DISCONTINUOUS GALERKIN APPROACH FOR OBTAINING GORLOV TURBINE EXTRACTED POWER

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Abstract. *A 2D discontinuous Galerkin approach using LDG flux is used to count the efficiency of Gorlov turbine extracted power. The flow is assumed to be incompressible and has Re = 10000. The model is based on the experimentally proven dimension with simplification on helical approach and modification on NACA blade number due to the angle of the flow. The result of computation shows an agreement to the experiment result. They have efficiency of 35%, which is the maximum of efficiency of extracting energy from free-flow using Gorban's model.*

Keywords: *Gorlov Turbine, Discontinuous Galerkin Methods.*

1 Introduction

Computation becomes an emerging tool for technology and science on fluid dynamics. Moreover it will support most of the research on this area. Computational model will cut the cycle of research and the cost [1]. Computational also becomes the only way for atmosphere and oceanic modeling which is an impossible thing to solve analytically.

One of the problems which have not been solved analytically yet is Navier–Stokes Equations. These equations consist of mass conservation, linear momentum conservation and energy conservation. For incompressible fluid model, the problem comes to mass conservation and linear momentum conservation. The energy conservation is neglected due to assumption of no energy transfer during process. In this kind problem, it confines difficulty of obtaining stable solution due to convection like character which is a 1st order problem. This problem needs special method to cope. Discontinuous Galerkin naturally has method to face the problem [5].

The advance of Discontinuous Galerkin will be used to predict the power which can be extracted using Gorlov Turbine. This turbine is designed to be used in free flow or low head flow. It is very promising technology of extracting energy from abundant energy resources such as ocean flow and wind. The computation of this turbine will support the application which is based on special case.

This work will be done on the based of experimental idealized case. The turbine is assumed in incompressible flow; and its wake which is generated during operation is neglected. The power extracted on operation of the turbine is assumed only from frontal flow and can be defined from the pressure which generate drag and lift forces onto the blades. Both variables are achieved using discontinuous finite elements. The method is done in time domain which is actually an approach to get deceive convergent condition.

2 Governing equation of the incompressible flow

Incompressible flow is a flow which takes place in low Reynold's number. Due to the assumption of no heat transfer during process, the Navier-Stokes equations will be dropped into [5]

$$\nabla.\mathbf{u} = 0 \in \Omega \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{u} + \mathbf{f} \in \Omega \tag{2}$$

with $u = (u_x, u_y)$ as the velocity and as the force. In condition without force as in free flow, **f** will be zero.

The equation (1) is equation of mass conservation which is called continuity equation and the other equations (2) are equation of linear momentum conservation. Mass conservation explains that during the process the compressibility which makes change of density does not exist.

The linear momentum conservation equations tell us that fluid moving is in accordance with linear momentum and viscous effect. They also show that pressure exists due to the change of momentum linear and viscous effect. The equations have pressure parts which are 1st order and velocities parts which are 2nd order. The existence of both brings the spurious problem on computation [5].

In a discontinuous galerkin forms, the (2) equation splited into [5]

$$\underline{\tau} = \frac{1}{\mathrm{Re}} \nabla \mathbf{u} \tag{3}$$

and

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \nabla.\underline{\tau} + \mathbf{f} \tag{4}$$

Therefore, all of the equations are in the same order.

3 Discontinuous galerkin representation of incompressible navier-stokes

For incompressible flow, therefore three equations exist that are equation (1), (3) and (4). The 1st equation is continuity equation. The 2nd and 3rd are continuity of momentum linear. Discontinuous Finite Element formulation of these equations will be [5]

$$-\int_{\Omega_j} \mathbf{u}.\nabla v dA + \int_{\partial\Omega_j} \mathbf{u}.\mathbf{n} v dl = 0 \tag{5}$$

$$\int_{\Omega_j} \underline{\tau}.\underline{\sigma} dA = -\frac{1}{\mathrm{Re}} \int_{\Omega_j} \mathbf{u}\nabla.\underline{\sigma} dA + \frac{1}{\mathrm{Re}} \int_{\partial\Omega_j} \mathbf{u}.\underline{\sigma}.\mathbf{n} v dl \tag{6}$$

Discontinuous galerkin approach for obtaining gorlov turbine extracted power

$$\begin{aligned} &\int_{\Omega_j} \mathbf{u}_t.\mathbf{v} dA - \int_{\Omega_j} \mathbf{u}.\mathbf{v}.(\mathbf{v} \otimes \mathbf{u}) dA + \int_{\partial\Omega_j} \mathbf{u}.\mathbf{n}\mathbf{u}.\mathbf{v} dl - \int_{\Omega_j} p\nabla.\mathbf{v} dA \\ &+ \int_{\partial\Omega_j} p\mathbf{v}.\mathbf{n} dl + \int_{\Omega_j} \underline{\tau} : \nabla \mathbf{v} dA - \int_{\partial\Omega_j} \underline{\tau} : (\mathbf{v} \otimes \mathbf{n}) dl = \mathbf{f}.\mathbf{v} dA \end{aligned} \tag{7}$$

They have weighted functions v and $\underline{\sigma}$. The 1st function works as scalar. The 2nd weighted function works as vector.

