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INTERNATIONAL CONFERENCE ON MATHEMATICS, SCIENCE, AND EDUCATION

Theme :

Optimizing The Role of Science and Science Education in Global Cooperation

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INTERNATIONAL CONFERENCE ON MATHEMATICS, SCIENCE, AND EDUCATION

”Optimizing The Role of Science and Science Education in Global Cooperation”

Reviewers:

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FACULTY OF MATHEMATICS AND NATURAL SCIENCES
SEMARANG STATE UNIVERSITY

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PREFACE

Thanks to God Almighty this International Conference Proceeding could be completed. All articles in this proceeding are presented in International Conference On Mathematics, Science, and Education – Optimizing The Role of Science and Science Education in Global Cooperation on September 19-21st, 2014 at Patra Jasa Semarang Convention Hotel. This Conference is organized by Faculty of Mathematics and Natural Science. This proceeding has been reviewed of Mathematics and Science experts before it is published.

This conference is designed to improve the discussion and research scope in mathematics, science, and education area in the international level. Sub topics in this proceeding cover mathematics, applied mathematics, and mathematics education in accelerating character building. Enhancing biology and biology education research for a better life. Green chemistry in research and education. Physics and physics education for trending research.

Hopefully this publication of proceeding will be profitable for all of us.

Semarang, 3 December 2014

Regards
Committee of ICMSE 2014

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MESSAGE FROM THE DEAN OF FMIPA UNNES

Dear Participants of ICMSE 2014,

It is a pleasure to welcome all of you in the first International Conference on Mathematics and Science Educations (ICMSE 2014) held by Faculty of Mathematics and Natural Sciences, Semarang State University.

Faculty of Mathematics and Natural Science Semarang State University or more popularly known as FMIPA Unnes has 6 departments and 11 study programs of Mathematics and Natural Sciences education backgrounds and non education backgrounds. FMIPA Unnes has the mission of being an excellent and meaningful faculty by improving human resources through scientific activity.

One of efforts to result excellent and meaningful human resources through scientific activity is by performing discussion and knowledge sharing. To widen discussion of science and research development in mathematics and science educations scopes in national and international level, ICMSE 2014 was initiated as the medium of that discussion. I believe that ICMSE 2014 as the first international conference held by FMIPA Unnes can facilitate the knowledge sharing in mathematics and science educations area in order to establish a global cooperation among experts and researchers.

With the hope that this conference will be the medium to optimize the role of Mathematics, Science and Education in global cooperation, I am proud to welcome all of you and I wish you a pleasant sharing and discussion in this conference and enjoyable stay in Semarang, Indonesia.

Prof. Dr. Wiyanto, M.Si.
Dean of Faculty of Mathematics and Natural Sciences
Semarang State University

MESSAGE FROM CONFERENCE CHAIRMAN

My pleasure, welcome to you today on the occasion of this International Conference on Mathematics, Science, and Education (ICMSE 2014). I would like to extend my warmest welcome to all of the distinguished participants, especially those who have travelled long distances to be present here. This conference has already established itself as a key event to offer various thoughts and knowledge in enhancing our understanding in fundamental sciences and education.

This conference focus on "Optimizing The Role of Science and Science Education in Global Cooperation", offers all of us the opportunity to explore exciting information. The aim of the conference is to provide an interdisciplinary forum for scientist engaged in the full spectrum of research and development activities. The meeting intends to bring together researchers, scientists, and scholars to exchange and share their experiences, new ideas, and research result in related fields and discuss the practical challenges encountered and the solutions adopted. I invite all of you to approach this year's events to take advantage of the many ways in which you too might explore the unfamiliar - and discover a great deal in the process.

First, the various sessions that have been organized for the next day promise exciting revelation for all who attend them. Each speakers who are experts in their respective fields, will address a major topic or issue related to Fundamental Sciences,. You might learn more about a topic with which you were already familiar; or you might also find yourself discovering a whole new world of ideas and information you didn't know existed. Either way, you'll have many opportunities to explore fascinating new terrain with these reputable speakers.

Second, the key note speakers will provide, for all of us, an important window into the world of the future. We are privileged to have them as our key note speakers Prof. Barke, Munster University Germany, Prof. Martin Stein, Munster University Germany, Prof. Simone Krees, Munster University Germany, Prof. Matthias Ludwig, University Frankfurt Germany, Prof. Van Horssen, Delf Univesity Netherland, Prof. Rahim Sahar, UTM Malaysia and Dr. Margareta Rahayuningsih, M.Si experience has taken them through the whole cycle of Life and General science.

