

# Metric Dimension of Circulant Graphs with 5 Consecutive Generators

Volume 12 · Issue 9 May-1 2024

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$$R^0: R^0, L^0, L^1$$

$$L^1: L^1, R^4, R^0$$

$$R^1: R^1, L^4, L^0$$

$$L^2: L^2, R^3, R^4$$

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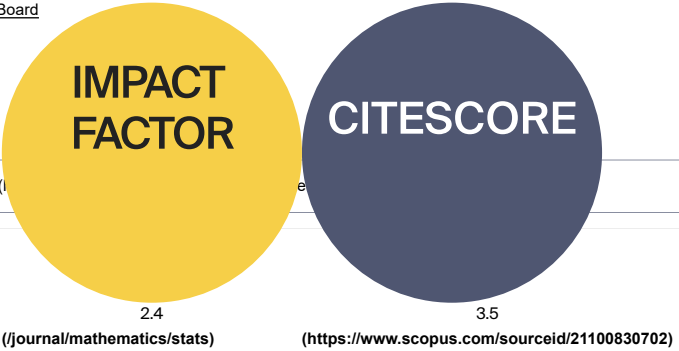
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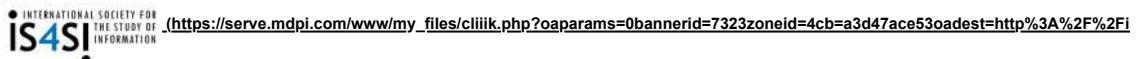
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


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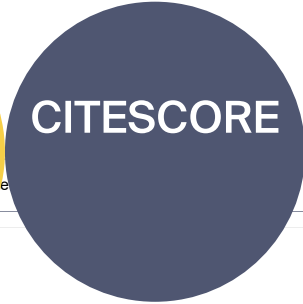
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# Mathematics, Volume 12, Issue 9 (May-1 2024) – 67 articles

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## [Depth-Optimized Quantum Circuits for ASCON: AEAD and HASH \(\(2227-7390/12/9/1337\)\)](https://doi.org/10.3390/math12091337)

by Yujin Oh, Kyungbae Jang, Anubhab Baksi and Hwajeong Seo

*Mathematics* **2024**, *12*(9), 1337; <https://doi.org/10.3390/math12091337> (registering DOI) - 27 Apr 2024

**Abstract** Quantum computing advancements pose security challenges for cryptography. Specifically, Grover's search algorithm affects the reduction in the search complexity of symmetric-key encryption and hash functions. Recent efforts have been made to estimate the complexity of Grover's search and evaluate post-quantum security. In this [...] [Read more](#).

(This article belongs to the Special Issue [Quantum Cryptography and Applications](https://journal/mathematics/special_issues/M5Q329TJ8) ([/journal/mathematics/special\\_issues/M5Q329TJ8](https://journal/mathematics/special_issues/M5Q329TJ8)))

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18 pages, 465 KIB [\(\(2227-7390/12/9/1336/pdf?version=1714224986\)\)](https://doi.org/10.3390/math12091336/pdf?version=1714224986)

## [Ensemble Approach Using k-Partitioned Isolation Forests for the Detection of Stock Market Manipulation \(\(2227-7390/12/9/1336\)\)](https://doi.org/10.3390/math12091336)

by Hugo Núñez Delafuente, César A. Astudillo and David Díaz

*Mathematics* **2024**, *12*(9), 1336; <https://doi.org/10.3390/math12091336> (registering DOI) - 27 Apr 2024

**Abstract** Stock market manipulation, defined as any attempt to artificially influence stock prices, poses significant challenges by causing financial losses and eroding investor trust. The prevalent reliance on supervised learning models for detecting such manipulations, while showing promise, faces notable hurdles due to the [...] [Read more](#).

(This article belongs to the Special Issue [Machine Learning and Finance](https://journal/mathematics/special_issues/9VEED7XUF6) ([/journal/mathematics/special\\_issues/9VEED7XUF6](https://journal/mathematics/special_issues/9VEED7XUF6)))

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24 pages, 358 KIB [\(\(2227-7390/12/9/1335/pdf?version=1714222682\)\)](https://doi.org/10.3390/math12091335/pdf?version=1714222682)

## [Existence of Solutions to a System of Fractional q-Difference Boundary Value Problems \(\(2227-7390/12/9/1335\)\)](https://doi.org/10.3390/math12091335)

by Alexandru Tudorache and Rodica Luca

*Mathematics* **2024**, *12*(9), 1335; <https://doi.org/10.3390/math12091335> (registering DOI) - 27 Apr 2024

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**Abstract** We are investigating the existence of solutions to a system of two fractional  $q$ -difference equations containing fractional  $q$ -integral terms, subject to multi-point boundary conditions that encompass  $q$ -derivatives and fractional  $q$ -derivatives of different orders. In our main results, we rely [...] [Read more](#).

(This article belongs to the Special Issue [Advances in Differential and Difference Equations and Their Applications](https://journal/mathematics/special_issues/8597U1432T) ([/journal/mathematics/special\\_issues/8597U1432T](https://journal/mathematics/special_issues/8597U1432T)))

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8 pages, 243 KIB [\(\(2227-7390/12/9/1334/pdf?version=1714213849\)\)](https://doi.org/10.3390/math12091334/pdf?version=1714213849)

## [Green Measures for a Class of Non-Markov Processes \(\(2227-7390/12/9/1334\)\)](https://doi.org/10.3390/math12091334)

by Herry P. Suryawan and José L. da Silva

*Mathematics* **2024**, *12*(9), 1334; <https://doi.org/10.3390/math12091334> (registering DOI) - 27 Apr 2024

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**Abstract** In this paper, we investigate the Green measure for a class of non-Gaussian processes in  $\mathbb{R}^d$ . These measures are associated with the family of generalized grey Brownian motions  $B_{\beta,\alpha}$ ,  $0 < \beta \leq 1$ , [...] [Read more](#).

(This article belongs to the Special Issue [New Advances in Applied Probability and Stochastic Processes](https://journal/mathematics/special_issues/O8TQ241A7J) ([/journal/mathematics/special\\_issues/O8TQ241A7J](https://journal/mathematics/special_issues/O8TQ241A7J)))

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## [μ-Integrable Functions and Weak Convergence of Probability Measures in Complete Paranormed Spaces \(\(2227-7390/12/9/1333\)\)](https://doi.org/10.3390/math12091333)

by Renying Zeng

*Mathematics* **2024**, *12*(9), 1333; <https://doi.org/10.3390/math12091333> (registering DOI) - 27 Apr 2024

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**Abstract** This paper works with functions defined in metric spaces and takes values in complete paranormed vector spaces or in Banach spaces, and proves some necessary and sufficient conditions for weak convergence of probability measures [...] [Full article \(\(2227-7390/12/9/1333\)\)](https://doi.org/10.3390/math12091333)

(This article belongs to the Special Issue [Functional Analysis and Mathematical Optimization](https://journal/mathematics/special_issues/2W4N957108) ([/journal/mathematics/special\\_issues/2W4N957108](https://journal/mathematics/special_issues/2W4N957108)))

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15 pages, 1335 KIB [\(\(2227-7390/12/9/1332/pdf?version=1714207451\)\)](https://doi.org/10.3390/math12091332/pdf?version=1714207451)

## [Embedding Secret Data in a Vector Quantization Codebook Using a Novel Thresholding Scheme \(\(2227-7390/12/9/1332\)\)](https://doi.org/10.3390/math12091332)

by Yijie Lin, Jui-Chuan Liu, Ching-Chun Chang and Chin-Chen Chang

*Mathematics* **2024**, *12*(9), 1332; <https://doi.org/10.3390/math12091332> (registering DOI) - 27 Apr 2024

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**Abstract** In recent decades, information security has become increasingly valued, including many aspects of privacy protection, copyright protection, and digital forensics. Therefore, many data hiding schemes have been proposed and applied to various carriers such as text, images, audio, and videos. Vector Quantization (VQ) [...] [Read more](#).

(This article belongs to the Special Issue [Advances in Mathematical Cryptography and Information Security toward Industry 5.0](https://journal/mathematics/special_issues/Math_Cryptogr_Inf_Secur) ([/journal/mathematics/special\\_issues/Math\\_Cryptogr\\_Inf\\_Secur](https://journal/mathematics/special_issues/Math_Cryptogr_Inf_Secur)))

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14 pages, 8767 KIB [\(\(2227-7390/12/9/1331/pdf?version=1714206582\)\)](https://doi.org/10.3390/math12091331/pdf?version=1714206582)

## [Enhanced YOLOX with United Attention Head for Road Detection When Driving \(\(2227-7390/12/9/1331\)\)](https://doi.org/10.3390/math12091331)

by Yuhuan Wu and Yonghong Wu

*Mathematics* **2024**, *12*(9), 1331; <https://doi.org/10.3390/math12091331> (registering DOI) - 27 Apr 2024

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**Abstract** Object detection plays a crucial role in autonomous driving assistance systems. It requires high accuracy for prediction, a small size for deployment on mobile devices, and real-time inference speed to ensure safety. In this paper, we present a compact and efficient algorithm called [...] [Read more](#).



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**Deep-Representation-Learning-Based Classification Strategy for Anticancer Peptides** ([/2227-7390/12/9/1330](https://doi.org/10.3390/math12091330))

by **Shujaat Khan**  
*Mathematics* **2024**, *12*(9), 1330; <https://doi.org/10.3390/math12091330> (registering DOI) - 27 Apr 2024  
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**Abstract** Cancer, with its complexity and numerous origins, continues to provide a huge challenge in medical research. *Anticancer peptides* are a potential treatment option, but identifying and synthesizing them on a large scale requires accurate prediction algorithms. This study presents an intuitive classification strategy, [...][Read more](#). (This article belongs to the Special Issue [Artificial Intelligence for Biomedical Applications](#) ([/journal/mathematics/special\\_issues/88EX01TH8V](#)))

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**Unraveling the Complexity of Inverting the Sturm–Liouville Boundary Value Problem to Its Canonical Form** ([/2227-7390/12/9/1329](https://doi.org/10.3390/math12091329))

by **Natanael Karjanto** and **Peter Sadhani**  
*Mathematics* **2024**, *12*(9), 1329; <https://doi.org/10.3390/math12091329> (registering DOI) - 26 Apr 2024  
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**Abstract** The Sturm–Liouville boundary value problem (SLBVP) stands as a fundamental cornerstone in the realm of mathematical analysis and physical modeling. Also known as the Sturm–Liouville problem (SLP), this paper explores the intricacies of this classical problem, particularly the relationship between its canonical and [...][Read more](#). (This article belongs to the Special Issue [Differential Equations with Boundary Value Problems: Theory and Applications](#) ([/journal/mathematics/special\\_issues/differential\\_equations\\_boundary\\_value\\_problems](#)))

Open Access Article 14 pages, 1897 KIB ([/2227-7390/12/9/1328/pdf?version=1714145379](https://doi.org/10.3390/math12091328/pdf?version=1714145379))

**Efficient List Intersection Algorithm for Short Documents by Document Reordering** ([/2227-7390/12/9/1328](https://doi.org/10.3390/math12091328))

by **Lianyin Jia**, **Dongyang Li**, **Haihe Zhou** and **Fengling Xia**  
*Mathematics* **2024**, *12*(9), 1328; <https://doi.org/10.3390/math12091328> (registering DOI) - 26 Apr 2024  
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**Abstract** List intersection plays a pivotal role in various domains such as search engines, database systems, and social networks. Efficient indexes and query strategies can significantly enhance the efficiency of list intersection. Existing inverted index-based algorithms fail to utilize the length information of documents [...][Read more](#). (This article belongs to the Special Issue [Advances of Computer Algorithms and Data Structures](#) ([/journal/mathematics/special\\_issues/369W6733O2](#)))

Open Access Article 21 pages, 537 KIB ([/2227-7390/12/9/1327/pdf?version=1714140282](https://doi.org/10.3390/math12091327/pdf?version=1714140282))