Equations (5), (6), (7) can be put in form of

$$-\int_{\Omega_j} \mathbf{u}_h.\nabla v dA + \int_{\partial\Omega_j} \hat{\mathbf{u}}_h^p.\mathbf{n} v dl = 0 \tag{8}$$

$$\int_{\Omega_j} \underline{\tau}_h.\underline{\sigma} dA = -\frac{1}{\mathrm{Re}} \int_{\Omega_j} \mathbf{u}_h.\nabla.\underline{\sigma} dA + \frac{1}{\mathrm{Re}} \int_{\partial\Omega_j} \hat{\mathbf{u}}_h^\sigma.\underline{\sigma}.\mathbf{n} v dl \tag{9}$$

$$\begin{aligned} &\int_{\Omega_j} \mathbf{u}_t.\mathbf{v} dA - \int_{\Omega_j} \mathbf{u}_h.\mathbf{v}.(\mathbf{v} \otimes \mathbf{u}_h) dA + \int_{\partial\Omega_j} \mathbf{u}_h.\mathbf{n}\hat{\mathbf{u}}_h^c.\mathbf{v} dl \\ &-\int_{\Omega_j} p_h \nabla.\mathbf{v} dA + \int_{\partial\Omega_j} \hat{p}_h \mathbf{v}.\mathbf{n} dl + \int_{\Omega_j} \underline{\tau}_h : \nabla \mathbf{v} dA \\ &-\int_{\partial\Omega_j} \hat{\underline{\tau}}_h : (\mathbf{v} \otimes \mathbf{n}) dl = \mathbf{f}.\mathbf{v} dA \end{aligned} \tag{10}$$

with $\hat{\mathbf{u}}_h^c$, $\hat{\mathbf{u}}_h^p$ and $\hat{p}_h$, $\hat{\mathbf{u}}_h^\sigma$ and $\hat{\underline{\tau}}_h$ are numerical fluxes of convective transfer, incompressibility condition and diffusive transfer respectively. For the case, the flux which is used, is the linear ones. It is called Local Discontinuous Galerkin flux [2].

The LDG consists of two parts. The 1st part is mean value of the neighboring elements. The other part is the jump value. Therefore each numerical flux will consist of both parts. To put into a stable condition, there is a need to choose the weighting number for each part in diffusive and incompressibility condition flux. The convective flux will be in a form of sign of the velocity between elements. It means the direction of the material derivative.

4 Model of gorlov turbine

Gorlov Turbine has helical blades with airfoil NACA 0020 shape. An experiment based model has 3 blades with helical degree of 30. The model was used in water flow of 1 – 2 m/s velocity [4]. Its dimension brings it a 100000 Reynold's number flow as it is put in water flow of 1 m/s.

Due to its helical degree, the NACA 0020 shape will be put into NACA 0017 as projection into 2 dimensions. This model assumes the laminar flow in vertical, which actually does not take place.

The helical projection into 2 dimensions makes the blade angles of attack from 0 – 2π for full submerged turbine. To simplify this condition, it will be computed on degree of 10.

5 Discontinuous galerkin approach on gorlov turbine

The model of Gorlov Turbine is computed using Discontinuous Galerkin in low order. The time is assumed to be constant as it is integrated using 2nd order Runge-Kutta to get its convergence in static condition. Runge-Kutta approach is conducted into equation (8), (9), (10) in a local form. It is done element by element as the advance of discontinuous approach [2, 5]. The boundary condition is put into non-slip condition. The velocity on the surface of the turbine is set to be 0. The far velocity is put into the boundary elements of computation.

The convergence in a static condition will bring pressure field. This field is used to compute the Drag Coefficient and Lift Coefficient in relation to the horizontal and vertical cross-sections respectively. Drag coefficient and its complement (Lift Coefficient) are components to compute Drag Force (FD) and Lift Force (FL). The Drag Force are computed using

$$FD = CD\frac{1}{2}\rho U^2 A$$

and the Lift Force are extracted from

$$FL = CL\frac{1}{2}\rho U^2 A\,.$$

Both forces are exploited to compute torques of the turbine which has relation with extracted power from the fluid. The Torque of the drag is computed using

$$T_D = FDh.r(\theta)/12$$

and its complement, the Torque of the lift is computed using

$$T_L = FLh.r(\theta)/12$$

with T_D = Torque of the drag, T_L = Torque of the lift, h = height of the turbine, and $r(\theta)$ = projection of radii of the blade from the axes of the turbine into x-axes for the lift and y-axes for the drag.

