PERTURBED INVARIANT UNDER A CYCLIC PERMUTATION WITH TRACE OF NEUTRINO MASS MATRIX REMAIN CONSTANT

Asan Damanik[1]

Department of Physics, Faculty of Science and Technology,
Sanata Dharma University,
Kampus III USD Paingan, Maguwoharjo, Sleman,Yogyakarta, Indonesia

Abstract

We construct a neutrino mass matrix M_ν via a seesaw mechanism with perturbed invariant under a cyclic permutation by introducing one parameter δ into the diagonal elements of M_ν with assumption that trace of the perturbed M_ν is equal to trace of the unperturbed M_ν. We found that the perturbed neutrino mass matrices M_ν can predicts the mass-squared difference $\Delta m_{ij}^2 \neq 0$ with the possible hierarchy of neutrino mass is normal or inverted hierarchy. By using the advantages of the mass-squared differences and mixing parameters data from neutrino oscillation experiments, we then have neutrino masses in inverted hierarchy with masses: $|m_1| = 0.101023$ eV, $|m_2| = 0.101428$ eV, and $|m_3| = 0.084413$ eV.

1 Introduction

Recently, there is a convincing evidence that neutrinos have a tiny non-zero mass. The evidence of neutrino mass is based on the experimental facts that both solar and atmospheric neutrinos undergo oscillations.[1]-[6] A global analysis of neutrino oscillations data gives the best fit value to solar neutrino mass-squared differences,[7]

$$\Delta m_{21}^2 = (8.2_{-0.3}^{+0.3}) \times 10^{-5} \text{ eV}^2 \tag{1}$$

with

$$\tan^2 \theta_{21} = 0.39_{-0.04}^{+0.05}, \tag{2}$$

and for the atmospheric neutrino mass-squared differences

$$\Delta m_{32}^2 = (2.2_{-0.4}^{+0.6}) \times 10^{-3} \text{ eV}^2 \tag{3}$$

---

[1]E-mail: d.asan@lycos.com

1

with
$$\tan^2 \theta_{32} = 1.0^{+0.35}_{-0.26}, \tag{4}$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ $(i, j = 1, 2, 3)$ with m_i is the neutrino mass in eigenstates basis ν_i $(i = 1, 2, 3)$, and θ_{ij} is the mixing angle between ν_i and ν_j. The mass eigenstates basis are related to the weak (flavor) eigenstates basis $(\nu_e, \nu_\mu, \nu_\tau)$ as follows,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{5}$$

where V is the mixing matrix.

It is also known that neutrino masses are very small compared to its corresponding charged lepton masses and mixing does exist in neutrino sector. Charged lepton mass has a normal hierarchy, but neutrino mass can have either a normal or an inverted hierarchy. Thus, neutrinos have some different properties from charged leptons. From the theoretical side, it has been a guiding principle that the presence of hierarchies or of tiny quantities imply a certain protection symmetry in underlying physics. The candidates of such symmetry in neutrino physics may include $U(1)_{L'}$ based on the conservation of $L_e - L_\mu - L_\tau = L'$ and a $\mu - \tau$ symmetry based on the invariance of flavor neutrino mass term underlying the interchange of ν_μ and ν_τ.

To accommodate a tiny non-zero neutrino mass that can produce the mass-squared differences and the neutrino mixing, several models for the neutrino mass matrices together with the responsible mechanisms for generating it patterns have been proposed by many authors. One of the interesting mechanism which can generate a small neutrino mass is the seesaw mechanism, in which the right-handed neutrino ν_R has a large Majorana mass M_N and the left-handed neutrino ν_L obtain a mass through leakage of the order of (m/M_N) with m is the Dirac mass.[8]

According to seesaw mechanism,[9] the neutrino mass matrix M_ν is given by,

$$M_\nu \approx -M_D M_N^{-1} M_D^T \tag{6}$$

where M_D and M_N are the Dirac and Majorana mass matrices respectively. The mass matrix model of a massive Majorana neutrino M_N which is constrained by the solar and atmospheric neutrinos deficit and incorporating the seesaw mechanism and Peccei-Quinn symmetry have been reported by Fukuyama and Nishiura.[10] Neutrino mass matrix patterns together with its underlying symmetry become an interesting research topic during the last few years. In related to the seesaw mechanism, Ma[11] pointed out that it is more sense to consider the structure of M_N for its imprint on M_ν.

