PTEP

Prog. Theor. Exp. Phys. **2024** 093B02 (43 pages)
DOI: 10.1093/ptep/ptae127

The Third Family Quark Mass Hierarchy and FCNC in the Universal Seesaw Model

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Received July 2, 2024; Revised August 7, 2024; Accepted August 13, 2024; Published August 13, 2024

We present the study of the quark sector of the universal seesaw model with $SU(2)_L \times SU(2)_R \times U(1)_{Y'}$ gauge symmetry in the massless case of the two lightest quark families. This model aims to explain the mass hierarchy of the third family quark by introducing a vector-like quark (VLQ) partner for each quark. In this model, we introduce $SU(2)_L$ and $SU(2)_R$ Higgs doublets. We derive explicitly the Lagrangian for the quark sector, Higgs sector, and kinetic terms of the gauge fields, starting from the Lagrangian, which is invariant under $SU(2)_L \times SU(2)_R \times U(1)_{Y'}$ gauge symmetry. At each stage of the symmetry breaking, we present the Lagrangian with the remaining gauge symmetry. Additionally, we investigate the flavor-changing neutral currents (FCNCs) of the Higgs (h) and Z-bosons in the interaction with the top, heavy top, bottom, and heavy bottom quarks.

Subject Index B40, B55

1. Introduction

The seesaw mechanism is a well-known approach to explain the smallness of neutrino masses [1–7]. It introduces heavy right-handed neutrinos that mix with the light left-handed neutrinos, giving them a small mass. This inspired the construction of a similar model that can be applied to other cases. One problem that the Standard Model (SM) cannot explain is the fermion mass hierarchy. In this paper, we study the quark sector of the universal seesaw model [8–23], an extension of the SM that applies a seesaw-like mechanism to the quark sector to solve the mass hierarchy problem. In the quark sector, an interesting aspect is the large mass of the top quark compared to the other quarks. Our focus is on the third family of quarks, and within our framework, the two lightest quark families are massless.

Introducing vector-like quarks (VLQs) into this model is essential. VLQs have left- and right-handed components that transform identically under some gauge group. Using this property, they can mix with SM quarks, resulting in modified mass matrices that can be diagonalized and generate a tiny seesaw-like mass. Various studies about adding VLQs to the SM have been explored, e.g. introducing one down-type isosinglet VLQ [24], one up-type isosinglet VLQ [25],

Funded by SCOAP³

and both one up-type and one down-type isosinglet VLQ [26]. The presence of VLQs also has implications for flavor physics, as they can introduce flavor-changing neutral currents (FCNCs) [27], and weak-basis invariants have been analyzed to understand the flavor structures [28,29]. Effective field theory approaches to VLQs have been studied to understand their contributions to low-energy observables [30,31]. A review of the theory and phenomenology of isosinglet VLQs can be found in Ref. [32].

This paper aims to study the quark sector within the universal seesaw model in the massless case of the two lightest quark families. We derive the Lagrangian, including the quark and Higgs sectors, and gauge kinetic terms. We also demonstrate how the model can naturally explain the observed quark mass hierarchies in the third family, particularly the significant mass of the top quark. We also explore the phenomenological implications of this model by investigating the interaction of the Higgs and Z-bosons with quarks, which includes FCNC processes.

The outline of this paper is as follows. In Section 2, we introduce the model with the particle contents and the Lagrangian. Section 3 focuses on the quark sector and Yukawa interactions. We explain the derivation of the Lagrangian of the kinetic terms and Yukawa interactions. Starting with the Lagrangian, which is invariant under $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$, in each stage of the symmetry breaking, we present the Lagrangian with the remaining gauge symmetry. The quark mass eigenvalues and the identification of FCNC within the massive third family quarks and their VLQ partners are discussed.

Section 4 discusses the Higgs sector of this model. The kinetic terms and Higgs potential are also derived step by step. In the end, we classify the terms based on the number of fields in the term as linear, quadratic, cubic, and quartic, ensuring a clear understanding of the interactions of the gauge sector. In addition, we also provide the exact diagonal mass of $Z - Z'$ bosons and $h - H$ bosons.

The kinetic terms of gauge fields are discussed in Section 5. In the final derivation, we show the difference between our model and the SM. Finally, in Section 6, we present some phenomenological implications of our model. We start the discussion with the hierarchy of VLQ's mass parameters, the nonzero vacuum expectation value (vev) of the $\mathrm{SU(2)_L}$ Higgs doublet (v_L), and the nonzero vev of the $\mathrm{SU(2)_R}$ Higgs doublet (v_R). Then, we analyze the interaction of the Higgs (h) and Z-bosons with the quarks. This leads to a discussion about FCNCs in this model.

2. The model

We consider an extension of the SM with $\mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$ gauge symmetry in the massless case of the two lightest quark families. In addition to the $\mathrm{SU(2)_L}$ SM Higgs doublet (ϕ_L), we have a $\mathrm{SU(2)_R}$ Higgs doublet (ϕ_R). We also introduce one up-type and one down-type isosinglet VLQ, denoted by T and B, respectively. The charge convention we use in this model is

$$Q = I_L^3 + I_R^3 + Y', \tag{2.1}$$

where Q, $I^3_{L(R)}$, and Y' are electromagnetic charge, left(right) weak-isospin, and $\mathrm{U(1)_{Y'}}$ hypercharge, respectively. The particle contents and their charge assignments under the model's gauge group are given in Table 1.

Table 1. Quark and Higgs fields with their quantum numbers under the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{Y'}$ gauge groups, where $i \in \{1, 2, 3\}$ is the family index.

Quark and Higgs fields	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	$U(1)_{Y'}$
$q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	**3**	**2**	**1**	1/6
$q_R^i = \begin{pmatrix} u_R^i \\ d_R^i \end{pmatrix}$	**3**	**1**	**2**	1/6
$T_{L,R}$	**3**	**1**	**1**	2/3
$B_{L,R}$	**3**	**1**	**1**	−1/3
$\phi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}$	**1**	**2**	**1**	1/2
$\phi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}$	**1**	**1**	**2**	1/2

The Lagrangian of this model (excluding the quantum chromodynamics (QCD) part) is as follows:

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_H + \mathcal{L}_{\text{gauge}}, \tag{2.2}$$

$$\begin{aligned}\mathcal{L}_q = {} & \overline{q_L^i} i\gamma^\mu D_{L\mu} q_L^i + \overline{q_R^i} i\gamma^\mu D_{R\mu} q_R^i + \overline{T} i\gamma^\mu D_{T\mu} T + \overline{B} i\gamma^\mu D_{B\mu} B \\ & - Y_{u_L}^3 \overline{q_L^3} \tilde{\phi}_L T_R - Y_{u_R}^3 \overline{T_L} \tilde{\phi}_R^\dagger q_R^3 - \overline{q_L^i} y_{d_L}^i \phi_L B_R - \overline{B_L} y_{d_R}^{i*} \phi_R^\dagger q_R^i - \text{h.c.} \\ & - \overline{T_L} M_T T_R - \overline{B_L} M_B B_R - \text{h.c.},\end{aligned} \tag{2.3}$$

$$\mathcal{L}_H = (D_L^\mu \phi_L)^\dagger (D_{L\mu} \phi_L) + (D_R^\mu \phi_R)^\dagger (D_{R\mu} \phi_R) - V(\phi_L, \phi_R), \tag{2.4}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{L\mu\nu}^a F_L^{a\mu\nu} - \frac{1}{4} F_{R\mu\nu}^a F_R^{a\mu\nu} - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu}, \tag{2.5}$$

where

$$V(\phi_L, \phi_R) = \mu_L^2 \phi_L^\dagger \phi_L + \mu_R^2 \phi_R^\dagger \phi_R + \lambda_L (\phi_L^\dagger \phi_L)^2 + \lambda_R (\phi_R^\dagger \phi_R)^2 + 2\lambda_{LR} (\phi_L^\dagger \phi_L)(\phi_R^\dagger \phi_R), \tag{2.6}$$

$$D_{L(R)\mu} q_{L(R)}^i = \left(\partial_\mu + i g_{L(R)} \frac{\tau^a}{2} W_{L(R)\mu}^a + i g'_1 Y'_q B'_\mu \right) q_{L(R)}^i, \tag{2.7}$$

$$D_{L(R)\mu} \phi_{L(R)} = \left(\partial_\mu + i g_{L(R)} \frac{\tau^a}{2} W_{L(R)\mu}^a + i g'_1 Y'_\phi B'_\mu \right) \phi_{L(R)}, \tag{2.8}$$

$$D_{T\mu} T = \left(\partial_\mu + i g'_1 Y'_T B'_\mu \right) T, \tag{2.9}$$

$$D_{B\mu} B = \left(\partial_\mu + i g'_1 Y'_B B'_\mu \right) B, \tag{2.10}$$

$$F_{L\mu\nu}^a = \partial_\mu W_{L\nu}^a - \partial_\nu W_{L\mu}^a - g_L \epsilon^{abc} W_{L\mu}^b W_{L\nu}^c, \tag{2.11}$$

$$F_{R\mu\nu}^a = \partial_\mu W_{R\nu}^a - \partial_\nu W_{R\mu}^a - g_R \epsilon^{abc} W_{R\mu}^b W_{R\nu}^c, \tag{2.12}$$

$$B'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu. \tag{2.13}$$

The Lagrangian in Eq. (2.2) is divided into three parts. The first part is the kinetic terms of quark doublet and isosinglet VLQs, Yukawa interactions, and mass terms of isosinglet VLQs, which are contained in Eq. (2.3). The second part is the kinetic terms and potential of the Higgs doublet, which are contained in Eq. (2.4). The third part is the kinetic terms of the gauge fields, which are written in Eq. (2.5).

The first line of Eq. (2.3) is the kinetic terms of quark doublet and isosinglet VLQs where the definitions of the covariant derivatives are written in Eqs. (2.7), (2.9), and (2.10), respectively, where $g_{L(R)}$ is $\mathrm{SU(2)_{L(R)}}$ gauge coupling, τ^a is the Pauli matrix, g'_1 is $\mathrm{U(1)_{Y'}}$ gauge coupling, and Y' is the corresponding $\mathrm{U(1)_{Y'}}$ hypercharge. For the Yukawa interaction part, one can choose a weak-basis where the Yukawa couplings of the up-type quark doublet ($Y^3_{u_L}$ and $Y^3_{u_R}$) are real positive numbers. In contrast, the Yukawa couplings of the down-type quark are general complex vectors as shown in the second line of Eq. (2.3). The derivation of this weak-basis is briefly explained in Appendix A. The family index for SM quarks is denoted as $i \in \{1, 2, 3\}$ and the charge conjugation of Higgs fields is defined as $\tilde{\phi}_{L(R)} = i\tau^2\phi^*_{L(R)}$. In the third line of Eq. (2.3), M_T and M_B are isosinglet VLQ mass parameters that we take as real numbers.

The first two terms of Eq. (2.4) are the kinetic terms of the Higgs doublet where the definition of the covariant derivatives is written in Eq. (2.8). The third term is the Higgs potential, which is shown in Eq. (2.6), containing the mass terms and quartic interactions of the Higgs doublet, including the interaction between ϕ_L and ϕ_R. Later ϕ_R and ϕ_L acquire nonzero vevs denoted as v_R and v_L that break $\mathrm{SU(2)_R}$ and $\mathrm{SU(2)_L}$, respectively, and satisfy $v_R \gg v_L$.

3. Quark sector and Yukawa interactions

In this section, we derive the kinetic terms of quark doublet and isosinglet VLQs, Yukawa interactions, and mass terms of isosinglet VLQs that are contained in Eq. (2.3) with the $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$ symmetric Lagrangian. After the $\mathrm{SU(2)_R}$ Higgs doublet acquires nonzero vev, we obtain the Lagrangian, which is invariant under SM gauge symmetry. Furthermore, the SM gauge group is broken into $\mathrm{U(1)_{em}}$ after the $\mathrm{SU(2)_L}$ Higgs doublet acquires nonzero vev. Finally, we obtain the masses of the top and bottom quark, and those of their heavy partners, Z, Z', h, and H. FCNC and the Cabibbo–Kobayashi–Maskawa (CKM) matrix are also generated.

3.1. $\mathbf{SU(2)_R \times U(1)_{Y'} \to U(1)_Y}$

In this stage, the neutral scalar component of the $\mathrm{SU(2)_R}$ Higgs doublet acquires nonzero vev and is expanded around the vev as follows:

$$\phi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\chi_R^+ \\ v_R + h_R + i\chi_R^3 \end{pmatrix}, \tag{3.1}$$

where v_R is the nonzero vev, h_R is the neutral CP-even state, and χ_R^3 is the neutral CP-odd state. The charged component is denoted as $\chi_R^+ = \frac{1}{\sqrt{2}}(\chi_R^1 + i\chi_R^2)$. In addition, we rotate the gauge fields with the following transformation:

$$\begin{pmatrix} B'_\mu \\ W^3_{R\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_R & -\sin\theta_R \\ \sin\theta_R & \cos\theta_R \end{pmatrix} \begin{pmatrix} B_\mu \\ Z_{R\mu} \end{pmatrix}, \tag{3.2}$$

where the mixing angle,

$$\sin\theta_R = \frac{g_1'}{\sqrt{g_R^2 + g_1'^2}}, \qquad \cos\theta_R = \frac{g_R}{\sqrt{g_R^2 + g_1'^2}}. \tag{3.3}$$

We also define the SM $\mathrm{U(1)_Y}$ gauge coupling as,

$$g' = g_1' \cos\theta_R = g_R \sin\theta_R. \tag{3.4}$$

After this spontaneous symmetry breaking, the Lagrangian in Eq. (2.3) becomes

$$\begin{aligned}
\mathcal{L}_q = {} & \overline{q_L^i} i\gamma^\mu D_{\mathrm{SM}\mu} q_L^i + \overline{T_L} i\gamma^\mu D_{\mathrm{SM}\mu} T_L + \overline{B_L} i\gamma^\mu D_{\mathrm{SM}\mu} B_L \\
& + \overline{u_R^i} i\gamma^\mu D_{\mathrm{SM}\mu} u_R^i + \overline{d_R^i} i\gamma^\mu D_{\mathrm{SM}\mu} d_R^i + \overline{T_R} i\gamma^\mu D_{\mathrm{SM}\mu} T_R \\
& + \overline{B_R} i\gamma^\mu D_{\mathrm{SM}\mu} B_R - \frac{g_R}{\sqrt{2}} \overline{u_R^i} \gamma^\mu d_R^i W_{R\mu}^+ - \mathrm{h.c.} \\
& + g' \tan\theta_R \left(\overline{q_L^i} \gamma^\mu Y_q q_L^i + \frac{2}{3} \overline{T_L} \gamma^\mu T_L - \frac{1}{3} \overline{B_L} \gamma^\mu B_L \right) Z_{R\mu} \\
& - \left\{ \frac{g_R}{2\cos\theta_R} \left(\overline{u_R^i} \gamma^\mu u_R^i - \overline{d_R^i} \gamma^\mu d_R^i \right) - g' \tan\theta_R \left(\frac{2}{3} \left(\overline{u_R^i} \gamma^\mu u_R^i + \overline{T_R} \gamma^\mu T_R \right) \right.\right. \\
& \left.\left. - \frac{1}{3} \left(\overline{d_R^i} \gamma^\mu d_R^i + \overline{B_R} \gamma^\mu B_R \right) \right) \right\} Z_{R\mu} \\
& - Y_{u_L}^3 \overline{q_L^3} \tilde{\phi}_L T_R - Y_{u_R}^3 \frac{v_R}{\sqrt{2}} \overline{T_L} u_R^3 - \overline{T_L} M_T T_R - \mathrm{h.c.} \\
& - Y_{u_R}^3 \overline{T_L} \left(\frac{1}{\sqrt{2}} u_R^3 \left(h_R + i\chi_R^3 \right) - d_R^3 \chi_R^+ \right) - \mathrm{h.c.} \\
& - \overline{q_L^i} y_{d_L}^i \phi_L B_R - \overline{B_L} y_{d_R}^{i*} \frac{v_R}{\sqrt{2}} d_R^i - \overline{B_L} M_B B_R - \mathrm{h.c.} \\
& - \overline{B_L} y_{d_R}^{i*} \left(\frac{1}{\sqrt{2}} d_R^i \left(h_R - i\chi_R^3 \right) + u_R^i \chi_R^- \right) - \mathrm{h.c.},
\end{aligned} \tag{3.5}$$

where $i \in \{1, 2, 3\}$ is the family index, and the SM covariant derivatives have the following expressions:

$$D_{\mathrm{SM}\mu} q_L^i = \left(\partial_\mu + i g_L \frac{\tau^a}{2} W_{L\mu}^a + i g' Y_{q_L} B_\mu \right) q_L^i, \tag{3.6}$$

$$D_{\mathrm{SM}\mu} f_u = \left(\partial_\mu + \frac{2}{3} i g' B_\mu \right) f_u, \tag{3.7}$$

$$D_{\mathrm{SM}\mu} f_d = \left(\partial_\mu - \frac{1}{3} i g' B_\mu \right) f_d, \tag{3.8}$$

where $f_u \in \left\{ u_R^i, T_{L,R} \right\}$ and $f_d \in \left\{ d_R^i, B_{L,R} \right\}$. At this stage, the $\mathrm{U(1)_Y}$ hypercharge can be obtained as following Eq. (2.1), $Y = I_R^3 + Y'$. In Eqs. (3.7) and (3.8), we write the $\mathrm{U(1)_Y}$ hypercharge of the corresponding fields explicitly. Next, we follow several steps to reach the Lagrangian invariant under $\mathrm{SU(2)_L \times U(1)_Y}$ gauge symmetry.