The efficiency is computed using ratio power from average torque that can be extracted and power of fluid flow. The power of the fluid flow is assumed to be kinetic energy of the flow which is

$$P = \frac{1}{2}\rho A u^3$$

A is cross-section of the turbine, ρ is fluid density and is the velocity of the fluid flow.

The computational result are conducted using Discontinuous Galerkin Approach for the model of experimental based which has diameter of 1 m and height 0.85 m in a flow of Re=10000. Drag Torque and Lift Torque of the model are shown in figure 1. The Lift Torque tends to be positive and the Drag Torque tends to be negative. It comes from the combination of the radii projection into axes and each force. The resultant of the Torque for turbine is shown on figure 2.

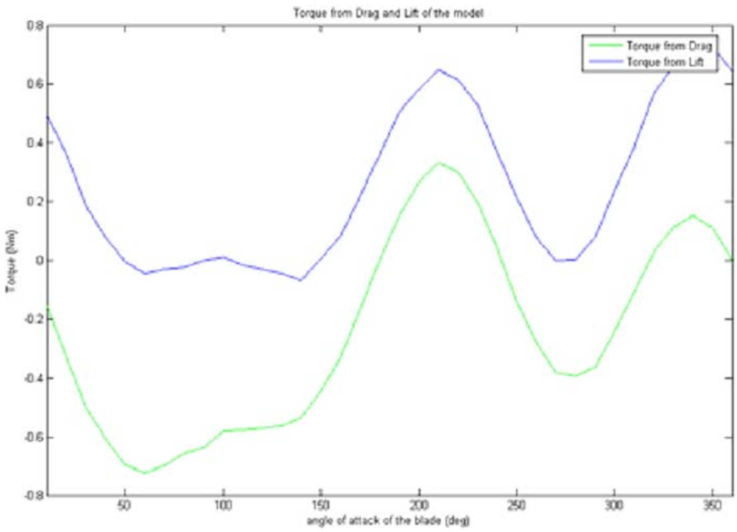

Figure 1. Torque from FD and FL

From the result which is shown on figure 2, it can be seen that the resultant of Torque which works on the model, brings a possibility of self starting. In every position it has non-zero Torque. Negative number of the Torque means that blade will swing in counter clock-wise direction and it is also possible for the turbine to be self-starting turbine. That the resultants of the Torque for every position of the blades are around -1.5 Nm, means that only small pulsation exists.

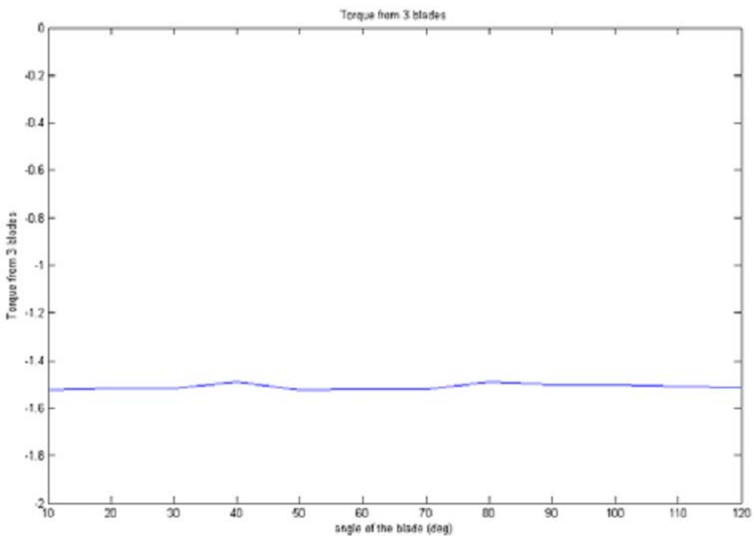

Figure 2. Resultant of the Torque from 3 blades brings only small pulsation.

Power which is extracted by the turbine is about 34.8 % of the total power of the flow. It is so closed to the experiment of Gorlov which brings 35% efficiency [3, 4]. It is also the limit of the turbine efficiency for free fluid flow according to Gorban [3].

6 Conclusion

The Galerkin Approach can predict the extracted power of the turbine in incompressible flow. The result shows accordance between experiment and computational work. It is a self-starting free – flow turbine and has small pulsating problem during its operation.

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