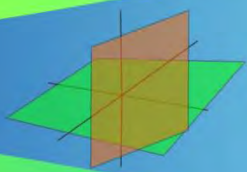
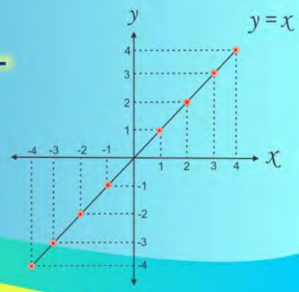
Proceeding

Dept of Mathematics Education
Dept of Math Education Graduate School
Yogyakarta State University





International Seminar on Innovation in Mathematics and Mathematics Education

1st ISIM-MED 2014

" Innovation and Technology for Mathematics and Mathematics Education"

Joint Conference on Innovation and Technology For Mathematics and Mathematics Education :

19th ATCM 1st ISIM-MED 3rd ISMEI 2014 2nd SeNdiMat

NOVEMBER, 26 - 30, 2014
YOGYAKARTA STATE UNIVERSITY

Organized by :

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Asian Technology Conference in Mathematics

P4TK Matematika, Yogyakarta

SEAMEO QITEP in Mathematics

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Department of Mathematics Education
Faculty of Sciences
Yogyakarta State University
2014

Preface

Assalaamu'alaikum Warahmatullaahi Wabarakaatuh.

First of all, we would like to praise *alhamdulillah*, praise is only to Allah SWT, the most gracious and the most merciful, for blessing and giving us energy in conducting a Joint Conference on *Innovation and Technology for Mathematics and Mathematics Education* from 26 to 30 of November 2014, under coordination of Asian Technology Conference in Mathematics (ATCM), Department of Mathematics of Yogyakarta State University, SEAMEO QITEP in Mathematics, and PPPPTK Matematika.

The conference is a joint conference between the 19th ATCM, 1st ISIMMED of Department of Mathematics of Yogyakarta State University, 3rd ISMEI of SEAMEO QITEP in Mathematics, and 2nd SeNdiMat of PPPPTK Matematika. During the joint conference, there are 170 papers are presented – 7 papers presented on plenary sessions, 10 papers presented on invited speech sessions, 33 ATCM papers presented on parallel sessions, 120 papers of ISIMMED and ISMEI presented on parallel sessions. There are also 20 workshop papers and 3 posters presented in the joint conference. In addition, more than 50 SeNdiMat papers are presented separately at PPPPTK Matematika. Each conference has its own proceeding. We are honored to present the proceeding of 1st ISIMMED 2014. This proceeding contains 110 papers consist of 77 papers on mathematics education and 33 papers on mathematics and/or computer.

Presenters in this conference come from more than 60 universities and institutions from 20 countries around the world: Australia, Belgium, Brunei Darussalam, China, France, Hong Kong, India, Indonesia, Japan, Luxembourg, Malaysia, Netherlands, Oman, Papua New Guinea, Philippines, Singapore, Taiwan, Thailand, United Kingdom, and United State of America. We are fortunate to have presenters from a wide spectrum of scientists and educators, whose presentations and workshops demonstrate the most current innovation and technology for mathematics and mathematics education. Papers and presentations address a very wide spectrum of topics and ideas. We can find papers concentrating on using computer software in teaching mathematics, papers on using Internet, multimedia, and other tools for interactive and online mathematics courses delivery, as well as research papers from pure mathematics where technology was used to produce some new results.

Thanks to evolving technological tools, we are able to explore more interdisciplinary areas such as science, technology, and engineering with Mathematics which we could not before. Therefore, integrating technology into mathematics teaching, learning and research will definitely allow us to expand our knowledge horizon in mathematics.

We would like to express our appreciation to all members of the local organizers from Yogyakarta State University, SEAMEO QITEP in Mathematics, and PPPTK Matematika for the enormous task of planning and preparation of this joint conference. We thank also the International Program Committee of ATCM, especially Prof. Wei-Chi YANG for giving opportunity to host the 19th ATCM 2014, one of the most enjoyable and instructive conference in the world. We are very grateful to all plenary and invited speakers for their inspiring papers as well as all reviewers for their great contribution in reviewing papers.

We hope that the papers in this proceeding become very useful references to enrich the creative and innovative ideas that can support the advancement of mathematics education, especially in Indonesia.

Wassalaamu'alaikum Warahmatullaahi Wabarakaatuh.

Yogyakarta, November 26, 2014
Chairman of the YSU Local Committee
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This Event is presented by :

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SEAMEO QITEP in Mathematics

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A – 1

Fuzzy Number Min-Plus Algebra and Matrix

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Abstract

This paper discusses the algebra of the set of all fuzzy numbers with the operations minimum and addition (plus) and matrix over its algebra. This algebra is an extension of the min-plus algebra through interval min-plus algebra and Decomposition Theorem in fuzzy sets. It can be shown that the minimum and addition operation defined by a closed interval alpha-cut in the set of all fuzzy numbers. The set of all fuzzy numbers with minimum and addition operations is a commutative idempotent semiring. Furthermore, the set of all matrices over this algebra is a semimodule. Given also an example calculation using MATLAB program.