- **Step 1**: Rotate d_R^i by the following transformation:

$$d_R^i = (V_{d_R})^{ij} (d_R')^j, \tag{3.9}$$

where V_{d_R} is a 3×3 unitary matrix, which is related to Yukawa coupling parameterization as shown in Eq. (A.3),

$$y_{d_R} = \begin{pmatrix} \sin\theta_R^d \sin\phi_R^d e^{i\alpha_{d_R}^1} \\ \sin\theta_R^d \cos\phi_R^d e^{i\alpha_{d_R}^2} \\ \cos\theta_R^d e^{i\alpha_{d_R}^3} \end{pmatrix} Y_{d_R}^3 = \mathbf{e}_{R_3}^d Y_{d_R}^3, \tag{3.10}$$

$$V_{d_R} = \begin{pmatrix} \mathbf{e}_{R_1}^d & \mathbf{e}_{R_2}^d & \mathbf{e}_{R_3}^d \end{pmatrix}. \tag{3.11}$$

If we multiply Eq. (3.11) by the Hermitian conjugate of Eq. (3.10) from the left, it can be shown that the terms in Eq. (3.5) which are proportional to the complex vector $y_{d_R}^*$ are replaced by a real positive number $Y_{d_R}^3$ multiplied with δ^{3j}. Then, we can extract the mass terms from the Lagrangian as follows:

$$\begin{aligned} \mathcal{L}_q \supset \mathcal{L}_{\text{mass}} = &-\overline{T_L} \begin{pmatrix} Y_{u_R}^3 \frac{v_R}{\sqrt{2}} & M_T \end{pmatrix} \begin{pmatrix} u_R^3 \\ T_R \end{pmatrix} - \text{h.c.} \\ &- \overline{B_L} \begin{pmatrix} Y_{d_R}^3 \frac{v_R}{\sqrt{2}} & M_B \end{pmatrix} \begin{pmatrix} \left(d'_R\right)^3 \\ B_R \end{pmatrix} - \text{h.c.}. \end{aligned} \tag{3.12}$$

After doing transformation in Eq. (3.9), V_{d_R} appears as a CKM-like matrix in the right-handed charged current term,

$$\mathcal{L}_q \supset \mathcal{L}_{\text{RCC}} = -\frac{g_R}{\sqrt{2}} \sum_{i,j=1}^{3} \overline{u_R^i} \gamma^\mu \left(V_{d_R}\right)^{ij} \left(d'_R\right)^j W_{R\mu}^+ - \text{h.c.}. \tag{3.13}$$

Equation (3.12) shows that the first and second families are decoupled from the Yukawa coupling. This leads to the fact that we have the freedom to do another U(2) transformation for the right-handed quark fields. This rotation should keep the third family unchanged.

- **Step 2:** Rotate u_R^i and $(d'_R)^i$ by the following transformations:

$$u_R^i = \sum_{j=1}^{3} \left(\widetilde{U}_{u_R}\right)^{ij} \left(\tilde{u}_R\right)^j, \tag{3.14}$$

$$\left(d'_R\right)^i = \sum_{j=1}^{3} \left(\widetilde{W}_{d_R}\right)^{ij} \left(\tilde{d}'_R\right)^j, \tag{3.15}$$

where $\widetilde{U}_{u_R}$ and $\widetilde{W}_{d_R}$ are 3×3 unitary matrices and written in matrix form as follows:

$$\widetilde{U}_{u_R} = \begin{pmatrix} \multicolumn{2}{c}{\multirow{2}{*}{U_{u_R}}} & 0 \\ & & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.16}$$

$$\widetilde{W}_{d_R} = \begin{pmatrix} \multicolumn{2}{c}{\multirow{2}{*}{W_{d_R}}} & 0 \\ & & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.17}$$

where U_{u_R} and W_{d_R} are 2×2 unitary matrices that rotate (u_R^1, u_R^2) and $((d'_R)^1, (d'_R)^2)$, respectively. By applying the transformations in Eqs. (3.14) and (3.15) to the charged current in Eq. (3.13), we further define

$$\widetilde{V}_{d_R} = \widetilde{U}_{u_R}^\dagger V_{d_R} \widetilde{W}_{d_R}. \tag{3.18}$$

As shown in Eq. (B.6), by choosing $\widetilde{U}_{u_R}$ and $\widetilde{W}_{d_R}$ properly, the unphysical phases and angles in V_{d_R} are removed and $\widetilde{V}_{d_R}$ has the following matrix form:

$$\widetilde{V}_{d_R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta^d_R & \sin\theta^d_R e^{i\frac{\alpha^3_{d_R}}{2}} \\ 0 & -\sin\theta^d_R e^{i\frac{\alpha^3_{d_R}}{2}} & \cos\theta^d_R e^{i\alpha^3_{d_R}} \end{pmatrix}. \tag{3.19}$$

The details of the parameterization and the procedure for the removal of unphysical phases and angles of V_{d_R} are shown in Appendix B.

- **Step 3**: Rotate $(\tilde{u}_R)^\alpha$ and $(\tilde{d}'_R)^\alpha$ by the following transformations:

$$(\tilde{u}_R)^\alpha = \sum_{\beta=1}^{4} \left(\widetilde{W}_{T_R}\right)^{\alpha\beta} \left(\tilde{u}'_R\right)^\beta , \tag{3.20}$$

$$\left(\tilde{d}'_R\right)^\alpha = \sum_{\beta=1}^{4} \left(\widetilde{W}_{B_R}\right)^{\alpha\beta} \left(\tilde{d}''_R\right)^\beta , \tag{3.21}$$

where $\alpha = \{1, 2, 3, 4\}$, $(\tilde{u}_R)^4 = T_R$, and $(\tilde{d}'_R)^4 = B_R$. The 4×4 unitary matrices $\widetilde{W}_{T_R}$ and $\widetilde{W}_{B_R}$ are expressed as follows:

$$\widetilde{W}_{T_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & W_{T_R} \end{pmatrix}, \tag{3.22}$$

$$\widetilde{W}_{B_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & W_{B_R} \end{pmatrix}, \tag{3.23}$$

where I_2 and 0_2 are the 2×2 identity matrix and zero matrix, respectively. The 2×2 submatrices W_{T_R} and W_{B_R} rotate $((\tilde{u}_R)^3, (\tilde{u}_R)^4)$ and $((\tilde{d}'_R)^3, (\tilde{d}'_R)^4)$, respectively, by the following expressions:

$$(\tilde{u}_R)^i = \sum_{j=3}^{4} (W_{T_R})^{ij} \left(\tilde{u}'_R\right)^j , \tag{3.24}$$

$$\left(\tilde{d}'_R\right)^i = \sum_{j=3}^{4} (W_{B_R})^{ij} \left(\tilde{d}''_R\right)^j , \tag{3.25}$$

where $i \in \{3, 4\}$. The explicit matrix forms of W_{T_R} and W_{B_R} are as follows:

$$W_{T_R} = \begin{pmatrix} \cos\theta_{T_R} & \sin\theta_{T_R} \\ -\sin\theta_{T_R} & \cos\theta_{T_R} \end{pmatrix}, \tag{3.26}$$

$$W_{B_R} = \begin{pmatrix} \cos\theta_{B_R} & \sin\theta_{B_R} \\ -\sin\theta_{B_R} & \cos\theta_{B_R} \end{pmatrix}, \tag{3.27}$$

where the mixing angles have the following expressions:

$$\cos\theta_{T_R} = \frac{M_T}{m_{u_4}}, \quad \sin\theta_{T_R} = \frac{Y^3_{u_R}}{m_{u_4}} \frac{v_R}{\sqrt{2}}, \quad \cos\theta_{B_R} = \frac{M_B}{m_{d_4}}, \quad \sin\theta_{B_R} = \frac{Y^3_{d_R}}{m_{d_4}} \frac{v_R}{\sqrt{2}},$$

$$m_{u_4} = \sqrt{\frac{\left(Y^3_{u_R}\right)^2 v_R^2}{2} + M_T^2}, \qquad m_{d_4} = \sqrt{\frac{\left(Y^3_{d_R}\right)^2 v_R^2}{2} + M_B^2}. \tag{3.28}$$

By using Eqs. (3.24) and (3.25), the mass terms in Eq. (3.12) transform into

$$\mathcal{L}_q \supset \mathcal{L}_{\text{mass}} = -m_{u_4}\overline{T_L}\left(\tilde{u}'_R\right)^4 - m_{d_4}\overline{B_L}\left(\tilde{d}''_R\right)^4 - \text{h.c.}. \tag{3.29}$$

The right-handed charged current in Eq. (3.13) becomes

$$\mathcal{L}_q \supset \mathcal{L}_{\text{RCC}} = -\frac{g_R}{\sqrt{2}}\sum_{\alpha,\beta=1}^{4}\overline{\left(\tilde{u}'_R\right)^\alpha}\gamma^\mu\left(V_R^{\text{CKM}}\right)^{\alpha\beta}\left(\tilde{d}''_R\right)^\beta W^+_{R\mu} - \text{h.c.}, \tag{3.30}$$

where

$$\left(V_R^{\text{CKM}}\right)^{\alpha\beta} = \sum_{i,j=1}^{3}\left(\widetilde{W}^\dagger_{T_R}\right)^{\alpha i}\left(\tilde{V}_{d_R}\right)^{ij}\left(\widetilde{W}_{B_R}\right)^{j\beta}; \quad \alpha,\beta \in \{1,2,3,4\} \tag{3.31}$$

is a 4×4 "intermediate" right-handed CKM-like matrix. We call this matrix intermediate because it is not the final expression of the right-handed CKM-like matrix. The explicit matrix form of V_R^{CKM} is shown in Eq. (D.1).

In addition, we define the right-handed weak isospin current in Eq. (3.5) as

$$j^\mu_{3R} \equiv \overline{u^i_R}\gamma^\mu u^i_R - \overline{d^i_R}\gamma^\mu d^i_R. \tag{3.32}$$

Following steps 1–3, Eq. (3.32) transforms into

$$\begin{aligned} j^\mu_{3R} = &\sum_{i=1}^{2}\overline{\left(\tilde{u}'_R\right)^i}\gamma^\mu\left(\tilde{u}'_R\right)^i + \sum_{j,k=3}^{4}\overline{\left(\tilde{u}'_R\right)^j}\gamma^\mu\left(Z_{T_R}\right)^{jk}\left(\tilde{u}'_R\right)^k \\ &- \sum_{i=1}^{2}\overline{\left(\tilde{d}''_R\right)^i}\gamma^\mu\left(\tilde{d}''_R\right)^i - \sum_{j,k=3}^{4}\overline{\left(\tilde{d}''_R\right)^j}\gamma^\mu\left(Z_{B_R}\right)^{jk}\left(\tilde{d}''_R\right)^k, \end{aligned} \tag{3.33}$$

where the tree-level FCNC couplings are generated with the following definitions:

$$\left(Z_{T_R}\right)^{jk} \equiv \left(W^\dagger_{T_R}\right)^{j3}\left(W_{T_R}\right)^{3k}, \tag{3.34}$$

$$\left(Z_{B_R}\right)^{jk} \equiv \left(W^\dagger_{B_R}\right)^{j3}\left(W_{B_R}\right)^{3k}, \tag{3.35}$$

with $j,k \in \{3,4\}$. Furthermore, Eqs. (3.34) and (3.35) can be expressed explicitly in 2×2 matrix form as follows:

$$Z_{T_R} = \begin{pmatrix} \cos^2\theta_{T_R} & \sin\theta_{T_R}\cos\theta_{T_R} \\ \sin\theta_{T_R}\cos\theta_{T_R} & \sin^2\theta_{T_R} \end{pmatrix}, \tag{3.36}$$

$$Z_{B_R} = \begin{pmatrix} \cos^2\theta_{B_R} & \sin\theta_{B_R}\cos\theta_{B_R} \\ \sin\theta_{B_R}\cos\theta_{B_R} & \sin^2\theta_{B_R} \end{pmatrix}. \tag{3.37}$$

These tree-level FCNC couplings are generated due to mixing between the third flavor of up and down quarks and their corresponding isosinglet right-handed VLQs.

After following steps 1–3, the Lagrangian in Eq. (3.5) becomes

$$\begin{aligned}
\mathcal{L}_q =& \overline{q_L^i} i\gamma^\mu D_{\mathrm{SM}\mu} q_L^i + \overline{T_L} i\gamma^\mu D_{\mathrm{SM}\mu} T_L + \overline{B_L} i\gamma^\mu D_{\mathrm{SM}\mu} B_L \\
&+ \overline{\left(\tilde{u}'_R\right)^\alpha} i\gamma^\mu D_{\mathrm{SM}\mu} \left(\tilde{u}'_R\right)^\alpha + \overline{\left(\tilde{d}''_R\right)^\alpha} i\gamma^\mu D_{\mathrm{SM}\mu} \left(\tilde{d}''_R\right)^\alpha \\
&- \frac{g_R}{\sqrt{2}} \sum_{\alpha,\beta=1}^{4} \overline{\left(\tilde{u}'_R\right)^\alpha} \gamma^\mu \left(V_R^{\mathrm{CKM}}\right)^{\alpha\beta} \left(\tilde{d}''_R\right)^\beta W_{R\mu}^+ - \mathrm{h.c.} \\
&+ g' \tan\theta_R \left(\overline{q_L^i} \gamma^\mu Y_q q_L^i + \frac{2}{3} \overline{T_L} \gamma^\mu T_L - \frac{1}{3} \overline{B_L} \gamma^\mu B_L \right) Z_{R\mu} \\
&- \left\{ \frac{g_R}{2\cos\theta_R} \left(j_{3_R}^\mu \right) - g' \tan\theta_R \left(\frac{2}{3} \overline{\left(\tilde{u}'_R\right)^\alpha} \gamma^\mu \left(\tilde{u}'_R\right)^\alpha - \frac{1}{3} \overline{\left(\tilde{d}''_R\right)^\alpha} \gamma^\mu \left(\tilde{d}''_R\right)^\alpha \right) \right\} Z_{R\mu} \\
&- Y_{u_L}^3 \overline{q_L^3} \tilde{\phi}_L \left(\sum_{j=3}^{4} \left(W_{T_R}\right)^{4j} \left(\tilde{u}'_R\right)^j \right) - m_{u_4} \overline{T_L} \left(\tilde{u}'_R\right)^4 - \mathrm{h.c.} \\
&- \frac{m_{u_4}}{v_R} \overline{T_L} \left[\left(\sum_{j=3}^{4} \left(Z_{T_R}\right)^{4j} \left(\tilde{u}'_R\right)^j \right) \left(h_R + i\chi_R^3\right) - \sqrt{2} \left(\sum_{\beta=2}^{4} \left(V_R^{\mathrm{CKM}}\right)^{4\beta} \left(\tilde{d}''_R\right)^\beta \right) \chi_R^+ \right] - \mathrm{h.c.} \\
&- \overline{q_L^i} y_{d_L}^i \phi_L \left(\sum_{j=3}^{4} \left(W_{B_R}\right)^{4j} \left(\tilde{d}''_R\right)^j \right) - m_{d_4} \overline{B_L} \left(\tilde{d}''_R\right)^4 - \mathrm{h.c.} \\
&- \frac{m_{d_4}}{v_R} \overline{B_L} \left[\left(\sum_{j=3}^{4} \left(Z_{B_R}\right)^{4j} \left(\tilde{d}''_R\right)^j \right) \left(h_R - i\chi_R^3\right) + \sqrt{2} \left(\sum_{\beta=2}^{4} \left(V_R^{\mathrm{CKM}\dagger}\right)^{4\beta} \left(\tilde{u}'_R\right)^\beta \right) \chi_R^- \right] - \mathrm{h.c.},
\end{aligned} \tag{3.38}$$

where $i = \{1, 2, 3\}$, $\alpha = \{1, 2, 3, 4\}$, and the definitions of W_{T_R}, W_{B_R}, m_{u_4}, m_{d_4}, V_R^{CKM}, Z_{T_R}, and Z_{B_R} are written in Eqs. (3.26), (3.27), (3.28), (D.1), (3.36), and (3.37), respectively. One can show that the Lagrangian in Eq. (3.38) is invariant under $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ gauge symmetry.