**Fast Eigenvalue Decomposition of Arrowhead and Diagonal-Plus-Rank-*k* Matrices of Quaternions** ([/2227-7390/12/9/1327](https://doi.org/10.3390/math12091327))

by **Thaniporn Chaysri**, **Nevena Jakovčević Stor** and **Ivan Slapničar**  
*Mathematics* **2024**, *12*(9), 1327; <https://doi.org/10.3390/math12091327> (<https://doi.org/10.3390/math12091327>) - 26 Apr 2024  
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**Abstract** Quaternions are a non-commutative associative number system that extends complex numbers, first described by Hamilton in 1843. We present algorithms for solving the eigenvalue problem for arrowhead and DPRK (diagonal-plus-rank-*k*) matrices of quaternions. The algorithms use the Rayleigh Quotient Iteration with [...][Read more](#). (This article belongs to the Section [Computational and Applied Mathematics](#) ([/journal/mathematics/sections/computational\\_mathematics](#)))

Open Access Feature Paper Article 17 pages, 1476 KIB ([/2227-7390/12/9/1326/pdf?version=1714140073](https://doi.org/10.3390/math12091326/pdf?version=1714140073))

**A Three-Dimensional Velocity Field Related to a Generalized Third-Grade Fluid Model** ([/2227-7390/12/9/1326](https://doi.org/10.3390/math12091326))

by **Fernando Carapau**, **Paulo Correia** and **Gracino Rodrigues**  
*Mathematics* **2024**, *12*(9), 1326; <https://doi.org/10.3390/math12091326> (<https://doi.org/10.3390/math12091326>) - 26 Apr 2024  
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**Abstract** In this work, we propose a new three-dimensional constitutive equation related to a third-grade fluid. This proposal is based on experimental work for which the viscosity term and the terms related to viscoelasticity may depend on the shear rate, in accordance with a [...][Read more](#). (This article belongs to the Section [Mathematical Physics](#) ([/journal/mathematics/sections/mathematical\\_physics](#)))

Open Access Article 15 pages, 329 KIB ([/2227-7390/12/9/1325/pdf?version=1714139027](https://doi.org/10.3390/math12091325/pdf?version=1714139027))

**Initial Coefficient Bounds Analysis for Novel Subclasses of Bi-Univalent Functions Linked with Lucas-Balancing Polynomials** ([/2227-7390/12/9/1325](https://doi.org/10.3390/math12091325))

by **Sondekola Rudra Swamy**, **Daniel Breaz**, **Kala Venugopal**, **Mamatha Paduvalapattana Kempegowda**, **Luminita-Ioana Cotîrlă** and **Eleonora Rapeanu**  
*Mathematics* **2024**, *12*(9), 1325; <https://doi.org/10.3390/math12091325> (<https://doi.org/10.3390/math12091325>) - 26 Apr 2024  
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**Abstract** We investigate some subclasses of regular and bi-univalent functions in the open unit disk that are associated with Lucas-Balancing polynomials in this work. For functions that belong to these subclasses, we obtain upper bounds on their initial coefficients. The Fekete–Szegő problem is also [...][Read more](#).

Open Access Article 15 pages, 499 KIB ([/2227-7390/12/9/1324/pdf?version=1714136836](https://doi.org/10.3390/math12091324/pdf?version=1714136836))



**Abstract** In practical applications, the temporal completeness of knowledge graphs is of great importance. However, previous studies have mostly focused on static knowledge graphs, generally neglecting the dynamic evolutionary properties of facts. Moreover, the unpredictable and limited availability of temporal knowledge graphs, together with [...] [Read more](#).

(This article belongs to the Topic [Complex Networks and Social Networks \(topics/social\\_network\)](#))

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29 pages, 778 KIB ([/2227-7390/12/9/1324/pdf?version=1714133704](https://doi.org/10.3390/math12091324/pdf?version=1714133704))

**Intellectual Capital Evaluation Index Based on a Hybrid Multi-Criteria Decision-Making Technique ((2227-7390/12/9/1323)**

by Chao Liu, Qichen Liao, Wenyan Gao, Shuxian Li, Peng Jiang and Ding Li

Mathematics 2024, 12(9), 1323; <https://doi.org/10.3390/math12091323> (<https://doi.org/10.3390/math12091323>) - 26 Apr 2024

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**Abstract** In the context of a burgeoning knowledge economy, enterprise intellectual capital has emerged as a pivotal asset for organizational growth. Evaluating it requires a comprehensive and robust index, yet there is no standard methodology for such assessments. Here, we propose an index for [...] [Read more](#).

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20 pages, 2130 KIB ([/2227-7390/12/9/1322/pdf?version=1714130894](https://doi.org/10.3390/math12091322/pdf?version=1714130894))

**Mathematical Modeling of the Displacement of a Light-Fuel Self-Moving Automobile with an On-Board Liquid Crystal Elastomer Propulsion Device ((2227-7390/12/9/1322)**

by Yunlong Qiu, Jiaying Chen, Yuntong Dai, Lin Zhou, Yong Yu and Kai Li

Mathematics 2024, 12(9), 1322; <https://doi.org/10.3390/math12091322> (<https://doi.org/10.3390/math12091322>) - 26 Apr 2024

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**Abstract** The achievement and control of desired motions in active machines often involves precise manipulation of artificial muscles in a distributed and sequential manner, which poses significant challenges. A novel motion control strategy based on self-oscillation in active machines offers distinctive benefits, such as [...] [Read more](#).

(This article belongs to the Special Issue [Mathematical Modeling, Asymptotic Analysis and Stability of Solutions of Nonlinear Dynamical Systems \(Journal/mathematics/special\\_issues/4BQ5OT780B\)](#).)

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17 pages, 946 KIB ([/2227-7390/12/9/1321/pdf?version=1714126833](https://doi.org/10.3390/math12091321/pdf?version=1714126833))

**Bifurcation Analysis for an OSN Model with Two Delays ((2227-7390/12/9/1321)**

by Liancheng Wang and Min Wang

Mathematics 2024, 12(9), 1321; <https://doi.org/10.3390/math12091321> (<https://doi.org/10.3390/math12091321>) - 26 Apr 2024

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**Abstract** In this research, we introduce and analyze a mathematical model for online social networks, incorporating two distinct delays. These delays represent the time it takes for active users within the network to begin disengaging, either with or without contacting non-users of online social [...] [Read more](#).

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**Essential Norm of  $t$ -Generalized Composition Operators from  $F(p, q, s)$  to Iterated Weighted-Type Banach Space ((2227-7390/12/9/1320)**

by Shams Alyusof and Nacir Hmidouch

Mathematics 2024, 12(9), 1320; <https://doi.org/10.3390/math12091320> (<https://doi.org/10.3390/math12091320>) - 26 Apr 2024

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**Abstract** In this work, we characterize the boundedness of  $t$ -generalized composition operators from  $F(p, q, s)$  spaces to iterated weighted-type Banach space. We also provide estimates of the norm and the essential norm of  $t$ -generalized [...] [Read more](#).

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18 pages, 2563 KIB ([/2227-7390/12/9/1319/pdf?version=1714121063](https://doi.org/10.3390/math12091319/pdf?version=1714121063))

**Neural Network-Based Distributed Consensus Tracking Control for Nonlinear Multi-Agent Systems with Mismatched and Matched Disturbances ((2227-7390/12/9/1319)**

by Linxi Xu and Kaiyu Qin

Mathematics 2024, 12(9), 1319; <https://doi.org/10.3390/math12091319> (<https://doi.org/10.3390/math12091319>) - 26 Apr 2024

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**Abstract** In practice, disturbances, including model uncertainties and unknown external disturbances, are always widely present and have a significant impact on the cooperative control performance of a networked multi-agent system. In this work, the distributed consensus tracking control problem for a class of multi-agent [...] [Read more](#). (This article belongs to the Special Issue [Advance in Control Theory and Optimization \(Journal/mathematics/special\\_issues/6846MP2NSF\)](#).)

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([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g007-550.jpg?1714121144](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g007-550.jpg?1714121144)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g008-550.jpg?1714121145](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g008-550.jpg?1714121145)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g009-550.jpg?1714121146](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g009-550.jpg?1714121146)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g010-550.jpg?1714121147](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g010-550.jpg?1714121147)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g011-550.jpg?1714121148](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g011-550.jpg?1714121148)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g012-550.jpg?1714121149](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g012-550.jpg?1714121149)) 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([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g022-550.jpg?1714121159](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g022-550.jpg?1714121159)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g023-550.jpg?1714121160](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g023-550.jpg?1714121160)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g024-550.jpg?1714121161](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g024-550.jpg?1714121161)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g025-550.jpg?1714121162](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g025-550.jpg?1714121162)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g026-550.jpg?1714121163](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g026-550.jpg?1714121163)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g027-550.jpg?1714121164](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g027-550.jpg?1714121164)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g028-550.jpg?1714121165](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g028-550.jpg?1714121165)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g029-550.jpg?1714121166](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g029-550.jpg?1714121166)) ([https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article\\_deploy/html/images/mathematics-12-01319-g030-550.jpg?1714121167](https://pub.mdpi-res.com/mathematics/mathematics-12-01319/article_deploy/html/images/mathematics-12-01319-g030-550.jpg?1714121167)) 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**[A Novel Method for Predicting the Behavior of a Sucker Rod Pumping Unit Based on the Polished Rod Velocity](#)** (/2227-7390/12/9/1318)

by Jiaojian Yin and Hongzhang Ma

*Mathematics* 2024, 12(9), 1318; <https://doi.org/10.3390/math12091318> (<https://doi.org/10.3390/math12091318>) - 25 Apr 2024

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**Abstract** Fault dynamometer cards are the basis of the diagnosis technique for sucker rod pumping systems. Predicting fault cards with a pumping condition model is an economical and effective method. The usual model is described by a mixed function of the pump displacement and [...] [Read more](#).

(This article belongs to the Special Issue [Mathematical Modeling and Simulation in Mechanics and Dynamic Systems, 3rd Edition](#) (/journal/mathematics/special\_issues/Math\_Model\_Simul\_Mech\_Dyn\_Syst\_3))

**[The Blow-Up of the Local Energy Solution to the Wave Equation with a Nontrivial Boundary Condition](#)** (/2227-7390/12/9/1317)

by Yulong Liu

*Mathematics* 2024, 12(9), 1317; <https://doi.org/10.3390/math12091317> (<https://doi.org/10.3390/math12091317>) - 25 Apr 2024

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**Abstract** In this study, we examine the wave equation with a nontrivial boundary condition. The main target of this study is to prove the local-in-time existence and the blow-up in finite time of the energy solution. Through the construction of an auxiliary function and [...] [Read more](#).

(This article belongs to the Special Issue [Advances in Differential and Difference Equations and Their Applications](#) (/journal/mathematics/special\_issues/8597U1432T))

**[Meshless Generalized Finite Difference Method Based on Nonlocal Differential Operators for Numerical Simulation of Elastostatics](#)** (/2227-7390/12/9/1316)

by Yeying Zhou, Chunguang Li, Xinshan Zhuang and Zhifen Wang

*Mathematics* 2024, 12(9), 1316; <https://doi.org/10.3390/math12091316> (<https://doi.org/10.3390/math12091316>) - 25 Apr 2024

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**Abstract** This study proposes an innovative meshless approach that merges the peridynamic differential operator (PDDO) with the generalized finite difference method (GFDM). Based on the PDDO theory, this method introduces a new nonlocal differential operator that aims to reduce the pre-assumption required for the [...] [Read more](#).