Finally, as you attend these various events, keep in mind that other people can also serve as doorways to new worlds. Hearing of someone else's background and experiences can often make for fascinating discoveries that can educate and profoundly affect us. So take advantage of this rare gathering of hundreds of people working in various fields to meet one another, talk with one another, and learn from one another.

In conclusion, I hope that you will find your time with us exciting. We have a great agenda for you with esteemed speakers and presenters from our profession. I do hope you will enjoy the next couple of days. I would like to once again extend my gratitude to all the participants, generous sponsor and I look forward to a most successful and fruitful conference.

Professor Dr. YL.Sukestiyarno
Chairman of ICMSE 2014

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A MAX-MIN ALGEBRA APPROACH TO MAXIMUM FUZZY CAPACITY ANALYSIS

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ABSTRACT

The capacities in a network are seldom precisely known, and then could be represented into the fuzzy number, that is called fuzzy capacity. This paper aims to determine the maximum fuzzy capacity of a path in the network with fuzzy capacity, using max-min algebra approach. The finding shows that the network with fuzzy capacity can be represented as a matrix over fuzzy number max-min algebra. The maximum fuzzy capacity of a path between two points in the network can be determined using the power operation, especially using star operation, for the matrix above. Furthermore, given a MATLAB program to compute the especially using star operation, for the matrix over fuzzy number max-min algebra.

Keywords: max-min algebra, fuzzy number, fuzzy capacity, maximum capacity

INTRODUCTION

Max-min algebra, the set of all real numbers **R** with the operation □ max (maximum) and min (minimum), have been used to determine the maximum capacity of a path with crisp capacity, in the form of real numbers (Gondran and Minoux, 2008)

In the problem of modeling and analysis of network capacity is sometimes unknown, for example because it is still at the design stage. These capacities can be estimated based on the experience and opinion of the experts and the network operator. In this case the capacity of the network can be modeled by a fuzzy number and its capacity is called *fuzzy capacity*.

Modeling and analysis of the maximum capacity path problem with capacity in the form of fuzzy numbers, as far as researchers know, no one has discussed, especially by using the approach of max-min algebra just as has been done for deterministic and probabilistic models. As has been known to approach problem solving network using max-min algebra can provide analytical results and further simplify the computation.

This article will discuss the fuzzy analysis of the determination of the maximum capacity of a path in the network by using fuzzy number max-min algebra approach. To facilitate the numerical calculations, we will use a computer program using MATLAB. From the discussion, this paper is expected as early theory for the next issue to the more complex, such as determining the flow in a network with a maximum fuzzy capacity.

MAX-MIN ALGEBRA

The following will be reviewed some basic concepts of max-min algebra and matrix algebra over the max-min, which can be seen in Baccelli, et al. (2001), Gondran and Minoux (2008), Rudhito (2013a, 2013b).

Let $\mathbf{R}_{\varepsilon}^{+} := \mathbf{R}^{+} \cup \{\varepsilon\}$ with $\mathbf{R}^{+}$ the set of all nonnegative real numbers and $\varepsilon : = +\infty$. In $\mathbf{R}_{\varepsilon}^{+}$ defined two operations: $\forall a,b \in \mathbf{R}_{\varepsilon}^{+}$, $a \oplus b := \max(a, b)$ and $a \otimes b : = \min(a, b)$. We can show that $(\mathbf{R}_{\varepsilon}^{+}, \oplus, \otimes)$ is a commutative idempotent semiring with neutral element $0 = 0$ and unity element $\varepsilon = +\infty$. Moreover $(\mathbf{R}_{\varepsilon}^{+}, \oplus, \otimes)$ is called *max-min algebra*, and is written as $\mathbf{R}_{\varepsilon}^{+}$. Since $\mathbf{R}_{\varepsilon}^{+}$ is an idempotent semiring, then operation $\oplus$ and $\otimes$ *consistent* to order $\preceq_{\mathrm{m}}$, that is $\forall a, b, c \in \mathbf{R}_{\varepsilon}^{+}$, if $a \preceq_{\mathrm{m}} b$, then $a \oplus c \preceq_{\mathrm{m}} b \oplus c$, and $a \otimes c \preceq_{\mathrm{m}} b \otimes c$. Max-min algebra $\mathbf{R}_\varepsilon^+$ *t does not contain a zero divisor* that is $\forall$ x, $y \in \mathbf{R}_\varepsilon^+$ hold: if $x \otimes y = \min(x, y) = 0$, then $x = 0$ or $y = 0$.

INTERVAL MAX-MIN ALGEBRA

The following will review the basic concepts of max-min interval algebra that is a generalization of max-min algebra and will be used as the basic for discussion of the fuzzy number max-min algebra through the Decomposition Theorem. A more complete discussion can be seen in Rudhito (2013a, 2013b).