3.2. $\mathbf{SU(2)_L \times U(1)_Y \to U(1)}_{em}$

In this stage, the neutral scalar component of the $\mathrm{SU(2)_L}$ Higgs doublet acquires nonzero vev and is expanded around vev's as follows:

$$\phi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\chi_L^+ \\ v_L + h_L + i\chi_L^3 \end{pmatrix}, \tag{3.39}$$

where v_L is the nonzero vev, h_L is the neutral CP-even state, and χ_L^3 is the neutral CP-odd state. The charged component, $\chi_L^+ = \frac{1}{\sqrt{2}}(\chi_L^1 + i\chi_L^2)$. In addition, we rotate the gauge fields with the following transformation:

$$\begin{pmatrix} B_\mu \\ W_{L\mu}^3 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_{L\mu} \end{pmatrix}, \tag{3.40}$$

where the mixing angles are defined as

$$\cos\theta_W = \frac{g_L}{\sqrt{g_L^2 + g'^2}}, \quad \sin\theta_W = \frac{g'}{\sqrt{g_L^2 + g'^2}}. \tag{3.41}$$

We also define the electromagnetic $U(1)_{em}$ gauge coupling as

$$e = g' \cos\theta_W = g_L \sin\theta_W . \tag{3.42}$$

After this breaking, the Lagrangian in Eq. (3.38) becomes

$$\begin{aligned}
\mathcal{L}_q = {} & \overline{u^i_L} i\gamma^\mu D_{\mathrm{em}\mu} u^i_L + \overline{T_L} i\gamma^\mu D_{\mathrm{em}\mu} T_L + \overline{d^i_L} i\gamma^\mu D_{\mathrm{em}\mu} d^i_L + \overline{B_L} i\gamma^\mu D_{\mathrm{em}\mu} B_L \\
& + \overline{(\tilde{u}'_R)}^\alpha i\gamma^\mu D_{\mathrm{em}\mu} \left(\tilde{u}'_R\right)^\alpha + \overline{\left(\tilde{d}''_R\right)}^\alpha i\gamma^\mu D_{\mathrm{em}\mu} \left(\tilde{d}''_R\right)^\alpha - \frac{g_L}{\sqrt{2}} \overline{u^i_L} \gamma^\mu d^i_L W^+_{L\mu} - \mathrm{h.c.} \\
& - \left(\frac{g_L}{2\cos\theta_W} \left(j^\mu_{3L}\right) - e\tan\theta_W \left(j^\mu_{\mathrm{em}}\right) \right) Z_{L\mu} \\
& - \frac{g_R}{\sqrt{2}} \sum_{\alpha,\beta=1}^{4} \overline{(\tilde{u}'_R)}^\alpha \gamma^\mu \left(V_R^{\mathrm{CKM}}\right)^{\alpha\beta} \left(\tilde{d}''_R\right)^\beta W^+_{R\mu} - \mathrm{h.c.} \\
& - \left\{ \frac{g_R}{2\cos\theta_R} \left(j^\mu_{3_R}\right) - g' \tan\theta_R \left(\left(j^\mu_{\mathrm{em}}\right) - \frac{1}{2} \left(j^\mu_{3_L}\right) \right) \right\} Z_{R\mu} \\
& - Y^3_{u_L} \frac{v_L}{\sqrt{2}} \overline{u^3_L} \left(\sum_{j=3}^{4} (W_{T_R})^{4j} \left(\tilde{u}'_R\right)^j \right) - m_{u_4} \overline{T_L} \left(\tilde{u}'_R\right)^4 - \mathrm{h.c.} \\
& - Y^3_{u_L} \left(\frac{1}{\sqrt{2}} \overline{u^3_L} \left(\sum_{j=3}^{4} (W_{T_R})^{4j} \left(\tilde{u}'_R\right)^j \right) \left(h_L - i\chi^3_L\right) - \overline{d^3_L} \left(\sum_{j=3}^{4} (W_{T_R})^{4j} \left(\tilde{u}'_R\right)^j \right) \chi^-_L \right) - \mathrm{h.c.} \\
& - \frac{m_{u_4}}{v_R} \overline{T_L} \left[\left(\sum_{j=3}^{4} (Z_{T_R})^{4j} \left(\tilde{u}'_R\right)^j \right) \left(h_R + i\chi^3_R\right) - \sqrt{2} \left(\sum_{\beta=2}^{4} \left(V_R^{\mathrm{CKM}}\right)^{4\beta} \left(\tilde{d}''_R\right)^\beta \right) \chi^+_R \right] - \mathrm{h.c.} \\
& - y^i_{d_L} \frac{v_L}{\sqrt{2}} \overline{d^i_L} \left(\sum_{j=3}^{4} (W_{B_R})^{4j} \left(\tilde{d}''_R\right)^j \right) - m_{d_4} \overline{B_L} \left(\tilde{d}''_R\right)^4 - \mathrm{h.c.} \\
& - y^i_{d_L} \left(\frac{1}{\sqrt{2}} \overline{d^i_L} \left(\sum_{j=3}^{4} (W_{B_R})^{4j} \left(\tilde{d}''_R\right)^j \right) \left(h_L + i\chi^3_L\right) + \overline{u^i_L} \left(\sum_{j=3}^{4} (W_{B_R})^{4j} \left(\tilde{d}''_R\right)^j \right) \chi^+_L \right) - \mathrm{h.c.} \\
& - \frac{m_{d_4}}{v_R} \overline{B_L} \left[\left(\sum_{j=3}^{4} (Z_{B_R})^{4j} \left(\tilde{d}''_R\right)^j \right) \left(h_R - i\chi^3_R\right) + \sqrt{2} \left(\sum_{\beta=2}^{4} \left(V_R^{\mathrm{CKM}\dagger}\right)^{4\beta} \left(\tilde{u}'_R\right)^\beta \right) \chi^-_R \right] - \mathrm{h.c.},
\end{aligned} \tag{3.43}$$

where the covariant derivatives are

$$D_{\mathrm{em}\mu} f'_u = \left(\partial_\mu + \frac{2}{3} ieA_\mu \right) f'_u, \tag{3.44}$$

$$D_{\mathrm{em}\mu} f'_d = \left(\partial_\mu - \frac{1}{3} ieA_\mu \right) f'_d. \tag{3.45}$$

The left-handed weak isospin current and electromagnetic current are

$$j^\mu_{3L} = \overline{u^i_L} \gamma^\mu u^i_L - \overline{d^i_L} \gamma^\mu d^i_L, \tag{3.46}$$

$$j^{\mu}_{\text{em}} = \frac{2}{3}\left(\overline{u^i_L}\gamma^{\mu}u^i_L + \overline{T_L}\gamma^{\mu}T_L + \overline{(\tilde{u}'_R)}^{\alpha}\gamma^{\mu}\left(\tilde{u}'_R\right)^{\alpha}\right)$$
$$-\frac{1}{3}\left(\overline{d^i_L}\gamma^{\mu}d^i_L + \overline{B_L}\gamma^{\mu}B_L + \overline{\left(\tilde{d}''_R\right)}^{\alpha}\gamma^{\mu}\left(\tilde{d}''_R\right)^{\alpha}\right), \tag{3.47}$$

where $f'_u \in \{u^i_L, (\tilde{u}'_R)^{\alpha}, T_L\}$, $f'_d \in \left\{d^i_L, (\tilde{d}''_R)^{\alpha}, B_L\right\}$, $i \in \{1, 2, 3\}$, $\alpha \in \{1, 2, 3, 4\}$, and the right-handed weak isospin current j^{μ}_{3R} is written in Eq. (3.33). Our main goal is to obtain the mass eigenvalues of the top and bottom quarks and their heavy partners. The following steps outline our approach (the number of counting steps continues from the previous subsection).

- **Step 4:** Rotate d^i_L by the following transformation:

$$d^i_L = (V_{d_L})^{ij}\left(d'_L\right)^j, \tag{3.48}$$

where V_{d_L} is a 3×3 unitary matrix, associated with the parameterization of Yukawa couplings as demonstrated in Eq. (A.3),

$$y_{d_L} = \begin{pmatrix} \sin\theta^d_L \sin\phi^d_L e^{i\alpha^1_{d_L}} \\ \sin\theta^d_L \cos\phi^d_L e^{i\alpha^2_{d_L}} \\ \cos\theta^d_L e^{i\alpha^3_{d_L}} \end{pmatrix} Y^3_{d_L} = \mathbf{e}^d_{L_3} Y^3_{d_L}, \tag{3.49}$$

$$V_{d_L} = \left(\mathbf{e}^d_{L_1}\ \mathbf{e}^d_{L_2}\ \mathbf{e}^d_{L_3}\right). \tag{3.50}$$

If we multiply Eq. (3.49) by the Hermitian conjugate of Eq. (3.50) from the left, it can be shown that the terms in Eq. (3.43) that are proportional to the complex vector y_{d_L} are replaced by the product of a real positive number $Y^3_{d_L}$ and δ^{j3}. The mass terms can be extracted from the Lagrangian and written as follows:

$$\mathcal{L}_q \supset \mathcal{L}_{\text{mass}} = -\begin{pmatrix}\overline{u^3_L} & \overline{T_L}\end{pmatrix}\begin{pmatrix} Y^3_{u_L}\frac{v_L}{\sqrt{2}}(W_{T_R})^{43} & Y^3_{u_L}\frac{v_L}{\sqrt{2}}(W_{T_R})^{44} \\ 0 & m_{u_4}\end{pmatrix}\begin{pmatrix}(\tilde{u}'_R)^3 \\ (\tilde{u}'_R)^4\end{pmatrix} - \text{h.c.}$$
$$-\begin{pmatrix}\overline{(d'_L)^3} & \overline{B_L}\end{pmatrix}\begin{pmatrix} Y^3_{d_L}\frac{v_L}{\sqrt{2}}(W_{B_R})^{43} & Y^3_{d_L}\frac{v_L}{\sqrt{2}}(W_{B_R})^{44} \\ 0 & m_{d_4}\end{pmatrix}\begin{pmatrix}\left(\tilde{d}''_R\right)^3 \\ \left(\tilde{d}''_R\right)^4\end{pmatrix} - \text{h.c.}. \tag{3.51}$$

Additionally, an important outcome of the transformation in Eq. (3.48) is that V_{d_L} appears as a CKM-like matrix in the left-handed charged current term,

$$\mathcal{L}_q \supset \mathcal{L}_{\text{LCC}} = -\frac{g_L}{\sqrt{2}}\sum_{i,j=1}^{3}\overline{u^i_L}\gamma^{\mu}(V_{d_L})^{ij}\left(d'_L\right)^j W^+_{L\mu} - \text{h.c.}. \tag{3.52}$$

From Eq. (3.51), we have freedom to apply another U(2) transformation to the left-handed quark fields while keeping the third family unchanged.

- **Step 5:** Rotate u^i_L and $(d'_L)^i$ by the following transformations:

$$u^i_L = \sum_{j=1}^{3}\left(\widetilde{U}_{u_L}\right)^{ij}(\tilde{u}_L)^j, \tag{3.53}$$

$$\left(d'_L\right)^i = \sum_{j=1}^{3}\left(\widetilde{W}_{d_L}\right)^{ij}\left(\tilde{d}'_L\right)^j, \tag{3.54}$$

where $\widetilde{U}_{u_L}$ and $\widetilde{W}_{d_L}$ are 3×3 unitary matrices and written in the matrix form as follows:

$$\widetilde{U}_{u_L} = \begin{pmatrix} \multicolumn{2}{c}{\multirow{2}{*}{U_{u_L}}} & 0 \\ & & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.55}$$

$$\widetilde{W}_{d_L} = \begin{pmatrix} \multicolumn{2}{c}{\multirow{2}{*}{W_{d_L}}} & 0 \\ & & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.56}$$

where U_{u_L} and W_{d_L} are 2×2 unitary matrices which rotate (u_L^1, u_L^2) and $((d'_L)^1, (d'_L)^2)$, respectively. By applying the transformations in Eqs. (3.53) and (3.54) to the charged current in Eq. (3.52), we further define

$$\widetilde{V}_{d_L} = \widetilde{U}_{u_L}^{\dagger} V_{d_L} \widetilde{W}_{d_L}. \tag{3.57}$$

By properly choosing $\widetilde{U}_{u_L}$ and $\widetilde{W}_{d_L}$, the unphysical phases and angles in V_{d_L} are eliminated, resulting in $\widetilde{V}_{d_L}$, which has the same matrix form as Eq. (3.19), with the R index replaced by L.

- **Step 6:** Rotate $(\tilde{u}_L)^\alpha$, $(\tilde{u}'_R)^\alpha$, $(\tilde{d}'_L)^\alpha$, and $(\tilde{d}''_R)^\alpha$ into the mass basis by the following transformations:

$$(\tilde{u}_L)^\alpha = \sum_{\beta=1}^{4} \left(\widetilde{K}_{T_L}\right)^{\alpha\beta} \left(u_L^m\right)^\beta, \tag{3.58}$$

$$\left(\tilde{u}'_R\right)^\alpha = \sum_{\beta=1}^{4} \left(\widetilde{K}_{T_R}\right)^{\alpha\beta} \left(u_R^m\right)^\beta, \tag{3.59}$$

$$\left(\tilde{d}'_L\right)^\alpha = \sum_{\beta=1}^{4} \left(\widetilde{K}_{B_L}\right)^{\alpha\beta} \left(d_L^m\right)^\beta, \tag{3.60}$$

$$\left(\tilde{d}''_R\right)^\alpha = \sum_{\beta=1}^{4} \left(\widetilde{K}_{B_R}\right)^{\alpha\beta} \left(d_R^m\right)^\beta, \tag{3.61}$$

where $\alpha \in \{1, 2, 3, 4\}$, $(\tilde{u}_L)^4 = T_L$, and $(\tilde{d}'_L)^4 = B_L$. The 4×4 unitary matrices $\widetilde{K}_{T_L}$, $\widetilde{K}_{T_R}$, $\widetilde{K}_{B_L}$, and $\widetilde{K}_{B_R}$ are expressed as follows:

$$\widetilde{K}_{T_L} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{T_L} \end{pmatrix}, \tag{3.62}$$

$$\widetilde{K}_{T_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{T_R} \end{pmatrix}, \tag{3.63}$$

$$\widetilde{K}_{B_L} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{B_L} \end{pmatrix}, \tag{3.64}$$

$$\widetilde{K}_{B_R} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & K_{B_R} \end{pmatrix}, \tag{3.65}$$

where I_2 and 0_2 are the 2×2 identity matrix and zero matrix, respectively. The 2×2 unitary submatrices K_{T_L}, K_{T_R}, K_{B_L}, and K_{B_R} rotate $((\tilde{u}_L)^3, (\tilde{u}_L)^4)$, $((\tilde{u}'_R)^3, (\tilde{u}'_R)^4)$, $((\tilde{d}'_L)^3, (\tilde{d}'_L)^4)$, and $((\tilde{d}''_R)^3, (\tilde{d}''_R)^4)$ pairs, respectively, where the explicit forms are written in Eqs. (C.19), (C.20), (C.24), and (C.25).

