$ and $b^\alpha = [\underline{b}^\alpha, \overline{b}^\alpha]$, where \underline{a}^α and \overline{a}^α are lower bound and upper bound of interval a^α respectively, for \underline{b}^α and \overline{b}^α analog,

- i) Minimum \tilde{a} and \tilde{b} , i.e. $\tilde{a} \oplus \tilde{b}$ is fuzzy set with its α -cut is interval $[\underline{a}^\alpha \oplus \underline{b}^\alpha, \overline{a}^\alpha \oplus \overline{b}^\alpha]$, for every $\alpha \in [0, 1]$.
- ii) Addition \tilde{a} and \tilde{b} , i.e. $\tilde{a} \otimes \tilde{b}$ is fuzzy set with its α -cut is interval $[\underline{a}^\alpha \otimes \underline{b}^\alpha, \overline{a}^\alpha \otimes \overline{b}^\alpha]$, for every $\alpha \in [0, 1]$.

To obtain the membership function of the results of operations on fuzzy numbers as above, can be used with Decomposition Theorem. In a manner analogous to the fuzzy number max-plus algebra in [5] and [6], we can show α -cut is defined in the above operations satisfied as α -cut family of a fuzzy number. Furthermore, using the decomposition theorem is obtained that $\tilde{a} \oplus \tilde{b} = \tilde{c} = \bigcup_{\alpha \in [0,1]} \tilde{c}^\alpha$, where \tilde{c}^α is a fuzzy set in

\mathbf{R} with membership function $\mu_{\tilde{c}^\alpha}(x) = \alpha \chi_{(a \oplus b)^\alpha}(x)$, where $\chi_{(a \oplus b)^\alpha}$ is the characteristic function of set $(a \oplus b)^\alpha$. For the operation \otimes can be done with an analog way.

Example 1. Given two trapezoidal fuzzy numbers $\tilde{a} = \text{BKT}(a_1, a_2, a_3, a_4)$ and $\tilde{b} = \text{BKT}(b_1, b_2, b_3, b_4)$, then $a^\alpha = [(a_2 - a_1)\alpha + a_1, (a_3 - a_4)\alpha + a_4]$ and $b^\alpha = [\underline{b}^\alpha, \overline{b}^\alpha] = b^\alpha = [(b_2 - b_1)\alpha + b_1, (b_3 - b_4)\alpha + b_4]$. Then α -cuts of $\tilde{a} \oplus \tilde{b}$ and $\tilde{a} \otimes \tilde{b}$ respectively $[((a_2 - a_1)\alpha + a_1) \oplus ((b_2 - b_1)\alpha + b_1), ((a_3 - a_4)\alpha + a_4) \oplus ((b_3 - b_4)\alpha + b_4)]$ and $[((a_2 - a_1)\alpha + a_1) \otimes ((b_2 - b_1)\alpha + b_1), ((a_3 - a_4)\alpha + a_4) \otimes ((b_3 - b_4)\alpha + b_4)]$.

Example 2 Given two triangle fuzzy numbers $\tilde{a} = \text{BKS}(2, 3, 4)$ and $\tilde{b} = \text{BKT}(1, 4, 5, 6)$, then

$$a^\alpha = [(3 - 2)\alpha + 2, (3 - 4)\alpha + 4] = [\alpha + 2, -\alpha + 4] \text{ and}$$

$$b^\alpha = [(4 - 1)\alpha + 1, (5 - 6)\alpha + 6] = [3\alpha + 1, -\alpha + 6].$$

Using *MATLAB* program, the following given bounds of α -cuts \tilde{a} , \tilde{b} (Figure 1 upper) and bounds of α -cuts $\tilde{a} \oplus \tilde{b}$ and $\tilde{a} \otimes \tilde{b}$ for $\alpha = 0, 0.05, 0.1, \dots, 1$ (Figure 1 lower).