Keywords: semiring, min-plus algebra, fuzzy number, matrix, semimodule.

1. Introduction

Min-plus algebra, which is the set of all real numbers **R** equipped with operation min (minimum) and plus (addition), has been used well to model and analyze the shortest path problem [1] and [2]. In the problem of modeling and analysis of a network sometimes travel time unknown, for example because it is still at the design stage. Travel time can be estimated based on the experience and opinion of the experts and the network operator. In this case the travel time of the network can be modeled by a fuzzy number, and its travel time is called fuzzy travel time.

Modeling and analysis of the shortest path problem with fuzzy travel time, as far as we know, no one has discussed using min-plus algebra approach such as has been done for deterministic and probabilistic models. As has been known to approach network solution using min-plus algebra can provide analytical results and more ease in computation, compared to other approaches which tend to be heuristic.

Min-plus algebra approach to solve the shortest path problem is also using the basic concepts in the min-plus algebra, such as matrix on min-plus algebra and system of min-plus linear equations, as has been discussed in [1] and [2]. Thus, to resolve the fuzzy shortest path problem, the min-plus algebra approach, first min-plus algebra needs to be generalized into fuzzy numbers min-plus algebra, where the elements of the set are discussed in the form of fuzzy numbers and concepts related thereto, such as matrix over fuzzy numbers min-plus algebra. Therefore in this article will discuss the definition and basic concepts in min-plus algebra and matrix fuzzy numbers on min-plus algebra fuzzy numbers. We will first review some basic concepts and results in the interval min-plus algebra and matrix. A more complete discussion can be found in [7] and [8].

2. Interval Min-Plus Algebra and Matrix

Let $\mathbf{R}_\varepsilon := \mathbf{R} \cup \{\varepsilon\}$ with **R** the set of all real numbers and $\varepsilon := \infty$. In $\mathbf{R}_\varepsilon$ defined two operations : $\forall a,b \in \mathbf{R}_\varepsilon$, $a \oplus b := \min(a, b)$ and $a \otimes b := a + b$. We can show

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that ($\mathbf{R}_{\varepsilon}$, ⊕, ⊗) is a commutative idempotent semiring [1] with neutral element $\varepsilon = \infty$ and unity element $e = 0$. Moreover, ($\mathbf{R}_{\varepsilon}$, ⊕, ⊗) is a semifield, that is ($\mathbf{R}_{\varepsilon}$, ⊕, ⊗) is a commutative semiring [1], where for every $a \in \mathbf{R}$ there exist $-a$ such that $a \otimes (-a) = 0$. Thus, ($\mathbf{R}_{\varepsilon}$, ⊕, ⊗) is a *min-plus algebra*, and is written as $\mathbf{R}_{\min}$ [2]. One can define $x^{\otimes 0} :=$ 0, $x^{\otimes k} := x \otimes x^{\otimes k-1}$, $\varepsilon^{\otimes 0} := 0$ and $\varepsilon^{\otimes k} := \varepsilon$, for $k = 1, 2, \ldots$.

The operations ⊕ and ⊗ in $\mathbf{R}_{\min}$ can be extend to the matrices operations in $\mathbf{R}_{\min}^{m\times n}$, with $\mathbf{R}_{\min}^{m\times n} := \{A = (A_{ij}) \mid A_{ij} \in \mathbf{R}_{\min}$, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n\}$, the set of all matrices over max-plus algebra. Specifically, for $A, B \in \mathbf{R}_{\min}^{n\times n}$ we define $(A \oplus B)_{ij} = A_{ij} \oplus B_{ij}$ and $(A \otimes B)_{ij} = \bigoplus_{k=1}^{n} A_{ik} \otimes B_{kj}$. We also define matrix $E \in \mathbf{R}_{\min}^{n\times n}$, $(E)_{ij} := \begin{cases} 0, & \text{if } i = j \\ \varepsilon, & \text{if } i \neq j \end{cases}$ and $\mathcal{E} \in \mathbf{R}_{\min}^{m\times n}$, $(\varepsilon)_{ij} := \varepsilon$ for every i and j . For any matrices $A \in \mathbf{R}_{\min}^{n\times n}$, one can define $A^{\otimes 0} = E_n$ and $A^{\otimes k} = A \otimes A^{\otimes k-1}$ for $k = 1, 2, \ldots$. We can show that ($\mathbf{R}_{\min}^{m\times n}$, ⊕) is an idempotent commutative semigroup, ($\mathbf{R}_{\min}^{n\times n}$, ⊕, ⊗) is an idempotent semiring with matrix $\mathcal{E}$ as neutral element and matrix E as unity element, and $\mathbf{R}_{\min}^{m\times n}$ is a semimodule over $\mathbf{R}_{\min}$ [1] and [6].

