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Left-Right Symmetry Model with Two Bidoublets and one Doublet Higgs Fields for Electroweak Interaction

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Abstract : Using the left-right symmetry model based on $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group with two bidoublets and one doublet Higgs fields for electroweak interaction, and the lepton fields to be doublet of SU(2) for both left and right fields, we evaluate the structure of the electroweak interaction. The model can can explained the parity violation which is known maximally in the weak interaction at low energy due to the very large of the W_R boson mass compared to the W_L boson mass. A tiny neutrino mass can be obtained from the model by choosing the value of the $e << h$ and the coupling $G^*_{2l} = 0$.

Keywords : Left-right symmetry, electroweak interaction, bidoublets and doublet Higgs fields

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1 INTRODUCTION

The Glashow-Weinberg-Salam (GWS) model for electroweak interaction (standard model for electroweak interaction) based on $SU(2)_L \otimes U(1)_Y$ gauge symmetry group has been in succes phenomenologically. Even though the GWS model in success phenomenologically, but it still far from a complete theory because the model blinds on many fundamental problems such as neutrino mass, and fermions (lepton and quark) mass hierarchy [1]. Recent experimental data on atmosphere and solar neutrinos indicate strongly that the neutrinos have a tiny mass and mixed up one another [2, 3, 4, 5, 6, 7, 8]. Many theories or models have been proposed to extend the standard model of electroweak interaction. One of the interesting model is the left-right symmetry model based on the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge group which is proposed by Senjanovic and Mohapatra [9].

Using the left-right symmetry model based on $SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge group with two doublets and one bidoublet Higgs fields, Senjanovic and Mohapatra found that the Higgs potential having a minimum value when we choose the asymmetric solution for the doublet Higgs fields vacuum expectation values. Within this scheme, the presence of the spontaneous parity violation at low energy is natural and the masses of neutrinos could be arbitraryly small.

Siringo [10] noticed that any viable gauge model for electroweak interactions must give an answer to the two quite different problems: (i) the breaking of symmetry from the full gauge group into electromagnetic Abelian group $U(1)_{em}$ giving a mass to the gauge bosons and then explains the known structure of weak interactions, and (ii) the mass matrices for fermions. In his other paper [11], by imposing the $O(2)$ custodial symmetry with left and right Higgs fields chosen to be a doublet of $SU(2)$, Siringo obtained the known *up-down* structure of the doublet fermions masses by inserting of *ad hoc* fermion-Higgs interactions. Meanwhile, Montero and Pleitez [12] used the approximate custodial $SU(2)_{L+R}$ global symmetry to extend the GWS model. But, the $O(2)$ custodial symmetry leads to five dimensions operator in the mass term of the Lagrangian density which is not renormalizable.

In this paper, we use the left-right symmetry model based on $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group by using two bidoublets and one doublet Higgs fields to break the left-right symmetry based on $SU(2)_L \otimes SU(R) \otimes U(1)_{B-L}$ gauge group down to $U(1)_{em}$ and then we study the predictive power of the model on the gauge bosons and fermions masses. Both left and right fermion fields are represented as doublet of $SU(2)$ group. Thus, in Section 2 we introduce explicitly our model and evaluate the minimum value of Higgs field potential by choosing the appropriate vacuum expectation values of the Higgs fileds. In section 3, we then explicitly give a systematic calculations of the gauge bosons and fermions masses. In section 4, we discuss our results and its implications to the fundamental processes, and finally in section 5 we present a conclusion.

2 THE MODEL

The left-right symmetry model based on $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ with the following lepton fields assignment,

$$\psi_L = \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L, \ \psi_R = \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_R, \tag{1}$$

where $l = e, \mu, \tau$, and three Higgs fields (two bidoublets and one doublet fields) with its electromagnetic charges reads,

$$\phi_1 = \begin{pmatrix} a^0 & b^+ \\ c^- & d^0 \end{pmatrix}, \ \phi_2 = \begin{pmatrix} e^0 & f^+ \\ g^- & h^0 \end{pmatrix},$$