It is known that $\mathbf{R}_\varepsilon^+$ is an partial ordered set with relation $\preceq_m$. Interval in $\mathbf{R}_\varepsilon^+$ has a form $x = [\underline{x}, \overline{x}] = \{ x \in \mathbf{R}_\varepsilon^+ \mid \underline{x} \preceq_m x \preceq_m \overline{x} \}$. Number $x \in \mathbf{R}_\varepsilon^+$ can be expressed with using interval $x = [x, x]$. Interval in $\mathbf{R}_\varepsilon^+$ for example are $[3, 5]$, $[0, 2]$, $[0, 0] = 0$ and $[\varepsilon, \varepsilon] = \varepsilon$.

It is also known that $(\mathbf{R}_\varepsilon^+, \oplus, \otimes)$ is an idempoten semiring and does not contain a zero divisor, with elemen netral ε. Defined $\mathbf{I}(\mathbf{R}^+)_\varepsilon = \{ x = [\underline{x}, \overline{x}] \mid \underline{x}, \overline{x} \in \mathbf{R}^+,\ \varepsilon \prec_m \underline{x} \preceq_m \overline{x} \} \cup \{[\varepsilon, \varepsilon]\}$. In $\mathbf{I}(\mathbf{R}^+)_\varepsilon$ defined operation $\overline{\oplus}$ and $\overline{\otimes}$ as follow $x \,\overline{\oplus}\, y = [\underline{x} \oplus \underline{y}, \overline{x} \oplus \overline{y}]$ and $x \,\overline{\otimes}\, y = [\underline{x} \otimes \underline{y}, \overline{x} \otimes \overline{y}]$ for every $x, y \in \mathbf{I}(\mathbf{R}^+)_\varepsilon$. We can show $(\mathbf{I}(\mathbf{R}^+)_\varepsilon, \overline{\oplus}, \overline{\otimes})$ is an idempotent semiring with neutral element $0 = [0, 0]$ and unity element satuan $\varepsilon = [\varepsilon, \varepsilon]$. Moreover since $(\mathbf{R}_\varepsilon, \oplus, \otimes)$ is an comutative idempotent semiring then $(\mathbf{I}(\mathbf{R})_\varepsilon, \overline{\oplus}, \overline{\otimes})$ is also comutative idempotent semiring. Futhermore $(\mathbf{I}(\mathbf{R}^+)_\varepsilon, \overline{\oplus}, \overline{\otimes})$ is called *interval max-min algebra* , which is written $\mathbf{I}(\mathbf{R}^+)_\varepsilon$.

Further, operations $\overline{\oplus}$ and $\overline{\otimes}$ in $\mathbf{I}(\mathbf{R}^+)_\varepsilon$ above can be extended for matrix operations in $\mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times n}$. Defined $\mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times n} := \{A = (A_{ij}) \mid A_{ij} \in \mathbf{I}(\mathbf{R}^+)_\varepsilon$, for $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n\}$. Matrix in $\mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times n}$ is called max-min *interval matrix*. Matrix A, $B \in \mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times n}$ is equal if $A_{ij} = B_{ij}$.

i) Let $\alpha \in \mathbf{I}(\mathbf{R}^+)_\varepsilon$, $A, B \in \mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times n}$. Defined scalar multiplication operation $\overline{\otimes}$ with $\alpha \,\overline{\otimes}\, A$ is a matrix where: $(\alpha \,\overline{\otimes}\, A)_{ij} = \alpha \,\overline{\otimes}\, A_{ij}$, and addition operation $\overline{\oplus}$ with $A \,\overline{\oplus}\, B$ is a matrix where: $(A \,\overline{\oplus}\, B)_{ij} = A_{ij} \,\overline{\oplus}\, B_{ij}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

ii) Let $A \in \mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times p}$, $B \in \mathbf{I}(\mathbf{R}^+)_\varepsilon^{p\times n}$. Defined multiplication operation $\overline{\otimes}$ with $A \,\overline{\otimes}\, B$ is a matrix where: $(A \,\overline{\otimes}\, B)_{ij} = \overline{\bigoplus_{k=1}^{p}} A_{ik} \,\overline{\otimes}\, B_{kj}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