We denote the top and bottom quarks as the third component of the fields in the mass basis, while the heavy top and bottom quarks are the fourth component. We can diagonalize the mass matrices in Eq. (3.51), which are defined as

$$\mathbb{M}_t \equiv \begin{pmatrix} Y^3_{u_L} \frac{v_L}{\sqrt{2}} (W_{T_R})^{43} & Y^3_{u_L} \frac{v_L}{\sqrt{2}} (W_{T_R})^{44} \\ 0 & m_{u_4} \end{pmatrix}, \tag{3.66}$$

$$\mathbb{M}_b \equiv \begin{pmatrix} Y^3_{d_L} \frac{v_L}{\sqrt{2}} (W_{B_R})^{43} & Y^3_{d_L} \frac{v_L}{\sqrt{2}} (W_{B_R})^{44} \\ 0 & m_{d_4} \end{pmatrix}, \tag{3.67}$$

by using the appropriate submatrices in Eqs. (3.58)–(3.61), resulting in:

$$K^\dagger_{T_L} \mathbb{M}_t K_{T_R} = \left(m_t^{\text{diag}} \right) = \text{diag}\,(m_t, m_{t'}), \tag{3.68}$$

$$K^\dagger_{B_L} \mathbb{M}_b K_{B_R} = \left(m_b^{\text{diag}} \right) = \text{diag}\,(m_b, m_{b'}). \tag{3.69}$$

From this diagonalization process, we obtain

$$m_{t(b)} = -\frac{\sqrt{M^2_{T(B)} + (m_{u(d)_R} - m_{u(d)_L})^2}}{2} + \frac{\sqrt{M^2_{T(B)} + (m_{u(d)_R} + m_{u(d)_L})^2}}{2}, \tag{3.70}$$

$$m_{t'(b')} = \frac{\sqrt{M^2_{T(B)} + (m_{u(d)_R} - m_{u(d)_L})^2}}{2} + \frac{\sqrt{M^2_{T(B)} + (m_{u(d)_R} + m_{u(d)_L})^2}}{2}, \tag{3.71}$$

where $m_{t(b)}$ and $m_{t'(b')}$ are the exact mass eigenvalues for the top(bottom) and heavy top(bottom), respectively. The definitions of m_{u_L}, m_{u_R}, m_{d_L}, and m_{d_R} are shown in Eqs. (C.16) and (C.23). The diagonalization procedure is explained in Appendix C. The mass eigenvalues for t and t' in Eqs. (3.70) and (3.71) agree with Eq. (10) of Ref. [31].

Moreover, the left-handed and right-handed charged currents in Eqs. (3.52) and (3.30) now become

$$\begin{aligned} \mathcal{L}_q \supset \mathcal{L}_{\text{CC}} &= \mathcal{L}_{\text{LCC}} + \mathcal{L}_{\text{RCC}} \\ &= -\frac{g_L}{\sqrt{2}} \sum_{\alpha,\beta=1}^{4} \overline{(u^m_L)^\alpha} \gamma^\mu \left(\mathcal{V}^{\text{CKM}}_L\right)^{\alpha\beta} \left(d^m_L\right)^\beta W^+_{L\mu} - \text{h.c.} \\ &\quad - \frac{g_R}{\sqrt{2}} \sum_{\alpha,\beta=1}^{4} \overline{(u^m_R)^\alpha} \gamma^\mu \left(\mathcal{V}^{\text{CKM}}_R\right)^{\alpha\beta} \left(d^m_R\right)^\beta W^+_{R\mu} - \text{h.c.}, \end{aligned} \tag{3.72}$$

where

$$\left(\mathcal{V}^{\text{CKM}}_L\right)^{\alpha\beta} = \sum_{i,j=1}^{3} \left(\widetilde{K}^\dagger_{T_L}\right)^{\alpha i} \left(\widetilde{V}_{d_L}\right)^{ij} \left(\widetilde{K}_{B_L}\right)^{j\beta}, \tag{3.73}$$

$$\left(\mathcal{V}^{\text{CKM}}_R\right)^{\alpha\beta} = \sum_{\rho,\eta=1}^{4} \left(\widetilde{K}^\dagger_{T_R}\right)^{\alpha\rho} \left(V^{\text{CKM}}_R\right)^{\rho\eta} \left(\widetilde{K}_{B_R}\right)^{\eta\beta} \tag{3.74}$$

are the left-handed and right-handed CKM-like matrices, respectively. The matrix forms are shown in Eqs. (D.3) and (D.5), respectively. However, there are some unphysical phases which can be eliminated from the left-handed and right-handed CKM-like matrices. We have the freedom to rephase the quark fields with the following transformations:

$$\left(u^m_{L(R)}\right)^\alpha = \left(\theta_{u_{L(R)}}\right)^\alpha \delta^{\alpha\beta} \left(\hat{u}^m_{L(R)}\right)^\beta, \tag{3.75}$$

$$\left(d^m_{L(R)}\right)^\alpha = \left(\theta_{d_{L(R)}}\right)^\alpha \delta^{\alpha\beta} \left(\hat{d}^m_{L(R)}\right)^\beta, \tag{3.76}$$

where

$$\theta_{u_{L(R)}} = \mathrm{diag}(e^{i\theta_{u_{L(R)1}}}, e^{i\theta_{u_{L(R)2}}}, e^{i\theta_{u_3}}, e^{i\theta_{u_4}}), \tag{3.77}$$

$$\theta_{d_{L(R)}} = \mathrm{diag}(e^{i\theta_{d_{L(R)1}}}, e^{i\theta_{d_{L(R)2}}}, e^{i\theta_{d_3}}, e^{i\theta_{d_4}}). \tag{3.78}$$

After rephasing the quark fields, the left-handed and right-handed CKM-like matrices become the final versions denoted as $\hat{\mathcal{V}}^{\mathrm{CKM}}_L$ and $\hat{\mathcal{V}}^{\mathrm{CKM}}_R$, whose matrix forms are as follows:

$$\hat{\mathcal{V}}^{\mathrm{CKM}}_L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta^d_L} & s_{\theta^d_L} c_{\phi_{B_L}} & -s_{\theta^d_L} s_{\phi_{B_L}} \\ 0 & -c_{\phi_{T_L}} s_{\theta^d_L} & c_{\phi_{T_L}} c_{\theta^d_L} c_{\phi_{B_L}} & -c_{\phi_{T_L}} c_{\theta^d_L} s_{\phi_{B_L}} \\ 0 & s_{\phi_{T_L}} s_{\theta^d_L} & -s_{\phi_{T_L}} c_{\theta^d_L} c_{\phi_{B_L}} & s_{\phi_{T_L}} c_{\theta^d_L} s_{\phi_{B_L}} \end{pmatrix}, \tag{3.79}$$

$$\hat{\mathcal{V}}^{\mathrm{CKM}}_R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta^d_R} & -s_{\theta^d_R} c_{\beta_{B_R}} e^{i\frac{\delta}{2}} & s_{\theta^d_R} s_{\beta_{B_R}} e^{i\frac{\delta}{2}} \\ 0 & c_{\beta_{T_R}} s_{\theta^d_R} e^{i\frac{\delta}{2}} & c_{\beta_{T_R}} c_{\theta^d_R} c_{\beta_{B_R}} e^{i\delta} & -c_{\beta_{T_R}} c_{\theta^d_R} s_{\beta_{B_R}} e^{i\delta} \\ 0 & -s_{\beta_{T_R}} s_{\theta^d_R} e^{i\frac{\delta}{2}} & -s_{\beta_{T_R}} c_{\theta^d_R} c_{\beta_{B_R}} e^{i\delta} & s_{\beta_{T_R}} c_{\theta^d_R} s_{\beta_{B_R}} e^{i\delta} \end{pmatrix}, \tag{3.80}$$

where

$$\begin{aligned} c_{\theta^d_L} &= \cos\theta^d_L, \quad s_{\theta^d_L} = \sin\theta^d_L, \quad c_{\phi_{T_L}} = \cos\phi_{T_L}, \\ s_{\phi_{T_L}} &= \sin\phi_{T_L}, \quad c_{\phi_{B_L}} = \cos\phi_{B_L}, \quad s_{\phi_{B_L}} = \sin\phi_{B_L}, \\ c_{\theta^d_R} &= \cos\theta^d_R, \quad s_{\theta^d_R} = \sin\theta^d_R, \quad c_{\beta_{T_R}} = \cos\beta_{T_R}, \\ s_{\beta_{T_R}} &= \sin\beta_{T_R}, \quad c_{\beta_{B_R}} = \cos\beta_{B_R}, \quad s_{\beta_{B_R}} = \sin\beta_{B_R}, \\ \beta_{T_R} &= \theta_{T_R} - \phi_{T_R}, \quad \beta_{B_R} = \theta_{B_R} - \phi_{B_R}, \quad \delta = \alpha^3_{d_R} - \alpha^3_{d_L}. \end{aligned} \tag{3.81}$$

The number of the CP-violating phase in this model is one. This agrees with the result in Ref. [18] for the $N = 1$ case. The details of the rephasing process are explained in Appendix D. In addition, the final expressions of the left-handed FCNC couplings, which appear in the left-handed weak isospin current in Eq. (3.46), are defined as follows:

$$(\mathcal{Z}_{T_L})^{ij} \equiv \left(K^\dagger_{T_L}\right)^{i3} (K_{T_L})^{3j}, \tag{3.82}$$

$$(\mathcal{Z}_{B_L})^{ij} \equiv \left(K^\dagger_{B_L}\right)^{i3} (K_{B_L})^{3j}, \tag{3.83}$$

where $i, j \in \{3, 4\}$. These have explicit matrix form as follows:

$$\mathcal{Z}_{T_L} = \begin{pmatrix} \cos^2\phi_{T_L} & -\sin\phi_{T_L}\cos\phi_{T_L} \\ -\sin\phi_{T_L}\cos\phi_{T_L} & \sin^2\phi_{T_L} \end{pmatrix}, \tag{3.84}$$

$$\mathcal{Z}_{B_L} = \begin{pmatrix} \cos^2\phi_{B_L} & -\sin\phi_{B_L}\cos\phi_{B_L} \\ -\sin\phi_{B_L}\cos\phi_{B_L} & \sin^2\phi_{B_L} \end{pmatrix}. \tag{3.85}$$

Similarly, for the right-handed weak isospin current from Eq. (3.33), the intermediate right-handed FCNC couplings transform into their final expressions as

$$(\mathcal{Z}_{T_R})^{ij} \equiv \sum_{k,l=3}^{4} \left(K_{T_R}^{\dagger}\right)^{ik} (Z_{T_R})^{kl} (K_{T_R})^{lj}, \tag{3.86}$$

$$(\mathcal{Z}_{B_R})^{ij} \equiv \sum_{k,l=3}^{4} \left(K_{B_R}^{\dagger}\right)^{ik} (Z_{B_R})^{kl} (K_{B_R})^{lj}, \tag{3.87}$$

where $i, j \in \{3, 4\}$. These can be expressed in matrix form as follows:

$$\mathcal{Z}_{T_R} = \begin{pmatrix} \cos^2 \beta_{T_R} & -\sin \beta_{T_R} \cos \beta_{T_R} \\ -\sin \beta_{T_R} \cos \beta_{T_R} & \sin^2 \beta_{T_R} \end{pmatrix}, \tag{3.88}$$

$$\mathcal{Z}_{B_R} = \begin{pmatrix} \cos^2 \beta_{B_R} & -\sin \beta_{B_R} \cos \beta_{B_R} \\ -\sin \beta_{B_R} \cos \beta_{B_R} & \sin^2 \beta_{B_R} \end{pmatrix}, \tag{3.89}$$

with $\beta_{T_R} = \theta_{T_R} - \phi_{T_R}$ and $\beta_{B_R} = \theta_{B_R} - \phi_{B_R}$.

Finally, we obtain the expression of the Lagrangian for the quark and Yukawa interaction after following all steps as follows:

$$\begin{aligned}
\mathcal{L}_q = & \sum_{\alpha=1}^{4} \overline{(\hat{u}^m)^\alpha} i\gamma^\mu D_{\mathrm{em}\mu} (\hat{u}^m)^\alpha + \sum_{\alpha=1}^{4} \overline{(\hat{d}^m)^\alpha} i\gamma^\mu D_{\mathrm{em}\mu} (\hat{d}^m)^\alpha \\
& - \frac{g_L}{\sqrt{2}} \left(\sum_{\alpha,\beta=1}^{4} \overline{\left(\hat{u}_L^m\right)^\alpha} \gamma^\mu \left(\hat{\mathcal{V}}_L^{\mathrm{CKM}}\right)^{\alpha\beta} \left(\hat{d}_L^m\right)^\beta W_{L\mu}^+ + \mathrm{h.c.} \right) \\
& - \left(\frac{g_L}{2\cos\theta_W} \left(j_{3L}^\mu\right) - e \tan\theta_W \left(j_{\mathrm{em}}^\mu\right) \right) Z_{L\mu} \\
& - \frac{g_R}{\sqrt{2}} \left(\sum_{\alpha,\beta=1}^{4} \overline{\left(\hat{u}_R^m\right)^\alpha} \gamma^\mu \left(\hat{\mathcal{V}}_R^{\mathrm{CKM}}\right)^{\alpha\beta} \left(\hat{d}_R^m\right)^\beta W_{R\mu}^+ + \mathrm{h.c.} \right) \\
& - \left\{ \frac{g_R}{2\cos\theta_R} \left(j_{3_R}^\mu\right) - g' \tan\theta_R \left(\left(j_{\mathrm{em}}^\mu\right) - \frac{1}{2} \left(j_{3_L}^\mu\right) \right) \right\} Z_{R\mu} \\
& - \sum_{j=3}^{4} \left(m_t^{\mathrm{diag}}\right)^{jj} \overline{(\hat{u}^m)^j} (\hat{u}^m)^j - \sum_{j=3}^{4} \left(m_b^{\mathrm{diag}}\right)^{jj} \overline{(\hat{d}^m)^j} (\hat{d}^m)^j \\
& - \frac{1}{v_L} \sum_{k,i=3}^{4} \left(\left(\mathcal{Z}_{T_L} m_t^{\mathrm{diag}}\right)^{ki} \overline{\left(\hat{u}_L^m\right)^k} \left(\hat{u}_R^m\right)^i + \left(m_t^{\mathrm{diag}} \mathcal{Z}_{T_L}\right)^{ki} \overline{\left(\hat{u}_R^m\right)^k} \left(\hat{u}_L^m\right)^i \right. \\
& \left. + \left(\mathcal{Z}_{B_L} m_b^{\mathrm{diag}}\right)^{ki} \overline{\left(\hat{d}_L^m\right)^k} \left(\hat{d}_R^m\right)^i + \left(m_b^{\mathrm{diag}} \mathcal{Z}_{B_L}\right)^{ki} \overline{\left(\hat{d}_R^m\right)^k} \left(\hat{d}_L^m\right)^i \right) h_L \\
& - \frac{\sqrt{2}}{v_L} \left[\sum_{k=3}^{4} \sum_{\alpha=2}^{4} \left(\overline{\left(\hat{u}_L^m\right)^\alpha} \left(\hat{\mathcal{V}}_L^{\mathrm{CKM}} m_b^{\mathrm{diag}}\right)^{\alpha k} \left(\hat{d}_R^m\right)^k - \overline{\left(\hat{u}_R^m\right)^k} \left(m_t^{\mathrm{diag}} \hat{\mathcal{V}}_L^{\mathrm{CKM}}\right)^{k\alpha} \left(\hat{d}_L^m\right)^\alpha \right) \chi_L^+ + \mathrm{h.c.} \right]
\end{aligned} \tag{3.90}$$