(This article belongs to the Special Issue [Recent Advances in Numerical Methods for Scientific and Engineering Applications, 2nd Edition](#) (/journal/mathematics/special\_issues/Numer\_Methods\_Sci\_Eng\_Appl))

**[Efficient Large-Scale IoT Botnet Detection through GraphSAINT-Based Subgraph Sampling and Graph Isomorphism Network](#)** (/2227-7390/12/9/1315)

by Lihua Yin, Weizhe Chen, Xi Luo and Hongyu Yang

*Mathematics* 2024, 12(9), 1315; <https://doi.org/10.3390/math12091315> (<https://doi.org/10.3390/math12091315>) - 25 Apr 2024

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**Abstract** In recent years, with the rapid development of the Internet of Things, large-scale botnet attacks have occurred frequently and have become an important challenge to network security. As artificial intelligence technology continues to evolve, intelligent detection solutions for botnets are constantly emerging. Although [...] [Read more](#).

(This article belongs to the Special Issue [Advanced Research on Information System Security and Privacy](#) (/journal/mathematics/special\_issues/93YQ7N975L))

**[A Privacy Protection Scheme of Certificateless Aggregate Ring Signcryption Based on SM2 Algorithm in Smart Grid](#)** (/2227-7390/12/9/1314)

by Hongna Song, Zhentao Liu, Teng Wang, Ling Zhao, Haonan Guo and Shuanggen Liu

*Mathematics* 2024, 12(9), 1314; <https://doi.org/10.3390/math12091314> (<https://doi.org/10.3390/math12091314>) - 25 Apr 2024

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**Abstract** With the rapid increase in smart grid users and the increasing cost of user data transmission, proposing an encryption method that does not increase the construction cost while increasing the user ceiling has become the focus of many scholars. At the same time, [...] [Read more](#).

(This article belongs to the Topic [Recent Advances in Security, Privacy, and Trust](#) (/topics/ELY82W061L))

**[Enhancing Surveillance Vision with Multi-Layer Deep Learning Representation](#)** (/2227-7390/12/9/1313)

by Dong-Min Son and Sung-Hak Lee

*Mathematics* 2024, 12(9), 1313; <https://doi.org/10.3390/math12091313> (<https://doi.org/10.3390/math12091313>) - 25 Apr 2024

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**Abstract** This paper aimed to develop a method for generating sand-dust removal and dehazed images utilizing CycleGAN, facilitating object identification on roads under adverse weather conditions such as heavy dust or haze, which severely impair visibility. Initially, the study addressed the scarcity of paired [...] [Read more](#).

(This article belongs to the Special Issue [New Advances and Applications in Image Processing and Computer Vision](#) (/journal/mathematics/special\_issues/F737IH6LF9))

**[Deep Learning Based Fall Recognition and Forecasting for Reconfigurable Stair-Accessing Service Robots](#)** (/2227-7390/12/9/1312)

by Jun Hua Ong, Abdullah Aamir Hayat, Braulio Felix Gomez, Mohan Rajesh Elara and Kristin Lee Wood

*Mathematics* 2024, 12(9), 1312; <https://doi.org/10.3390/math12091312> (<https://doi.org/10.3390/math12091312>) - 25 Apr 2024

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**Abstract** This paper presents a comprehensive study on fall recognition and forecasting for reconfigurable stair-accessing robots by leveraging deep learning techniques. The proposed framework integrates machine learning algorithms and recurrent neural networks (RNNs), specifically Long Short-Term Memory (LSTM) and Bidirectional LSTM (BiLSTM), for fall [...] [Read more](#).

**[A Few Similarity Measures on the Class of Trapezoidal-Valued Intuitionistic Fuzzy Numbers and Their Applications in Decision Analysis](#)** (/2227-7390/12/9/1311)

by Jeevaraj Selvaraj and Melfi Alrasheedi

*Mathematics* 2024, 12(9), 1311; <https://doi.org/10.3390/math12091311> (<https://doi.org/10.3390/math12091311>) - 25 Apr 2024

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**Abstract** Similarity measures on trapezoidal-valued intuitionistic fuzzy numbers (TrVIFNs) are functions that measure the closeness between two TrVIFNs, which has a lot of applications in the area of pattern recognition, clustering, decision-making, etc. Researchers around the world are proposing various similarity measures on the [...] [Read more](#).

(This article belongs to the Special Issue [Various Generalizations of Fuzzy Sets and Their Applications in Engineering and Management](#) (/journal/mathematics/special\_issues/Gen\_Fuzzy\_Sets))

**[Robust Bias Compensation Method for Sparse Normalized Quasi-Newton Least-Mean with Variable Mixing-Norm Adaptive Filtering](#)** (/2227-7390/12/9/1310)

by Ying-Ren Chien, Han-En Hsieh and Guobing Qian

**Abstract** Input noise causes inescapable bias to the weight vectors of the adaptive filters during the adaptation processes. Moreover, the impulse noise at the output of the unknown systems can prevent bias compensation from converging. This paper presents a robust bias compensation method for [...] [Read more](#).  
(This article belongs to the Special Issue [Advanced Research in Data-Centric AI \( /journal/mathematics/special\\_issues/04597CV762 \)](#))

Open Access Article 19 pages, 642 KIB ([/2227-7390/12/9/1309/pdf?version=1714049829](#))

**Study of Random Walk Invariants for Spiro-Ring Network Based on Laplacian Matrices ( /2227-7390/12/9/1309 )**

by Yasir Ahmad, Umar Ali, Daniele Ettore Otera and Xiang-Feng Pan

Mathematics 2024, 12(9), 1309; <https://doi.org/10.3390/math12091309> (<https://doi.org/10.3390/math12091309>) - 25 Apr 2024

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**Abstract** The use of the global mean first-passage time (GMFPT) in random walks on networks has been widely explored in the field of statistical physics, both in theory and practical applications. The GMFPT is the estimated interval of time needed to reach a state [...] [Read more](#).  
(This article belongs to the Special Issue [Geometry and Topology with Applications \( /journal/mathematics/special\\_issues/5652O9S898 \)](#))

Open Access Article 18 pages, 414 KIB ([/2227-7390/12/9/1308/pdf?version=1714049415](#))

**Semi-Analytical Closed-Form Solutions for Dynamical Rossler-Type System ( /2227-7390/12/9/1308 )**

by Remus-Daniel Ene and Nicolina Pop

Mathematics 2024, 12(9), 1308; <https://doi.org/10.3390/math12091308> (<https://doi.org/10.3390/math12091308>) - 25 Apr 2024

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**Abstract** Mathematical models and numerical simulations are necessary to understand the functions of biological rhythms, to comprehend the transition from simple to complex behavior and to delineate the conditions under which they arise. The aim of this work is to investigate the R [...] [Read more](#).  
(This article belongs to the Special Issue [Analytical Approaches to Nonlinear Dynamical Systems and Applications II \( /journal/mathematics/special\\_issues/Analytical\\_Approaches\\_Nonlinear\\_Dynamical\\_Systems\\_Applications\\_II \)](#))

Open Access Article 24 pages, 3827 KIB ([/2227-7390/12/9/1307/pdf?version=1714049411](#))

**Monitoring the Wear Trends in Wind Turbines by Tracking Fourier Vibration Spectra and Density Based Support Vector Machines ( /2227-7390/12/9/1307 )**

by Claudiu Bisu, Adrian Olaru, Serban Olaru, Adrian Alexei, Niculae Mihai and Haleema Ushaq

Mathematics 2024, 12(9), 1307; <https://doi.org/10.3390/math12091307> (<https://doi.org/10.3390/math12091307>) - 25 Apr 2024

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**Abstract** To make wind power more competitive, it is necessary to reduce turbine downtime and reduce costs associated with wind turbine operation and maintenance (O&M). Incorporating machine learning in the development of condition-based predictive maintenance methodologies for wind turbines can enhance their efficiency and [...] [Read more](#).

(This article belongs to the Special Issue [Dynamic Modeling and Simulation for Control Systems, 2nd Edition \( /journal/mathematics/special\\_issues/Dyn\\_Modeling\\_Simul\\_Control\\_Syst\\_II \)](#))

Open Access Article 18 pages, 4855 KIB ([/2227-7390/12/9/1306/pdf?version=1714049289](#))

**A Reentry Trajectory Planning Algorithm via Pseudo-Spectral Convexification and Method of Multipliers ( /2227-7390/12/9/1306 )**

by Haizhao Liang, Yunhao Luo, Haohui Che, Jingxian Zhu and Jianying Wang

Mathematics 2024, 12(9), 1306; <https://doi.org/10.3390/math12091306> (<https://doi.org/10.3390/math12091306>) - 25 Apr 2024

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**Abstract** The reentry trajectory planning problem of hypersonic vehicles is generally a continuous and nonconvex optimization problem, and it constitutes a critical challenge within the field of aerospace engineering. In this paper, an improved sequential convexification algorithm is proposed to solve it and achieve [...] [Read more](#).  
(This article belongs to the Special Issue [Advanced Guidance and Control of Flight Vehicle: Theory and Application, 2nd Edition \( /journal/mathematics/special\\_issues/YX9Z9A8HE1 \)](#))

Open Access Article 10 pages, 260 KIB ([/2227-7390/12/9/1305/pdf?version=1714046581](#))

**On  $\alpha$ -Pseudo Spirallike Functions Associated with Exponential Pareto Distribution (EPD) and Libera Integral Operator ( /2227-7390/12/9/1305 )**

by Hamzat Jamiu Olusegun, Oluwayemi Matthew Olanrewaju, Oladipo Abiodun Tinuoye and Alb Lupas Alina

Mathematics 2024, 12(9), 1305; <https://doi.org/10.3390/math12091305> (<https://doi.org/10.3390/math12091305>) - 25 Apr 2024

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**Abstract** The present study aims at investigating some characterizations of a new subclass  $G_{\alpha}(\mu, \tau)$  and obtaining the bounds on the first two Taylor–Maclaurin coefficients for functions belonging to the newly introduced subclass. In order to achieve this, a [...] [Read more](#).  
(This article belongs to the Special Issue [New Trends in Complex Analysis Research, 2nd Edition \( /journal/mathematics/special\\_issues/35JBAPIUMD \)](#))

Open Access Article 14 pages, 443 KIB ([/2227-7390/12/9/1304/pdf?version=1714046664](#))

**Sparsity-Constrained Vector Autoregressive Moving Average Models for Anomaly Detection of Complex Systems with Multisensory Signals ( /2227-7390/12/9/1304 )**

by Meng Ma, Zhongyi Zhang, Zhi Zhai and Zhirong Zhong

Mathematics 2024, 12(9), 1304; <https://doi.org/10.3390/math12091304> (<https://doi.org/10.3390/math12091304>) - 25 Apr 2024

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**Abstract** Detecting anomalies in large, complex systems is a critical and challenging task, and this is especially true for high-dimensional anomaly detection due to the underlying dependency structures among sensors. To incorporate the interrelationships among various sensors, a novel sparsity-constrained vector autoregressive moving average [...] [Read more](#).

(This article belongs to the Special Issue [Functional Statistics: Outliers Detection and Quality Control, 2nd Edition \( /journal/mathematics/special\\_issues/Functional\\_Statistics\\_Outliers\\_Detection\\_Quality\\_Control\\_2021 \)](#))

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Open Access Article 22 pages, 971 KIB ([/2227-7390/12/9/1303/pdf?version=1714046315](#))

**TPTM-HANN-GA: A Novel Hyperparameter Optimization Framework Integrating the Taguchi Method, an Artificial Neural Network, and a Genetic Algorithm for the Precise Prediction of Cardiovascular Disease Risk ( /2227-7390/12/9/1303 )**

by Chia-Ming Lin and Yu-Shiang Lin

Mathematics 2024, 12(9), 1303; <https://doi.org/10.3390/math12091303> (<https://doi.org/10.3390/math12091303>) - 25 Apr 2024

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**Abstract** The timely and precise prediction of cardiovascular disease (CVD) risk is essential for effective prevention and intervention. This study proposes a novel framework that integrates the two-phase Taguchi method (TPTM), the hyperparameter artificial neural network (HANN), and a genetic algorithm (GA) called TPTM-HANN-GA. [...] [Read more](#).

