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Neutrino Mass Matrix from a Seesaw Mechanism with Heavy Majorana Mass Matrix Subject to Texture Zero and Invariant Under a Cyclic Permutation

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Abstract : We evaluate the invariant forms of the neutrino mass matrices which arise from a seesaw mechanism with heavy Majorana mass matrices subject to texture zero and invariant under a cyclic permutation. From eight possible patterns of the heavy Majorana neutrino mass matrices, we found that there is no heavy Majorana neutrino mass matrix which is invariant in form under a cyclic permutation. But, by imposing an additional assumption that at least one of the 2×2 sub-matrices of the heavy Majorana neutrino mass matrix inverse has zero determinant, we found two of the heavy Majorana neutrino mass matrices have to be invariant under a cyclic permutation. One of these two invariant heavy Majorana neutrino mass matrices can be used to explain the neutrino mixing phenomena for both solar and atmospheric neutrinos qualitatively

Keywords : Neutrino mass matrix, seesaw mechanism, texture zero, cyclic permutation
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1 INTRODUCTION

The Glashow-Weinberg-Salam (GWS) model for electroweak interaction which is based on $SU(2)_L \otimes U(1)_Y$ gauge symmetry group (see for example Peskin and Schroeder [1]) has been successful phenomenologically. Even though the GWS model successful phenomenologically, it stil still far from a complete theory because the GWS model is blind to many fundamental problems such as neutrino mass problem and fermions (lepton and quark) mass hierarchy [2]. For more than two decades, the solar neutrino flux measured on Earth has been much less than predicted by the solar model [3]. The solar neutrino deficit can be explained if neutrino undergoes oscillation during its propagation to earth. Neutrino oscillation implies that neutrinos have a non-zero mass or at least one of the three known neutrino flavors has a non-zero mass and neutrino mixing does exist. Recently, there is a convincing evidence that neutrinos have a non-zero mass. This evidence was based on the experimental facts that both solar and atmospheric neutrinos undergo oscillation [4, 5, 6, 7, 8, 9].

The global analysis of neutrino oscillations data gives the best fit value to the solar neutrino mass-squared differences [10]

$$\Delta m^2_{21} = (8.2^{+0.3}_{-0.3}) \times 10^{-5}\ eV^2 \tag{1}$$

with

$$\tan^2 \theta_{21} = 0.39^{+0.05}_{-0.04}, \tag{2}$$

and for the atmospheric neutrino mass-squared differences

$$\Delta m^2_{32} = (2.2^{+0.6}_{-0.4}) \times 10^{-3}\ eV^2 \tag{3}$$

with

$$\tan^2 \theta_{32} = 1.0^{+0.35}_{-0.26}, \tag{4}$$

where $\Delta m^2_{ij} = m^2_i - m^2_j$ $(i, j = 1, 2, 3)$ with m_i as the neutrino mass eigenstates basis ν_i $(i = 1, 2, 3)$ and θ_{ij}

is the mixing angle between ν_i and ν_j. The relation between neutrino mass eigenstates and neutrino weak (flavor) eigenstates basis $(\nu_e, \nu_\mu, \nu_\tau)$ is given by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (5)$$

where V is the mixing matrix.

To explain a non-zero neutrino mass-squared differences and neutrino mixing, several models for neutrino mass and its underlying family symmetries, and the possible mechanism for generating a neutrino mass have been proposed [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. One of the interesting mechanism to generate neutrino mass is the seesaw mechanism [27, 28, 29, 30], in which the right-handed neutrino ν_R has a large Majorana mass M_N and the left-handed neutrino ν_L is given a mass through leakage of the order of (m^2/M_N) with m the Dirac mass. Thus, the seesaw mechanism could also be used to explains the smallness of the neutrino mass at the electro-weak energy scale. The mass matrix model of a massive Majorana neutrino M_N which is constrained by the solar and atmospheric neutrinos deficit and incorporate the seesaw mechanism and Peccei-Quinn symmetry have already been reported by Fukuyama and Nishiura [31]. By using an $SU(2)_L \otimes SU(2)_R \otimes U(1)_Y$ gauge group with assumption that the form of the mass matrix is invariant under a cyclic permutation among fermions, Koide [12] obtained a unified mass matrix model for leptons and quarks. By choosing a specific neutrino mixing matrix, Koide [32] obtained a result that can be used to explain maximal mixing between μ_ν and μ_τ as suggested by the atmospheric neutrino data.