$$+\frac{1}{v_L}\sum_{k,i=3}^{4}\left(\left(\mathcal{Z}_{T_L}m_t^{\text{diag}}\right)^{ki}\overline{\left(\hat{u}_L^m\right)^k}\left(\hat{u}_R^m\right)^i-\left(m_t^{\text{diag}}\mathcal{Z}_{T_L}\right)^{ki}\overline{\left(\hat{u}_R^m\right)^k}\left(\hat{u}_L^m\right)^i\right.$$

$$\left.-\left(\mathcal{Z}_{B_L}m_b^{\text{diag}}\right)^{ki}\overline{\left(\hat{d}_L^m\right)^k}\left(\hat{d}_R^m\right)^i+\left(m_b^{\text{diag}}\mathcal{Z}_{B_L}\right)^{ki}\overline{\left(\hat{d}_R^m\right)^k}\left(\hat{d}_L^m\right)^i\right)i\chi_L^3$$

$$-\frac{1}{v_R}\sum_{k,i=3}^{4}\left(\left((1-\mathcal{Z}_{T_L})\,m_t^{\text{diag}}\mathcal{Z}_{T_R}\right)^{ki}\overline{\left(\hat{u}_L^m\right)^k}\left(\hat{u}_R^m\right)^i+\left(\mathcal{Z}_{T_R}m_t^{\text{diag}}\left(1-\mathcal{Z}_{T_L}\right)\right)^{ki}\overline{\left(\hat{u}_R^m\right)^k}\left(\hat{u}_L^m\right)^i\right.$$

$$\left.+\left((1-\mathcal{Z}_{B_L})\,m_b^{\text{diag}}\mathcal{Z}_{B_R}\right)^{ki}\overline{\left(\hat{d}_L^m\right)^k}\left(\hat{d}_R^m\right)^i+\left(\mathcal{Z}_{B_R}m_b^{\text{diag}}\left(1-\mathcal{Z}_{B_L}\right)\right)^{ki}\overline{\left(\hat{d}_R^m\right)^k}\left(\hat{d}_L^m\right)^i\right)h_R$$

$$-\frac{\sqrt{2}}{v_R}\left[\sum_{k=3}^{4}\sum_{\alpha=2}^{4}\left(\overline{\left(\hat{u}_R^m\right)^\alpha}\left(\hat{\mathcal{V}}_R^{\text{CKM}}m_b^{\text{diag}}\left(1-\mathcal{Z}_{B_L}\right)\right)^{\alpha k}\left(\hat{d}_L^m\right)^k\right.\right.$$

$$\left.\left.-\overline{\left(\hat{u}_L^m\right)^k}\left((1-\mathcal{Z}_{T_L})\,m_t^{\text{diag}}\hat{\mathcal{V}}_R^{\text{CKM}}\right)^{k\alpha}\left(\hat{d}_R^m\right)^\alpha\right)\chi_R^+ + \text{h.c.}\right]$$

$$+\frac{1}{v_R}\sum_{k,i=3}^{4}\left(\left((1-\mathcal{Z}_{B_L})\,m_b^{\text{diag}}\mathcal{Z}_{B_R}\right)^{ki}\overline{\left(\hat{d}_L^m\right)^k}\left(\hat{d}_R^m\right)^i-\left(\mathcal{Z}_{B_R}m_b^{\text{diag}}\left(1-\mathcal{Z}_{B_L}\right)\right)^{ki}\overline{\left(\hat{d}_R^m\right)^k}\left(\hat{d}_L^m\right)^i\right.$$

$$\left.-\left((1-\mathcal{Z}_{T_L})\,m_t^{\text{diag}}\mathcal{Z}_{T_R}\right)^{ki}\overline{\left(\hat{u}_L^m\right)^k}\left(\hat{u}_R^m\right)^i+\left(\mathcal{Z}_{T_R}m_t^{\text{diag}}\left(1-\mathcal{Z}_{T_L}\right)\right)^{ki}\overline{\left(\hat{u}_R^m\right)^k}\left(\hat{u}_L^m\right)^i\right)i\chi_R^3,$$

where we define $\hat{u}^m = \hat{u}_L^m + \hat{u}_R^m$ and $\hat{d}^m = \hat{d}_L^m + \hat{d}_R^m$. As mentioned before, the top and bottom quarks are the third component of the fields in the mass basis, while the heavy partners are the fourth component,

$$\left(\hat{u}_{L(R)}^m\right)^3 = t_{L(R)},\quad \left(\hat{u}_{L(R)}^m\right)^4 = t'_{L(R)},\quad \left(\hat{d}_{L(R)}^m\right)^3 = b_{L(R)},\quad \left(\hat{d}_{L(R)}^m\right)^4 = b'_{L(R)}. \tag{3.91}$$

The left-handed, right-handed weak isospin, and electromagnetic current in Eq. (3.90) now have the following final expressions:

$$j_{3L}^\mu = \sum_{i=1}^{2}\overline{\left(\hat{u}_L^m\right)^i}\gamma^\mu\left(\hat{u}_L^m\right)^i+\sum_{l,j=3}^{4}\overline{\left(\hat{u}_L^m\right)^l}\gamma^\mu\left(\mathcal{Z}_{T_L}\right)^{lj}\left(\hat{u}_L^m\right)^j$$

$$-\sum_{i=1}^{2}\overline{\left(\hat{d}_L^m\right)^i}\gamma^\mu\left(\hat{d}_L^m\right)^i-\sum_{l,j=3}^{4}\overline{\left(\hat{d}_L^m\right)^l}\gamma^\mu\left(\mathcal{Z}_{B_L}\right)^{lj}\left(\hat{d}_L^m\right)^j, \tag{3.92}$$

$$j_{3R}^\mu = \sum_{i=1}^{2}\overline{\left(\hat{u}_R^m\right)^i}\gamma^\mu\left(\hat{u}_R^m\right)^i+\sum_{l,j=3}^{4}\overline{\left(\hat{u}_R^m\right)^l}\gamma^\mu\left(\mathcal{Z}_{T_R}\right)^{lj}\left(\hat{u}_R^m\right)^j$$

$$-\sum_{i=1}^{2}\overline{\left(\hat{d}_R^m\right)^i}\gamma^\mu\left(\hat{d}_R^m\right)^i+\sum_{l,j=3}^{4}\overline{\left(\hat{d}_R^m\right)^l}\gamma^\mu\left(\mathcal{Z}_{B_R}\right)^{lj}\left(\hat{d}_R^m\right)^j, \tag{3.93}$$

$$j_{\text{em}}^\mu = \frac{2}{3}\sum_{\alpha=1}^{4}\overline{\left(\hat{u}^m\right)^\alpha}\gamma^\mu\left(\hat{u}^m\right)^\alpha-\frac{1}{3}\sum_{\alpha=1}^{4}\overline{(\hat{d}^m)^\alpha}\gamma^\mu(\hat{d}^m)^\alpha, \tag{3.94}$$

where the definitions and matrix forms of the FCNC couplings are shown in Eqs. (3.82)–(3.89). It should be noted that the Lagrangian, written in Eq. (3.90), can be expressed in the mass eigenstate of the Higgs and Z-bosons. We will discuss this in Section 4.

4. Higgs sector

In this section, we derive the kinetic terms and potential of the Higgs, which are contained in Eq. (2.4). In the same way as in Section 3, we derive these step by step from the $\mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_{\mathrm{Y}'}$ breaking into $\mathrm{U}(1)_\mathrm{Y}$ and finally $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ breaking into $\mathrm{U}(1)_\mathrm{em}$.

4.1. $\mathbf{SU(2)_R \times U(1)_{Y'} \to U(1)_Y}$

This stage occurs after the $\mathrm{SU}(2)_\mathrm{R}$ Higgs doublet acquires nonzero vev and takes the parameterization in Eq. (3.1). In addition, there is a mixing between B'_μ and $W^3_{R\mu}$ into B_μ and $Z_{R\mu}$ following the transformation shown in Eq. (3.2). We will analyze the kinetic terms and potential separately. Furthermore, we classify the terms based on the number of fields in the term as linear, quadratic, cubic, and quartic. The gauge fields inside the covariant derivatives are not counted as fields.

4.1.1. *Kinetic terms.* The kinetic terms in Eq. (2.4) become

$$\begin{aligned}
\mathcal{L}_H \supset \mathcal{L}_{\mathrm{kin}} = {} & \left(D^\mu_{\mathrm{SM}}\phi_L\right)^\dagger \left(D_{\mathrm{SM}\mu}\phi_L\right) - ig' Y_\phi \tan\theta_R Z_{R\mu} \left\{\left(D_{\mathrm{SM}\mu}\phi_L\right)^\dagger \phi_L - \phi_L^\dagger \left(D^\mu_{\mathrm{SM}}\phi_L\right)\right\} \\
& + g'^2 Y_\phi^2 \tan^2\theta_R Z^\mu_R Z_{R\mu}\phi_L^\dagger\phi_L + \left(D^\mu_{\mathrm{SM}}\chi_R^-\right)\left(D_{\mathrm{SM}\mu}\chi_R^+\right) \\
& + i\frac{g_R v_R}{2}\left\{W_R^{+\mu}\left(D_{\mathrm{SM}\mu}\chi_R^-\right) - W_R^{-\mu}\left(D_{\mathrm{SM}\mu}\chi_R^+\right)\right\} \\
& + \frac{g_R^2 v_R^2}{4} W_R^{-\mu} W_{R\mu}^+ + \frac{1}{2}\left(\partial_\mu h_R\right)^2 + \frac{1}{2}\left(\partial_\mu \chi_R^3 - \frac{g_R v_R}{2\cos\theta_R} Z_{R\mu}\right)^2 \\
& - \frac{g_R}{2}\chi_R^3\left\{\left(W_R^{+\mu} D_{\mathrm{SM}\mu}\chi_R^-\right) + W_R^{-\mu}\left(D_{\mathrm{SM}\mu}\chi_R^+\right)\right\} \\
& + i\frac{g_R}{2}\left\{W_R^{+\mu}\left(D_{\mathrm{SM}\mu}\chi_R^-\right) - W_R^{-\mu}\left(D_{\mathrm{SM}\mu}\chi_R^+\right)\right\} h_R + \frac{g_R^2 v_R}{2} h_R W_R^{-\mu} W_{R\mu}^+ \\
& + i\frac{g_R}{2}\frac{\cos 2\theta_R}{\cos\theta_R} Z_R^\mu \left\{\chi_R^+\left(D_{\mathrm{SM}\mu}\chi_R^-\right) - \chi_R^-\left(D_{\mathrm{SM}\mu}\chi_R^+\right)\right\} \\
& + \frac{g_R^2 v_R}{4}\left(\frac{\cos 2\theta_R - 1}{\cos\theta_R}\right)\left(W_{R\mu}^+\chi_R^- + W_{R\mu}^-\chi_R^+\right) Z_R^\mu \\
& + \frac{g_R}{2}\left(W_{R\mu}^+\chi_R^- + W_{R\mu}^-\chi_R^+\right)\partial^\mu\chi_R^3 - i\frac{g_R}{2}\left(W_{R\mu}^+\chi_R^- - W_{R\mu}^-\chi_R^+\right)\partial^\mu h_R \\
& + \frac{g_R}{2\cos\theta_R}\left\{\chi_R^3\left(\partial^\mu h_R\right) - \left(\partial^\mu\chi_R^3\right) h_R\right\} Z_{R\mu} + \left(\frac{g_R}{2\cos\theta_R}\right)^2 v_R h_R Z_R^\mu Z_{R\mu} \\
& + \frac{g_R^2\left(\cos 2\theta_R\right) - 1}{4\cos\theta_R}\left\{\left(W_{R\mu}^+\chi_R^- - W_{R\mu}^-\chi_R^+\right) i\chi_R^3 + \left(W_{R\mu}^+\chi_R^- + W_{R\mu}^-\chi_R^+\right) h_R\right\} Z_R^\mu \\
& + \frac{g_R^2}{4}\left(\frac{1}{2\cos^2\theta_R} Z_{R\mu} Z_R^\mu + W_R^{+\mu} W_{R\mu}^-\right)\left(\left(\chi_R^3\right)^2 + h_R^2\right) \\
& + \frac{g_R^2}{2}\left(W_R^{+\mu} W_{R\mu}^- + \frac{\cos^2 2\theta_R}{2\cos^2\theta_R} Z_R^\mu Z_{R\mu}\right)\left(\chi_R^-\chi_R^+\right),
\end{aligned} \tag{4.1}$$

where

$$D_{\mathrm{SM}\mu}\phi_L = \left(\partial_\mu + ig_L W^a_{L\mu}\frac{\tau^a_L}{2} + ig' Y_\phi B_\mu\right)\phi_L, \tag{4.2}$$

$$D_{\mathrm{SM}\mu}\chi_R^+ = (\partial_\mu + ig' B_\mu)\chi_R^+, \tag{4.3}$$

are the definition of the SM covariant derivatives for ϕ_L and χ_R^+, respectively.

4.1.2. *Higgs potential.* The Higgs potential which is written in Eq. (2.6) now becomes

$$\begin{aligned} V(\phi_L, \phi_R) = &\left(\mu_L^2 + \lambda_{LR} v_R^2\right)\phi_L^\dagger \phi_L + \lambda_L \left(\phi_L^\dagger \phi_L\right)^2 \\ &+ 2\lambda_{LR} v_R \left(\phi_L^\dagger \phi_L\right) h_R + 2\lambda_{LR}\left(\phi_L^\dagger \phi_L\right)\left(\chi_R^- \chi_R^+ + \frac{1}{2}\left(h_R^2 + \left(\chi_R^3\right)^2\right)\right) \\ &+ \frac{\mu_R^2}{2} v_R^2 + \frac{\lambda_R}{4} v_R^4 + h_R\left(\mu_R^2 v_R + \lambda_R v_R^3\right) \\ &+ \frac{h_R^2}{2}\left(\mu_R^2 + 3\lambda_R v_R^2\right) + \left(\mu_R^2 + \lambda_R v_R^2\right)\left(\chi_R^- \chi_R^+ + \frac{1}{2}\left(\chi_R^3\right)^2\right) \\ &+ 2 v_R \lambda_R h_R \left(\chi_R^- \chi_R^+ + \frac{1}{2}\left(h_R^2 + \left(\chi_R^3\right)^2\right)\right) \\ &+ \lambda_R \left(\chi_R^- \chi_R^+ + \frac{1}{2}\left(h_R^2 + \left(\chi_R^3\right)^2\right)\right)^2. \end{aligned} \tag{4.4}$$

4.2. $\mathbf{SU(2)_L \times U(1)_Y \to U(1)_{em}}$

This stage occurs after the $\mathrm{SU(2)_L}$ Higgs doublet acquires nonzero vev and takes the parameterization that is written in Eq. (3.39). As happens in the SM, there is a mixing between B_μ and $W_{L\mu}^3$ into A_μ and $Z_{L\mu}$ following the transformation shown in Eq. (3.40).