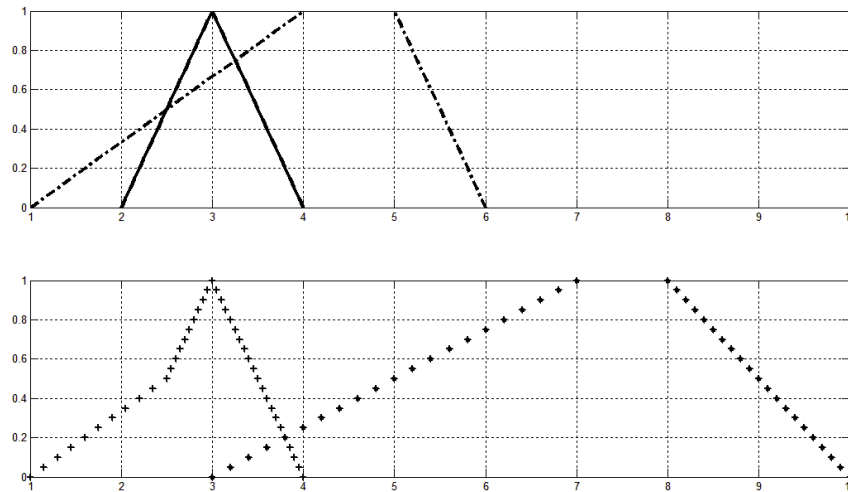


Figure 1. Graph of membership function of operation result BKS(2, 3, 4) and BKT(1, 4, 5, 6).

Remarks of Figure 1 : $- : \tilde{a}$, $-.- : \tilde{b}$, $+ : \tilde{a} \oplus \tilde{b}$, $* : \tilde{a} \otimes \tilde{b}$.

With regard Figure 1 and the intersection point of $\mu_{\tilde{a}}(x) = x - 2$ and $\mu_{\tilde{b}}(x) = \frac{x-1}{3}$ is (2.5, 0.5), then we obtain following membership function

$$\mu_{\tilde{a} \oplus \tilde{b}}(x) = \begin{cases} \frac{x}{5} , & 0 \leq x \leq 2.5 \\ x - 2 , & 2.5 < x < 3 \\ 1 , & x = 3 \\ 4 - x , & 3 < x \leq 4 \\ 0 , & \text{other} \end{cases} \quad \mu_{\tilde{a} \otimes \tilde{b}}(x) = \begin{cases} \frac{x-3}{4} , & 3 \leq x < 7 \\ 1 , & 7 \leq x \leq 8 \\ \frac{10-x}{2} , & 8 < x \leq 10 \\ 0 , & \text{other} \end{cases} .$$

Meanwhile $\tilde{a} \otimes \tilde{b} = \text{BKT}(3, 7, 8, 10)$ with membership function $\mu_{\tilde{a} \otimes \tilde{b}}$ above. From example 2 above it appears that the minimum operating results of two triangular fuzzy numbers is not always a fuzzy triangular number.

Given $\mathbf{F}(\mathbf{R})_{\tilde{\varepsilon}} := \mathbf{F}(\mathbf{R}) \cup \{\tilde{\varepsilon}\}$ with $\mathbf{F}(\mathbf{R})$ is the set of all positive fuzzy number and $\tilde{\varepsilon} := \{-\infty\}$, with $\varepsilon^\alpha = [-\infty, -\infty]$ for every $\alpha \in [0, 1]$. In $(\mathbf{F}(\mathbf{R}))_{\tilde{\varepsilon}}$ defined maximum $\tilde{\oplus}$ and addition $\tilde{\otimes}$ operation as given above. With a manner analogous with the case of max-plus algebra as seen in the [6], we can show that structure of $(\mathbf{F}(\mathbf{R}))_{\tilde{\varepsilon}}$, $(\tilde{\oplus}, \tilde{\otimes})$ is a comutative idempoten semiring with neutral elemen $\tilde{e} = \{0\}$, with $e^\alpha = [0, 0]$ and unity elemen $\tilde{\varepsilon} := \{-\infty\}$, with $\varepsilon^\alpha = [-\infty, -\infty]$, for every $\alpha \in [0, 1]$. This comutative idempoten semiring ini is called *fuzzy number min-plus algebra*, and is written as $\mathbf{F}(\mathbf{R})_{\tilde{\varepsilon}}$.

