MODIFIED NEUTRINO MASS MATRIX SUBJECT TO 2-3 SYMMETRY WITH VANISHING Tr M

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ABSTRACT

We modified neutrino mass matrix subject to 2-3 symmetry by imposing the requirement that its trace vanishing. The modified neutrino mass matrix prediction on neutrino mass hierarchy is evaluated by setting the value of parameters in eigenvalues. We find that the modified neutrino mass matrix can predict either normal or inverted hierarchy. We also discuss its phenomenological implication qualitatively on neutrino oscillation experimental data.

Keywords: neutrino mass matrix, 2-3 symmetry, neutrino mass.

I. INTRODUCTION

According to Standard Model of Electroweak interaction (which also known as Glashow-Weinberg-Salam model), neutrinos are massless and only left-handed neutrinos or right-handed anti-neutrinos participate in weak interactions (Bilenky and Petcov, 1987). Recently, there is a convincing evidence that neutrinos have a tiny mass and mixing does exist in neutrino sector. This evidence is based on the experimental facts that both solar and atmospheric neutrinos undergo oscillations (Fukuda *et al.*, 1998; Fukuda *et al.*, 1999; Ahmad *et al.*, 2002; Ahn *et al.*, 2003).

In order to explain neutrino oscillations and its implications, several models have been proposed by many authors to extend the standard model of electroweak interactions. One of the most interesting models is the model that put the neutrino flavor (weak) eigenstates (ν_e, ν_μ, ν_τ) as superposition of mass eigenstates (ν_1, ν_2, ν_3). The neutrino flavor eigenstates are related to neutrino mass eigenstates as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{1}$$

where V is mixing matrix. From Eq. (1), one can see that the neutrino mass matrix in flavor basis (M_ν) is given by

$$M_\nu = V^T M V \tag{2}$$

where M is the neutrino mass matrix in mass eigenstates basis. In general, neutrino mass matrix M is given by

$$M = \begin{pmatrix} A & B & C \\ B & D & E \\ C & E & F \end{pmatrix} \tag{3}$$

and the mixing matrix V as following (Maki *et al.*, 1962; Pontecorvo, 1968)

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & z^* \\ -s_{12}c_{23} - c_{12}z & c_{12}c_{23} - s_{12}s_{23}z & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}z & -c_{12}s_{23} - c_{12}s_{23}z & c_{23}c_{13} \end{pmatrix} \tag{4}$$

where c_{ij} and s_{ij} stand for $\cos\theta_{ij}$ and $\sin\theta_{ij}$ respectively, and $z = s_{13}e^{i\varphi}$.

Theoretically, the texture and the underlying symmetry of the neutrino mass matrix M are two of the interesting problems in neutrino physics. In this paper, we only consider the underlying symmetry

of neutrino mass matrix. One of the proposed underlying symmetry of neutrino mass matrix M is a 2-3 symmetry (also known as $\mu-\tau$ symmetry). The 2-3 symmetry is based on the invariance of the mass terms in the Lagrangian under the interchange of $2 \leftrightarrow 3$ or $2 \leftrightarrow -3$.

By imposing the 2-3 symmetry into neutrino mass matrix of Eq. (3), then we have two possible patterns of neutrino mass matrices. For the interchange of $2 \leftrightarrow 3$ we have

$$M = \begin{pmatrix} A & B & B \\ B & D & E \\ B & E & D \end{pmatrix} \tag{5}$$

and for the interchange of $2 \leftrightarrow -3$ the neutrino mass matrix reads

$$M = \begin{pmatrix} A & B & -B \\ B & D & E \\ -B & E & D \end{pmatrix} \tag{6}$$

The neutrino mass matrix subject to 2-3 symmetry can accommodate the $\mu-\tau$ maximal mixing in atmospheric neutrino (Lam, 2001; Fuki & Yassue, 2006). But, the neutrino mass matrix subject to 2-3 symmetry has four parameters that are not easy to be determined theoretically. It implies that complicated calculations are needed to see the neutrino mass predictions on neutrino mass hierarchy. Neutrino mass hierarchy is also interesting problem in neutrino physics. As shown by neutrino oscillation experimental data, there are two possibilities for neutrino mass hierarchy: normal hierarchy and inverted hierarchy.

In order to reduce the number of parameters in neutrino mass matrix, in this paper we modified the neutrino mass matrix subject to 2-3 symmetry by imposing the requirement that trace of the modified neutrino mass matrix vanish. Explicitly, the aims of this paper are: (i) to modify neutrino mass matrix subject to 2-3 symmetry by imposing the requirement that the trace of modified neutrino mass matrix to be vanish and (ii) to evaluate the neutrino masses hierarchy that predicted by modified neutrino mass matrix.