Definitions and concepts in the interval min-plus algebra are analogous to the concepts in the interval max-plus algebra which can be seen in [6]. The (closed) interval x in $\mathbf{R}_{\min}$ is a subset of $\mathbf{R}_{\min}$ of the form $\mathrm{x} = [\underline{x}, \overline{x}] = \{x \in \mathbf{R}_{\min} \mid \underline{x} \preceq_m x \preceq_m \overline{x}\}$.
The interval x in $\mathbf{R}_{\min}$ is called *min-plus interval*, which is in short is called *interval*. Define

$$\mathbf{I}(\mathbf{R})_{\varepsilon} := \{ \mathrm{x} = [\underline{x}, \overline{x}] \mid \underline{x}, \overline{x} \in \mathbf{R},\ \varepsilon \prec_m \underline{x} \preceq_m \overline{x} \} \cup \{ \varepsilon \}, \text{ where } \varepsilon := [\varepsilon, \varepsilon].$$

In the $\mathbf{I}(\mathbf{R})_{\varepsilon}$, define operation $\overline{\oplus}$ and $\overline{\otimes}$ as

$$\mathrm{x} \,\overline{\oplus}\, \mathrm{y} = [\underline{x} \oplus \underline{y}, \overline{x} \oplus \overline{y}] \text{ and } \mathrm{x} \,\overline{\otimes}\, \mathrm{y} = [\underline{x} \otimes \underline{y}, \overline{x} \otimes \overline{y}], \forall\, \mathrm{x}, \mathrm{y} \in \mathbf{I}(\mathbf{R})_{\varepsilon}.$$

Since ($\mathbf{R}_{\varepsilon}$, ⊕, ⊗) is an idempotent semiring and it has no zero divisors, with neutral element ε, we can show that $\mathbf{I}(\mathbf{R})_{\varepsilon}$ is closed with respect to the operation $\overline{\oplus}$ and $\overline{\otimes}$. Moreover, ($\mathbf{I}(\mathbf{R})_{\varepsilon}, \overline{\oplus}, \overline{\otimes}$) is a comutative idempotent semiring with neutral element ε = [ε, ε] and unity element 0 = [0, 0]. This comutative idempotent semiring ($\mathbf{I}(\mathbf{R})_{\varepsilon}, \overline{\oplus}, \overline{\otimes}$) is called *interval min-plus algebra* which is written as $\mathbf{I}(\mathbf{R})_{\min}$.

Define $\mathbf{I}(\mathbf{R})_{\min}^{m\times n} := \{\mathrm{A} = (\mathrm{A}_{ij}) \mid \mathrm{A}_{ij} \in \mathbf{I}(\mathbf{R})_{\min}$, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ }. The element of $\mathbf{I}(\mathbf{R})_{\min}^{m\times n}$ are called *matrix over interval min-plus algebra*. Furthermore, this matrix is called *interval matrix*. The operations $\overline{\oplus}$ and $\overline{\otimes}$ in $\mathbf{I}(\mathbf{R})_{\min}$ can be extended to the matrices operations of in $\mathbf{I}(\mathbf{R})_{\max}^{m\times n}$. Specifically, for A, B ∈ $\mathbf{I}(\mathbf{R})_{\min}^{n\times n}$ and α ∈ $\mathbf{I}(\mathbf{R})_{\min}$ we define

$$(\alpha \,\overline{\otimes}\, \mathrm{A})_{ij} = \alpha \,\overline{\otimes}\, \mathrm{A}_{ij},\ (\mathrm{A} \,\overline{\oplus}\, \mathrm{B})_{ij} = \mathrm{A}_{ij} \,\overline{\oplus}\, \mathrm{B}_{ij} \text{ and } (\mathrm{A} \,\overline{\otimes}\, \mathrm{B})_{ij} = \overline{\bigoplus_{k=1}^{n}} \mathrm{A}_{ik} \,\overline{\otimes}\, \mathrm{B}_{kj}.$$

Matrices A, B $\in$ $\mathbf{I}(\mathbf{R})_{\min}^{m\times n}$ are *equal* if $A_{ij} = B_{ij}$, that is if $\underline{A}_{ij} = \underline{B}_{ij}$ and $\overline{A}_{ij} = \overline{B}_{ij}$ for every *i* and *j*. We can show that $(\mathbf{I}(\mathbf{R})_{\min}^{n\times n}, \overline{\oplus}, \overline{\otimes})$ is a idempotent semiring with neutral element is matrix ε, with $(\varepsilon)_{ij} := \varepsilon$ for every *i* and *j*, and unity element is matrix E, with $(E)_{ij} := \begin{cases} 0 & \text{, if } i = j \\ \varepsilon & \text{, if } i \neq j \end{cases}$. We can also show that $\mathbf{I}(\mathbf{R})_{\min}^{m\times n}$ is a semi-module over $\mathbf{I}(\mathbf{R})_{\min}$.