In this paper, we construct the neutrino mass matrices which are generated via a seesaw mechanism with the heavy Majorana neutrino mass matrix M_N subject to texture zero and invariant in form under a cyclic permutation. We then evaluate the resulted neutrino mass matrices predictions on neutrino mixing phenomena. This paper is organized as follows: In Section 2, we determine the possible patterns for heavy Majorana neutrino mass matrices M_N subject to texture zero and investigate it whether invariant under a cyclic permutation. The M_N matrices which are invariant in form under a cyclic permutation will be used to obtain the neutrino mass matrices M_ν via seesaw mechanism. In Section 3, we evaluate and discuss the predictive power of the resulted neutrino mass matrices M_ν against experimental results. Finally, in Section 4 we give the conclusion.

2 TEXTURE ZERO AND INVARIANT UNDER A CYCLIC PERMUTATION

According to the seesaw mechanism, the neutrino mass matrix M_ν is given by

$$M_\nu \approx -M_D M_N^{-1} M_D^T, \quad (6)$$

where M_D and M_N are the Dirac and Majorana mass matrices respectively. If we take M_D to be diagonal, then the pattern of the neutrino mass matrix M_ν depends only on the pattern of the M_N matrix. From eq. (6), one can see that the pattern of the M_N^{-1} matrix will be preserved in M_ν matrix when M_D matrix is diagonal.

Following standard convention, let us denote the neutrino current eigenstates coupled to the charged leptons by the W boson as $\nu_\alpha(\alpha = e, \mu, \tau)$, and the neutrino mass eigenstates as $\nu_i(i = 1, 2, 3)$, then the mixing matrix V has the form as shown in eq. (5). As we have stated explicitly above, we use the seesaw mechanism as the responsible mechanism for generating the neutrino mass. The Majorana mass term in Lagrangian density is given by

$$L = -\nu_\alpha M_{\alpha\beta} C \nu_\beta + h.c. \quad (7)$$

where C is the charge conjugation matrix. For the sake of simplicity, we will assume that CP is conserve such that $M_{\alpha\beta}$ is real. Within this simplification, the neutrino mass matrix $M_{\alpha\beta}$ is diagonalized by the matrix V as

$$V^T M_{\alpha\beta} V = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \quad (8)$$

From recent experimental results, the explicit values of the mixing matrix moduli V can be found in [10]. According to the requirement that the mixing matrix V must be orthogonal, together with putting V in a nice simple looking numbers, to a first approximation we can take the mixing matrix V to be

$$V = \begin{pmatrix} -2/\sqrt{6} & 1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix}. \quad (9)$$

We can obtain the neutrino mass matrix M_ν by using the relation

$$M_\nu = V(V^T M_{\alpha\beta} V)V^T. \quad (10)$$

If we substitute eqs. (8) and (9) into eq. (10), we then have

$$M_\nu = \begin{pmatrix} P & Q & Q \\ Q & R & S \\ Q & S & R \end{pmatrix}, \quad (11)$$

where $P = (2m_1 + m_2)/3$, $Q = (m_2 - m_1)/3$, $R = (m_1 + 2m_2 + 3m_3)/6$, and $S = (m_1 + 2m_2 - m_3)/6$.

If the general neutrino mass matrix M_ν in eq.(11) is taken as the neutrino mass matrix arise from the seesaw mechanism, then we can write M_ν as

$$M_\nu = \begin{pmatrix} P & Q & Q \\ Q & R & S \\ Q & S & R \end{pmatrix} = M_D M_N^{-1} M_D. \quad (12)$$

From eq.(12), and putting M_D matrix to be diagonal

$$M_D = \begin{pmatrix} m_1^D & 0 & 0 \\ 0 & m_2^D & 0 \\ 0 & 0 & m_3^D \end{pmatrix}, \quad (13)$$

with the constraint that $m_2^D = m_3^D = m^D$ at high energy (CP is conserved), then the pattern of M_N^{-1} matrix is preserved in M_ν [24]. Thus, the M_N^{-1} matrix patterns can be written as

$$M_N^{-1} = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix}, \quad (14)$$

and then the M_N reads

$$M_N = X \begin{pmatrix} Y & Z & Z \\ Z & W & K \\ Z & K & W \end{pmatrix}, \quad (15)$$

where $X = 1/(A(C^2 - D^2) + 2B^2(D - C))$, $Y = C^2 - D^2$, $Z = B(D - C)$,$W = AC - B^2$, and $K = -AD + B^2$.