Open Access Article 15 pages, 322 KIB ([/2227-7390/12/9/1302/pdf?version=1714046957](#))

**Abstract** This paper combines the concept of an arbitrary binary connection with the widely recognized principle of  $\theta$ -contraction to investigate the innovative features of vector-valued metric spaces. This methodology demonstrates the existence of fixed points for both single- and multi-valued mappings within complete [...] [Read more](#).

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25 pages, 1132 KIB ([/2227-7390/12/9/1301/pdf?version=1714044811](https://doi.org/10.3390/math12091302/pdf?version=1714044811))

### [Synergising an Advanced Optimisation Technique with Deep Learning: A Novel Method in Fault Warning Systems. \(2227-7390/12/9/1301\)](#)

by Jia Tian, Xingqin Zhang, Shuangqing Zheng, Zhiyong Liu and Changshu Zhan

Mathematics 2024, 12(9), 1301; <https://doi.org/10.3390/math12091301> (<https://doi.org/10.3390/math12091301>) - 25 Apr 2024

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**Abstract** In the realm of automated industry and smart production, the deployment of fault warning systems is crucial for ensuring equipment reliability and enhancing operational efficiency. Although there are a multitude of existing methodologies for fault warning, the proficiency of these systems in processing [...] [Read more](#).  
(This article belongs to the Special Issue [Computational Intelligence and Machine Learning with Applications \( /journal/mathematics/special\\_issues/4EPTL99AK2 \)](#))

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16 pages, 312 KIB ([/2227-7390/12/9/1300/pdf?version=1714040495](https://doi.org/10.3390/math12091300/pdf?version=1714040495))

### [Estimating the Complier Average Causal Effect with Non-Ignorable Missing Outcomes Using Likelihood Analysis. \(2227-7390/12/9/1300\)](#)

by Jierui Du, Gao Wen and Xin Liang

Mathematics 2024, 12(9), 1300; <https://doi.org/10.3390/math12091300> (<https://doi.org/10.3390/math12091300>) - 25 Apr 2024

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**Abstract** Missing data problems arise in randomized trials, which complicates the inference of causal effects if the missing mechanism is non-ignorable. We tackle the challenge of identifying and estimating the complier average causal effect parameters under non-ignorable missingness by increasing the covariates to mitigate [...] [Read more](#).  
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18 pages, 1170 KIB ([/2227-7390/12/9/1299/pdf?version=1714038698](https://doi.org/10.3390/math12091299/pdf?version=1714038698))

### [Robust Design Optimization of Electric Machines with Isogeometric Analysis. \(2227-7390/12/9/1299\)](#)

by Theodor Komann, Michael Wiesheu, Stefan Ulbrich and Sebastian Schöps

Mathematics 2024, 12(9), 1299; <https://doi.org/10.3390/math12091299> (<https://doi.org/10.3390/math12091299>) - 25 Apr 2024

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**Abstract** In electric machine design, efficient methods for the optimization of the geometry and associated parameters are essential. Nowadays, it is necessary to address the uncertainty caused by manufacturing or material tolerances. This work presents a robust optimization strategy to address uncertainty in the [...] [Read more](#).  
(This article belongs to the Special Issue [Numerical Optimization for Electromagnetic Problems \( /journal/mathematics/special\\_issues/Numer\\_Optim\\_Electromagn\\_Probl \)](#))

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22 pages, 13881 KIB ([/2227-7390/12/9/1298/pdf?version=1714037409](https://doi.org/10.3390/math12091298/pdf?version=1714037409))

### [Mining Trajectory Planning of Unmanned Excavator Based on Machine Learning. \(2227-7390/12/9/1298\)](#)

by Zhong Jin, Mingde Gong, Dingxuan Zhao, Shaomeng Luo, Guowang Li, Jiaheng Li, Yue Zhang and Wenbin Liu

Mathematics 2024, 12(9), 1298; <https://doi.org/10.3390/math12091298> (<https://doi.org/10.3390/math12091298>) - 25 Apr 2024

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**Abstract** Trajectory planning plays a crucial role in achieving unmanned excavator operations. The quality of trajectory planning results heavily relies on the level of rules extracted from objects such as scenes and optimization objectives, using traditional theoretical methods. To address this issue, this study [...] [Read more](#).

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17 pages, 6027 KIB ([/2227-7390/12/9/1297/pdf?version=1714034754](https://doi.org/10.3390/math12091297/pdf?version=1714034754))

### [Exploring an Intelligent Classification Model for the Recognition of Automobile Sounds Based on EEG Physiological Signals. \(2227-7390/12/9/1297\)](#)



**Abstract** The advancement of an intelligent automobile sound switching system has the potential to elevate the market standing of automotive products, with the pivotal prerequisite being the selection of automobile sounds based on the driver's subjective perception. The subjective responses of diverse individuals to [...] [Read more](#).

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19 pages, 2006 KB ([/2227-7390/12/9/1296/pdf?version=1714033082](https://pub.mdpi-res.com/mathematics/mathematics-12-01296/pdf?version=1714033082))

**Shearlet Transform Applied to a Prostate Cancer Radiomics Analysis on MR Images** ([/2227-7390/12/9/1296](https://pub.mdpi-res.com/mathematics/mathematics-12-01296/pdf?version=1714033082))

by Rosario Corso, Alessandro Stefano, Giuseppe Salvaggio and Albert Comelli

Mathematics 2024, 12(9), 1296; <https://doi.org/10.3390/math12091296> (<https://doi.org/10.3390/math12091296>) - 25 Apr 2024

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**Abstract** For decades, wavelet theory has attracted interest in several fields in dealing with signals. Nowadays, it is acknowledged that it is not very suitable to face aspects of multidimensional data like singularities and this has led to the development of other mathematical tools. [...] [Read more](#).

(This article belongs to the Special Issue **Artificial Intelligence for Biomedical Image Processing and Data Analysis** ([/journal/mathematics/special\\_issues/T635A6WNH6](https://pub.mdpi-res.com/mathematics/mathematics-12-01296/pdf?version=1714033082)))

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17 pages, 363 KB ([/2227-7390/12/9/1295/pdf?version=1714031812](https://pub.mdpi-res.com/mathematics/mathematics-12-01295/pdf?version=1714031812))

**Feynman Diagrams beyond Physics: From Biology to Economy** ([/2227-7390/12/9/1295](https://pub.mdpi-res.com/mathematics/mathematics-12-01295/pdf?version=1714031812))

by Nicolò Cangini

Mathematics 2024, 12(9), 1295; <https://doi.org/10.3390/math12091295> (<https://doi.org/10.3390/math12091295>) - 25 Apr 2024

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**Abstract** Feynman diagrams represent one of the most powerful and fascinating tools developed in theoretical physics in the last century. Introduced within the framework of quantum electrodynamics as a suitable method for computing the amplitude of a physical process, they rapidly became a fundamental [...] [Read more](#).

(This article belongs to the Special Issue **Algebraic Structures and Graph Theory, 2nd Edition** ([/journal/mathematics/special\\_issues/1RKT503SSL](https://pub.mdpi-res.com/mathematics/mathematics-12-01295/pdf?version=1714031812)))

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14 pages, 5340 KB ([/2227-7390/12/9/1294/pdf?version=1714008914](https://pub.mdpi-res.com/mathematics/mathematics-12-01294/pdf?version=1714008914))

**Improvement of Distributed Denial of Service Attack Detection through Machine Learning and Data Processing** ([/2227-7390/12/9/1294](https://pub.mdpi-res.com/mathematics/mathematics-12-01294/pdf?version=1714008914))

by Fray L. Becerra-Suarez, Ismael Fernández-Roman and Manuel G. Forero

Mathematics 2024, 12(9), 1294; <https://doi.org/10.3390/math12091294> (<https://doi.org/10.3390/math12091294>) - 25 Apr 2024

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**Abstract** The early and accurate detection of Distributed Denial of Service (DDoS) attacks is a fundamental area of research to safeguard the integrity and functionality of organizations' digital ecosystems. Despite the growing importance of neural networks in recent years, the use of classical techniques [...] [Read more](#).

(This article belongs to the Special Issue **Network Security in Artificial Intelligence Systems** ([/journal/mathematics/special\\_issues/S5N4U43HG4](https://pub.mdpi-res.com/mathematics/mathematics-12-01294/pdf?version=1714008914)))

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**Comparisons of Numerical and Solitary Wave Solutions for the Stochastic Reaction–Diffusion Biofilm Model including Quorum Sensing** (2227-7390/12/9/1293)

by Muhammad Zafarullah Baber, Nauman Ahmed, Muhammad Waqas Yasin, Muhammad Sajid Iqbal, Ali Akgül, Alicia Cordero and Juan R. Torregrosa  
*Mathematics* 2024, 12(9), 1293; <https://doi.org/10.3390/math12091293> (https://doi.org/10.3390/math12091293) - 24 Apr 2024

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**Abstract** This study deals with a stochastic reaction–diffusion biofilm model under quorum sensing. Quorum sensing is a process of communication between cells that permits bacterial communication about cell density and alterations in gene expression. This model produces two results: the bacterial concentration, which over [...] [Read more](#).

**Intelligent Low-Consumption Optimization Strategies: Economic Operation of Hydropower Stations Based on Improved LSTM and Random Forest Machine Learning Algorithm** (2227-7390/12/9/1292)

by Hong Pan, Jie Yang, Yang Yu, Yuan Zheng, Xiaonan Zheng and Chenyang Hang  
*Mathematics* 2024, 12(9), 1292; <https://doi.org/10.3390/math12091292> (https://doi.org/10.3390/math12091292) - 24 Apr 2024

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**Abstract** The economic operation of hydropower stations has the potential to increase water use efficiency. However, there are some challenges, such as the fixed and unchangeable flow characteristic curve of the hydraulic turbines, and the large number of variables in optimal load distribution, which [...] [Read more](#).

(This article belongs to the Special Issue [Computational Methods and Applications for Numerical Analysis, 2nd Edition](#) (/journal/mathematics/special\_issues/0K108J6FA8))

**Multi-Objective Portfolio Optimization Using a Quantum Annealer** (2227-7390/12/9/1291)

by Esteban Aguilera, Jins de Jong, Frank Phillipson, Skander Taamallah and Mischa Vos  
*Mathematics* 2024, 12(9), 1291; <https://doi.org/10.3390/math12091291> (https://doi.org/10.3390/math12091291) - 24 Apr 2024

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**Abstract** In this study, the portfolio optimization problem is explored, using a combination of classical and quantum computing techniques. The portfolio optimization problem with specific objectives or constraints is often a quadratic optimization problem, due to the quadratic nature of, for example, risk measures. [...] [Read more](#).

(This article belongs to the Section [Mathematics and Computer Science](#) (/journal/mathematics/sections/mathematics\_computers\_science))

**Traffic Prediction with Self-Supervised Learning: A Heterogeneity-Aware Model for Urban Traffic Flow Prediction Based on Self-Supervised Learning** (2227-7390/12/9/1290)

by Min Gao, Yingmei Wei, Yuxiang Xie and Yitong Zhang  
*Mathematics* 2024, 12(9), 1290; <https://doi.org/10.3390/math12091290> (https://doi.org/10.3390/math12091290) - 24 Apr 2024

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**Abstract** Accurate traffic prediction is pivotal when constructing intelligent cities to enhance urban mobility and to efficiently manage traffic flows. Traditional deep learning-based traffic prediction models primarily focus on capturing spatial and temporal dependencies, thus overlooking the existence of spatial and temporal heterogeneities.

Heterogeneity [...] [Read more](#).

**Genetic Algorithms Application for Pricing Optimization in Commodity Markets** (2227-7390/12/9/1289)

by Yiyu Li, Qingjie Xu, Ying Wang and Bin Liu  
*Mathematics* 2024, 12(9), 1289; <https://doi.org/10.3390/math12091289> (https://doi.org/10.3390/math12091289) - 24 Apr 2024

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**Abstract** The perishable nature of vegetable commodities poses challenges for superstores, as reselling them is often unfeasible due to their short freshness period. Reliable market demand analysis is crucial for boosting revenue. This study simplifies the pricing and replenishment decision-making process by making reasonable [...] [Read more](#).