For any matrix A $\in$ $\mathbf{I}(\mathbf{R})_{\min}^{m\times n}$, define the matrices $\underline{A} = (\underline{A}_{ij}) \in \mathbf{R}_{\min}^{m\times n}$ and $\overline{A} = (\overline{A}_{ij}) \in \mathbf{R}_{\min}^{m\times n}$, which is called *lower bound matrices* and *upper bound matrices of* A, respectively. Define *matrices interval of* A, that is

$$[\underline{A}, \overline{A}] = \{ A \in \mathbf{R}_{\min}^{m\times n} | \underline{A} \preceq_m A \preceq_m \overline{A} \} \text{ and } \mathbf{I}(\mathbf{R}_{\min}^{m\times n})^* = \{ [\underline{A}, \overline{A}] \mid A \in \mathbf{I}(\mathbf{R})_{\min}^{n\times n} \}.$$

Specifically, for $[\underline{A}, \overline{A}], [\underline{B}, \overline{B}] \in \mathbf{I}(\mathbf{R}_{\min}^{m\times n})^*$ and $\alpha \in \mathbf{I}(\mathbf{R})_{\min}$ we define

$$\alpha \,\overline{\otimes}\, [\underline{A}, \overline{A}] = [\underline{\alpha} \otimes \underline{A},\ \overline{\alpha} \otimes \overline{A}],\ [\underline{A}, \overline{A}] \,\overline{\oplus}\, [\underline{B}, \overline{B}] = [\underline{A} \oplus \underline{B}, \overline{A} \oplus \overline{B}]$$

$$\text{and } [\underline{A}, \overline{A}] \,\overline{\otimes}\, [\underline{B}, \overline{B}] = [\underline{A} \otimes \underline{B}, \overline{A} \otimes \overline{B}].$$

The matrices interval $[\underline{A}, \overline{A}]$ and $[\underline{B}, \overline{B}] \in \mathbf{I}(\mathbf{R}_{\min}^{m\times n})^*$ are *equal* if $\underline{A} = \underline{B}$ and $\overline{A} = \overline{B}$. We can show that $(\mathbf{I}(\mathbf{R}_{\min}^{n\times n})^*, \overline{\oplus}, \overline{\otimes})$ is an idempotent semiring with neutral element matrix interval [ε, ε] and the unity element is matrix interval [E, E]. We can also show that $\mathbf{I}(\mathbf{R}_{\min}^{n\times n})^*$ is a semimodule over $\mathbf{I}(\mathbf{R})_{\min}$.

The semiring $(\mathbf{I}(\mathbf{R})_{\min}^{n\times n}, \overline{\oplus}, \overline{\otimes})$ is isomorfic with semiring $(\mathbf{I}(\mathbf{R}_{\min}^{n\times n})^*, \overline{\oplus}, \overline{\otimes})$. We can define a mapping *f*, where $f(A) = [\underline{A}, \overline{A}]$, $\forall A \in \mathbf{I}(\mathbf{R})_{\min}^{n\times n}$. Also, the semimodule $\mathbf{I}(\mathbf{R})_{\min}^{n\times n}$ is isomorfic with semimodule $\mathbf{I}(\mathbf{R}_{\min}^{n\times n})^*$. So, for every matrices interval A $\in$ $\mathbf{I}(\mathbf{R}_{\min}^{n\times n})^*$ we can determine matrices interval $[\underline{A}, \overline{A}] \in \mathbf{I}(\mathbf{R}_{\min}^{n\times n})^*$. Conversely, for every $[\underline{A}, \overline{A}] \in \mathbf{I}(\mathbf{R}_{\min}^{n\times n})^*$, then $\underline{A}, \overline{A} \in \mathbf{R}_{\min}^{n\times n}$, such that $[\underline{A}_{ij}, \overline{A}_{ij}] \in \mathbf{I}(\mathbf{R})_{\min}$, $\forall i$ and *j*. The matrix interval $[\underline{A}, \overline{A}]$ is called *matrix interval associated with the interval matrix* A and which is written $A \approx [\underline{A}, \overline{A}]$. So we have $\alpha \overline{\otimes} A \approx [\underline{\alpha} \otimes \underline{A}, \overline{\alpha} \otimes \overline{A}]$, $A \overline{\oplus} B \approx [\underline{A} \oplus \underline{B}, \overline{A} \oplus \overline{B}]$ and $A \overline{\otimes} B \approx [\underline{A} \otimes \underline{B}, \overline{A} \otimes \overline{B}]$.