Now, we are in position to impose the requirement of texture zero into M_N matrix. The M_N matrix in eq. (15) will have texture zero if one or more of the following constraints are satisfied: (i) $C = -D$, (ii) $AD - B^2 = 0$, (iii) $AC - B^2 = 0$, and (iv) $B = 0$. If M_N matrix in eq. (15) has one or more of its entries to be zero (texture zero), then M_N^{-1} matrix has one or more of its 2×2 sub-matrices with zero determinants. The texture zero of the mass matrix indicates the existence of additional symmetries beyond the Standard Model of Particle Physics of electro-weak interaction. After lenghty calculations, we obtain eight possible patterns for the heavy Majorana M_N matrices with texture zero as one can read in [33]. The eight possible patterns of M_N heavy Majorana neutrino matrices read

$$M_N = \begin{pmatrix} 0 & a & a \\ a & b & c \\ a & c & b \end{pmatrix}, M_N = \begin{pmatrix} a & b & b \\ b & c & 0 \\ b & 0 & c \end{pmatrix},$$

$$M_N = \begin{pmatrix} a & b & b \\ b & 0 & c \\ b & c & 0 \end{pmatrix}, M_N = \begin{pmatrix} a & 0 & 0 \\ 0 & b & c \\ 0 & c & b \end{pmatrix},$$

$$M_N = \begin{pmatrix} 0 & a & a \\ a & b & 0 \\ a & 0 & b \end{pmatrix}, M_N = \begin{pmatrix} 0 & a & a \\ a & 0 & b \\ a & b & 0 \end{pmatrix},$$

$$M_N = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{pmatrix}, M_N = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}. \quad (16)$$

By checking the invariant form of the resulting neutrino mass matrices M_N with texture zero under a cyclic permutation, we found that there is no M_N matrix with texture zero invariant under a cyclic permutation. But, with additional assumption, there is a possibility to put the M_N matrices with texture zero to be invariant under a cyclic permutation, those are the M_N matrices with the patterns:

$$M_N = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{pmatrix}, \quad (17)$$

and

$$M_N = \begin{pmatrix} 0 & a & a \\ a & 0 & b \\ a & b & 0 \end{pmatrix}, \quad (18)$$

if we impose an additional assumption that $a = b$. With the above assumption, we finally obtain two M_N matrices invariant under a cyclic permutation. The M_N matrices which are invariant under a cyclic permutation read

$$M_N = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (19)$$

and

$$M_N = a \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}. \quad (20)$$

By substituting eqs. (13), (19) and (20) into eq. (6), we then have the neutrino mass matrices (M_ν) becomes

$$M_\nu = -\frac{1}{m_N} \begin{pmatrix} (m_1^D)^2 & 0 & 0 \\ 0 & (m^D)^2 & 0 \\ 0 & 0 & (m^D)^2 \end{pmatrix}, \quad (21)$$

with mass eigenvalues

$$m_1 = -\frac{(m_1^D)^2}{m_N},\ m_2 = -\frac{(m^D)^2}{m_N},$$
$$m_3 = -\frac{(m^D)^2}{m_N}, \quad (22)$$

and

$$M_\nu = \frac{1}{m_N^{'}}$$
$$\otimes \begin{pmatrix} (m_1^D)^2 & -m_1^D m^D & -m_1^D m^D \\ -m_1^D m^D & (m^D)^2 & -(m^D)^2 \\ -m_1^D m^D & -(m^D)^2 & (m^D)^2 \end{pmatrix}, \quad (23)$$