In order to consider the structure of the M_N for its imprint on M_ν, in this paper we construct the neutrino mass matrices M_ν arise from a seesaw mechanism with both heavy Majorana and Dirac neutrino mass matrix are invariant under a cyclic permutation. As we have already knew that neutrino mass matrix which is invariant under a cyclic permutation gives $m_1 = m_3$ and then it fails to predict mass-squared difference $\Delta m_{31}^2 \neq 0$. The charged-lepton mass matrix which is invariant under a cyclic permutation have been analyzed by Koide[12] that also suggested to break the invariant under a cyclic permutation if we want to obtain the charged-lepton mass spectrum compatible with the empirical fact.

To overcome the weakness of cyclic permutation on predicting mass-squared differences, in this paper we introduce a perturbation into neutrino mass matrix with

the assumption that the perturbed cyclic permutation mass matrix has the same trace with the unperturbed neutrino mass matrix. This paper is organized as follows: In Section 2, we construct the heavy Majorana and Dirac neutrino mass matrices which are invariant under a cyclic permutation. The resulted neutrino mass matrices to be used for obtaining the neutrino mass matrix M_ν in the scheme of seesaw mechanism. In Section 3, we use a seesaw mechanism for obtaining neutrino mass matrix and evaluate its phenomenological consequences. Finally, the Section 4 is devoted to a conclusion.

2   Neutrino Mass Matrix with Invariant under Cyclic Permutation

As we have already stated above, the aim of this paper is to study the phenomenological consequences of the perturbed cyclic permutation neutrino mass matrices arise from a seesaw mechanism with both heavy Majorana and Dirac neutrino mass matrices are invariant under a cyclic permutation. The seesaw mechanism to be considered in this paper is the type-I seesaw. In order to realize the goals of this section, first we write down the the possible patterns for heavy Majorana neutrino mass matrices M_N is invariant under a cyclic permutation. Second, we write down the possible patterns for Dirac neutrino mass matrices by taking into account the same constraints that we have imposed on heavy Majorana neutrino mass matrix.

We consider the Majorana neutrino mass matrix M_N in Eq. (6) to be symmetric in form and that matrix is given by

$$M_N = \begin{pmatrix} A & B & C \\ B & D & E \\ C & E & F \end{pmatrix}. \tag{7}$$

In order to obtain the M_N that invariant under a cyclic permutation among neutrino fields: $\nu_1 \to \nu_2 \to \nu_3 \to \nu_1$, first we define

$$\nu_i' = U_{ij}\nu_j, \tag{8}$$

where U_{ij} are the entries of the cyclic permutation matrix U. From Eq. (8), one can see that the M_N matrix satisfy

$$M_N' = U^T M_N U. \tag{9}$$

If the M_N matrix is invariant under a cyclic permutation, then the pattern of the M_N' is the same with the M_N pattern.

By imposing the requirement that the form of the M_N matrix in Eq. (7) must be invariant under a cyclic permutation together with the requirement that the M_N is non-singular matrix such that the M_N has a M_N^{-1}, then we have three possible patterns for heavy Majorana neutrino mass matrix M_N as follow

$$M_N = \begin{pmatrix} A & B & B \\ B & A & B \\ B & B & A \end{pmatrix}, \tag{10}$$

$$M_N = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{pmatrix}, \tag{11}$$

$$M_N = \begin{pmatrix} 0 & B & B \\ B & 0 & B \\ B & B & 0 \end{pmatrix}. \tag{12}$$

From Eqs. (10), (11), and (12), one can see that the patterns of neutrino mass matrices in Eqs. (11) and (12) are special cases of the neutrino mass matrix pattern in Eq. (10). Thus, the neutrino mass matrix given by Eq. (10) is the most general pattern, and we will consider it as a good candidate for neutrino mass matrix M_N.

To obtain the neutrino mass matrices M_ν arise from a seesaw mechanism (using Eq. (6)), we should know the patterns of the Dirac neutrino mass matrices M_D. Because the heavy neutrino fields are part of the Dirac mass term, according to Eqs. (8) and (9), the possible patterns for Dirac neutrino mass matrices are given by

$$M_D = \begin{pmatrix} a & a & a \\ a & a & a \\ a & a & a \end{pmatrix}, \tag{13}$$

$$M_D = \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix}, \tag{14}$$

$$M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}, \tag{15}$$

$$M_D = \begin{pmatrix} 0 & b & b \\ b & 0 & b \\ b & b & 0 \end{pmatrix}. \tag{16}$$

It is apparent from Eqs. (13)-(16) that the pattern of neutrino mass matrix M_D in Eq. (14) is the most general pattern. Thus, we will consider neutrino mass matrix M_D in Eq. (14) as a good candidate for Dirac neutrino matrix.