(This article belongs to the Special Issue [Mathematical Modeling and Machine Learning with Application to Economics and Finance](#) (/journal/mathematics/special\_issues/mathmodel\_economic\_fina))

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**Local  $C^{0,1}$ -Regularity for the Parabolic  $p$ -Laplacian Equation on the Group  $SU(3)$**  (2227-7390/12/9/1288)

by Yongming He, Chengwei Yu and Hongqing Wang  
*Mathematics* 2024, 12(9), 1288; <https://doi.org/10.3390/math12091288> (https://doi.org/10.3390/math12091288) - 24 Apr 2024

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**Abstract** In this article, when  $2 \leq p \leq 4$ , we establish the  $C^{0,1}_{loc}$ -regularity of weak solutions to the degenerate parabolic  $p$ -Laplacian equation [...] [Read more](#).

(This article belongs to the Special Issue [Research on Dynamical Systems and Differential Equations](#) (/journal/mathematics/special\_issues/S83FDFT2H7))

**A Weighted Skew-Logistic Distribution with Applications to Environmental Data** (2227-7390/12/9/1287)

by Isaac Cortés, Jimmy Reyes and Yuri A. Iriarte  
*Mathematics* 2024, 12(9), 1287; <https://doi.org/10.3390/math12091287> (https://doi.org/10.3390/math12091287) - 24 Apr 2024

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**Abstract** Skewness and bimodality properties are frequently observed when analyzing environmental data such as wind speeds, precipitation levels, and ambient temperatures. As an alternative to modeling data exhibiting these properties, we propose a flexible extension of the skew-logistic distribution. The proposal corresponds to a [...] [Read more](#).

(This article belongs to the Section [Probability and Statistics](#) (/journal/mathematics/sections/probability\_and\_statistics\_theory))

**VTT-LLM: Advancing Vulnerability-to-Tactic-and-Technique Mapping through Fine-Tuning of Large Language Model** (2227-7390/12/9/1286)

by Chenhui Zhang, Le Wang, Dunqiu Fan, Junyi Zhu, Tang Zhou, Liyi Zeng and Zhaohua Li



**Abstract** Vulnerabilities are often accompanied by cyberattacks. CVE is the largest repository of open vulnerabilities, which keeps expanding. ATT&CK models known multi-step attacks both tactically and technically and remains up to date. It is valuable to correlate the vulnerability in CVE with the corresponding [Read more](#).  
(This article belongs to the Special Issue [Advanced Research on Information System Security and Privacy \( /journal/mathematics/special\\_issues/93YQ7N975L \)](#))

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25 pages, 8813 KIB ( /2227-7390/12/9/1285/pdf?version=1713956392 )

**Spatial Network Analysis of Coupling Coordination between Digital Financial Inclusion and Common Prosperity in the Yangtze River Delta Urban Agglomeration ( /2227-7390/12/9/1285 )**

by Fanlong Zeng and Huaping Sun

Mathematics 2024, 12(9), 1285; <https://doi.org/10.3390/math12091285> (https://doi.org/10.3390/math12091285) - 24 Apr 2024  
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**Abstract** Digital financial inclusion and common prosperity are pivotal elements in promoting the sustainable socioeconomic development of China. This study introduces a novel Multi-Criteria Decision Analysis (MCDA) method to evaluate the Common Prosperity Index (CPI). Using this index, alongside the Digital Financial Inclusion Index [Read more](#).  
(This article belongs to the Special Issue [Mathematical Modelling of Economics and Regional Development \( /journal/mathematics/special\\_issues/BIM75R3XZ9 \)](#))

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13 pages, 251 KIB ( /2227-7390/12/9/1284/pdf?version=1713953406 )

**Asymptotic Behavior of Stochastic Reaction–Diffusion Equations ( /2227-7390/12/9/1284 )**

by Hao Wen, Yuanjing Wang, Guangyuan Liu and Dawei Liu

Mathematics 2024, 12(9), 1284; <https://doi.org/10.3390/math12091284> (https://doi.org/10.3390/math12091284) - 24 Apr 2024  
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**Abstract** In this paper, we concentrate on the propagation dynamics of stochastic reaction–diffusion equations, including the existence of travelling wave solution and asymptotic wave speed. Based on the stochastic Feynman–Kac formula and comparison principle, the boundedness of the solution of stochastic reaction–diffusion equations can [Read more](#).  
(This article belongs to the Special Issue [Dynamics of Predator-Prey and Infectious Disease Models \( /journal/mathematics/special\\_issues/H617262S6K \)](#))

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19 pages, 11263 KIB ( /2227-7390/12/9/1283/pdf?version=1713950576 )

**Inter-Channel Correlation Modeling and Improved Skewed Histogram Shifting for Reversible Data Hiding in Color Images ( /2227-7390/12/9/1283 )**

by Dan He, Zhanchuan Cai, Dujuan Zhou and Zhihui Chen

Mathematics 2024, 12(9), 1283; <https://doi.org/10.3390/math12091283> (https://doi.org/10.3390/math12091283) - 24 Apr 2024  
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**Abstract** Reversible data hiding (RDH) is an advanced data protection technology that allows the embedding of additional information into an original digital medium while maintaining its integrity. Color images are typical carriers for information because of their rich data content, making them suitable for [Read more](#).  
(This article belongs to the Special Issue [Advanced Research on Information System Security and Privacy \( /journal/mathematics/special\\_issues/93YQ7N975L \)](#))

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15 pages, 4676 KIB ( /2227-7390/12/9/1282/pdf?version=1713946143 )

**Numerical Investigation of Supersonic Flow over a Wedge by Solving 2D Euler Equations Utilizing the Steger–Warming Flux Vector Splitting (FVS) Scheme ( /2227-7390/12/9/1282 )**

by Mitch Wolff, Hashim H. Abada and Hussein Awad Kurdi Saad

Mathematics 2024, 12(9), 1282; <https://doi.org/10.3390/math12091282> (https://doi.org/10.3390/math12091282) - 24 Apr 2024  
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**Abstract** Supersonic flow over a half-angle wedge ( $\theta = 15^\circ$ ) with an upstream Mach number of 2.0 was investigated using 2D Euler equations where sea level conditions were considered. The investigation employed the Steger–Warming flux vector splitting (FVS) method executed in MATLAB 9.13.0 (R2022b) [Read more](#).  
(This article belongs to the Special Issue [Modeling, Simulation and Control of Dynamical Systems \( /journal/mathematics/special\\_issues/Modeling\\_Simulation\\_Control\\_Dynamical\\_Systems \)](#))

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Open Access Article 22 pages, 3459 KIB ([/2227-7390/12/9/1281/pdf?version=1714036288](https://doi.org/10.3390/math12091281))

**MDER-Net: A Multi-Scale Detail-Enhanced Reverse Attention Network for Semantic Segmentation of Bladder Tumors in Cystoscopy Images** ([/2227-7390/12/9/1281](https://doi.org/10.3390/math12091281))

by Chao Nie, Chao Xu and Zhengping Li  
*Mathematics* **2024**, *12*(9), 1281; <https://doi.org/10.3390/math12091281> (<https://doi.org/10.3390/math12091281>) - 24 Apr 2024

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**Abstract** White light cystoscopy is the gold standard for the diagnosis of bladder cancer. Automatic and accurate tumor detection is essential to improve the surgical resection of bladder cancer and reduce tumor recurrence. At present, Transformer-based medical image segmentation algorithms face challenges in restoring [...] **Read more.** (This article belongs to the Special Issue **New Advances and Applications in Image Processing and Computer Vision** ([/journal/mathematics/special\\_issues/F737IH6LF9](https://journal/mathematics/special_issues/F737IH6LF9)))

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Open Access Article 13 pages, 286 KIB ([/2227-7390/12/9/1280/pdf?version=1713945578](https://doi.org/10.3390/math12091280))

**On an Anisotropic Logistic Equation** ([/2227-7390/12/9/1280](https://doi.org/10.3390/math12091280))

by Leszek Gasiński and Nikolaos S. Papageorgiou  
*Mathematics* **2024**, *12*(9), 1280; <https://doi.org/10.3390/math12091280> (<https://doi.org/10.3390/math12091280>) - 24 Apr 2024

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**Abstract** We consider a nonlinear Dirichlet problem driven by the  $(p(z), q)$ -Laplacian and with a logistic reaction of the equidiffusive type. Under a nonlinearity condition on a quotient map, we show existence and uniqueness of positive solutions [...] **Read more.** (This article belongs to the Special Issue **Problems and Methods in Nonlinear Analysis** ([/journal/mathematics/special\\_issues/K0IXM3KOT6](https://journal/mathematics/special_issues/K0IXM3KOT6)))

Open Access Article 16 pages, 325 KIB ([/2227-7390/12/9/1279/pdf?version=1713939186](https://doi.org/10.3390/math12091279))

**Statistical Solitonic Impact on Submanifolds of Kenmotsu Statistical Manifolds** ([/2227-7390/12/9/1279](https://doi.org/10.3390/math12091279))

by Abdullah Ali H. Ahmadini, Mohd. Danish Siddiqi and Aliya Naaz Siddiqui  
*Mathematics* **2024**, *12*(9), 1279; <https://doi.org/10.3390/math12091279> (<https://doi.org/10.3390/math12091279>) - 24 Apr 2024

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**Abstract** In this article, we delve into the study of statistical solitons on submanifolds of Kenmotsu statistical manifolds, introducing the presence of concircular vector fields. This investigation is further extended to study the behavior of almost quasi-Yamabe solitons on submanifolds with both concircular and [...] **Read more.** (This article belongs to the Special Issue **Differentiable Manifolds and Geometric Structures** ([/journal/mathematics/special\\_issues/48F13IX0FT](https://journal/mathematics/special_issues/48F13IX0FT)))

Open Access Article 18 pages, 321 KIB ([/2227-7390/12/9/1278/pdf?version=1714005960](https://doi.org/10.3390/math12091278))

**A One-Parameter Family of Methods with a Higher Order of Convergence for Equations in a Banach Space** ([/2227-7390/12/9/1278](https://doi.org/10.3390/math12091278))

by Ramandeep Behl, Ioannis K. Argyros and Sattam Alharbi  
*Mathematics* **2024**, *12*(9), 1278; <https://doi.org/10.3390/math12091278> (<https://doi.org/10.3390/math12091278>) - 23 Apr 2024

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**Abstract** The conventional approach of the local convergence analysis of an iterative method on  $\mathbb{R}^m$ , with  $m$  a natural number, depends on Taylor series expansion. This technique often requires the calculation of high-order derivatives. However, those derivatives may not be part of [...] **Read more.** (This article belongs to the Special Issue **Numerical Analysis and Modeling** ([/journal/mathematics/special\\_issues/4V3W111X5](https://journal/mathematics/special_issues/4V3W111X5)))

Open Access Article 33 pages, 2107 KIB ([/2227-7390/12/9/1277/pdf?version=1713882911](https://doi.org/10.3390/math12091277))

**Geometric Control and Structure-at-Infinity Control for Disturbance Rejection and Fault Compensation Regarding Buck Converter-Based LED Driver** ([/2227-7390/12/9/1277](https://doi.org/10.3390/math12091277))

by Jesse Y. Rumbo-Morales, Jair Gómez-Radilla, Gerardo Ortiz-Torres, Felipe D. J. Sorcia-Vázquez, Hector M. Buenabad-Arias, María A. López-Osorio, Carlos A. Torres-Cantero, Moises Ramos-Martinez, Mario A. Juárez, Manuela Calixto-Rodriguez, Jorge A. Brizuela-Mendoza and Jesús E. Valdez-Resendiz  
*Mathematics* **2024**, *12*(9), 1277; <https://doi.org/10.3390/math12091277> (<https://doi.org/10.3390/math12091277>) - 23 Apr 2024

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**Abstract** Currently, various light-emitting diode (LED) lighting systems are being developed because LEDs are one of the most used lighting sources for work environments, buildings, homes, and public roads in terms of some of their applications. Similarly, they have low energy consumption, quick responses, [...] **Read more.** (This article belongs to the Special Issue **System Modeling, Control Theory, and Their Applications** ([/journal/mathematics/special\\_issues/SJ09SBJD20](https://journal/mathematics/special_issues/SJ09SBJD20)))

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**Fixed/Preassigned-Time Synchronization of Fuzzy Memristive Fully Quaternion-Valued Neural Networks Based on Event-Triggered Control (2227-7390/12/9/1276)**

by Shichao Jia, Cheng Hu and Haijun Jiang

*Mathematics* 2024, 12(9), 1276; <https://doi.org/10.3390/math12091276> (https://doi.org/10.3390/math12091276) - 23 Apr 2024

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**Abstract** In this paper, the fixed-time and preassigned-time synchronization issues of fully quaternion-valued fuzzy memristive neural networks are studied based on the dynamic event-triggered control mechanism. Initially, the fuzzy rules are defined within the quaternion domain and the relevant properties are established through rigorous [...] [Read more](#).