with mass eigenvalues

$$m_1 = \frac{(m_1^D)^2 - m_1^D\sqrt{(m_1^D)^2 + 8(m^D)^2}}{2m_N^{'}},$$
$$m_2 = \frac{(m_1^D)^2 + m_1^D\sqrt{(m_1^D)^2 + 8(m^D)^2}}{2m_N^{'}},$$
$$m_3 = \frac{2(m^D)^2}{m_N^{'}}, \quad (24)$$

where $m_N = a$ and $m_N^{'} = 2a$ respectively. The neutrino mass matrix in eq.(21) will give $\Delta m_{32}^2 = 0$, it then implies that this neutrino mass matrices fail to predict the neutrino mixing phenomena for the atmospheric neutrino at low energy. Thus, we obtain only one of the neutrino mass matrix M_ν (eq. (23) which is produced from the seesaw mechanism with heavy Majorana mass matrix subject to texture zero and invariant under a cyclic permutation that can be used to account the neutrino mixing for both solar and atmospheric neutrinos.

3 DISCUSSIONS

Without imposing an additional assumption for the M_N^{-1} matrices, we have no M_N matrix which is invariant under a cyclic permutation. But, by imposing an additional assumption that M_N^{-1} matrices have at least one of its 2×2 sub-matrices has zero determinant, then we have two M_N matrices which are invariant under a cyclic permutation.

By inspecting eq.(23), one can see that neutrino mass matrix M_ν arising from seesaw mechanism, with M_N subject to texture zero and invariant under a cyclic permutation, could be used to explain the neutrino mixing for both solar and atmospheric neutrinos data. From eq. (24), we obtain the neutrino masses $|m_1| = |m_2| \approx \sqrt{2} m_1^D m^D/m_N^{'}$ and $m_3 = 2(m^D)^2/m_N^{'}$ for case $m_1^D << m^D$, $|m_1| \approx (m^D)^2/m_N^{'}, m_2 \approx 2(m^D)^2/m_N^{'}$, and $m_3 = 2(m^D)^2/m_N^{'}$ for the case $m_1^D \approx m^D$, and $m_1 \approx 0, m_2 \approx (m_1^D)^2/m_N^{'}$, and $m_3 = 2(m^D)^2/m_N^{'}$ for the case $m_1^D >> m^D$. The case $m_1^D >> m^D$ gives a normal hierarcy for neutrino masses and can predict qualitatively the masses square differences in eqs. (1) and (3).

In really, using the heavy Majorana mass matrix M_N subject to texture zero in a seesaw mechanism will produces naturally a neutrino mass matrix without additional requirement that the M_ν is invariant under a cyclic permutation. This fact can be read in Ref.[33] for the cases that M_N matrix having one, two, or three of its entries to be zero. The M_N matrices which have three zero entries will lead to the tri-maximal mixing. One of the M_N matrices which have three zero entries (all of the diagonal M_N are zero) produces the neutrino mass matrix M_ν whose eigenvalues have normal hierarchy that can be used to explain the experimental result qualitatively. If the heavy neutrino Majorana M_N matrix has six of its entries to be zero (all of the M_N off-diagonal entries to be zero), then we obtain M_ν matrix as shown in eq. (21) arises from the seesaw mechanism that can not be used to explain neutrino mixing phenomena.

4 CONCLUSION

The eight possible patterns of heavy Majorana neutrino matrix M_N subject to texture zero could produce the neutrino mass matrices M_ν using seesaw mechanism as one can read in Ref.[33] and it could account for bi- and tri-maximal mixing in neutrino sector without additional assumption that these matrices are invariant in form under a cyclic permutation. When we evaluate the M_N invariance on the cyclic permutation, we found that there is no M_N matrix to be invariant in form. But, by imposing an additional assumption that the M_N^{-1} matrix has at least one of its 2×2 sub-matrices having zero determinant, we obtain two of the M_N matrices invariant in form under a cyclic permutation. One of the two M_N matrices, which are invariant under a cyclic permutation, could produces a neutrino mass mixing matrix M_ν as shown in Eq. (23) with eigenvalues $|m_1| = |m_2| \approx \sqrt{2} m_1^D m^D / m_N^{'}$ and $m_3 = 2(m^D)^2/m_N^{'}$ for case $m_1^D << m^D$.

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JTCS

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