3. Fuzzy Number Min-Plus Algebra

In this section we assume that readers have some knowledge of basic concepts of fuzzy set and fuzzy number. Further details can be found in [3] and Susilo [9]. In this article, fuzzy number operations defined by α-cuts. Firstly, we will review a theorem in the fuzzy set that we will use to following discussion.

Theorem 1. (Decomposition Theorem). If A^α is a α-cut of fuzzy set $\tilde{A}$ in *X* and $\tilde{A}^\alpha$ is a fuzzy set in *X* with membership function $\mu_{\tilde{A}^\alpha}(x) = \alpha\ \chi_{A^\alpha}(x)$, where χ_{A^α} is the characteristic function of set A^α, then $\tilde{A} = \bigcup_{\alpha\in[0,1]} \tilde{A}^\alpha$.

Proof: [6].

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Definition 1. Let $\tilde{a}$ and $\tilde{b}$ are fuzzy number with $a^{\alpha} = [\underline{a^{\alpha}}, \overline{a^{\alpha}}]$ and $b^{\alpha} = [\underline{b^{\alpha}}, \overline{b^{\alpha}}]$, where $\underline{a^{\alpha}}$ and $\overline{a^{\alpha}}$ are lower bound and upper bound of interval a^{α} respectively, for $\underline{b^{\alpha}}$ and $\overline{b^{\alpha}}$ analog,

i) Minimum $\tilde{a}$ and $\tilde{b}$, i.e. $\tilde{a} \tilde{\oplus} \tilde{b}$ is fuzzy set with its α-cut is interval $[\underline{a^{\alpha}} \oplus \underline{b^{\alpha}}, \overline{a^{\alpha}} \oplus \overline{b^{\alpha}}]$, for every $\alpha \in [0, 1]$.

ii) Addition $\tilde{a}$ and $\tilde{b}$, i.e. $\tilde{a} \tilde{\otimes} \tilde{b}$ is fuzzy set with its α-cut is interval $[\underline{a^{\alpha}} \otimes \underline{b^{\alpha}}, \overline{a^{\alpha}} \otimes \overline{b^{\alpha}}]$, for every $\alpha \in [0, 1]$.

To obtain the membership function of the results of operations on fuzzy numbers as above, can be used with Decomposition Theorem. In a manner analogous to the fuzzy number max-plus algebra in [5] and [6], we can show α-cut is defined in the above operations satisfied as α-cut family of a fuzzy number. Furthermore, using the decomposition theorem is obtained that $\tilde{a} \tilde{\oplus} \tilde{b} = \tilde{c} = \bigcup_{\alpha \in [0,1]} \tilde{c}^{\alpha}$, where $\tilde{c}^{\alpha}$ is a fuzzy set in **R** with membership function $\mu_{\tilde{c}^{\alpha}}(x) = \alpha \chi_{(a \oplus b)^{\alpha}}(x)$, where $\chi_{(a \oplus b)^{\alpha}}$ is the characteristic function of set $(a \oplus b)^{\alpha}$. For the operation $\tilde{\otimes}$ can be done with an analog way.

Example 1. Given two trapezoidal fuzzy numbers $\tilde{a}$ = BKT(a_1, a_2, a_3, a_4) and $\tilde{b}$ = BKT(b_1, b_2, b_3, b_4), then $a^{\alpha} = [(a_2 - a_1)\alpha + a_1, (a_3 - a_4)\alpha + a_4]$ and $b^{\alpha} = [\underline{b}^{\alpha}, \overline{b}^{\alpha}] = b^{\alpha} = [(b_2 - b_1)\alpha + b_1, (b_3 - b_4)\alpha + b_4]$. Then α-cuts of $\tilde{a} \tilde{\oplus} \tilde{b}$ and $\tilde{a} \tilde{\otimes} \tilde{b}$ respectively
$[((a_2 - a_1)\alpha + a_1) \oplus ((b_2 - b_1)\alpha + b_1), ((a_3 - a_4)\alpha + a_4) \oplus ((b_3 - b_4)\alpha + b_4)]$ and
$[((a_2 - a_1)\alpha + a_1) \otimes ((b_2 - b_1)\alpha + b_1), ((a_3 - a_4)\alpha + a_4) \otimes ((b_3 - b_4)\alpha + b_4)]$.

Example 2 Given two triangle fuzzy numbers $\tilde{a}$ = BKS(2, 3, 4) and $\tilde{b}$ = BKT(1, 4, 5, 6), then
$a^{\alpha} = [(3 - 2)\alpha + 2, (3 - 4)\alpha + 4] = [\alpha + 2, -\alpha + 4]$ and
$b^{\alpha} = [(4 - 1)\alpha + 1, (5 - 6)\alpha + 6] = [3\alpha + 1, -\alpha + 6]$.
Using *MATLAB* program, the following given bounds of α-cuts $\tilde{a}$, $\tilde{b}$ (Figure 1 upper) and bounds of α-cuts $\tilde{a} \tilde{\oplus} \tilde{b}$ and $\tilde{a} \tilde{\otimes} \tilde{b}$ for $\alpha = 0, 0.05, 0.1, ..., 1$ (Figure 1 lower).