$$\Phi = \begin{pmatrix} p^+ \\ q^0 \end{pmatrix}, \tag{2}$$

which break the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ down to $U(1)_{em}$. The bidoublet Higgs ϕ_1 transforms as $\phi_1(2,2,-2)$, ϕ_2 transforms as $\phi_2(2,2,-2)$, and Φ transforma as $\Phi(0,2,0)$ under $SU(2)$ respectively. The general potential of the Higgs fields, which consistent with the renormalizability, the gauge invariance, and the discrete left-right symmetry, is given by,

$$\begin{aligned} V(\phi_1,\phi_2,\Phi) = -\mu^2 \left[Tr(\phi_1^+\phi_1 + \phi_2^+\phi_2)\right] \\ +\lambda_1[Tr((\phi_1^+\phi_1)^2 + (\phi_2^+\phi_2)^2)] \\ +\lambda_2[Tr(\phi_1^+\phi_1)Tr(\phi_2^+\phi_2)] - \alpha^2(\Phi^+\Phi) \\ +\beta(\Phi^+\Phi)^2 + \gamma\left[Tr(\phi_1^+\phi_1 + \phi_2^+\phi_2)\right](\Phi^+\Phi) \\ +\delta\left[Tr(\phi_1^+\Phi\Phi^+\phi_1 + \phi_2^+\Phi\Phi^+\phi_2)\right] + H.c.\,. \end{aligned} \tag{3}$$

After explicitly performing the calculation to find out the minimum value of the Higgs potential in Eq.(3), we then obtain the following constraints,

$$\begin{aligned} Tr(\phi_1^+\phi_1) &= Tr(\phi_2^+\phi_2) \\ &= \frac{2\mu^2 - 2(\gamma+\delta)\Phi^+\Phi}{2\lambda_1 + \lambda_2}, \end{aligned} \tag{4}$$

and

$$\Phi^+\Phi = \frac{\left[\alpha^2 - 2(\beta+\gamma)\right] Tr(\phi_1^+\phi_1)}{2\beta}. \tag{5}$$

From Eqs.(4) and (5), we can see that the minimum potential $V(\phi_1,\phi_2,\Phi)$ in Eq.(3) can be made to be minimum by arranging the values of the parameters $\lambda_1, \lambda_2, \beta, \gamma$, and δ.

We can choose the minimum so that the Higgs fields develop its vacuum expectation values, for the domain of parameters $\mu^2 > \alpha^2, \lambda_2 > 2\lambda_1$, and $e << h$, as follow,

$$\langle\phi_1\rangle = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \ \langle\phi_2\rangle = \begin{pmatrix} e & 0 \\ 0 & h \end{pmatrix},$$

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ q \end{pmatrix}. \tag{6}$$

These set of vacuum expectation values are known as an asymmetric solution. The asymmetric solution guarantee the presence of parity violation at low energy as known today in the electroweak interactions phenomenologically.

The complete Lagrangian density L in our model is given by,

$$\begin{aligned} L = -\frac{1}{4}\left(W_{\mu\nu L}.W^{\mu\nu L} + W_{\mu\nu R}.W^{\mu\nu R}\right) \\ -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ +\bar{\psi_L}\gamma^\mu(i\partial_\mu - g\frac{1}{2}\tau.W_{\mu L} - g'\frac{I}{2}B_\mu)\psi_L \\ +\bar{\psi_R}\gamma^\mu(i\partial_\mu - g\frac{1}{2}\tau.W_{\mu R} - g'\frac{I}{2}B_\mu)\psi_R \\ +Tr|(i\partial_\mu - g\frac{1}{2}\tau.W_{\mu L})\phi_1 \\ -g\frac{1}{2}\phi_1\tau.W_{\mu R} - g'\frac{I}{2}B_\mu\phi_1|^2 \\ +Tr|(i\partial_\mu - g\frac{1}{2}\tau.W_{\mu L})\phi_2 \\ -g\frac{1}{2}\phi_2\tau.W_{\mu R} - g'\frac{I}{2}B_\mu\phi_2|^2 \\ +|(i\partial_\mu - g\frac{1}{2}\tau.W_{\mu R} - g'\frac{I}{2}B_\mu)\Phi|^2 \\ -V(\phi_1,\phi_2,\Phi) - (m_o\bar{\psi_L}\psi_R \\ +G_{1l}\bar{\psi_L}\phi_1\psi_R + G_{1l}^*\bar{\psi_L}\tau_2\phi_1^*\tau_2\psi_R \\ +G_{2l}\bar{\psi_L}\phi_2\psi_R + G_{2l}^*\bar{\psi_L}\tau_2\phi_2^*\tau_2\psi_R + H.c.). \end{aligned} \tag{7}$$