To simplify the technical operation of interval matrix following given interval matrix concept of a matrix interval. Given $A \in \mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times n}$, defined matrix $\underline{A} = (\underline{A_{ij}}) \in \mathbf{R}_\varepsilon^{+m\times n}$ and $\overline{A} = (\overline{A_{ij}}) \in \mathbf{R}_\varepsilon^{+m\times n}$ are called *lower bound matrix* and *upper bound matrix* of interval matrix A, respectively. Defined *matrix interval* of A, that is $[\underline{A}, \overline{A}] = \{A \in \mathbf{R}_\varepsilon^{+m\times n} \mid \underline{A} \preceq_m A \preceq_m \overline{A} \}$ and $\mathbf{I}(\mathbf{R}_\varepsilon^{+m\times n})_b = \{ [\underline{A}, \overline{A}] \mid A \in \mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times n} \}$. Interval matrix $[\underline{A}, \overline{A}], [\underline{B}, \overline{B}] \in \mathbf{I}(\mathbf{R}_\varepsilon^{+m\times n})_b$ is *equal* if $\underline{A} = \underline{B}$ and $\overline{A} = \overline{B}$. Defined operations of matrix interval following.

i) Given $\alpha \in \mathbf{I}(\mathbf{R}^+)_\varepsilon$, $[\underline{A}, \overline{A}]$, $[\underline{B}, \overline{B}] \in \mathbf{I}(\mathbf{R}_\varepsilon^{+m\times n})_b$. Defined $\alpha \,\overline{\otimes}\, [\underline{A}, \overline{A}] := [\underline{\alpha} \otimes \underline{A}, \overline{\alpha} \otimes \overline{A}]$ and $[\underline{A}, \overline{A}] \,\overline{\oplus}\, [\underline{B}, \overline{B}] := [\underline{A} \oplus \underline{B}, \overline{A} \oplus \overline{B}]$

ii) Given $[\underline{A}, \overline{A}] \in \mathbf{I}(\mathbf{R}_\varepsilon^{+m\times p})_b$, $[\underline{B}, \overline{B}] \in \mathbf{I}(\mathbf{R}_\varepsilon^{+p\times n})_b$. Defined $[\underline{A}, \overline{A}] \,\overline{\otimes}\, [\underline{B}, \overline{B}] := [\underline{A} \otimes \underline{B}, \overline{A} \otimes \overline{B}]$.

We can show for every interval matrix $A \in \mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times n}$ always can be determined uniquely *matrix interval*$[\underline{A}, \overline{A}] \in \mathbf{I}(\mathbf{R}_\varepsilon^{+m\times n})_b$, and conversly. So interval matrix $A \in \mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times n}$ can be seen as matrix interval $[\underline{A}, \overline{A}] \in \mathbf{I}(\mathbf{R}_\varepsilon^{+m\times n})_b$. Interval matrix $A \in \mathbf{I}(\mathbf{R}^+)_\varepsilon^{m\times n}$ is said *corespond with* matrix interval $[\underline{A}, \overline{A}] \in \mathbf{I}(\mathbf{R}_\varepsilon^{+m\times n})_b$, and written "$A \approx [\underline{A}, \overline{A}]$". We also can be shown $\alpha \,\overline{\otimes}\, A \approx [\underline{\alpha} \otimes \underline{A}, \overline{\alpha} \otimes \overline{A}]$, $A \,\overline{\oplus}\, B \approx [\underline{A} \oplus \underline{B}, \overline{A} \oplus \overline{B}]$ and $A \,\overline{\otimes}\, B \approx [\underline{A} \otimes \underline{B}, \overline{A} \otimes \overline{B}]$.

FUZZY NUMBER MAX-MIN ALGEBRA

In this section we assume that readers have some knowledge of basic concepts of fuzzy set and fuzzy number. Further details can be found in Lee (2005) and Susilo (2006). In this article, fuzzy number operations defined by α-cuts. Firstly, we will review a theorem in the fuzzy set that we will use to following discussion.

Theorem 1. (Decomposition Theorem). If A^α is a α-cut of fuzzy set $\widetilde{A}$ in X and $\widetilde{A}^\alpha$ is a fuzzy set in X with membership function $\mu_{\widetilde{A}^\alpha}(x) = \alpha\ \chi_{A^\alpha}(x)$, where χ_{A^α} is the charateristic function of set A^α, then $\widetilde{A} = \bigcup_{\alpha\in[0,1]} \widetilde{A}^\alpha$.

Proof: (Zimmermann, 1991).