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**Relation-Theoretic Nonlinear Almost Contractions with an Application to Boundary Value Problems (2227-7390/12/9/1275)**

by Salma Aljawi and Izhar Uddin

*Mathematics* 2024, 12(9), 1275; <https://doi.org/10.3390/math12091275> (https://doi.org/10.3390/math12091275) - 23 Apr 2024

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**Abstract** This article investigates certain fixed-point results enjoying nonlinear almost contraction conditions in the setup of relational metric space. Some examples are constructed in order to indicate the profitability of our results. As a practical use of our findings, we demonstrate the existence of [...] [Read more](#).

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**Joins, Secant Varieties and Their Associated Grassmannians (2227-7390/12/9/1274)**

by Edoardo Ballico

*Mathematics* 2024, 12(9), 1274; <https://doi.org/10.3390/math12091274> (https://doi.org/10.3390/math12091274) - 23 Apr 2024

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**Abstract** We prove a strong theorem on the partial non-defectivity of secant varieties of embedded homogeneous varieties developing a general set-up for families of subvarieties of Grassmannians. We study this type of problem in the more general set-up of joins of embedded varieties. Joins [...] [Read more](#).

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**The Optimal Stopping Problem under a Random Horizon (2227-7390/12/9/1273)**

by Tahir Choulli and Safa' Alsheyab

*Mathematics* 2024, 12(9), 1273; <https://doi.org/10.3390/math12091273> (https://doi.org/10.3390/math12091273) - 23 Apr 2024

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**Abstract** This paper considers a pair  $(\mathbb{F}, \tau)$ , where  $\mathbb{F}$  is a filtration representing the "public" flow of information that is available to all agents over time, and  $\tau$  is a random time that might not be an  $\mathbb{F}$ -stopping [...] [Read more](#).

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**Requirement Dependency Extraction Based on Improved Stacking Ensemble Machine Learning (2227-7390/12/9/1272)**

by Hui Guan, Hang Xu and Lie Cai

*Mathematics* 2024, 12(9), 1272; <https://doi.org/10.3390/math12091272> (https://doi.org/10.3390/math12091272) - 23 Apr 2024

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**Abstract** To address the cost and efficiency issues of manually analysing requirement dependency in requirements engineering, a requirement dependency extraction method based on part-of-speech features and an improved stacking ensemble learning model (P-Stacking) is proposed. Firstly, to overcome the problem of singularity in the [...] [Read more](#).

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Open Access Feature Paper Article 18 pages, 15710 KIB (2227-7390/12/9/1271/pdf?version=1713866479)

**Unveiling Fall Triggers in Older Adults: A Machine Learning Graphical Model Analysis (2227-7390/12/9/1271)**

by Tho Nguyen, Ladda Thiamwong, Qian Lou and Rui Xie

*Mathematics* 2024, 12(9), 1271; <https://doi.org/10.3390/math12091271> (https://doi.org/10.3390/math12091271) - 23 Apr 2024





**Abstract** While existing research has identified diverse fall risk factors in adults aged 60 and older across various areas, comprehensively examining the interrelationships between all factors can enhance our knowledge of complex mechanisms and ultimately prevent falls. This study employs a novel approach—a *mixed [...]* [Read more](#).  
(This article belongs to the Special Issue [Computational Intelligence in Addressing Data Heterogeneity](#) ([/journal/mathematics/special\\_issues/CIADH2022](#)))

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# Green Measures for a Class of Non-Markov Processes

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Herry P. Suryawan and José L. da Silva

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# Green Measures for a Class of Non-Markov Processes

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† These authors contributed equally to this work.

**Abstract:** In this paper, we investigate the Green measure for a class of non-Gaussian processes in  $\mathbb{R}^d$ . These measures are associated with the family of generalized grey Brownian motions  $B_{\beta,\alpha}$ ,  $0 < \beta \leq 1$ ,  $0 < \alpha \leq 2$ . This family includes both fractional Brownian motion, Brownian motion, and other non-Gaussian processes. We show that the perpetual integral exists with probability 1 for  $d\alpha > 2$  and  $1 < \alpha \leq 2$ . The Green measure then generalizes those measures of all these classes.

**Keywords:** fractional Brownian motion; generalized grey Brownian motion; green measure; subordination

**MSC:** 60G22; 65N80; 47A30

## 1. Introduction

In recent years, there has been a significant amount of research devoted to fractional dynamics related to fractional Brownian motion and related processes. These processes lack both the Markov and semimartingale properties from a mathematical standpoint. As a result, many traditional approaches in stochastic analysis do not apply, making their analysis more challenging. These processes are capable of modeling systems that exhibit long-range self-interaction and memory effects.

In 1992, Schneider introduced the grey Brownian motion [1], a class of non-Gaussian processes, to solve the time-fractional diffusion equation with a Caputo–Djrbashian derivative of fractional order. During the 1990s, Mainardi and their co-authors conducted a systematic investigation into fractional differential equations; see [2] and the references therein. They introduced the notion of generalized grey Brownian motion (ggBm for short), and the corresponding time-fractional differential equations governing its densities. This family of processes is denoted by  $B_{\beta,\alpha}$  with parameters  $0 < \beta \leq 1$  and  $0 < \alpha \leq 2$ . If  $\beta \neq 1$ , the process  $B_{\beta,\alpha}$  is non-Gaussian with stationary increments and  $\alpha/2$ -self-similar; see Section 2 for details. The process  $B_{\beta,\alpha}$  admits different representations (cf. (12) and (13) below) in terms of other known processes, which are useful for simulation and to derive other properties. In a recent work, Grothaus et al. [3] elaborated an infinite dimensional analysis for (non-Gaussian) measures of the Mittag-Leffler type. They used ggBm to solve the time-fractional heat equation, extending the fractional Feynman–Kac formula of Schneider [1].

The goal of this paper (see Theorem 1 and Corollary 1 below) is to prove the existence of the Green measure for the class of non-Gaussian processes ggBm in  $\mathbb{R}^d$ . This result will extend the results of Kondratiev et al. [4]. More precisely, for a Borel function  $f : \mathbb{R}^d \rightarrow \mathbb{R}$ , the potential of  $f$  (see [5,6] for details) is defined as

$$V_{\beta,\alpha}(f, x) = \int_0^\infty \mathbb{E}[f(x + B_{\beta,\alpha}(t))] dt, \quad x \in \mathbb{R}^d. \quad (1)$$



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We would like to investigate the class of functions  $f$  for which the potential of  $f$  has the representation

$$V_{\beta,\alpha}(f, x) = \int_{\mathbb{R}^d} f(y) \mathcal{G}_{\beta,\alpha}(x, dy), \tag{2}$$

where  $\mathcal{G}(x, \cdot) := \mathcal{G}_{\beta,\alpha}(x, \cdot)$  is a Radon measure on  $\mathbb{R}^d$  called the Green measure corresponding to the ggBm  $B_{\beta,\alpha}$ ; see Definition 2 below. If  $B_{\beta,\alpha}$  admits a generator  $L_{\beta,\alpha}$ , then the potential  $V(x, f)$  can be obtained from the equation

$$-LV = f.$$

The Green measure can be seen as the fundamental solution for the generator  $L_{\beta,\alpha}$  of the process  $B_{\beta,\alpha}$ . First, we establish the existence of the perpetual integral (cf. Theorem 1):

$$\int_0^\infty f(x + B_{\beta,\alpha}(t)) dt \tag{3}$$

with probability one. This leads to an explicit representation of the Green measure for ggBm, namely (cf. Corollary 1)

$$\mathcal{G}_{\beta,\alpha}(x, dy) = \frac{D}{|x - y|^{d-2/\alpha}} dy, \quad d\alpha > 2, \quad 1 < \alpha \leq 2,$$

where  $D$  is a constant that depends on  $\beta, \alpha$ , and the dimension  $d$ ; see (17) for the explicit expression. Note that as  $d\alpha > 2$  and  $1 < \alpha \leq 2$ , the Green measure  $\mathcal{G}_{\beta,\alpha}(x, \cdot)$  exists for  $d \geq 2$ , since  $d > 2/\alpha \in [1, 2)$ . The Brownian case ( $\alpha = 1$ ) is covered only for  $d \geq 3$ . We emphasize that the existence of the Green measure for a given process  $X$  is not always guaranteed. In addition, finding a proper space of functions  $f : \mathbb{R}^d \rightarrow \mathbb{R}$  that guarantees the existence of (1) is crucial. As an example, the  $d$ -dimensional Bm starting at  $x \in \mathbb{R}^d$  has a density given by  $p_t(x, y) = (2\pi t)^{-d/2} \exp(-|x - y|^2/(2t))$ ,  $y \in \mathbb{R}^d$ . It is not difficult to see that  $\int_0^\infty p_t(x, y) dt$  does not exist for  $d = 1, 2$ . Hence, the Green measure of Bm for  $d = 1, 2$  does not exist. On the other hand, for  $d \geq 3$ , the Green measure of Bm on  $\mathbb{R}^d$  exists and is given by  $\mathcal{G}(x, dy) = C(d)|x - y|^{2-d} dy$ , where  $C(d)$  is a constant depending on the dimension  $d$ ; see [4] and the references therein for more details. In a two-dimensional space, the Green measure of ggBm is determined by the parameter  $\alpha$  that is related to the roughness of the path. The Green measure of ggBm for  $d = 1$  requires further analysis (for Bm, see [7], Ch. 4), which we will postpone for a future paper.

This paper is organized as follows. In Section 2, we recall the definition and main properties of ggBm that will be needed later. In Section 3, we show the existence of the perpetual integral with probability one, which leads to the explicit formula for the Green measure for ggBm. In Section 4, we discuss the obtained results, connect them with other topics, and draw conclusions.

## 2. Generalized Grey Brownian Motion

We recall the class of non-Gaussian processes, called generalized grey Brownian motion, which we study below. This class of processes was first introduced by Schneider [8,9], and was generalized by Mura et al. (see [10,11]) as a stochastic model for slow/fast anomalous diffusion described by the time-fractional diffusion equation.

### 2.1. Definition and Properties

For  $0 < \beta \leq 1$ , the (entire) Mittag-Leffler function  $E_\beta$  is defined by the Taylor series

$$E_\beta(z) := \sum_{n=0}^\infty \frac{z^n}{\Gamma(\beta n + 1)}, \quad z \in \mathbb{C}, \tag{4}$$

where

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt, \quad z \in \mathbb{C}, \operatorname{Re}(z) \geq 0$$

is the Euler gamma function.