4. Matrix over Fuzzy Number Min-Plus Algebra

Concepts in matrix over the min-plus algebra, also can be generalized to the matrix over the fuzzy numbers min-plus algebra. The discussion is based on the results of the matrix and vector over intervals min-plus algebra.

Definition 2. Defined $\mathbf{F}(\mathbf{R})_{\varepsilon}^{m \times n} := \{ \tilde{A} = (\tilde{A}_{ij}) \mid \tilde{A}_{ij} \in \mathbf{F}(\mathbf{R})_{\varepsilon}, \text{ for } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n \}$. Matrix in $\mathbf{F}(\mathbf{R})_{\min}^{m \times n}$ is called matrix over fuzzy number min-plus algebra. Furthermore, the above matrix, is called *fuzzy number matrix*. Matrix \tilde{A} and $\tilde{B} \in \mathbf{F}(\mathbf{R})_{\varepsilon}^{m \times n}$ is equal if $\tilde{A}_{ij} = \tilde{B}_{ij}$ for every i and j .

Operation $\tilde{\oplus}$ and $\tilde{\otimes}$ in $\mathbf{F}(\mathbf{R})_{\varepsilon}$ can be extended into fuzzy number matrices operations in $\mathbf{F}(\mathbf{R})_{\varepsilon}^{m \times n}$.

- i) Let $\tilde{\lambda} \in \mathbf{F}(\mathbf{R})_{\varepsilon}$, $\tilde{A}, \tilde{B} \in \mathbf{F}(\mathbf{R})_{\varepsilon}^{m \times n}$. Defined $\tilde{\lambda} \tilde{\otimes} \tilde{A}$ is a matrix where $(\tilde{\lambda} \tilde{\otimes} \tilde{A})_{ij} = \tilde{\lambda} \tilde{\otimes} \tilde{A}_{ij}$ for $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$ and $\tilde{A} \tilde{\oplus} \tilde{B}$ is a matrix where $(\tilde{A} \tilde{\oplus} \tilde{B})_{ij} = \tilde{A}_{ij} \tilde{\oplus} \tilde{B}_{ij}$ for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.
- ii) Let $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\varepsilon}^{m \times p}$, $\tilde{B} \in \mathbf{F}(\mathbf{R})_{\varepsilon}^{p \times n}$. Defined $\tilde{A} \tilde{\otimes} \tilde{B}$ is a matrix where $(\tilde{A} \tilde{\otimes} \tilde{B})_{ij} = \bigoplus_{k=1}^p \tilde{A}_{ik} \tilde{\otimes} \tilde{B}_{kj}$ for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$

Definition 3. For every $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\varepsilon}^{m \times n}$ and $\alpha \in [0, 1]$, defined a α -cut matrix of \tilde{A} , as the interval matrix $A^{\alpha} = (A_{ij}^{\alpha}) \in \mathbf{I}(\mathbf{R})_{\varepsilon}^{m \times n}$, with $A_{ij}^{\alpha} \in \mathbf{I}(\mathbf{R})_{\varepsilon}$. Matrix $\underline{A}^{\alpha} = (\underline{A}_{ij}^{\alpha}) \in \mathbf{R}_{\varepsilon}^{+m \times n}$ and $\overline{A}^{\alpha} = (\overline{A}_{ij}^{\alpha}) \in \mathbf{R}_{\varepsilon}^{+m \times n}$ which are called *lower bound* and *upper bound* of matrix A^{α} . For $\tilde{A}, \tilde{B} \in \mathbf{F}(\mathbf{R})_{\varepsilon}^{m \times n}$ is equal if $A^{\alpha} = B^{\alpha}$ for every $\alpha \in [0, 1]$, yaitu $A_{ij}^{\alpha} = B_{ij}^{\alpha}$ for every i and j .