Definition 1. Let $\widetilde{a}$ and $\widetilde{b}$ are fuzzy number with $a^\alpha = [\underline{a^\alpha}, \overline{a^\alpha}]$ and $b^\alpha = [\underline{b^\alpha}, \overline{b^\alpha}]$, where $\underline{a^\alpha}$ and $\overline{a^\alpha}$ are lower bound and upper bound of interval a^α respectively, for $\underline{b^\alpha}$ and $\overline{b^\alpha}$ analog,

i) Maximum $\widetilde{a}$ and $\widetilde{b}$, i.e. $\widetilde{a}\ \widetilde{\oplus}\ \widetilde{b}$ is fuzzy set with its α-cut is interval $[\underline{a^\alpha} \oplus \underline{b^\alpha}, \overline{a^\alpha} \oplus \overline{b^\alpha}]$, for every $\alpha \in [0, 1]$.

ii) Minimum $\widetilde{a}$ and $\widetilde{b}$, i.e. $\widetilde{a}\ \widetilde{\otimes}\ \widetilde{b}$ is fuzzy set with its α-cut is interval $[\underline{a^\alpha} \otimes \underline{b^\alpha}, \overline{a^\alpha} \otimes \overline{b^\alpha}]$, for every $\alpha \in [0, 1]$.

To obtain the membership function of the results of operations on fuzzy numbers as above, can be used with Decomposition Theorem. In a manner analogous to the fuzzy number max-plus algebra in Rudhito, et al. (2008) and Rudhito (2011), we can show α-cut is defined in the above operations satisfied as α-cut family of a fuzzy number. Furthermore, using the decomposition theorem is obtained that $\widetilde{a}\ \widetilde{\oplus}\ \widetilde{b} = \widetilde{c} = \bigcup_{\alpha\in[0,1]} \widetilde{c}^\alpha$, where $\widetilde{c}^\alpha$ is a fuzzy set in $\mathbf{R}$ with membership function $\mu_{\widetilde{c}^\alpha}(x) = \alpha\chi_{(a\oplus b)^\alpha}(x)$, where $\chi_{(a\oplus b)^\alpha}$ is the characteristic function of set $(a \oplus b)^\alpha$. For the operation $\widetilde{\otimes}$ can be done with an analog way.

Given $\mathbf{F}(\mathbf{R}^+)_{\widetilde{\varepsilon}} := \mathbf{F}(\mathbf{R}^+) \cup \{\widetilde{\varepsilon}\}$ with $\mathbf{F}(\mathbf{R}^+)$ is the set of all positive fuzzy number and $\widetilde{\varepsilon} := \{-\infty\}$, with $\varepsilon^\alpha = [-\infty,-\infty]$ for every $\alpha \in [0, 1]$. In $(\mathbf{F}(\mathbf{R}^+))_{\widetilde{\varepsilon}}$ defined maximum $\widetilde{\oplus}$ and minimum $\widetilde{\otimes}$ operation as given above. With a manner analogous with the case of max-plus algebra as seen in the Rudhito (2011), we can show that structure of $(\mathbf{F}(\mathbf{R}^+)_{\widetilde{\varepsilon}}, \widetilde{\oplus}, \widetilde{\otimes})$ is a comutative idempoten semiring with neutral elemen $\widetilde{e} = \{0\}$, with $e^\alpha = [0, 0]$ and unity elemen $\widetilde{\varepsilon} := \{-\infty\}$, with $\varepsilon^\alpha = [-\infty,-\infty]$, for every $\alpha \in [0, 1]$. This comutative idempoten semiring ini is called *fuzzy number max-min algebra*, and is written as $\mathbf{F}(\mathbf{R}^+)_{\widetilde{\varepsilon}}$.

Concepts in matrix over the max-min algebra, also can be generalized to the matrix over the fuzzy numbers max-min algebra. The discussion is based on the results of the matrix and vector over intervals max-min algebra.

Definition 2. Defined $\mathbf{F}(\mathbf{R}^+)_{\varepsilon}^{m\times n} := \{\widetilde{A} = (\widetilde{A}_{ij}) \mid \widetilde{A}_{ij} \in \mathbf{F}(\mathbf{R}^+)_{\widetilde{\varepsilon}}$, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ $\}$. Matrix in $\mathbf{F}(\mathbf{R})_{\min}^{m\times n}$ is called matrix over fuzzy number max-min algebra. Furthermore, the above matrix, is called *fuzzy number matrix*. Matrix $\widetilde{A}$ and $\widetilde{B} \in \mathbf{F}(\mathbf{R}^+)_{\varepsilon}^{m\times n}$ is equal if $\widetilde{A}_{ij} = \widetilde{B}_{ij}$ for every i and j.