The  $M$ -Wright function is a special case of the class of Wright functions  $W_{\lambda,\mu}$ ,  $\lambda > -1$ ,  $\mu \in \mathbb{C}$ , via

$$M_\beta(z) := W_{-\beta,1-\beta}(-z) = \sum_{n=0}^\infty \frac{(-z)^n}{n! \Gamma(-\beta n + 1 - \beta)}.$$

The special choice  $\beta = 1/2$  yields the Gaussian density on  $[0, \infty)$ :

$$M_{1/2}(z) = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{z^2}{4}\right). \tag{5}$$

The Mittag-Leffler function  $E_\beta$  is the Laplace transform of the  $M$ -Wright function, that is,

$$E_\beta(-s) = \int_0^\infty e^{-s\tau} M_\beta(\tau) d\tau. \tag{6}$$

The generalized moments of the density  $M_\beta$  of order  $\delta > -1$  are finite and are given (see [10]) by

$$\int_0^\infty \tau^\delta M_\beta(\tau) d\tau = \frac{\Gamma(\delta + 1)}{\Gamma(\beta\delta + 1)}. \tag{7}$$

**Definition 1.** Let  $0 < \beta \leq 1$  and  $0 < \alpha \leq 2$  be given. A  $d$ -dimensional continuous stochastic process  $B_{\beta,\alpha} = \{B_{\beta,\alpha}(t), t \geq 0\}$ , starting at  $0 \in \mathbb{R}^d$  and defined on a complete probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ , is a ggBm in  $\mathbb{R}^d$  (see [11] for  $d = 1$ ) if the following is satisfied:

1.  $\mathbb{P}(B_{\beta,\alpha}(0) = 0) = 1$ , that is,  $B_{\beta,\alpha}$  starts at zero  $\mathbb{P}$ -almost surely ( $\mathbb{P}$ -a.s.).
2. Any collection  $\{B_{\beta,\alpha}(t_1), \dots, B_{\beta,\alpha}(t_n)\}$  with  $0 \leq t_1 < t_2 < \dots < t_n < \infty$  has a characteristic function given, for any  $\theta = (\theta_1, \dots, \theta_n) \in (\mathbb{R}^d)^n$  with  $\theta_k = (\theta_{k,1}, \dots, \theta_{k,d})$ ,  $k = 1, \dots, n$ , by

$$\mathbb{E} \left[ \exp \left( i \sum_{k=1}^n (\theta_k, B_{\beta,\alpha}(t_k))_{\mathbb{R}^d} \right) \right] = E_\beta \left[ -\frac{1}{2} \sum_{j=1}^d (\theta_{\cdot,j}, \gamma_\alpha \theta_{\cdot,j})_{\mathbb{R}^n} \right], \tag{8}$$

where  $\mathbb{E}$  denotes the expectation with regard to  $\mathbb{P}$  and

$$\gamma_\alpha := \gamma_{\alpha,n} := (t_k^\alpha + t_j^\alpha - |t_k - t_j|^\alpha)_{k,j=1}^n.$$

3. The joint probability density function of  $(B_{\beta,\alpha}(t_1), \dots, B_{\beta,\alpha}(t_n))$  is equal to

$$\rho_\beta(\theta, \gamma_\alpha) = \frac{(2\pi)^{-nd/2}}{(\det \gamma_\alpha)^{d/2}} \int_0^\infty \tau^{-nd/2} e^{-\frac{1}{2\tau} \sum_{j=1}^d (\theta_{\cdot,j}, \gamma_\alpha^{-1} \theta_{\cdot,j})_{\mathbb{R}^n}} M_\beta(\tau) d\tau. \tag{9}$$

The following are the most important key properties of ggBm:

(P1). For each  $t \geq 0$ , the moments of any order of  $B_{\beta,\alpha}(t)$  are given by

$$\begin{cases} \mathbb{E}[|B_{\beta,\alpha}(t)|^{2n+1}] &= 0, \\ \mathbb{E}[|B_{\beta,\alpha}(t)|^{2n}] &= \frac{(2n)!}{2^n \Gamma(\beta n + 1)} t^{\alpha n}. \end{cases}$$

(P2). The covariance function has the form

$$\mathbb{E}[(B_{\beta,\alpha}(t), B_{\beta,\alpha}(s))] = \frac{d}{2\Gamma(\beta + 1)} (t^\alpha + s^\alpha - |t - s|^\alpha), \quad t, s \geq 0. \tag{10}$$

(P3). For each  $t, s \geq 0$ , the characteristic function of the increments is

$$\mathbb{E} \left[ e^{i(k, B_{\beta, \alpha}(t) - B_{\beta, \alpha}(s))} \right] = E_{\beta} \left( -\frac{|k|^2}{2} |t - s|^{\alpha} \right), \quad k \in \mathbb{R}^d. \tag{11}$$

- (P4). The process  $B_{\beta, \alpha}$  is non-Gaussian and  $\alpha/2$ -self-similar with stationary increments.
- (P5). The ggBm is not a semimartingale. Furthermore,  $B_{\alpha, \beta}$  cannot be of finite variation in  $[0, 1]$  and, by the scaling and stationarity of the increment, on any interval in  $\mathbb{R}^+$ .
- (P5). For  $n = 1$ , the density  $\rho_{\beta}(x, t)$ ,  $x \in \mathbb{R}^d$ ,  $t > 0$  is the fundamental solution of the following fractional differential equation (see [12]):

$$\mathbb{D}_t^{2\beta} \rho_{\beta}(x, t) = \Delta_x \rho_{\beta}(x, t),$$

where  $\Delta_x$  is the  $d$ -dimensional Laplacian in  $x$  and  $\mathbb{D}_t^{2\beta}$  is the Caputo–Dzherbashian fractional derivative; see [13] for the definition and properties.

### 2.2. Representations of Generalized Grey Brownian Motion

The ggBm admits different representations in terms of well-known processes. It follows from (8) that ggBm has an elliptical distribution; see Section 3 in [3]. On the other hand, ggBm is also given as a product (see [10] for  $d = 1$ ) of two processes, as follows:

$$\{B_{\beta, \alpha}(t), t \geq 0\} \stackrel{\mathcal{L}}{=} \{ \sqrt{Y_{\beta}} B^{\alpha/2}(t), t \geq 0 \}. \tag{12}$$

Here,  $\stackrel{\mathcal{L}}{=}$  means equality in law, the non-negative random variable  $Y_{\beta}$  has density  $M_{\beta}$ , and  $B^{\alpha/2}$  is a  $d$ -dimensional fBm with Hurst parameter  $\alpha/2$  and is independent of  $Y_{\beta}$ .

We give another representation of ggBm  $B_{\beta, \alpha}$  as a subordination of fBm (see Section 2.14 in [14] for  $d = 1$ ) which is used below. For completeness, we give a short proof.

**Proposition 1.** *The ggBm has the following representation:*

$$\{B_{\beta, \alpha}(t), t \geq 0\} \stackrel{\mathcal{L}}{=} \{B^{\alpha/2}(t Y_{\beta}^{1/\alpha}), t \geq 0\}. \tag{13}$$

**Proof.** We must show that both representations (12) and (13) have the same finite-dimensional distribution. For every  $\theta = (\theta_1, \dots, \theta_n) \in (\mathbb{R}^d)^n$ , we have

$$\begin{aligned} \mathbb{E} \left[ \exp \left( i \sum_{k=1}^n (\theta_k, B^{\alpha/2}(t_k Y_{\beta}^{1/\alpha})) \right) \right] &= \int_0^{\infty} \mathbb{E} \left[ \exp \left( i \sum_{k=1}^n (\theta_k, B^{\alpha/2}(t_k y^{1/\alpha})) \right) \right] M_{\beta}(y) dy \\ &= \int_0^{\infty} \mathbb{E} \left[ \exp \left( i \sum_{k=1}^n (\theta_k, y^{1/2} B^{\alpha/2}(t_k)) \right) \right] M_{\beta}(y) dy \\ &= \mathbb{E} \left[ \exp \left( i \sum_{k=1}^n (\theta_k, Y_{\beta}^{1/2} B^{\alpha/2}(t_k)) \right) \right]. \end{aligned}$$

In the second equality, we use the  $\alpha/2$ -self-similarity of fBm. This completes the proof.  $\square$

### 3. The Green Measure for Generalized Grey Brownian Motion

In this section, we show the existence of the Green measure for ggBm; see (1) and (2). Let us begin by discussing the existence of the Green measure for a general stochastic process  $X$ .

Let  $X = \{X(t), t \geq 0\}$  be a stochastic process in  $\mathbb{R}^d$  starting from  $x \in \mathbb{R}^d$ . If  $X(t)$ ,  $t \geq 0$ , has a probability distribution  $\rho_{X(t)}(x, \cdot)$ , then Equation (1) becomes

$$V_X(x, f) = \int_0^{\infty} \int_{\mathbb{R}^d} f(y) \rho_{X(t)}(x, dy) dt. \tag{14}$$

Then, applying the Fubini theorem, the Green measure  $\mathcal{G}_X(x, \cdot)$  of  $X$  is given by

$$\mathcal{G}_X(x, dy) = \int_0^\infty \rho_{X(t)}(x, dy) dt,$$

assuming the existence of  $\mathcal{G}_X(x, \cdot)$  as a Radon measure on  $\mathbb{R}^d$ . That is, for every bounded Borel set  $B \in \mathcal{B}_b(\mathbb{R}^d)$  we have

$$\mathcal{G}_X(x, B) = \int_0^\infty \rho_{X(t)}(x, B) dt < \infty.$$

If the probability distribution  $\rho_{X(t)}(x, \cdot)$  is also absolutely continuous with respect to the Lebesgue measure, say  $\rho_{X(t)}(x, dy) = \rho_t(x, y) dy$ , then the function

$$g_X(x, y) := \int_0^\infty \rho_t(x, y) dt, \quad \forall y \in \mathbb{R}^d, \tag{15}$$

is called the Green function of the stochastic process  $X$ . Moreover, the Green measure in this case is given by  $\mathcal{G}_X(x, dy) = g_X(x, y) dy$ .

This leads us to the following definition of the Green measure of a stochastic process  $X$ .

**Definition 2.** Let  $X = \{X(t), t \geq 0\}$  be a stochastic process on  $\mathbb{R}^d$  starting from  $x \in \mathbb{R}^d$  and  $\rho_{X(t)}(x, \cdot)$  be the probability distribution of  $X(t), t \geq 0$ . The Green measure of  $X$  is defined as a Radon measure on  $\mathbb{R}^d$  by

$$\mathcal{G}_X(x, B) := \int_0^\infty \rho_{X(t)}(x, B) dt, \quad B \in \mathcal{B}_b(\mathbb{R}^d),$$

or

$$\int_{\mathbb{R}^d} f(y) \mathcal{G}_X(x, dy) = \int_{\mathbb{R}^d} f(y) \int_0^\infty \rho_{X(t)}(x, dy) dt, \quad f \in C_0(\mathbb{R}^d)$$

whenever these integrals exist.

In other words,  $\mathcal{G}_X(x, B)$  is the expected length of time the process remains in  $B$ .

To state the main theorem that establishes the existence of the Green measure for ggBm, first, we introduce a proper Banach space of functions  $f : \mathbb{R}^d \rightarrow \mathbb{R}$  such that the perpetual integral (3) is finite  $\mathbb{P}$ -a.s. Without a loss of generality, we can assume that  $f \geq 0$  above. We define the space  $CL(\mathbb{R}^d)$ , of continuous real valued, on  $\mathbb{R}^d$  by

$$CL(\mathbb{R}^d) := \{f : \mathbb{R}^d \rightarrow \mathbb{R} \mid f \text{ is continuous, bounded and } f \in L^1(\mathbb{R}^d)\}.$$

The space  $CL(\mathbb{R}^d)$  becomes a Banach space with the norm

$$\|f\|_{CL} := \|f\|_\infty + \|f\|_1, \quad \forall f \in CL(\mathbb{R}^d),$$

where  $\|\cdot\|_\infty$  denotes the sup-norm and  $\|\cdot\|_1$  is the norm in  $L^1(\mathbb{R}^d)$ . The choice of  $CL(\mathbb{R}^d)$  allows us to show that the family of random variables (3) with  $f \in CL(\mathbb{R}^d)$  have finite expectations  $\mathbb{P}$ -a.s.