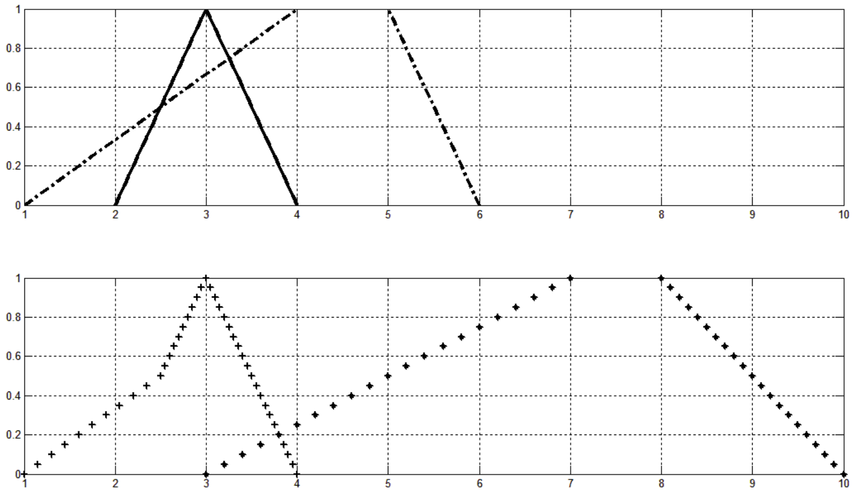

Figure 1. Graph of membership function of operation result BKS(2, 3, 4) and BKT(1, 4, 5, 6).

Remaks of Figure 1 : $-$: $\tilde{a}$, $-.-$: $\tilde{b}$, + : $\tilde{a} \tilde{\oplus} \tilde{b}$, * : $\tilde{a} \tilde{\otimes} \tilde{b}$.

With regard Figure 1 and the intersection point of $\mu_{\tilde{a}}(x) = x - 2$ and $\mu_{\tilde{b}}(x) = \dfrac{x-1}{3}$ is (2.5, 0.5), then we obtain following membership function

$$\mu_{\tilde{a}\tilde{\oplus}\tilde{b}}(x) = \begin{cases} \dfrac{x}{5} & , 0 \le x \le 2.5 \\ x-2 & , 2.5 < x < 3 \\ 1 & , \; x = 3 \\ 4-x & , 3 < x \le 4 \\ 0 & , \text{other} \end{cases} \qquad \mu_{\tilde{a}\tilde{\otimes}\tilde{b}}(x) = \begin{cases} \dfrac{x-3}{4} & , 3 \le x < 7 \\ 1 & , 7 \le x \le 8 \\ \dfrac{10-x}{2} & , 8 < x \le 10 \\ 0 & , \text{other} \end{cases}.$$

Meanwhile $\tilde{a} \tilde{\otimes} \tilde{b}$ = BKT(3, 7, 8, 10) with membership function $\mu_{\tilde{a}\tilde{\otimes}\tilde{b}}$ above. From example 2 above it appears that the minimum operating results of two triangular fuzzy numbers is not always a fuzzy triangular number.

Given $\mathbf{F}(\mathbf{R})_{\tilde{\varepsilon}} := \mathbf{F}(\mathbf{R}) \cup \{\tilde{\varepsilon}\}$ with $\mathbf{F}(\mathbf{R})$ is the set of all positive fuzzy number and $\tilde{\varepsilon} := \{-\infty\}$, with $\varepsilon^{\alpha} = [-\infty, -\infty]$ for every $\alpha \in [0, 1]$. In $(\mathbf{F}(\mathbf{R}))_{\tilde{\varepsilon}}$ defined maximum $\tilde{\oplus}$ and addition $\tilde{\otimes}$ operation as given above. With a manner analogous with the case of max-plus algebra as seen in the [6], we can show that structure of $(\mathbf{F}(\mathbf{R})_{\tilde{\varepsilon}}, \tilde{\oplus}, \tilde{\otimes})$ is a comutative idempoten semiring with neutral elemen $\tilde{e}$ = {0}, with e^{α} = [0, 0] and unity elemen $\tilde{\varepsilon} := \{-\infty\}$, with $\varepsilon^{\alpha} = [-\infty, -\infty]$, for every $\alpha \in [0, 1]$. This comutative idempoten semiring ini is called *fuzzy number min-plus algebra*, and is written as $\mathbf{F}(\mathbf{R})_{\tilde{\varepsilon}}$.

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4. Matrix over Fuzzy Number Min-Plus Algebra

Concepts in matrix over the min-plus algebra, also can be generalized to the matrix over the fuzzy numbers min-plus algebra. The discussion is based on the results of the matrix and vector over intervals min-plus algebra.