4.2.1. *Kinetic terms.* At this stage, one can show that the first line of Eq. (4.1) has a similar result, with $\mathrm{SU(2)_R \times U(1)_{Y'}}$ breaking by replacing $R \to L$, $\theta_R \to \theta_W$, and $D_{\mathrm{SM}} \to D_{\mathrm{em}}$. After computing all terms, the kinetic terms of the Higgs in Eq. (4.1) become

$$\begin{aligned} \mathcal{L}_H \supset \mathcal{L}_{\mathrm{kin}} = &\left(D_{\mathrm{em}}^\mu \chi_L^-\right)\left(D_{\mathrm{em}\mu}\chi_L^+\right) + \left(D_{\mathrm{em}}^\mu \chi_R^-\right)\left(D_{\mathrm{em}\mu}\chi_R^+\right) \\ &+ i\frac{g_L v_L}{2}\left\{W_L^{+\mu}\left(D_{\mathrm{em}\mu}\chi_L^-\right) - W_L^{-\mu}\left(D_{\mathrm{em}\mu}\chi_L^+\right)\right\} + \frac{g_L^2 v_L^2}{4} W_L^{-\mu} W_{L\mu}^+ \\ &+ i\frac{g_R v_R}{2}\left\{W_R^{+\mu}\left(D_{\mathrm{em}\mu}\chi_R^-\right) - W_R^{-\mu}\left(D_{\mathrm{em}\mu}\chi_R^+\right)\right\} + \frac{g_R^2 v_R^2}{4} W_R^{-\mu} W_{R\mu}^+ \\ &+ \frac{1}{2}\left(\partial_\mu h_L\right)^2 + \frac{1}{2}\left(\partial_\mu \chi_L^3 - \frac{g_L v_L}{2\cos\theta_W} Z_{L\mu}\right)^2 + \frac{1}{2}\left(\partial_\mu h_R\right)^2 + \frac{1}{2}\left(\partial_\mu \chi_R^3 - \frac{g_R v_R}{2\cos\theta_R} Z_{R\mu}\right)^2 \\ &+ \frac{1}{2} g' \tan\theta_R Z_{R\mu}\left\{-v_L\left(\partial^\mu \chi_L^3\right) + \frac{g_L v_L^2}{2\cos\theta_W} Z_L^\mu\right\} + \frac{1}{8} v_L^2 g'^2 \tan^2\theta_R Z_R^\mu Z_{R\mu} \\ &- \frac{g_L}{2}\chi_L^3\left\{W_L^{+\mu}\left(D_{\mathrm{em}\mu}\chi_L^-\right) + W_L^{-\mu}\left(D_{\mathrm{em}\mu}\chi_L^+\right)\right\} \end{aligned}$$

$$
\begin{aligned}
&- \frac{g_R}{2}\chi_R^3\left\{W_R^{+\mu}\left(D_{\mathrm{em}\mu}\chi_R^-\right)+W_R^{-\mu}\left(D_{\mathrm{em}\mu}\chi_R^+\right)\right\}\\
&+ i\frac{g_L}{2}\left\{W_L^{+\mu}\left(D_{\mathrm{em}\mu}\chi_L^-\right)-W_L^{-\mu}\left(D_{\mathrm{em}\mu}\chi_L^+\right)\right\}h_L+\frac{g_L^2v_L}{2}h_LW_L^{-\mu}W_{L\mu}^+\\
&+ i\frac{g_R}{2}\left\{W_R^{+\mu}\left(D_{\mathrm{em}\mu}\chi_R^-\right)-W_R^{-\mu}\left(D_{\mathrm{em}\mu}\chi_R^+\right)\right\}h_R+\frac{g_R^2v_R}{2}h_RW_R^{-\mu}W_{R\mu}^+\\
&+ i\frac{g_L}{2}\frac{\cos 2\theta_W}{\cos\theta_W}\left\{\chi_L^+\left(D_{\mathrm{em}\mu}\chi_L^-\right)-\chi_L^-\left(D_{\mathrm{em}\mu}\chi_L^+\right)\right\}Z_L^\mu\\
&+ i\frac{g_R}{2}\frac{\cos 2\theta_R}{\cos\theta_R}\left\{\chi_R^+\left(D_{\mathrm{em}\mu}\chi_R^-\right)-\chi_R^-\left(D_{\mathrm{em}\mu}\chi_R^+\right)\right\}Z_R^\mu\\
&+ \frac{g_L^2v_L}{4}\left(\frac{\cos 2\theta_W-1}{\cos\theta_W}\right)\left(W_{L\mu}^+\chi_L^-+W_{L\mu}^-\chi_L^+\right)Z_L^\mu\\
&+ \frac{g_R^2v_R}{4}\left(\frac{\cos 2\theta_R-1}{\cos\theta_R}\right)\left(W_{R\mu}^+\chi_R^-+W_{R\mu}^-\chi_R^+\right)Z_R^\mu\\
&+ \frac{g_L}{2}\left(W_{L\mu}^+\chi_L^-+W_{L\mu}^-\chi_L^+\right)\partial^\mu\chi_L^3-i\frac{g_L}{2}\left(W_{L\mu}^+\chi_L^--W_{L\mu}^-\chi_L^+\right)\partial^\mu h_L\\
&+ \frac{g_R}{2}\left(W_{R\mu}^+\chi_R^-+W_{R\mu}^-\chi_R^+\right)\partial^\mu\chi_R^3-i\frac{g_R}{2}\left(W_{R\mu}^+\chi_R^--W_{R\mu}^-\chi_R^+\right)\partial^\mu h_R\\
&+ \frac{g_L}{2\cos\theta_W}\left\{\chi_L^3\left(\partial_\mu h_L\right)-\left(\partial_\mu\chi_L^3\right)h_L\right\}Z_L^\mu+\left(\frac{g_L}{2\cos\theta_W}\right)^2v_Lh_LZ_L^\mu Z_{L\mu}\\
&+ \frac{g_R}{2\cos\theta_R}\left\{\chi_R^3\left(\partial_\mu h_R\right)-\left(\partial_\mu\chi_R^3\right)h_R\right\}Z_R^\mu+\left(\frac{g_R}{2\cos\theta_R}\right)^2v_Rh_RZ_R^\mu Z_{R\mu}\\
&- ie\tan\theta_W\left\{\chi_R^+\left(D_{\mathrm{em}\mu}\chi_R^-\right)-\chi_R^-\left(D_{\mathrm{em}\mu}\chi_R^+\right)\right\}Z_L^\mu\\
&- i\frac{1}{2}g'\tan\theta_R\left\{\chi_L^+\left(D_{\mathrm{em}\mu}\chi_L^-\right)-\chi_L^-\left(D_{\mathrm{em}\mu}\chi_L^+\right)\right\}Z_R^\mu\\
&- \frac{g_R}{2}v_Re\tan\theta_W\left(W_{R\mu}^+\chi_R^-+W_{R\mu}^-\chi_R^+\right)Z_L^\mu-\frac{g_L}{2}v_Lg'\tan\theta_R\left(W_{L\mu}^+\chi_L^-+W_{L\mu}^-\chi_L^+\right)Z_R^\mu\\
&+ g'\frac{1}{2}\tan\theta_R\left\{\left(\partial_\mu h_L\right)\chi_L^3-\left(\partial_\mu\chi_L^3\right)h_L\right\}Z_R^\mu\\
&+ g'\frac{1}{2}\tan\theta_R\frac{g_L}{\cos\theta_W}v_Lh_LZ_{R\mu}Z_L^\mu+v_Lg'^2\frac{1}{4}\tan^2\theta_Rh_LZ_{R\mu}Z_R^\mu\\
&+ \frac{g_L^2\left(\cos 2\theta_W-1\right)}{4\cos\theta_W}\left\{\left(W_{L\mu}^+\chi_L^--W_{L\mu}^-\chi_L^+\right)i\chi_L^3+\left(W_{L\mu}^+\chi_L^-+W_{L\mu}^-\chi_L^+\right)h_L\right\}Z_L^\mu\\
&+ \frac{g_R^2\left(\cos 2\theta_R-1\right)}{4\cos\theta_R}\left\{\left(W_{R\mu}^+\chi_R^--W_{R\mu}^-\chi_R^+\right)i\chi_R^3+\left(W_{R\mu}^+\chi_R^-+W_{R\mu}^-\chi_R^+\right)h_R\right\}Z_R^\mu\\
&- i\frac{g_R}{2}e\tan\theta_W\chi_R^3\left(W_{R\mu}^+\chi_R^--W_{R\mu}^-\chi_R^+\right)Z_L^\mu-i\frac{g_L}{2}g'\tan\theta_R\chi_L^3\left(W_{L\mu}^+\chi_L^--W_{L\mu}^-\chi_L^+\right)Z_R^\mu\\
&- \frac{g_R}{2}e\tan\theta_Wh_R\left(W_{R\mu}^+\chi_R^-+W_{R\mu}^-\chi_R^+\right)Z_L^\mu-\frac{g_L}{2}g'\tan\theta_Rh_L\left(W_{L\mu}^+\chi_L^-+W_{L\mu}^-\chi_L^+\right)Z_R^\mu
\end{aligned}
$$

$$+\frac{g_L^2}{4}\left(W_L^{+\mu}W_{L\mu}^-\right)\left(\left(\chi_L^3\right)^2+h_L^2\right)+\frac{g_R^2}{4}\left(W_R^{+\mu}W_{R\mu}^-\right)\left(\left(\chi_R^3\right)^2+h_R^2\right)$$
$$+\frac{g_L^2}{2}W_L^{+\mu}W_{L\mu}^-\chi_L^+\chi_L^-+\frac{g_R^2}{2}W_R^{+\mu}W_{R\mu}^-\chi_R^+\chi_R^-$$
$$+\frac{g_L^2}{2}\frac{\cos^2 2\theta_W}{2\cos^2\theta_W}\chi_L^+\chi_L^-Z_{L\mu}Z_L^\mu+\frac{g_R^2}{2}\frac{\cos^2 2\theta_R}{2\cos^2\theta_R}\chi_R^+\chi_R^-Z_{R\mu}Z_R^\mu$$
$$+e^2\tan^2\theta_W\chi_R^+\chi_R^-Z_{L\mu}Z_L^\mu+\frac{g'^2}{4}\tan^2\theta_R\chi_L^-\chi_L^+Z_{R\mu}Z_R^\mu$$
$$-\frac{g_R}{2}e\tan\theta_W\frac{\cos 2\theta_R}{\cos\theta_R}\left(2\chi_R^+\chi_R^-\right)Z_{L\mu}Z_R^\mu-\frac{g_L}{2}g'\tan\theta_R\frac{\cos 2\theta_W}{\cos\theta_W}\chi_L^-\chi_L^+Z_{L\mu}Z_R^\mu$$
$$+\frac{g_L^2}{4}\frac{1}{2\cos^2\theta_W}Z_{L\mu}Z_L^\mu\left(\left(\chi_L^3\right)^2+h_L^2\right)+\frac{g_R^2}{4}\frac{1}{2\cos^2\theta_R}Z_{R\mu}Z_R^\mu\left(\left(\chi_R^3\right)^2+h_R^2\right)$$
$$+\frac{g'^2}{4}\frac{1}{2}\tan^2\theta_R Z_{R\mu}Z_R^\mu\left(\left(\chi_L^3\right)^2+h_L^2\right), \tag{4.5}$$

where

$$D_{\mathrm{em}\mu}\chi_{L(R)}^+=(\partial_\mu+ieA_\mu)\chi_{L(R)}^+. \tag{4.6}$$

4.2.2. *Higgs potential.* At this stage, the Higgs potential in Eq. (4.4) becomes

$$V(\phi_L,\phi_R)=\frac{\mu_L^2}{2}v_L^2+\frac{\mu_R^2}{2}v_R^2+\frac{\lambda_L}{4}v_L^4+\frac{\lambda_R}{4}v_R^4+\frac{\lambda_{LR}}{2}v_R^2v_L^2$$
$$+h_L\left(\mu_L^2v_L+\lambda_Lv_L^3+\lambda_{LR}v_R^2v_L\right)+h_R\left(\mu_R^2v_R+\lambda_Rv_R^3+\lambda_{LR}v_Rv_L^2\right)$$
$$+h_L\left(2\lambda_{LR}v_Rv_L\right)h_R+\frac{h_L^2}{2}\left(\mu_L^2+3\lambda_Lv_L^2+\lambda_{LR}v_R^2\right)+\frac{h_R^2}{2}\left(\mu_R^2+3\lambda_Rv_R^2+\lambda_{LR}v_L^2\right)$$
$$+\left(\mu_L^2+\lambda_Lv_L^2+\lambda_{LR}v_R^2\right)\left(\chi_L^-\chi_L^++\frac{1}{2}\left(\chi_L^3\right)^2\right)$$
$$+\left(\mu_R^2+\lambda_Rv_R^2+\lambda_{LR}v_L^2\right)\left(\chi_R^-\chi_R^++\frac{1}{2}\left(\chi_R^3\right)^2\right)$$
$$+2v_L\left\{\lambda_L\left(\chi_L^-\chi_L^++\frac{1}{2}\left(h_L^2+\left(\chi_L^3\right)^2\right)\right)+\lambda_{LR}\left(\chi_R^-\chi_R^++\frac{1}{2}\left(h_R^2+\left(\chi_R^3\right)^2\right)\right)\right\}h_L$$
$$+2v_R\left\{\lambda_R\left(\chi_R^-\chi_R^++\frac{1}{2}\left(h_R^2+\left(\chi_R^3\right)^2\right)\right)+\lambda_{LR}\left(\chi_L^-\chi_L^++\frac{1}{2}\left(h_L^2+\left(\chi_L^3\right)^2\right)\right)\right\}h_R$$
$$+\lambda_L\left(\chi_L^-\chi_L^++\frac{1}{2}\left(h_L^2+\left(\chi_L^3\right)^2\right)\right)^2+\lambda_R\left(\chi_R^-\chi_R^++\frac{1}{2}\left(h_R^2+\left(\chi_R^3\right)^2\right)\right)^2$$
$$+2\lambda_{LR}\left(\chi_L^-\chi_L^++\frac{1}{2}\left(h_L^2+\left(\chi_L^3\right)^2\right)\right)\left(\chi_R^-\chi_R^++\frac{1}{2}\left(h_R^2+\left(\chi_R^3\right)^2\right)\right), \tag{4.7}$$

where μ_L^2 and μ_R^2 are negative. The minimization conditions of the potential are

$$v_L\left(\mu_L^2+\lambda_Lv_L^2+\lambda_{LR}v_R^2\right)=0, \tag{4.8}$$

$$v_R\left(\mu_R^2+\lambda_Rv_R^2+\lambda_{LR}v_L^2\right)=0. \tag{4.9}$$

We can obtain the expressions of the nonzero vevs as follows:

$$v_L = \sqrt{\frac{-\mu_L^2\lambda_R + \lambda_{LR}\mu_R^2}{\lambda_R\lambda_L - \lambda_{LR}^2}} \quad \text{and} \quad v_R = \sqrt{\frac{-\mu_R^2\lambda_L + \lambda_{LR}\mu_L^2}{\lambda_R\lambda_L - \lambda_{LR}^2}}, \tag{4.10}$$

where the vevs are taken to be positive. One can show that the linear terms of the Higgs fields and quadratic terms of $\chi^\pm_{L(R)}$ and $\chi^3_{L(R)}$ will vanish by using Eqs. (4.8) and (4.9).