**Theorem 1.** Let  $f \in CL(\mathbb{R}^d)$  and  $x \in \mathbb{R}^d$  be given and consider ggBm  $B_{\beta, \alpha}$  with  $d\alpha > 2$  and  $1 < \alpha \leq 2$ . Then, the perpetual integral functional  $\int_0^\infty f(x + B(t)) dt$  is finite  $\mathbb{P}$ -a.s. and its expectation equals

$$\mathbb{E} \left[ \int_0^\infty f(x + B_{\beta, \alpha}(t)) dt \right] = D \int_{\mathbb{R}^d} \frac{f(x + y)}{|y|^{d-2/\alpha}} dy, \tag{16}$$

where

$$D = D(\beta, \alpha, d) = \frac{1}{\alpha} 2^{-1/\alpha} \pi^{-\frac{d}{2}} \Gamma\left(\frac{d}{2} - \frac{1}{\alpha}\right) \frac{\Gamma(1 - \frac{1}{\alpha})}{\Gamma(1 - \frac{\beta}{\alpha})}. \tag{17}$$

**Proof.** Given that  $x \in \mathbb{R}^d$  and  $f \in CL(\mathbb{R}^d)$  are non-negative, let  $\rho_\beta(\cdot, t^\alpha)$  denote the density of  $B_{\beta,\alpha}(t)$ ,  $t \geq 0$ , which is given by (see (9) with  $n = 1$ )

$$\rho_\beta(y, t^\alpha) = \frac{1}{(2\pi t^\alpha)^{d/2}} \int_0^\infty \tau^{-d/2} e^{-\frac{|y|^2}{2t^\alpha \tau}} M_\beta(\tau) d\tau, \quad y \in \mathbb{R}^d.$$

First, we show equality (16). It follows from the above considerations that

$$\begin{aligned} \mathbb{E} \left[ \int_0^\infty f(x + B_{\beta,\alpha}(t)) dt \right] &= \int_0^\infty \int_{\mathbb{R}^d} f(x + y) \rho_\beta(y, t^\alpha) dy dt. \\ &= \int_0^\infty \int_{\mathbb{R}^d} f(x + y) \frac{1}{(2\pi t^\alpha)^{d/2}} \int_0^\infty \tau^{-d/2} M_\beta(\tau) e^{-\frac{|y|^2}{2t^\alpha \tau}} d\tau dy dt. \end{aligned}$$

Using Fubini’s Theorem, we first compute the  $t$ -integral and use the assumption  $d\alpha > 2$ . We obtain

$$\int_0^\infty \frac{1}{(2\pi t^\alpha \tau)^{d/2}} e^{-\frac{|y|^2}{2t^\alpha \tau}} dt = C(\alpha, d) \frac{\tau^{-\frac{1}{\alpha}}}{|y|^{d-2/\alpha}},$$

where

$$C(\alpha, d) := \frac{1}{\alpha} 2^{-1/\alpha} \pi^{-\frac{d}{2}} \Gamma\left(\frac{d}{2} - \frac{1}{\alpha}\right).$$

Next, we compute the  $\tau$ -integral using (7) so that

$$\int_0^\infty \tau^{-1/\alpha} M_\beta(\tau) d\tau = \frac{\Gamma(1 - \frac{1}{\alpha})}{\Gamma(1 - \frac{\beta}{\alpha})}, \quad \alpha > 1.$$

Combining them gives the equality (16) where  $D = D(\beta, \alpha, d) = C(\alpha, d) \frac{\Gamma(1 - \frac{1}{\alpha})}{\Gamma(1 - \frac{\beta}{\alpha})}$ .

Now, we show that the right-hand side of (16) is finite for every non-negative  $f \in CL(\mathbb{R}^d)$ . To see this, we can use the local integrability of  $|y|^{d-2/\alpha}$  in  $y$  and obtain

$$\begin{aligned} \int_{\mathbb{R}^d} \frac{f(x + y)}{|y|^{d-2/\alpha}} dy &= \int_{\{|y| \leq 1\}} \frac{f(x + y)}{|y|^{d-2/\alpha}} dy + \int_{\{|y| > 1\}} \frac{f(x + y)}{|y|^{d-2/\alpha}} dy \\ &\leq C_1 \|f\|_\infty + C_2 \|f\|_1 \leq C \|f\|_{CL}. \end{aligned}$$

Therefore, the integral in (16) is, in fact, well defined. In other words, the integral  $\int_0^\infty f(x + B_{\beta,\alpha}(t)) dt$  exists with probability one. This completes the proof.  $\square$

As a consequence of the above theorem, we immediately obtain the Green measure of ggBm  $B_{\beta,\alpha}$ , that is, comparing (2) and (16).

**Corollary 1.** *The Green measure of ggBm  $B_{\beta,\alpha}$  for  $d\alpha > 2$  is given by*

$$\mathcal{G}_{\beta,\alpha}(x, dy) = \frac{D}{|x - y|^{d-2/\alpha}} dy,$$

where  $D$  is given by (17).

**Remark 1.**

1. It is possible to show that, given  $f \neq 0$ , the perpetual integral (3) is a non-constant random variable. As a consequence, for  $f \geq 0$  the variance of the random variable (3) is strictly positive. The proof uses the notion of conditional full support of ggBm. We do not provide a detailed explanation of this result that closely follows the ideas of Theorem 2.2 in [4], to which we address interested readers.



2. Note also that the functional in (1),

$$V_{\beta,\alpha}(\cdot, x) : CL(\mathbb{R}^d) \longrightarrow \mathbb{R}$$

is continuous. In fact, from the proof of Theorem 1, any  $f \in CL(\mathbb{R}^d)$  yields

$$|V_{\beta,\alpha}(f, x)| \leq K\|f\|_{CL},$$

where  $K$  is a constant depending on the parameters  $\beta, \alpha$ , and  $d$ .

#### 4. Discussion and Conclusions

We derived the Green measure for the class of stochastic processes called generalized grey Brownian motion in Euclidean space  $\mathbb{R}^d$  for  $d \geq 2$ . This class includes, in particular, fractional Brownian motion and other non-Gaussian processes. To address the case where  $d = 1$ , a renormalization process is needed. However, this will be postponed to future work. For  $\beta = \alpha = 1$  ggBm,  $B_{1,1}$  is nothing but a Brownian motion. In this case, the Green measure exists for  $d \geq 3$ . Green measures and Green functions are well known to be intrinsically connected and applied to (stochastic partial) differential equations. In this context, the Green measures discussed in this paper play the same role for space-time-fractional derivatives. The presented method can be applied to other processes with sufficient information on the density and existence of the integrals. If we consider a Markov process  $X$  that admits a Green measure and  $T$ , a random time change given by an inverse subordinator, then the Green measure of the subordinated process  $X(T(t)), t \geq 0$  exists only after renormalization. Mixing different types of processes, e.g., fBm and scaled Bm, as described in [15], or Markovian and non-Markovian, as in [16], may lead us to a renormalization procedure to guarantee the existence of the Green measure.

The relationship between the Green measure and the local time of the ggBm can be described as follows. For any  $T > 0$  and a continuous function  $f : \mathbb{R}^d \longrightarrow \mathbb{R}$ , the integral functional

$$\int_0^T f(B_{\beta,\alpha}(t)) dt \tag{18}$$

is well defined. For  $d = 1$ , the integral (18) with  $f \in L^1(\mathbb{R})$  is represented as

$$\int_0^T f(B_{\beta,\alpha}(t)) dt = \int_{\mathbb{R}} f(x)L_{\beta,\alpha}(T, x) dx,$$

where  $L_{\beta,\alpha}(T, x)$  is the local time of ggBm up to time  $T$  at the point  $x$  (see [3]). The Green measure corresponds to the asymptotic behaviour in  $T$  of the expectation of local time  $L_{\beta,\alpha}(T, x)$ . The existence of this asymptotic depends on the dimension  $d$  and the transient or recurrent properties in the process.

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#### References

- Schneider, W.R. Grey noise. In *Ideas and Methods in Mathematical Analysis, Stochastics, and Applications (Oslo, 1988)*; Albeverio, S., Fenstad, J.E., Holden, H., Lindström, T., Eds.; Cambridge University Press: Cambridge, UK, 1992; pp. 261–282.
- Mainardi, F. *Fractional Calculus and Waves in Linear Viscoelasticity: An Introduction to Mathematical Models*; World Scientific: Singapore, 2010.

3. Grothaus, M.; Jahnert, F. Mittag-Leffler Analysis II: Application to the fractional heat equation. *J. Funct. Anal.* **2016**, *270*, 2732–2768. [[CrossRef](#)]
4. Kondratiev, Y.; Mishura, Y.; da Silva, J.L. Perpetual integral functionals of multidimensional stochastic processes. *Stochastics* **2021**, *93*, 1249–1260. [[CrossRef](#)]
5. Blumenthal, R.M.; Gettoor, R.K. *Markov Processes and Potential Theory*; Academic Press: New York, NY, USA; London, UK, 1968.
6. Revuz, D.; Yor, M. *Continuous Martingales and Brownian Motion*, 3rd ed.; Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]; Springer: Berlin, Germany, 1999; Volume 293, pp. xii+560. [[CrossRef](#)]
7. Armitage, D.H.; Gardiner, S.J. *Classical Potential Theory*; Springer Monographs in Mathematics; Springer Science & Business Media: London, UK, 2001.
8. Schneider, W.R. Fractional diffusion. In *Dynamics and Stochastic Processes (Lisbon, 1988)*; Lima, R., Streit, L., Vilela Mendes, R., Eds.; Lecture Notes in Physics; Springer: New York, NY, USA, 1990; Volume 355, pp. 276–286.
9. Schneider, W.R. Grey noise. In *Stochastic Processes, Physics and Geometry*; Albeverio, S., Casati, G., Cattaneo, U., Merlini, D., Moresi, R., Eds.; World Scientific Publishing: Teaneck, NJ, USA, 1990; pp. 676–681.
10. Mura, A.; Pagnini, G. Characterizations and simulations of a class of stochastic processes to model anomalous diffusion. *J. Phys. A Math. Theor.* **2008**, *41*, 285003. [[CrossRef](#)]
11. Mura, A.; Mainardi, F. A class of self-similar stochastic processes with stationary increments to model anomalous diffusion in physics. *Integral Transform. Spec. Funct.* **2009**, *20*, 185–198. [[CrossRef](#)]
12. Mentrelli, A.; Pagnini, G. Front propagation in anomalous diffusive media governed by time-fractional diffusion. *J. Comput. Phys.* **2015**, *293*, 427–441. [[CrossRef](#)]
13. Samko, S.G.; Kilbas, A.A.; Marichev, O.I. *Fractional Integrals and Derivatives*; Gordon and Breach Science Publishers: Yverdon, Switzerland, 1993; pp. xxxvi+976.
14. Erraoui, M.; Röckner, M.; da Silva, J.L. Cameron-Martin Type Theorem for a Class of non-Gaussian Measures. *arXiv* **2023**. [[CrossRef](#)]
15. Wang, W.; Metzler, R.; Cherstvy, A.G. Anomalous diffusion, aging, and nonergodicity of scaled Brownian motion with fractional Gaussian noise: Overview of related experimental observations and models. *Phys. Chem. Chem. Phys.* **2022**, *24*, 18482–18504. [[CrossRef](#)] [[PubMed](#)]
16. Qu, X.; Hu, Y.; Cai, W.; Xu, Y.; Ke, H.; Zhu, G.; Zihan, U. Semantic segmentation of anomalous diffusion using deep convolutional networks. *Phys. Rev. Res.* **2024**, *6*, 013054. [[CrossRef](#)]

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