Operation $\widetilde{\oplus}$ and $\widetilde{\otimes}$ in $\mathbf{F}(\mathbf{R}^+)_{\widetilde{\varepsilon}}$ can be extended into fuzzy number matrices operations in $\mathbf{F}(\mathbf{R}^+)_{\varepsilon}^{m\times n}$.

i) Let $\widetilde{\lambda} \in \mathbf{F}(\mathbf{R}^+)_{\widetilde{\varepsilon}}$, $\widetilde{A}, \widetilde{B} \in \mathbf{F}(\mathbf{R}^+)_{\varepsilon}^{m\times n}$. Defined $\widetilde{\lambda}\ \widetilde{\otimes}\ \widetilde{A}$ is a matrix where $(\widetilde{\lambda}\ \widetilde{\otimes}\ \widetilde{A})_{ij} = \widetilde{\lambda}\ \widetilde{\otimes}\ \widetilde{A}_{ij}$ for $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ and $\widetilde{A}\ \widetilde{\oplus}\ \widetilde{B}$ is a matrix where $(\widetilde{A}\ \widetilde{\oplus}\ \widetilde{B})_{ij} = \widetilde{A}_{ij}\ \widetilde{\oplus}\ \widetilde{B}_{ij}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

ii) Let $\widetilde{A} \in \mathbf{F}(\mathbf{R}^+)_{\varepsilon}^{m\times p}$, $\widetilde{B} \in \mathbf{F}(\mathbf{R}^+)_{\varepsilon}^{p\times n}$. Defined $\widetilde{A}\ \widetilde{\otimes}\ \widetilde{B}$ is a matrix where $(\widetilde{A}\ \widetilde{\otimes}\ \widetilde{B})_{ij} = \widetilde{\bigoplus_{k=1}^{p}} \widetilde{A}_{ik} \otimes \widetilde{B}_{kj}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$

Definisi 3. For every $\widetilde{A} \in \mathbf{F}(\mathbf{R}^+)_{\varepsilon}^{m\times n}$ and $\alpha \in [0, 1]$, defined a *α-cut matrix of* $\widetilde{A}$, as the interval matrix $A^\alpha = (A_{ij}^\alpha) \in \mathbf{I}(\mathbf{R}^+)_{\varepsilon}^{m\times n}$, *with* $A_{ij}^\alpha \in \mathbf{I}(\mathbf{R}^+)_{\varepsilon}$. Matrix $\underline{A^\alpha} = (\underline{A_{ij}^\alpha}) \in \mathbf{R}_{\varepsilon}^{+m\times n}$ and $\overline{A^\alpha} = (\overline{A_{ij}^\alpha}) \in \mathbf{R}_{\varepsilon}^{+m\times n}$ which are called *lower bound* and *upper bound* of matrix A^α. For $\widetilde{A}, \widetilde{B} \in \mathbf{F}(\mathbf{R}^+)_{\varepsilon}^{m\times n}$ is *equal* if $A^\alpha = B^\alpha$ for every $\alpha \in [0, 1]$, yaitu $A_{ij}^\alpha = B_{ij}^\alpha$ for every i and j.

We can show $A^\alpha \approx [\underline{A}^\alpha, \overline{A}^\alpha]$, $(\lambda \otimes A)^\alpha \approx [\underline{\lambda^\alpha} \otimes \underline{A^\alpha}, \overline{\lambda^\alpha} \otimes \overline{A^\alpha}]$, $(A \oplus B)^\alpha \approx [\underline{A^\alpha} \oplus \underline{B^\alpha}, \overline{A^\alpha} \oplus \overline{B^\alpha}]$ and $(A \otimes B)^\alpha \approx [\underline{A^\alpha} \otimes \underline{B^\alpha}, \overline{A^\alpha} \otimes \overline{B^\alpha}]$, for every $\alpha \in [0, 1]$.

MAXIMUM FUZZY CAPACITY ANALYIS

Let $\widetilde{\mathcal{G}} = (V, \widetilde{A})$ with $V = \{1, 2, \ldots, p\}$ is nonempty finite set which is its elements is called *node* and A is a set of ordered pairs of nodes. A directed graph $\widetilde{\mathcal{G}}$ is said to be *fuzzy number weighted* if every arch $(j, i) \in \widetilde{A}$ corresponds to a fuzzy number $\widetilde{A}_{ij} \in (\mathbf{F}(\mathbf{R}^{+})_{\widetilde{\varepsilon}} - \{\varepsilon\})$. The fuzzy number $\widetilde{A}_{ij}$ is called the fuzzy weight of arch (j, i), and is written as $fw(j, i)$. In pictorial representation of a fuzzy number weighted directed graph, arches are labelled by its weight. Define a *fuzzy number precedence graph* of a matrix $\widetilde{A} \in \mathbf{F}(\mathbf{R}^{+})_{\varepsilon}^{n \times n}$ as fuzzy number weighted directed graph $\widetilde{\mathcal{G}}(\widetilde{A}) = (V, \widetilde{A})$ with $V = \{1, 2, \ldots, n\}$, $\widetilde{A} = \{(j, i) \mid fw(i, j) = \widetilde{A}_{ij} \neq \varepsilon\}$. Conversely, for every fuzzy number weighted directed graph $\widetilde{\mathcal{G}} = (V, \widetilde{A})$ can be defined a matrix $\widetilde{A} \in \mathbf{F}(\mathbf{R}^{+})_{\varepsilon}^{n \times n}$, which is called the *fuzzy weighted matrix* of graph $\widetilde{\mathcal{G}}$, where $\widetilde{A}_{ij} = \begin{cases} fw(j,i), & \text{jika } (j,i) \in \widetilde{A} \\ \widetilde{\varepsilon}, & \text{jika } (j,i) \notin \widetilde{A}. \end{cases}$