4.3. *Boson mass*

We collect the quadratic terms from the kinetic terms in Eq. (4.5) and Higgs potential in Eq. (4.7) below:

$$\begin{aligned}
\mathcal{L}_H \supset \mathcal{L}_{\text{quad}} &= \left(D^\mu_{\text{em}}\chi_L^-\right)\left(D_{\text{em}\mu}\chi_L^+\right) + \left(D^\mu_{\text{em}}\chi_R^-\right)\left(D_{\text{em}\mu}\chi_R^+\right) \\
&+ i\frac{g_L v_L}{2}\left\{W_L^{+\mu}\left(D_{\text{em}\mu}\chi_L^-\right) - W_L^{-\mu}\left(D_{\text{em}\mu}\chi_L^+\right)\right\} + \frac{g_L^2 v_L^2}{4}W_L^{-\mu}W_{L\mu}^+ \\
&+ i\frac{g_R v_R}{2}\left\{W_R^{+\mu}\left(D_{\text{em}\mu}\chi_R^-\right) - W_R^{-\mu}\left(D_{\text{em}\mu}\chi_R^+\right)\right\} + \frac{g_R^2 v_R^2}{4}W_R^{-\mu}W_{R\mu}^+ \\
&+ \frac{1}{2}\left(\frac{g_L}{2}\frac{v_L}{\cos\theta_W}\right)^2 Z_L^\mu Z_{L\mu} + \frac{1}{2}\left\{\left(\frac{g_R}{2}\frac{v_R}{\cos\theta_R}\right)^2 + \left(\frac{g'}{2}v_L\tan\theta_R\right)^2\right\} Z_R^\mu Z_{R\mu} \\
&+ \frac{g' v_L}{2}\tan\theta_R\frac{g_L}{2}\frac{v_L}{\cos\theta_W}Z_L^\mu Z_{R\mu} + \frac{1}{2}\left(\partial_\mu\chi_L^3\right)^2 + \frac{1}{2}\left(\partial_\mu\chi_R^3\right)^2 \\
&- \frac{1}{2}\frac{g_L v_L}{2\cos\theta_W}Z_{L\mu}\left(\partial^\mu\chi_L^3\right) - \frac{1}{2}\frac{g_R v_R}{2\cos\theta_R}Z_{R\mu}\left(\partial^\mu\chi_R^3\right) - \frac{g' v_L}{2}\tan\theta_R Z_{R\mu}\left(\partial^\mu\chi_L^3\right) \\
&+ \frac{1}{2}\left(\partial_\mu h_L\right)^2 + \frac{1}{2}\left(\partial_\mu h_R\right)^2 \\
&- h_L\left(2\lambda_{LR}v_R v_L\right)h_R - \frac{h_L^2}{2}\left(2\lambda_L v_L^2\right) - \frac{h_R^2}{2}\left(2\lambda_R v_R^2\right).
\end{aligned} \tag{4.11}$$

From Eq. (4.11), we obtain the W_L and W_R mass,

$$M_{W_L} = \frac{g_L}{2}v_L, \tag{4.12}$$

$$M_{W_R} = \frac{g_R}{2}v_R. \tag{4.13}$$

Since there is mixing between Z_L and Z_R as well as h_L and h_R, then we need to diagonalize the mass matrices to obtain the mass eigenstate for the Z-bosons and the Higgs bosons. In line with that, the Nambu–Goldstone bosons χ_L^3 and χ_R^3 also mix.

4.3.1. *Z- and Z′-boson mass.*

We define the following transformation from the Z_L and Z_R basis into the mass eigenstates:

$$\begin{pmatrix} Z_{L\mu} \\ Z_{R\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} Z_\mu \\ Z'_\mu \end{pmatrix}. \tag{4.14}$$

From Eq. (4.11), the mass matrix in the Z_L and Z_R basis is as follows:

$$\mathbb{M}_Z^2 = \begin{pmatrix} \left(\frac{g_L v_L}{2\cos\theta_W}\right)^2 & \frac{1}{2}g' v_L\tan\theta_R\frac{g_L v_L}{2\cos\theta_W} \\ \frac{1}{2}g' v_L\tan\theta_R\frac{g_L v_L}{2\cos\theta_W} & \left(\frac{g_R v_R}{2\cos\theta_R}\right)^2 + \left(\frac{1}{2}g' v_L\tan\theta_R\right)^2 \end{pmatrix}. \tag{4.15}$$

The mass matrix $\mathbb{M}_Z^2$ can be diagonalized,

$$\mathcal{O}_Z^T \mathbb{M}_Z^2 \mathcal{O}_Z = \text{diag}\left(M_Z^2, M_{Z'}^2\right), \tag{4.16}$$

where $\mathcal{O}_Z$ is the mixing matrix in Eq. (4.14). The exact mass eigenvalues and mixing angles are as follows:

$$M_Z^2 = \frac{M_{W_R}^2}{2c_R^2}\left\{1 + \left(c_R^2 + t_W^2\right)\frac{M_{W_L}^2}{M_{W_R}^2} - \sqrt{1 - \frac{2M_{W_L}^2}{M_{W_R}^2}\left(\frac{c_R^2 - s_W^2 s_R^2}{c_W^2}\right) + \left(c_R^2 + t_W^2\right)^2\left(\frac{M_{W_L}^2}{M_{W_R}^2}\right)^2}\right\}, \tag{4.17}$$

$$M_{Z'}^2 = \frac{M_{W_R}^2}{2c_R^2}\left\{1 + \left(c_R^2 + t_W^2\right)\frac{M_{W_L}^2}{M_{W_R}^2} + \sqrt{1 - \frac{2M_{W_L}^2}{M_{W_R}^2}\left(\frac{c_R^2 - s_W^2 s_R^2}{c_W^2}\right) + \left(c_R^2 + t_W^2\right)^2\left(\frac{M_{W_L}^2}{M_{W_R}^2}\right)^2}\right\}, \tag{4.18}$$

$$\tan 2\theta = \frac{2c_R s_R^3 s_W \frac{v_L^2}{v_R^2}}{s_W^2 - s_R^2\left(s_W^2 \cos 2\theta_R + c_W^2 c_R^2\right)\frac{v_L^2}{v_R^2}}, \quad 0 \leq \theta \leq \frac{\pi}{4}, \tag{4.19}$$

where

$$c_R = \cos\theta_R, \quad s_R = \sin\theta_R, \quad c_W = \cos\theta_W, \quad s_W = \sin\theta_W, \quad t_W = \tan\theta_W. \tag{4.20}$$

When $M_{W_R} \gg M_{W_L}$, the masses of the Z- and Z'-bosons are approximately given as follows:

$$M_Z^2 \simeq \frac{M_{W_L}^2}{c_W^2}\left(1 - \frac{M_{W_L}^2}{M_{W_R}^2} s_R^2 t_W^2\right), \tag{4.21}$$

$$M_{Z'}^2 \simeq \frac{M_{W_R}^2}{c_R^2}\left(1 + \frac{M_{W_L}^2}{M_{W_R}^2} s_R^2 t_W^2\right). \tag{4.22}$$

4.3.2. *Higgs boson mass.* We define the following transformation from the h_L and h_R basis into the mass eigenstate:

$$\begin{pmatrix} h_L \\ h_R \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} h \\ H \end{pmatrix}. \tag{4.23}$$

The mass matrix of the Higgs in the h_L and h_R basis is as follows:

$$\mathbb{M}_h^2 = \begin{pmatrix} 2\lambda_L v_L^2 & 2\lambda_{LR} v_R v_L \\ 2\lambda_{LR} v_R v_L & 2\lambda_R v_R^2 \end{pmatrix}. \tag{4.24}$$

By defining the mixing matrix in Eq. (4.23) as $\mathcal{O}_h$, we can diagonalize $\mathbb{M}_h$ as

$$\mathcal{O}_h^T \mathbb{M}_h^2 \mathcal{O}_h = \text{diag}\left(m_h^2, m_H^2\right), \tag{4.25}$$

which gives the exact mass eigenvalues,

$$m_h^2 = \lambda_L v_L^2 + \lambda_R v_R^2 - \sqrt{\left(\lambda_L v_L^2 - \lambda_R v_R^2\right)^2 + 4\lambda_{LR}^2 v_L^2 v_R^2}, \tag{4.26}$$

$$m_H^2 = \lambda_L v_L^2 + \lambda_R v_R^2 + \sqrt{\left(\lambda_L v_L^2 - \lambda_R v_R^2\right)^2 + 4\lambda_{LR}^2 v_L^2 v_R^2}. \tag{4.27}$$

In addition, the mixing angle in Eq. (4.23) is given by

$$\tan 2\phi = \frac{2\lambda_{LR} v_R v_L}{\lambda_R v_R^2 - \lambda_L v_L^2}, \quad 0 \leq |\phi| \leq \frac{\pi}{4}. \tag{4.28}$$

Furthermore, the mass eigenvalues and mixing angle can be expressed in the following approximation forms:

$$m_h^2 \simeq 2\lambda_L \left(1 - \frac{\lambda_{LR}^2}{\lambda_L \lambda_R}\right) v_L^2, \tag{4.29}$$

$$m_H^2 \simeq 2\lambda_R v_R^2, \tag{4.30}$$

$$\tan 2\phi \simeq \frac{2\lambda_{LR}}{\lambda_R} \frac{v_L}{v_R}, \tag{4.31}$$

if we ignore the correction of $\mathcal{O}\left(v_L^2/v_R^2\right)$.

4.4. χ_L^3 *and* χ_R^3 *mixing*

From Eq. (4.11), we extract the following form:

$$\begin{aligned}\mathcal{L}_{\text{quad}} \supset \mathcal{L}_\chi = &\frac{1}{2}\left(\partial_\mu \chi_L^3\right)^2 + \frac{1}{2}\left(\partial_\mu \chi_R^3\right)^2 - \frac{1}{2}\frac{g_L v_L}{\cos\theta_W} Z_{L\mu}\left(\partial^\mu \chi_L^3\right) \\ &- \frac{1}{2}\frac{g_R v_R}{\cos\theta_R} Z_{R\mu}\left(\partial^\mu \chi_R^3\right) - \frac{g' v_L}{2}\tan\theta_R Z_{R\mu}\left(\partial^\mu \chi_L^3\right).\end{aligned} \tag{4.32}$$

By changing into the mass eigenstate using Eq. (4.14) and writing in terms of the diagonal mass eigenvalues (M_Z, $M_{Z'}$), Eq. (4.32) can be written as

$$\mathcal{L}_{\text{quad}} \supset \mathcal{L}_\chi = \frac{1}{2}(\partial_\mu \chi_Z)^2 + \frac{1}{2}(\partial_\mu \chi_{Z'})^2 - M_Z\,(\partial^\mu \chi_Z) Z_\mu - M_{Z'}(\partial^\mu \chi_{Z'}) Z'_\mu, \tag{4.33}$$

where

$$\begin{pmatrix}\chi_L^3 \\ \chi_R^3\end{pmatrix} = \begin{pmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\chi_Z \\ \chi_{Z'}\end{pmatrix}, \tag{4.34}$$

$$\cos\alpha = \frac{M_Z \cos\theta}{\sqrt{M_Z^2 \cos^2\theta + M_{Z'}^2 \sin^2\theta}}, \tag{4.35}$$

$$\sin\alpha = \frac{M_{Z'} \sin\theta}{\sqrt{M_Z^2 \cos^2\theta + M_{Z'}^2 \sin^2\theta}}. \tag{4.36}$$

Therefore, the quadratic terms in Eq. (4.11) are written in terms of the mass basis of the Z-bosons, Higgs bosons, and Nambu–Goldstone bosons,

$$\begin{aligned}\mathcal{L}_H \supset \mathcal{L}_{\text{quad}} = &\left(D_{\text{em}}^\mu \chi_L^- - iM_{W_L} W_L^{\mu -}\right)\left(D_{\text{em}\mu} \chi_L^+ + iM_{W_L} W_{L\mu}^+\right) \\ &+ \left(D_{\text{em}}^\mu \chi_R^- - iM_{W_R} W_R^{\mu -}\right)\left(D_{\text{em}\mu} \chi_R^+ + iM_{W_R} W_{R\mu}^+\right) \\ &+ \frac{1}{2}\left(\partial_\mu \chi_Z - M_Z Z_\mu\right)^2 + \frac{1}{2}\left(\partial_\mu \chi_{Z'} - M_{Z'} Z'_\mu\right)^2 \\ &+ \frac{1}{2}\left(\partial_\mu h\right)^2 - \frac{1}{2}m_h^2 h^2 + \frac{1}{2}\left(\partial_\mu H\right)^2 - \frac{1}{2}m_H^2 H^2,\end{aligned} \tag{4.37}$$

where the covariant derivatives of χ_L and χ_R are in Eq. (4.6). We have shown explicitly that χ_L^3 and χ_R^3 are mixed in this model. From Eq. (4.37), it is shown clearly that the degrees of freedom χ_Z and $\chi_{Z'}$ become the longitudinal components of the massive Z- and Z'-bosons, respectively.

5. Kinetic terms of gauge fields

In this section we derive the kinetic terms of the gauge fields starting from the Lagrangian in Eq. (2.5).

5.1. $\mathbf{SU(2)_R \times U(1)_{Y'} \to U(1)_Y}$

At this stage, the kinetic terms of the gauge fields change from the B'_μ and $W_{R\mu}$ basis into the B_μ and $Z_{R\mu}$ basis. Following transformation in Eq. (3.2), the Lagrangian in Eq. (2.5) becomes

$$\begin{aligned}\mathcal{L}_{\text{gauge}} = &-\frac{1}{4}F^a_{L\mu\nu}F^{a\mu\nu}_L - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}\left(\partial_\mu W^+_{R\nu} - \partial_\nu W^+_{R\mu}\right)\left(\partial^\mu W^{-\nu}_R - \partial^\nu W^{-\mu}_R\right)\\ &- i\left(\partial_\mu W^+_{R\nu} - \partial_\nu W^+_{R\mu}\right)\left(g_R\cos\theta_R Z^\nu_R + g' B^\nu\right)W^{-\mu}_R\\ &+ i\left(\partial^\mu W^{-\nu}_R - \partial^\nu W^{-\mu}_R\right)\left(g_R\cos\theta_R Z_{R\nu} + g' B_\nu\right)W^+_{R\mu}\\ &- \Big\{\left(g_R\cos\theta_R Z_{R\nu} + g' B_\nu\right)W^+_{R\mu}\left(g_R\cos\theta_R Z^\nu_R + g' B^\nu\right)W^{-\mu}_R\\ &- \left(g_R\cos\theta_R Z_{R\mu} + g' B_\mu\right)W^+_{R\nu}\left(g_R\cos\theta_R Z^\nu_R + g' B^\nu\right)W^{-\mu}_R\Big\}\\ &- \frac{1}{4}F^0_{Z_R\mu\nu}F^{0\mu\nu}_{Z_R} + iW^-_{R\mu}W^+_{R\nu}\left(g_R\cos\theta_R F^{0\mu\nu}_{Z_R} + g' B^{\mu\nu}\right)\\ &+ \frac{1}{2}g^2_R\left(W^-_{R\mu}W^+_{R\nu} - W^+_{R\mu}W^-_{R\nu}\right)\left(W^{-\mu}_R W^{+\nu}_R\right),\end{aligned} \tag{5.1}$$

where

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{5.2}$$

$$F^a_{L\mu\nu} = \partial_\mu W^a_{L\nu} - \partial_\nu W^a_{L\mu} - g_L\epsilon^{abc}W^b_{L\mu}W^c_{L\nu}, \tag{5.3}$$

$$F^0_{Z_R\mu\nu} = \partial_\mu Z_{R\nu} - \partial_\nu Z_{R\mu}. \tag{5.4}$$

5.2. $\mathbf{SU(2)_L \times U(1)_Y \to U(1)}_{em}$

At this stage, there is a mixing between B_μ and $W^3_{L\mu}$ into A_μ and $Z_{L\mu}$ following the transformation shown in Eq. (3.40). In addition, we also express in the diagonal basis of Z and Z' where the transformation is shown in Eq. (4.14). So the Lagrangian in Eq. (5.1) becomes

$$\begin{aligned}\mathcal{L}_{\text{gauge}} = &-\frac{1}{4}F^0_{Z\mu\nu}F^{0\mu\nu}_Z - \frac{1}{4}F^0_{Z'\mu\nu}F^{0\mu\nu}_{Z'} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\ &- \frac{1}{2}\left(\mathcal{D}_\mu W^+_{L\nu} - \mathcal{D}_\nu W^+_{L\mu}\right)\left(\mathcal{D}^\mu W^{-\nu}_L - \mathcal{D}^\nu W^{-\mu}_L\right)\\ &- \frac{1}{2}\left(\mathcal{D}_\mu W^+_{R\nu} - \mathcal{D}_\nu W^+_{R\mu}\right)\left(\mathcal{D}^\mu W^{-\nu}_R - \mathcal{D}^\nu W^{-\mu}_R\right)\\ &+ \frac{g^2_L}{2}\left((W^-_L\cdot W^-_L)(W^+_L\cdot W^+_L) - \left(W^-_L\cdot W^+_L\right)^2\right)\end{aligned}$$

$$
\begin{aligned}
&+ \frac{g_R^2}{2}\left((W_R^- \cdot W_R^-)(W_R^+ \cdot W_R^+) - \left(W_R^- \cdot W_R^+\right)^2\right) \\
&+ i\left\{g_L \cos\theta_W \cos\theta F_Z^{0\mu\nu} + g_L \cos\theta_W \sin\theta F_{Z'}^{0\mu\nu} + eF^{\mu\nu}\right\}\left(W_{L\mu}^- W_{L\nu}^+\right) \\
&+ i\left\{-(g_R \cos\theta_R \sin\theta + e\tan\theta_W \cos\theta)F_Z^{0\mu\nu}\right. \\
&\left.+ (g_R \cos\theta_R \cos\theta - e\tan\theta_W \sin\theta)F_{Z'}^{0\mu\nu} + eF^{\mu\nu}\right\}\left(W_{R\mu}^- W_{R\nu}^+\right),
\end{aligned}
\tag{5.5}
$$

where

$$
\begin{aligned}
F_{Z\mu\nu}^0 &= \partial_\mu Z_\nu - \partial_\nu Z_\mu, \\
F_{Z'\mu\nu}^0 &= \partial_\mu Z'_\nu - \partial_\nu Z'_\mu, \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\
\mathcal{D}_\mu W_{R\nu}^+ &= \left(D_{\mathrm{em}\mu} W_{R\nu}^+\right) - i\left(e\tan\theta_W Z_{L\mu} - g_R\cos\theta_R Z_{R\mu}\right) W_{R\nu}^+, \\
\mathcal{D}_\mu W_{L\nu}^+ &= \left(D_{\mathrm{em}\mu} W_{L\nu}^+\right) + ig_L \cos\theta_W Z_{L\mu} W_{L\nu}^+, \\
D_{\mathrm{em}\mu} G_\nu &= \left(\partial_\mu + ieA_\mu\right) G_\nu,
\end{aligned}
\tag{5.6}
$$

with $G_\nu \in \{W_{R\nu}^+, W_{L\nu}^+\}$.