where γ^ν is the usual Dirac matrices, τ is the Pauli spin matrices, g is the $SU(2)$ coupling , g' is the $U(1)$ coupling, m_o is the free lepton mass, G_{1l} and G_{2l} are the Yukawa couplings, and the $I = B - L$ quantum number is associated with the $U(1)$ generator in the left-right symmetry model. The relation of the I with respect to the electric charge Q in the left-right symmetry model is given by [13],

$$Q = T_{3L} + T_{3R} + \frac{I}{2}\,. \tag{8}$$

3 THE GAUGE BOSONS AND LEPTONS MASSES

3.1 Gauge Bosons Masses

The relevant mass terms for gauge bosons can be obtained from Eq.(7), they are the sixth, seventh, eighth, ninth, and the tenth terms without $i\partial_\mu$. By substituting Eq.(6) into these relevant mass terms, we then obtain,

$$\begin{aligned}
L_B = {} & Tr|(-\frac{g}{2}\tau.W_{\mu L} - g'\frac{I}{2}B_\mu)\left\langle\phi_1\right\rangle \\
& -\frac{g}{2}\left\langle\phi_1\right\rangle\tau.W_{\mu R}|^2 \\
& +Tr|(-\frac{g}{2}\tau.W_{\mu L} - g'\frac{I}{2}B_\mu)\left\langle\phi_2\right\rangle \\
& -\frac{g}{2}\left\langle\phi_2\right\rangle\tau.W_{\mu R}|^2 \\
& +|(-g\frac{1}{2}\tau.W_{\mu R} - \frac{g'}{2}B_\mu)\left\langle\Phi\right\rangle|^2 \\
= {} & \frac{1}{4}(2g^2[eh+q^2])[(W^1_{\mu R})^2 + (W^2_{\mu R})^2] \\
& +\frac{1}{4}(2g^2 eh)[(W^1_{\mu L})^2 + (W^2_{\mu L})^2] \\
& +\frac{1}{4}(2g^2(h^2+e^2))[W^1_{\mu R}W^{\mu L1} \\
& +W^2_{\mu R}W^{\mu L2} + W_{\mu L1}W^{\mu R2} \\
& -W_{\mu L2}W^{\mu R1}] + \frac{1}{4}e^2[gW^3_{\mu L} - 2g'B_\mu]^2 \\
& +\frac{1}{4}h^2[gW^3_{\mu L} + 2g'B_\mu]^2 + \frac{1}{4}(2h^2 gW^3_{\mu R}[gW^3_{\mu L} \\
& +2g'B_\mu]) + \frac{1}{4}(2e^2W^3_{\mu L}[gW^3_{\mu L} - 2g'B_\mu]) \\
& +\frac{1}{4}g^2(e^2+h^2+q^2)(W^3_{\mu R})^2,
\end{aligned} \tag{9}$$

where we have taken the value of $I = -2$ for both ϕ_1 and ϕ_2 and $I = 0$ for Φ to satisfy the requirement that the vacuum is invariant under $U(1)_{em}$ transformation and the photon remain massless.