We can show $A^{\alpha} \approx [\underline{A}^{\alpha}, \overline{A}^{\alpha}]$, $(\lambda \otimes A)^{\alpha} \approx [\lambda^{\alpha} \otimes \underline{A}^{\alpha}, \lambda^{\alpha} \otimes \overline{A}^{\alpha}]$, $(A \oplus B)^{\alpha} \approx [\underline{A}^{\alpha} \oplus \underline{B}^{\alpha}, \overline{A}^{\alpha} \oplus \overline{B}^{\alpha}]$ and $(A \otimes B)^{\alpha} \approx [\underline{A}^{\alpha} \otimes \underline{B}^{\alpha}, \overline{A}^{\alpha} \otimes \overline{B}^{\alpha}]$, for every $\alpha \in [0, 1]$.

Example 3. $\tilde{A} = \begin{bmatrix} (1,3,4) & (1,2,3) \\ (\varepsilon, \varepsilon, \varepsilon) & (-3,-1,1) \end{bmatrix}$ and $\tilde{B} = \begin{bmatrix} (2, 2.5,3) & (-5,-4,-2) \\ (1,2,4) & (-1,0,1) \end{bmatrix}$.

We will determine i) $\tilde{A} \tilde{\oplus} \tilde{B}$ and ii) $\tilde{A} \tilde{\otimes} \tilde{B}$.

$$i) \tilde{A} \tilde{\oplus} \tilde{B} = \begin{bmatrix} (1,3,4) & (1,2,3) \\ (\varepsilon, \varepsilon, \varepsilon) & (-3,-1,1) \end{bmatrix} \tilde{\oplus} \begin{bmatrix} (2, 2.5,3) & (-5,-4,-2) \\ (1,2,4) & (-1,0,1) \end{bmatrix} \\ = \begin{bmatrix} \tilde{c} & (-5,-4,-2) \\ (1,2,4) & (-3,-1,1) \end{bmatrix}$$

With regard graph of α -cut bounds of \tilde{c} for $\alpha = 0, 0.05, 0.1, \dots, 1$, as Figure 2 bellow,

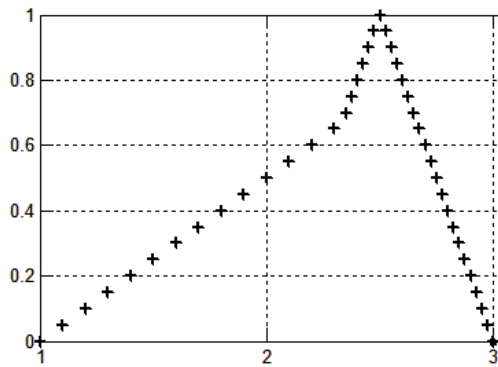


Figure 2. Graph of α -cuts bounds of $BKS(1, 3, 4) \oplus BKS(2, 2.5, 3)$

and intersection point of $\frac{x-2}{0,5}$ and $\frac{x-1}{2}$ is $(7/3, 2/3)$, we obtain

$$\mu_c(x) = \begin{cases} 0 & , x < 1 \\ \frac{x-1}{2} & , 1 \leq x \leq 7/3 \\ \frac{x-2}{0,5} & , 7/3 < x \leq 2,5 \\ 6-2x & , 2,5 < x \leq 3 \\ 0 & , x > 3 \end{cases}$$