Definition 2. Defined $\mathbf{F(R)}_{\varepsilon}^{m\times n} := \{ \tilde{A} = (\tilde{A}_{ij}) \mid \tilde{A}_{ij} \in \mathbf{F(R)}_{\tilde{\varepsilon}}$, for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$ $\}$. Matrix in $\mathbf{F(R)}_{\min}^{m\times n}$ is called matrix over fuzzy number min-plus algebra**.** Furthermore, the above matrix, is called *fuzzy number matrix*. Matrix $\tilde{A}$ and $\tilde{B} \in \mathbf{F(R)}_{\varepsilon}^{m\times n}$ is equal if $\tilde{A}_{ij} = \tilde{B}_{ij}$ for every i and j.

Operation $\tilde{\oplus}$ and $\tilde{\otimes}$ in $\mathbf{F(R)}_{\tilde{\varepsilon}}$ can be extended into fuzzy number matrices operations in $\mathbf{F(R)}_{\varepsilon}^{m\times n}$.

i) Let $\tilde{\lambda} \in \mathbf{F(R)}_{\tilde{\varepsilon}}$, $\tilde{A}, \tilde{B} \in \mathbf{F(R)}_{\varepsilon}^{m\times n}$. Defined $\tilde{\lambda} \tilde{\otimes} \tilde{A}$ is a matrix where $(\tilde{\lambda} \tilde{\otimes} \tilde{A})_{ij} = \tilde{\lambda} \tilde{\otimes} \tilde{A}_{ij}$ for $i = 1, 2, ..., m$, $j = 1, 2, ..., n$ and $\tilde{A} \tilde{\oplus} \tilde{B}$ is a matrix where $(\tilde{A} \tilde{\oplus} \tilde{B})_{ij} = \tilde{A}_{ij} \tilde{\oplus} \tilde{B}_{ij}$ for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$.

ii) Let $\tilde{A} \in \mathbf{F(R)}_{\varepsilon}^{m\times p}$, $\tilde{B} \in \mathbf{F(R)}_{\varepsilon}^{p\times n}$. Defined $\tilde{A} \tilde{\otimes} \tilde{B}$ is a matrix where $(\tilde{A} \tilde{\otimes} \tilde{B})_{ij} = \tilde{\bigoplus}_{k=1}^{p} \tilde{A}_{ik} \otimes \tilde{B}_{kj}$ for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$

Definition 3. For every $\tilde{A} \in \mathbf{F(R)}_{\varepsilon}^{m\times n}$ and $\alpha \in [0, 1]$, defined a *α-cut matrix of* $\tilde{A}$, as the interval matrix $A^{\alpha} = (A_{ij}^{\alpha}) \in \mathbf{I(R)}_{\varepsilon}^{m\times n}$, *with* $A_{ij}^{\alpha} \in \mathbf{I(R)}_{\varepsilon}$. Matrix $\underline{A^{\alpha}} = (\underline{A_{ij}^{\alpha}}) \in \mathbf{R}_{\varepsilon}^{+m\times n}$ and $\overline{A^{\alpha}} = (\overline{A_{ij}^{\alpha}}) \in \mathbf{R}_{\varepsilon}^{+m\times n}$ which are called *lower bound* and *upper bound* of matrix A^{α}. For $\tilde{A}$, $\tilde{B} \in \mathbf{F(R)}_{\varepsilon}^{m\times n}$ is *equal* if $A^{\alpha} = B^{\alpha}$ for every $\alpha \in [0, 1]$, yaitu $A_{ij}^{\alpha} = B_{ij}^{\alpha}$ for every i and j.

We can show $A^{\alpha} \approx [\underline{A^{\alpha}}, \overline{A^{\alpha}}]$, $(\lambda \otimes A)^{\alpha} \approx [\underline{\lambda^{\alpha}} \otimes \underline{A^{\alpha}}, \overline{\lambda^{\alpha}} \otimes \overline{A^{\alpha}}]$, $(A \oplus B)^{\alpha} \approx [\underline{A^{\alpha}} \oplus \underline{B^{\alpha}}, \overline{A^{\alpha}} \oplus \overline{B^{\alpha}}]$ and $(A \otimes B)^{\alpha} \approx [\underline{A^{\alpha}} \otimes \underline{B^{\alpha}}, \overline{A^{\alpha}} \otimes \overline{B^{\alpha}}]$, for every $\alpha \in [0, 1]$.

Example 3. $\tilde{A} = \begin{bmatrix} (1,3,4) & (1,2,3) \\ (\varepsilon,\varepsilon,\varepsilon) & (-3,-1,1) \end{bmatrix}$ and $\tilde{B} = \begin{bmatrix} (2, 2.5,3) & (-5,-4,-2) \\ (1,2,4) & (-1,0,1) \end{bmatrix}$.