With menggunakan hasil pembahasan dalam kasus pasitas real and interval in landasan teori di over, berikut dibahas kapasitas *fuzzy* maksimum suatu lintasan dalam jaringan with kapasitas *fuzzy*.

In the maximum fuzzy capacity path problem, $\widetilde{A}_{ij}$ is a nonnegative fuzzy number, i.e. the fuzzy number that every its α-cut is a closed interval which is its lower bound and upper bound are non-negative real numbers, and this fuzzy number is a *fuzzy capacity* of arc (j, i), i.e. the fuzzy maximum flow through the arc (j, i). Using the results of the discussion in the case of real capacities (Rudhito, 2013c) and intervals capacities (Rudhito, 2014), the following will be discussed fuzzy maximum capacity of a path in a network with fuzzy capacity.

RESULTS AND DISCUSSION

Teorema 1 If $\widetilde{A} \in \mathbf{F}(\mathbf{R}^{+})_{\varepsilon}^{n \times n}$ is a fuzzy weighted matrix of a fuzzy number weighted directed graph, where $\widetilde{A}_{ij}$ is a fuzzy capacity of arc (j, i), i.e. the fuzzy maximum flow through the arc (j, i), then entries $(\widetilde{A}^{*})_{ij}$ is maximum fuzzy capacity of a path with starting point j and end poin i.

Proof: The α-cut matrix of fuzzy matrix $\widetilde{A}$ are A^{α} for every $\alpha \in [0, 1]$, where $(A^{\alpha})_{ij}$ is interval capacities of arch (j, i). According to the Theorem 3 in Rudhito (2014), entries $((A^{\alpha})^{*})_{ij}$ are maximum interval capacity of path with starting point j and end point i, for every $\alpha \in [0, 1]$. Since operation in matrix is consisten, then can be conclude that entries $(\widetilde{A}^{*})_{ij}$ is a maximum fuzzy capacity of path with starting point j and end point i. ■

Here's an example of a fuzzy capacity network and the calculation of the fuzzy maximum capacity for each path.

Example 1 Given a fuzzy capacity network, in the form of triangular fuzzy numbers as in Figure 1 below.

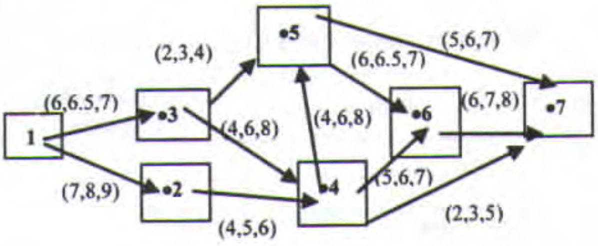


Figure 1. A Fuzzy Capacity Network

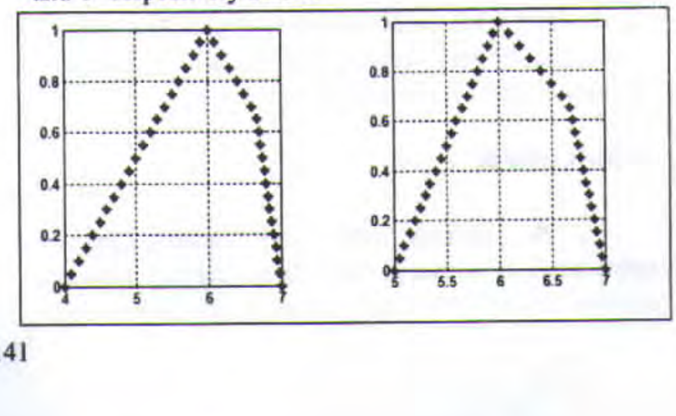
Fuzzy weighted matrix of directed graph in capacity network above is matrix $\widetilde{A}$ below.