3 Neutrino Mass matrix via a Seesaw Mechanism

Using the seesaw mechanism in Eq. (6), the heavy Majorana neutrino mass matrix in Eq. (10), and the Dirac neutrino mass matrix in Eq. (14), we then obtain a neutrino

mass matrix with pattern,

$$M_\nu = \begin{pmatrix} P & Q & Q \\ Q & P & Q \\ Q & Q & P \end{pmatrix}. \tag{17}$$

The eigenvalues of the neutrino mass matrix in Eq. (17) are given by

$$\lambda_1 = \lambda_2 = P - Q, \ \lambda_3 = P + 2Q. \tag{18}$$

It is easy to see that one of the eigenvectors of the M_ν is $(1,1,1)^T$ and this eigenvector corresponds to eigenvalue λ_3. Thus, the eigenvalue λ_3 should be identified as neutrino mass m_2, meanwhile λ_1 and λ_2 correspond to neutrino masses m_1 and m_2. Finally, the neutrino mass matrix in Eq. (17) gives neutrino masses,

$$m_1 = m_3 = P - Q, \ m_2 = P + 2Q. \tag{19}$$

From Eq. (19), we have $\Delta m_{21}^2 = |\Delta m_{32}^2| = 6PQ + 3Q^2$ which is contrary to the experimental fact. Thus, the neutrino mass matrix in Eq. (17) could not reproduce the mass-squared difference $\Delta m_{21}^2 << \Delta m_{32}^2$. It is also apparent that the resulted neutrino mass matrix in this scenario gives $m_1 + m_2 + m_3 = 3P$ is which is equal to $Tr(M_\nu)$.

Even though neutrino mass matrix that invariant under a cyclic permutation, as one can see in Eq. (17), could not predict correctly the experimental data, we can still use it as a neutrino mass matrix. In order to obtain neutrino mass matrix that can give correct predictions on mass-squared differences and mixing parameters, we modify the neutrino mass matrix M_ν in Eq. (17) by introducing one parameter δ to perturb the diagonal elements of M_ν such that the perturbed mass matrix satisfies the requirement $Tr(M_\nu) = 3P$. In this scenario, we then can put the neutrino mass matrix M_ν in form,

$$M_\nu = \begin{pmatrix} P + 2\delta & Q & Q \\ Q & P - \delta & Q \\ Q & Q & P - \delta \end{pmatrix}. \tag{20}$$

The eigenvalues of the neutrino mass matrix in Eq. (20) read

$$\beta_{1,2} = P + \frac{Q}{2} + \frac{\delta}{2} \mp \frac{\sqrt{9\delta^2 - 6Q\delta + 9Q^2}}{2}, \tag{21}$$

$$\beta_3 = P - Q - \delta. \tag{22}$$

If the neutrino mass matrices M_ν in Eq. (20) is diagonalized by mixing matrix V in Eq. (5) with V given by[11]

$$V = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & -1/\sqrt{2} \\ \sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{23}$$

then we obtain,

$$\tan^2(2\theta) = \frac{8Q^2}{(Q - 3\delta)^2}, \tag{24}$$

and neutrino masses as follow,

$$m_1 = P + \frac{Q}{2} + \frac{\delta}{2} - \frac{\sqrt{9\delta^2 - 6Q\delta + 9Q^2}}{2}, \tag{25}$$

$$m_2 = P + \frac{Q}{2} + \frac{\delta}{2} + \frac{\sqrt{9\delta^2 - 6Q\delta + 9Q^2}}{2}, \tag{26}$$

$$m_3 = P - Q - \delta. \tag{27}$$

One can see that the obtained neutrino masses in this scenario is an inverted hierarchy with masses: $|m_3| < |m_1| < |m_2|$.