In oreder to find out the gauge bosons masses explicitly, we define,

$$\begin{aligned}
W^\pm_{\mu L} &= \frac{1}{\sqrt{2}}(W^1_{\mu L} \mp iW^2_{\mu L}), \\
W^\pm_{\mu R} &= \frac{1}{\sqrt{2}}(W^1_{\mu R} \mp iW^2_{\mu R}), \\
Z_{\mu L} &= \frac{gW^3_{\mu L} - 2g'B_\mu}{\sqrt{g^2+4g'^2}} \\
&= W^3_{\mu L}\cos\theta_W - B_\mu\sin\theta_W, \\
Z'_\mu &= \frac{gW^3_{\mu L} + 2g'B_\mu}{\sqrt{g^2+4g'^2}} \\
&= W^3_{\mu L}\cos\theta_W + B_\mu\sin\theta_W, \\
A_{\mu L} &= \frac{2g'W^3_{\mu L} + gB_\mu}{\sqrt{g^2+4g'^2}} \\
&= W^3_{\mu L}\sin\theta_W + B_\mu\cos\theta_W, \\
A'_{\mu L} &= \frac{2g'W^3_{\mu L} - gB_\mu}{\sqrt{g^2+4g'^2}} \\
&= W^3_{\mu j}\sin\theta_W - B_\mu\cos\theta_W, \\
X_{\mu R} &= W^3_{\mu R} \qquad ,
\end{aligned} \tag{10}$$

where the θ_W is the weak mixing angle defined by,

$$\begin{aligned}
\cos\theta_W &= \frac{g}{\sqrt{g^2+4g'^2}}, \\
\sin\theta_W &= \frac{2g'}{\sqrt{g^2+4g'^2}}.
\end{aligned} \tag{11}$$

By substituting Eqs.(10) and (11) into Eq.(9), we finally obtain the relevant mass terms in the Lagrangian density of the physical $W^\pm_{\mu R}$, $W^\pm_{\mu L}$, $Z_{\mu L}$, $Z'_{\mu L}$, A_μ, A'_μ, and $X_{\mu R}$ fields, namely,

$$\begin{aligned}
L_B = {} & \frac{1}{2}m^2_{W_R}W^+_{\mu R}W^{-\mu}_R + \frac{1}{2}m^2_{W_L}W^+_{\mu L}W^{-\mu}_L \\
& +\frac{1}{2}m_{W_R W_L}W^+_{\mu L}W^{\mu R-} + \frac{1}{2}m^2_Z(Z_\mu)^2 \\
& +\frac{1}{2}m^2_{Z'}(Z'_\mu)^2 + \frac{1}{2}m^2_{Z'W_R}Z'_{\mu L}W^{\mu R3} \\
& +\frac{1}{2}m^2_{ZW_L}Z_{\mu L}W^{\mu L3} + \frac{1}{2}m^2_x(X_{\mu R})^2 \\
& +\frac{1}{2}m^2_\gamma A^2_\mu + \frac{1}{2}m'^2_\gamma(A'_\mu)^2,
\end{aligned} \tag{12}$$

where

$$m_{W_R} = g\sqrt{2eh+q^2},\ m_{W_L} = g\sqrt{2eh},$$
$$m_Z = \frac{e}{\sqrt{2}}\sqrt{g^2+4g'^2},$$
$$m_{Z'} = \frac{h}{\sqrt{2}}\sqrt{g^2+4g'^2},$$
$$m_X = \frac{g}{\sqrt{2}}\sqrt{e^2+h^2+q^2},$$
$$m_{W_LW_R} = 2g^2(e^2+h^2),$$
$$m^2_{Z'W_L} = gh^2\sqrt{g^2+4g'^2},$$
$$m^2_{ZW_R} = ge^2\sqrt{g^2+4g'^2},$$
$$m_\gamma = m'_\gamma = 0. \qquad (13)$$

From Eq.(13) we see that the photon is massless ($m_\gamma = m'_\gamma = 0$), and the bosons W, X, and Z are massive with $m_{W_R} > m_{W_L}$.

3.2 Leptons Masses

As one can read from Eq.(7), the leptons masses term in the Lagrangian density is,

$$L_{mass} = m_o\bar{\psi}_L\psi_R + G_{1l}\bar{\psi}_L\phi_1\psi_R$$
$$+G^*_{1l}\bar{\psi}_L\tau_2\phi^*_1\tau_2\psi_R + G_{2l}\bar{\psi}_L\phi_2\psi_R$$
$$+G^*_{2l}\bar{\psi}_L\tau_2\phi^*_2\tau_2\psi_R + H.c. \qquad (14)$$

From Eq.(14),one can see that the leptons mass terms in our model arise from two kinds of mass term, they are: (i) the free lepton mass term, and (ii) the Yukawa term. The free lepton mass term is the mass of the lepton as a free particle, and the Yukawa term is an additional mass to the free lepton mass which is resulted when the symmetry breaking take place.