$$\widetilde{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (7,8,9) & 0 & 0 & 0 & 0 & 0 & 0 \\ (6,6.5,7) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (4,5,6) & (4,6,8) & 0 & 0 & 0 & 0 \\ 0 & 0 & (2,3,4) & (4,6,8) & 0 & 0 & 0 \\ 0 & 0 & 0 & (5,6,7) & (6,6.5,7) & 0 & 0 \\ 0 & 0 & 0 & (2,3,5) & (5,6,7) & (6,7,8) & 0 \end{bmatrix}$$

Using *MATLAB* program, can be obtain matrix:

$$\widetilde{A}^{*} = \begin{bmatrix} \widetilde{\varepsilon} & 0 & 0 & 0 & 0 & 0 & 0 \\ (7,8,9) & \widetilde{\varepsilon} & 0 & 0 & 0 & 0 & 0 \\ (6,6.5,7) & 0 & \widetilde{\varepsilon} & 0 & 0 & 0 & 0 \\ \widetilde{a} & (4,5,6) & (4,6,8) & \widetilde{\varepsilon} & 0 & 0 & 0 \\ \widetilde{a} & (4,5,6) & (4,6,8) & (4,6,8) & \widetilde{\varepsilon} & 0 & 0 \\ \widetilde{a} & (4,5,6) & \widetilde{a} & \widetilde{b} & (6,6.5,7) & \widetilde{\varepsilon} & 0 \\ \widetilde{a} & (4,5,6) & \widetilde{a} & \widetilde{b} & (6,6.5,7) & (6,7,8) & \widetilde{\varepsilon} \end{bmatrix}$$

where graph of fuction and membership function of $\widetilde{a}$ and $\widetilde{b}$ respectively below.



$$\mu_{\widetilde{a}}(x)=\begin{cases}0 & ,\ x<4\\ \dfrac{x-4}{2} & ,\ 4\le x\le 6\\ \dfrac{8-x}{2} & ,\ 6<x\le \dfrac{20}{3}\\ 14-2x & ,\ \dfrac{20}{3}<x\le 7\\ 0 & ,\ x>7\end{cases}\qquad \mu_{\widetilde{b}}(x)=\begin{cases}0 & ,\ x<5\\ x-5 & ,\ 5\le x\le 6\\ \dfrac{8-x}{2} & ,\ 6<x\le \dfrac{20}{3}\\ 14-2x & ,\ \dfrac{20}{3}<x\le 7\\ 0 & ,\ x>7\end{cases}$$

Figure 2 Graph of Membership Function of $\widetilde{a}$ and $\widetilde{b}$

From matrix $\widetilde{A}^{*}$ can be seen that maximum capacity of path with starting poin **j** are the entries of colomn **j**th of matrix $\widetilde{A}^{*}$. Table 1 below given list of all path and its maximum fuzzy capacities.

Table 1 Maximum Fuzzy Capacity of Paths

No	Path	Maximum Fuzzy Capacity
1	$1\to 2$	(7, 8, 9)
2	$1\to 3$	(6, 6.6, 7)
3	$1\to 3\to 4$	$\widetilde{a}$
4	$1\to 3\to 4\to 5$	$\widetilde{a}$
5	$1\to 3\to 4\to 5\to 6$	$\widetilde{a}$
6	$1\to 3\to 4\to 5\to 6\to 7$	$\widetilde{a}$
7	$2\to 4$	(4, 5, 6)
8	$2\to 4\to 5$	(4, 5, 6)
9	$2\to 4\to 5\to 6$	(4, 5, 6)
10	$2\to 4\to 5\to 7$	(4, 5, 6)
11	$3\to 4$	(4, 6, 8)
12	$3\to 4\to 5$	(4, 6, 8)
13	$3\to 4\to 5\to 6$	$\widetilde{a}$
14	$3\to 4\to 5\to 6\to 7$	$\widetilde{a}$
15	$4\to 5$	(4, 6, 8)
16	$4\to 5\to 6$	$\widetilde{b}$
17	$4\to 5\to 6\to 7$	$\widetilde{b}$
18	$5\to 6$	(6, 6.5, 7)
19	$5\to 6\to 7$	(6, 6.5, 7)
20	$6\to 7$	(6, 7, 8)

Form the result of Example 1 above, can be seen that the maximum fuzzy capacity of the paths in the network is not always a fuzzy capacities of arc of path in a network is not like the real capacity that has always been one capacity of arch of path in its network.

CONCLUSION

The network with fuzzy capacity can be represented as a matrix over fuzzy number max-min algebra. The maximum fuzzy capacity of a path between two points in the network can be determined using the power operation, especially using star operation, for the matrix above. Furthermore, given a *MATLAB* program to compute the especially using star operation, for the matrix over fuzzy number max-min algebra.

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