If θ is the θ_{21} in Eq. (2), then from Eq. (24) we have $\delta = -0.1271Q$. If we insert this δ value into Eqs. (25)-(27), then we have the neutrino masses as follow

$$m_1 = P - 0.1374Q, \tag{28}$$

$$m_2 = P + 2.0103Q, \tag{29}$$

$$m_3 = P - 0.8729Q. \tag{30}$$

The plot of m_1, m_2, and m_3 as function of parameters P and Q are shown in Fig.1.
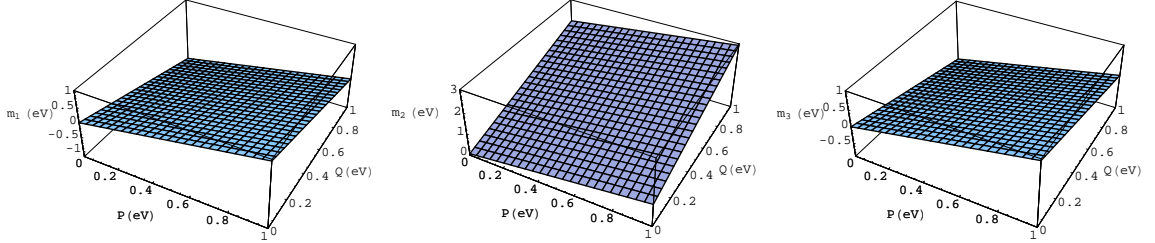

Figure 1: Neutrino masses m_1, m_2, and m_3 as function of parameters P and Q.

From Fig. 1 we can see that the neutrino mass can have normal, degenerate, or inverted hierarchy which it depends on the sign and values of of parameters P and Q. For example, if we put the values of $P = -0.8$ and $Q = 1$, then we have neutrino masses in normal hierarchy: $|m_1| < |m_2| < |m_3|$. The degenerate hierarchy: $|m_1| \approx |m_2| \approx |m_3|$ can be obtained if we put the value of parameter $Q \approx 0$, and the inverted hierarchy: $|m_3| < |m_1| < |m_2|$ is produced for $P > 0.8729Q$ and $Q > 0$.

If we use the advantages of the experimental data of neutrino oscillation in Eqs. (1) and (3), from Eqs. (25), (26), and (27), then we obtain the neutrino masses,

$$|m_1| = 0.101023 \text{ eV} , \ |m_2| = 0.101428 \text{ eV} , \ |m_3| = 0.084413 \text{ eV}, \tag{31}$$

for $Q = -0.06432$ eV and $P = 0.02827$ eV. One can see that the value of $\delta = 0.008176$ eV is smaller than the values of P and Q. Inserting the obtained values of P, Q, and δ into Eq. (20), we finally have neutrino mass matrix in eV unit as follow

$$M_\nu = \begin{pmatrix} 0.04463 & -0.06432 & -0.06432 \\ -0.06432 & 0.02010 & -0.06432 \\ -0.06432 & -0.06432 & 0.02010 \end{pmatrix}. \tag{32}$$

4 Conclusion

Neutrino mass matrix M_ν arise from a seesaw mechanism, with both heavy Majorana and Dirac neutrino mass matrices are invariant under a cyclic permutation, can not be used to explain the present experimental data of neutrino oscillation. By introducing one parameter δ to perturb the diagonal elements of M_ν with the assumption that the value of the trace of M_ν remain constant, we then have a neutrino mass matrix that can be used to explain mass-squared differences. In this scenario, the possible hierarchy of neutrino mass that can be used to explain mass-squared differences is normal or inverted hierarchy. By using the mass-squared differences and mixing parameters which obtained from neutrino oscillation experiments, we then have neutrino masses in inverted hierarchy with masses: $|m_1| = 0.101023$ eV, $|m_2| = 0.101428$ eV, and $|m_3| = 0.084413$ eV.

Acknowledgments

References

[1] Y. Fukuda *et al.*, *Phys. Rev. Lett.* **81**, 1158 (1998).

[2] Y. Fukuda *et al.*, *Phys. Rev. Lett.* **82**, 2430 (1999).

[3] M. H. Ahn *et al.*, *Phys. Rev. Lett.* **90**, 041801-1 (2003).

[4] T. Toshito *et al.*, hep-ex/0105023.

[5] G. Giacomelli and M. Giorgini, hep-ex/0110021.

[6] Q.R. Ahmad *et al.*, *Phys. Rev. Lett.* **89**, 011301 (2002).

[7] M.C. Gonzales-Garcia, *Phys. Scripta* **T121**, 72 (2005).

[8] M. Fukugita and T. Yanagida, *Physics of Neutrinos and Application to Astrophysics* (Springer-Verlag, 2003).

[9] M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity*, eds. P. van Niewenhuzen and D. Z. Freedman (North-Holland, 1979).

[10] T. Fukuyama and H. Nishiura, arXiv: hep-ph/9702253.

[11] E. Ma, *Phys. Rev.* **D71**, 111301 (2005).

[12] Y. Koide, arXiv: hep-ph/0005137v1.