$$\begin{aligned} ii) \tilde{A} \tilde{\otimes} \tilde{B} &= \begin{bmatrix} (1,3,4) & (1,2,3) \\ \tilde{\mathcal{E}} & (-3,-1,1) \end{bmatrix} \tilde{\otimes} \begin{bmatrix} (2, 2.5, 3) & (-5,-4,-2) \\ (1,2,4) & (-1,0,1) \end{bmatrix} \\ &= \begin{bmatrix} \{(1,3,4) \tilde{\otimes} (2, 2.5, 3)\} \tilde{\oplus} \{(1,2,3) \tilde{\otimes} (1,2,4)\} & \dots \\ \{(\varepsilon, \varepsilon, \varepsilon) \tilde{\otimes} (2, 2.5, 3)\} \tilde{\oplus} \{(-3,-1,1) \tilde{\otimes} (1,2,4)\} & \dots \\ \dots & \{(1,3,4) \tilde{\otimes} (-5,-4,-2)\} \tilde{\oplus} \{(1,2,3) \tilde{\otimes} (-1,0,1)\} \\ \dots & \{(\varepsilon, \varepsilon, \varepsilon) \tilde{\otimes} (-5,-4,-2)\} \tilde{\oplus} \{(-3,-1,1) \tilde{\otimes} (-1,0,1)\} \end{bmatrix} \\ &= \begin{bmatrix} (3, 5.5, 7) \tilde{\oplus} (2,4,7) & (-4,-1,2) \tilde{\oplus} (0,2,4) \\ (\varepsilon, \varepsilon, \varepsilon) \tilde{\oplus} (-2,1,4) & (\varepsilon, \varepsilon, \varepsilon) \tilde{\oplus} (-4,-1,2) \end{bmatrix} \\ &= \begin{bmatrix} (2, 4, 7) & (-4,-1,2) \\ (-2,1,4) & (-4,-1,2) \end{bmatrix}. \end{aligned}$$

We can show that $(\mathbf{F}(\mathbf{R})_{\max}^{m \times n}, \tilde{\oplus})$ is a commutative idempotent semigroup. Moreover, $\mathbf{F}(\mathbf{R})_{\max}^{m \times n}$ is a semimodule over $\mathbf{F}(\mathbf{R})_{\max}$, where $(\mathbf{F}(\mathbf{R})_{\max}^{n \times n}, \tilde{\oplus}, \tilde{\otimes})$ is an idempotent semiring with matrix $\tilde{\mathcal{E}}$ as the neutral element and matrix \tilde{E} as the unity element.

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References

- [1] Baccelli, F., Cohen, G., Olsder, G.J. and Quadrat, J.P. 2001. *Synchronization and Linearity*. New York: John Wiley & Sons..
- [2] Gondran, M and Minoux, M. 2008. *Graph, Dioids and Semirings*. New York: Springer.
- [3] Lee, K.H. 2005. *First Course on Fuzzy Theory and Applications*. Berlin: Spinger-Verlag.
- [4] Litvinov, G.L., Sobolevskii, A.N. 2001. Idempotent Interval Anaysis and Optimization Problems. *Reliab. Comput.*, 7, 353 – 377; arXiv: math.SC/010180.
- [5] Rudhito, Andy. Wahyuni, Sri. Suparwanto, Ari and Susilo, F. 2008. Aljabar Max-Plus Bilangan Fuzzy. *Berkala Ilmiah MIPA Majalah Ilmiah Matematika & Ilmu Pengetahuan Alam*. Vol. 18 (2): pp. 153-164.
- [6] Rudhito, Andy. 2011.. *Aljabar Max-Plus Bilangan Kabur and Penerapannya pada Masalah Penjadwalan and Jaringan Antrian Kabur*. Disertasi: Program Pascasarjana Universitas Gadjah Mada. Yogyakarta.
- [7] Rudhito, Andy. 2013. Aljabar Min-plus Interval. *Prosiding Seminar Nasional Penelitian, Pendidikan, and Penerapan MIPA*. Fakultas MIPA, Universitas Negeri Yogyakarta, 18 Mei 2013. pp. M-97 – M-102.
- [8] Rudhito, Andy. 2013. Matriks Atas Aljabar Min-plus Interval. *Prosiding Seminar Sains and Pendidikan Sains VIII*. FSM UKSW Salatiga 15 Juni 2013. ISSN : 2087-0922. pp: 130 – 136.
- [9] Susilo, F. 2006. *Himpunan and Logika Fuzzy serta Aplikasinya Edisi kedua*. Yogyakarta: Graha Ilmu.