II. RESEARCH METHOD

As one can see in the introduction, this paper is a theoretical research paper on modified neutrino mass matrix subject to 2-3 symmetry with the requirement that trace of modified neutrino mass matrix M vanish. In accordance with the background of theoretical research, the problem to be solved, and the aims of research, in this paper we then take the following steps:
1. To modify neutrino mass matrix of Eqs. (5) and (6) by imposing the requirement that the trace of modified neutrino mass matrices are vanishing.
2. To calculate eigenvalues of modified neutrino mass matrices which correspond to neutrino masses.
3. To set the eigenvalue parameters for determining neutrino masses hierarchy.

To discuss our results and to make some conclusions.

III. RESULTS AND DISCUSSIONS

Neutrino mass matrices of Eqs. (5) and (6) are modified such that it meets the requirement that trace of the modified neutrino mass matrices vanish: $trace\,M = 0$, it implies that $A = -2D$. If we replace A by $-2B$, then both neutrino mass matrices of Eqs. (5) and (6) have the forms

$$M = \begin{pmatrix} -2D & B & B \\ B & D & E \\ B & E & D \end{pmatrix} \tag{7}$$

for the interchange of $2 \leftrightarrow 3$, and

$$M = \begin{pmatrix} -2D & B & -B \\ B & D & E \\ -B & E & D \end{pmatrix} \tag{8}$$

for the interchange of $2 \leftrightarrow -3$ respectively.

The eigenvalues of modified neutrino mass matrix in Eq. (7) are given by

$$\lambda_1 = -\frac{D}{2} + \frac{E}{2} - \frac{\sqrt{9D^2 + 6DE + E^2 + 8B^2}}{2},$$

$$\lambda_2 = -\frac{D}{2} + \frac{E}{2} + \frac{\sqrt{9D^2 + 6DE + E^2 + 8B^2}}{2}, \tag{9}$$

$$\lambda_3 = D - E,$$

and for the modified neutrino mass matrix in Eq. (8) its eigenvalues are given by

$$\lambda_1 = -\frac{D}{2} - \frac{E}{2} - \frac{\sqrt{9D^2 - 6DE + E^2 + 8B^2}}{2},$$

$$\lambda_2 = -\frac{D}{2} - \frac{E}{2} + \frac{\sqrt{9d^2 - 6DE + E^2 + 8B^2}}{2}, \tag{10}$$

$$\lambda_3 = D + E.$$

To see the predictions of modified neutrino mass matrices qualitatively, for the first approximation, we take $B << D, E$ and $E < D$. By using this approximation, the eigenvalues of Eqs. (9) are given by

$$\lambda_1 \approx 2D - \frac{2B^2}{3D + E},$$

$$\lambda_2 \approx D + E + \frac{2B^2}{3D + E}, \tag{11}$$

$$\lambda_3 = D - E,$$

which give inverted hierarchy $|\lambda_3| < |\lambda_1| < |\lambda_2|$ and it implies that the hierarchy of modified neutrino mass matrix of Eq. (7) is inverted hierarchy

$$|m_3| < |m_1| < |m_2|. \tag{12}$$

For Eq. (10), using the same approximation, we have

$$\lambda_1 \approx -2D - \frac{2B^2}{3D - E},$$

$$\lambda_2 \approx D + \frac{2B^2}{3D - E}, \tag{13}$$

$$\lambda_3 = D + E,$$

which λ_2 corresponds to m_1, λ_1 corresponds to m_2, and λ_3 corresponds to m_3. This implies that the modified neutrino mass matrix of Eq. (8) gives neutrino masses in normal hierarchy

$$|m_1| < |m_2| < |m_3|. \tag{14}$$

Both patterns of modified neutrino mass matrices can predict qualitatively $\Delta m_{21}^2 \neq 0$ and $\Delta m_{31}^2 \neq 0$ which are qualitatively in agreement with the experimental data.

IV. CONCLUSION

Modified neutrino mass matrix subject to 2-3 symmetry by imposing the requirement that its trace vanishing can predict neutrino masses either in normal ($|m_1| < |m_2| < |m_3|$) or inverted ($|m_3| < |m_1| < |m_2|$) hierarchy. The neutrino oscillation experimental data, especially for the squared mass difference $\Delta m_{21}^2 \neq 0$ and $\Delta m_{31}^2 \neq 0$ can be predicted by modified neutrino mass matrix.

V. REFERENCES

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TANYA JAWAB

Gatot Wurdiyanto :

? *Bagaimana pemecahan bentuk massa menggunakan persamaan Lagrange?*

Asan Damanik :
@ *Lagrange untuk sistem partikel yang berinteraksi dengan medan Higgs terdiri dari suku energi kinetik, suku energi potensial, dan suku interaksi antar medan Higgs dan partikel. Interaksi antar medan Higgs dan partikel menghasilkan suku massa.*