We will determine *i)* $\tilde{A} \tilde{\oplus} \tilde{B}$ and *ii)* $\tilde{A} \tilde{\otimes} \tilde{B}$.

$$i)\ \tilde{A} \tilde{\oplus} \tilde{B} = \begin{bmatrix} (1,3,4) & (1,2,3) \\ (\varepsilon,\varepsilon,\varepsilon) & (-3,-1,1) \end{bmatrix} \tilde{\oplus} \begin{bmatrix} (2, 2.5,3) & (-5,-4,-2) \\ (1,2,4) & (-1,0,1) \end{bmatrix} = \begin{bmatrix} \tilde{c} & (-5,-4,-2) \\ (1,2,4) & (-3,-1,1) \end{bmatrix}.$$

With regard graph of α-cut bounds of $\tilde{c}$ for α = 0, 0.05, 0.1, ..., 1 , as Figure 2 bellow,

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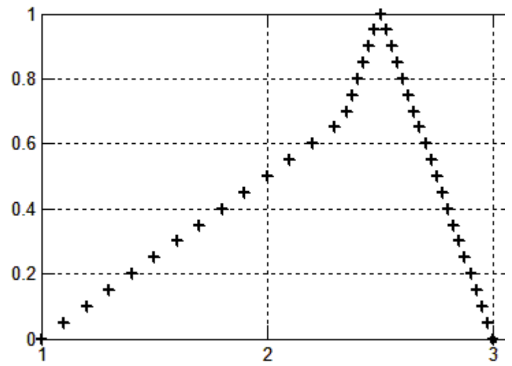

Figure 2. Graph of α-cuts bounds of BKS(1, 3, 4) $\tilde{\oplus}$ BKS(2, 2.5, 3)

and intersection point of $\frac{x-2}{0,5}$ and $\frac{x-1}{2}$ is (7/3 , 2/3), we obtain

$$\mu_{\tilde{c}}(x) = \begin{cases} 0 & , x < 1 \\ \frac{x-1}{2} & , 1 \le x \le 7/3 \\ \frac{x-2}{0,5} & , 7/3 < x \le 2,5 \\ 6-2x & , 2,5 < x \le 3 \\ 0 & , x > 3 \end{cases} .$$

ii) $\tilde{A} \tilde{\otimes} \tilde{B} = \begin{bmatrix} (1,3,4) & (1,2,3) \\ \tilde{\varepsilon} & (-3,-1,1) \end{bmatrix} \tilde{\otimes} \begin{bmatrix} (2,\ 2.5,\ 3) & (-5,-4,-2) \\ (1,2,4) & (-1,0,1) \end{bmatrix}$

$$= \begin{bmatrix} \{(1,3,4)\tilde{\otimes}(2,2.5,3)\}\tilde{\oplus}\{(1,2,3)\tilde{\otimes}(1,2,4)\} & \cdots \\ \{(\varepsilon,\varepsilon,\varepsilon)\tilde{\otimes}(2,2.5,3)\}\tilde{\oplus}\{(-3,-1,1)\tilde{\otimes}(1,2,4)\} & \cdots \end{bmatrix}$$

$$\begin{matrix} \cdots & \{(1,3,4)\tilde{\otimes}(-5,-4,-2)\}\tilde{\oplus}\{(1,2,3)\tilde{\otimes}(-1,0,1)\} \\ \cdots & \{(\varepsilon,\varepsilon,\varepsilon)\tilde{\otimes}(-5,-4,-2)\}\tilde{\oplus}(-3,-1,1)\tilde{\otimes}(-1,0,1)\} \end{matrix}\Bigg]$$

$$= \begin{bmatrix} (3,\ 5.5,\ 7)\tilde{\oplus}(2,4,7) & (-4,-1,2)\tilde{\oplus}(0,2,4) \\ (\varepsilon,\varepsilon,\varepsilon)\tilde{\oplus}(-2,1,4) & (\varepsilon,\varepsilon,\varepsilon)\tilde{\oplus}(-4,-1,2) \end{bmatrix}$$

$$= \begin{bmatrix} (2,\ 4,\ 7) & (-4,-1,2) \\ (-2,1,4) & (-4,-1,2) \end{bmatrix}.$$

We can show that $(\mathbf{F}(\mathbf{R})_{\max}^{m\times n},\ \tilde{\oplus})$ is a commutative idempotent semigroup. Moreover, $\mathbf{F}(\mathbf{R})_{\max}^{m\times n}$ is a semimodule over $\mathbf{F}(\mathbf{R})_{\max}$, where $(\mathbf{F}(\mathbf{R})_{\max}^{n\times n},\ \tilde{\oplus},\ \tilde{\otimes})$ is an idempotent semiring with matrix $\tilde{\mathcal{E}}$ as the neutral element and matrix $\tilde{E}$ as the unity element.

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