By substituting Eqs.(1) and (6) into Eq.(14), we then obtain,

$$L_{mass} = m_o(\bar{\nu}_{lL}\nu_{lR} + \bar{l}_Ll_R)$$
$$+G_{2l}(e\bar{\nu}_{lL}\nu_{lR} + h\bar{l}_Ll_R) + G^*_{2l}(h\bar{\nu}_{lL}\nu_{lR}$$
$$+e\bar{l}_Ll_R) + H.c. \qquad (15)$$

where $m_o + G_{2l}e + G^*_{2l}h = m_{\nu_l}$ is the neutrino mass, and $m_o + G_{2l}h + G^*_{2l}e = m_l$ is the electron or muon or tauon mass. We obtain the neutrino mass is equal to the charge lepton mass which is contrary to the experimental facts. But, the phenomenological of the leptons masses can be deduced from the model if we put $e << h$ and $G^*_{2l} = 0$.

4 DISCUSSION

By using the left-right symmetry model based on $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group with two bidoublets and one doublet Higgs fields, we obtain ten bosons (eight bosons to be massive and two are massless) after symmetry breaking. Two bosons, W_R charge boson with mass m_{W_R} and X neutral boson with mass m_X, are very massive. To see qualitatively the contribution of the W_R to the weak interaction at low energy, we can check it via Fermi coupling G_F. The effective interactions at low energy is proportional to the Fermi coupling G_F. The relation of the G_F to the W boson mass is given by,

$$G_F = \frac{\sqrt{2}g^2}{8m^2_W} \qquad (16)$$

Because the W_R boson is very massive compared to the W_L boson, then the contribution of the W_R boson (with mass m_{W_R}) interactions with leptons fields at low energy is very weak compared to the interactions of the W_L boson (with mass m_{W_L}) with the leptons fields, and it then can be neglected. As known today that the parity is violated maximally in the weak interaction. Thus, in this model, due to the very large of the m_{W_R} compared to the m_{W_L}, the parity violation in the weak interaction can be expalined. The X_R neutral boson with mass m_{X_R} is a new boson.

From Eq.(11), we obtain a relation,

$$\tan\theta_W = \frac{2g'}{g}. \qquad (17)$$

By using the left-right symmetry model, the neutrino masses arise naturally. The free lepton masses m_0 should be zero if we fully adopt the GWS model which dictate that all of the leptons acquire a mass after the symmetry breaking. The problem of the *up-down* doublet mass difference in the lepton sector can be understood qualitatively, as one can seen in Eq.(15), if we put $e << h$ and $G^*_{2l} = 0$. It also apparent that the *up-down* doublet lepton mass difference can be addreseed to the symmetry breaking. But, the value of the $G^*_{2l} = 0$ in our model, in order to accomodate the present experimental facts, imply that there is an additional mechanism which is not known today.

5 CONCLUSION

In the scheme of the left-right symmetry model based on $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group with two bidoublets and one doublet Higgs fields, and the lepton fields are represented as an $SU(2)$ doublet for both left and right fields, we obtain two very massive bosons after the symmetry breaking take place, the W_R charge boson with mass m_{W_R} and the new X_R neutral boson with mass m_x. Because the W_R bosons mass is very large compared to the W_L boson mass, thus its interactions with lepton fields are very weak compared to the interactions of the W_L boson (with mass m_{W_L}) with leptons fields. Thus, the structure of the electroweak interactions which is dominated by the V-A interactions can be understood as due to the very massiveness of the W_R boson such that the contribution of the V+A interactions are very small compared to the V-A interaction at low energy. The neutrino mass arises naturally in the left-right symmetry model and the smallnes of its mass can be obtained if we take $e << h$ and the coupling $G^*_{2l} = 0$. We also obtained that the weak mixing angle $\theta_W = \arctan\left(\frac{2g'}{g}\right)$.

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