6. Discussion

6.1. *Hierarchy of VLQ's mass parameters, v_L, and v_R*

In this subsection, we discuss the hierarchy of VLQ's mass parameters, v_L, and v_R. From Eqs. (3.70) and (3.71), we have the exact mass eigenvalues of the top and bottom quarks, as well as the heavy top and bottom quarks, respectively. One of the motivations for the universal seesaw model in the quark sector is to explain the mass hierarchy of quarks, particularly the third family quark mass hierarchy in our model. Therefore, the hierarchy of VLQ's mass parameters M_T, M_B, v_L, and v_R is essential to be studied. We give the analytical and numerical analysis.

6.1.1. *Analytical analysis.* The top quark exact mass eigenvalue in Eq. (3.70) can be written as follows:

$$
\begin{aligned}
m_t &= \frac{\sqrt{M_T^2 + m_{u_R}^2 + m_{u_L}^2 + 2m_{u_L}m_{u_R}}}{2} - \frac{\sqrt{M_T^2 + m_{u_R}^2 + m_{u_L}^2 - 2m_{u_L}m_{u_R}}}{2} \\
&\simeq \left(\frac{m_{u_R}}{\sqrt{M_T^2 + m_{u_R}^2}}\right) m_{u_L}.
\end{aligned}
\tag{6.1}
$$

From the first line to the second line of Eq. (6.1), we use $m_{u_L} < m_{u_R}$. We can express the second line of Eq. (6.1) in terms of Yukawa couplings using Eq. (C.16) as follows:

$$
m_t \simeq \left(\frac{\frac{Y_{u_R}^3 v_R}{\sqrt{2}}}{\sqrt{M_T^2 + \frac{\left(Y_{u_R}^3\right)^2 v_R^2}{2}}}\right) \frac{Y_{u_L}^3 v_L}{\sqrt{2}}.
\tag{6.2}
$$

If we assume $Y^3_{u_L} = Y^3_{u_R} \simeq \mathcal{O}(1)$ and the factor inside the parenthesis is $\mathcal{O}(1)$, we can obtain the top quark mass $m_t \simeq v_L$. This implies $M_T < v_R$. In order to determine the hierarchy between M_T and v_R for the large top quark mass, from Eq. (6.2) one can obtain the ratio M_T/v_R as follows:

$$\frac{M_T}{v_R} = \frac{Y^3_{u_L} Y^3_{u_R}}{\sqrt{2}} \sqrt{\frac{1}{\left(y_t^{\rm SM}\right)^2} - \frac{1}{\left(Y^3_{u_L}\right)^2}}, \tag{6.3}$$

where $y_t^{\rm SM}$ is the SM Yukawa coupling of the top quark and $Y^3_{u_L} \geq y_t^{\rm SM}$. If we further require that the Yukawa couplings are in the perturbative region, $y_t^{\rm SM} \leq Y^3_{u_L}, Y^3_{u_R} \leq 1$, the upper and lower limits of the ratio M_T/v_R are given by

$$0 \leq \frac{M_T}{v_R} \leq \frac{1}{\sqrt{2}} \sqrt{\frac{1}{\left(y_t^{\rm SM}\right)^2} - 1}. \tag{6.4}$$

If we take $y_t^{\rm SM} = 0.9912$, we obtain the upper limit of the ratio $M_T/v_R \leq 0.0944$. This shows how the seesaw mechanism accommodates the top quark mass and the hierarchy between M_T and v_R.

Similarly for the bottom sector, by using $m_{d_L} < m_{d_R}$ the bottom quark mass becomes

$$m_b \simeq \left(\frac{\frac{Y^3_{d_R} v_R}{\sqrt{2}}}{\sqrt{M_B^2 + \frac{\left(Y^3_{d_R}\right)^2 v_R^2}{2}}} \right) \frac{Y^3_{d_L} v_L}{\sqrt{2}}. \tag{6.5}$$

If we assume $Y^3_{d_L} = Y^3_{d_R} \simeq \mathcal{O}(1)$ and the factor inside the parenthesis is much smaller than $\mathcal{O}(1)$, we can obtain the light bottom quark mass. This implies $M_B \gg v_R$ and we can write Eq. (6.5) as follows:

$$m_b \simeq \frac{v_R Y^3_{d_R} Y^3_{d_L} v_L}{2 M_B}. \tag{6.6}$$

In order to determine the hierarchy between M_B and v_R for the light bottom quark mass, from Eq. (6.6) one can obtain the ratio M_B/v_R as follows:

$$\frac{M_B}{v_R} = \frac{Y^3_{d_L} Y^3_{d_R}}{\sqrt{2}} \frac{1}{y_b^{\rm SM}}, \tag{6.7}$$

where $y_b^{\rm SM}$ is the SM Yukawa coupling of the bottom quark. If we further require that the Yukawa couplings are in the perturbative region, $Y^3_{d_L}, Y^3_{d_R} \leq 1$, the upper limit of the ratio M_B/v_R is given by

$$\frac{M_B}{v_R} \leq \frac{1}{\sqrt{2}} \frac{1}{y_b^{\rm SM}}. \tag{6.8}$$

If we take $y_b^{\rm SM} = 2.4 \times 10^{-2}$, we obtain the upper limit of the ratio $M_B/v_R \leq 29.46$. The equality holds when the Yukawa couplings $Y^3_{d_L} = Y^3_{d_R} = 1$. This shows how the seesaw mechanism accommodates the bottom quark mass and the hierarchy between M_B and v_R. Therefore, when all the Yukawa couplings $Y^3_{d_L}$, $Y^3_{d_R}$, $Y^3_{u_L}$, and $Y^3_{u_R}$ are $\mathcal{O}(1)$, the hierarchy for the three scales is $M_T < v_R \ll M_B$. If we include the v_L, the hierarchy has two possibilities depending on the numerical inputs. The hierarchy can be $v_L < M_T < v_R \ll M_B$ or $M_T < v_L < v_R \ll M_B$.

To summarize, by using the hierarchy that we discussed before, from the exact mass eigenvalues in Eqs. (3.70) and (3.71) we can obtain the approximate form as follows:

$$m_t^{\text{approx}} \simeq \frac{v_R Y_{u_R}^3 Y_{u_L}^3 v_L}{2\sqrt{\frac{v_R^2}{2}\left(Y_{u_R}^3\right)^2 + M_T^2}}, \tag{6.9}$$

$$m_{t'}^{\text{approx}} \simeq \sqrt{\frac{v_R^2}{2}\left(Y_{u_R}^3\right)^2 + M_T^2}, \tag{6.10}$$

$$m_b^{\text{approx}} \simeq \frac{v_R Y_{d_R}^3 Y_{d_L}^3 v_L}{2M_B}, \tag{6.11}$$

$$m_{b'}^{\text{approx}} \simeq M_B. \tag{6.12}$$

Our results in Eqs. (6.9) and (6.10) agree with Eqs. (7) and (8) in Ref. [16], as well as Eqs. (3.19) and (3.17) in Ref. [17], respectively. In addition, our results in Eqs. (6.11) and (6.12) agree with Eqs. (14) and (15) in Ref. [16], as well as Eq. (3.9) in Ref. [17], respectively.

6.1.2. *Numerical analysis.* We start by analyzing the constraints in the top sector, as shown in Fig. 1(a). We consider an asymmetric left-right model with $g_L \neq g_R$. By assuming $g_R \simeq 1$ and using the value of $g' \simeq 0.357$, we obtain θ_R with Eq. (3.4). Additionally, we assume $Y_{u_R}^3 \simeq Y_{u_L}^3 \simeq 1$. The following are the constraints that we used [33]: (1) the top quark mass obtained by the direct measurement is $m_t = 172.57$ GeV; (2) the lower bound for the heavy top quark mass is set to be $m_{t'} > 1310$ GeV; (3) the lower bound for the Z'-boson mass is set to be $M_{Z'} > 5150$ GeV. Using the exact mass eigenvalue for the Z'-boson mass in Eq. (4.18), we compute the lower bound for the W_R-boson mass as $M_{W_R} \gtrsim 5$ TeV. Consequently, we find the constraint for v_R using Eq. (4.13), yielding $v_R \gtrsim 10$ TeV. At $v_R = 10$ TeV, M_T is 942.3 GeV as shown by the black dot in Fig. 1(a). Using these v_R and M_T values, we further calculate the heavy top quark mass with Eq. (3.71) and obtain $m_{t'} = 7.13$ TeV.

Next, we analyze the constraints in the bottom sector, as depicted in Fig. 1(b). Here, we also assume $Y_{d_R}^3 \simeq Y_{d_L}^3 \simeq 1$. The constraints are [33]: (1) the SM bottom quark mass we use is the running mass at bottom mass $m_b = 4.183$ GeV; (2) the lower bound for the heavy bottom quark mass is set to be $m_{b'} > 1390$ GeV; (3) the constraint for $v_R \gtrsim 10$ TeV is derived from the lower bound for the Z'-boson mass. For the bottom sector, at $v_R = 10$ TeV, M_B is 293.74 TeV as indicated by the black dot in Fig. 1(b). Using these v_R and M_B values, we further calculate the heavy bottom quark mass with Eq. (3.71) and obtain $m_{b'} = 293.82$ TeV. This result indicates that $m_{b'} \simeq M_B$.

From the above facts, the mass parameter of the top partner VLQ (M_T) is smaller than v_R but it could be larger or smaller than v_L depending on the other parameters. On the other hand, in the bottom sector, the mass parameter of the bottom partner VLQ (M_B) is significantly larger compared to v_R. This explains the mass hierarchy problem, where the smallness of the bottom

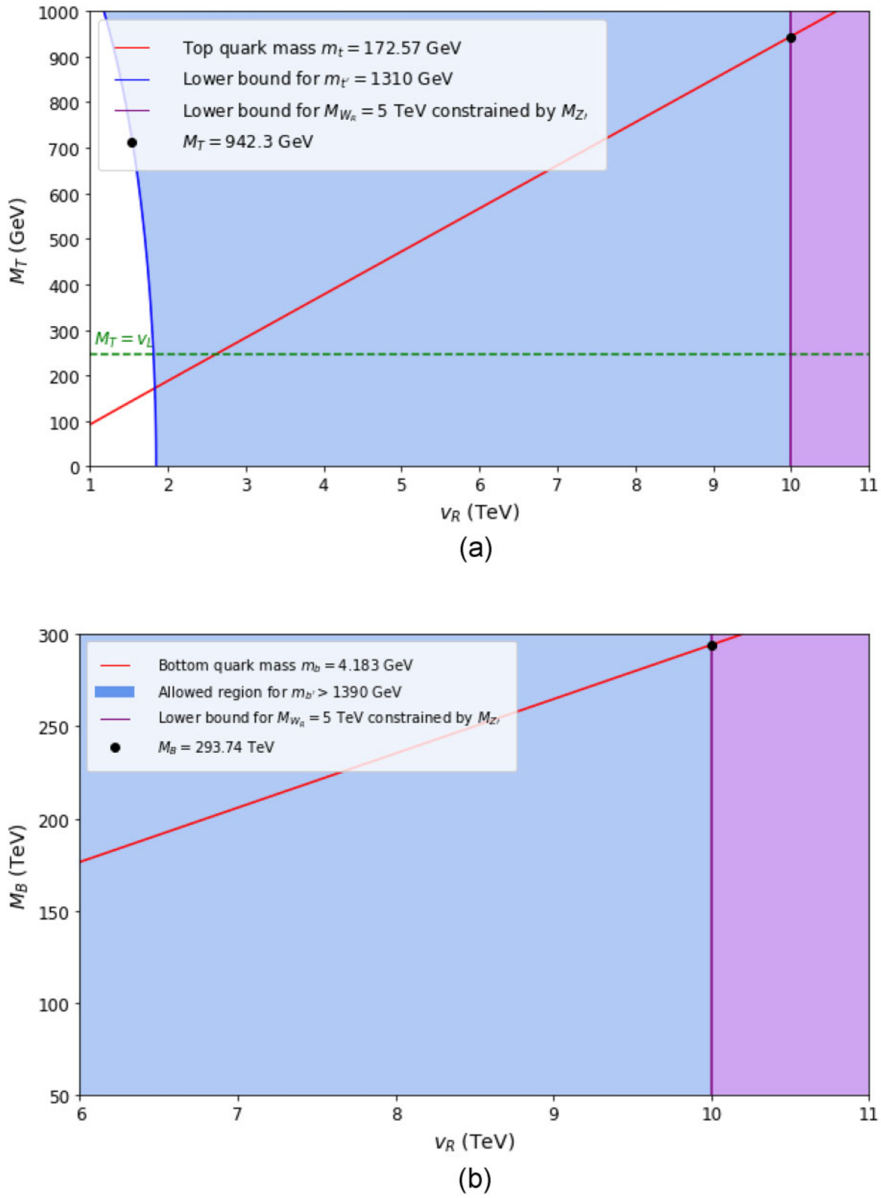

Fig. 1. Constraints on v_R and VLQ mass parameters of different sectors. (a) Top sector. (b) Bottom sector.

quark mass is suppressed by the large mass of the bottom VLQ through a seesaw mechanism. Mathematically, our choice of numerical input satisfies the following hierarchy: (1) for the top sector: $v_L < M_T < v_R$; (2) for the bottom sector: $v_L < v_R \ll M_B$.

One can compute the masses in the approximation form given in Eqs. (6.9), (6.10), (6.11), and (6.12) by using our choice of numerical input and obtain $m_t^{\text{approx}} = 172.58$ GeV, $m_{t'}^{\text{approx}} = 7.13$ TeV, $m_b^{\text{approx}} = 4.19$ GeV, and $m_{b'}^{\text{approx}} = 293.74$ TeV. These values are very close to the exact mass eigenvalues formula. We will use $v_R = 10$ TeV for the rest of our numerical analysis. This $v_R = 10$ TeV is also used in Ref. [22], although unlike this paper, they considered the model with left-right symmetry where $g_L = g_R$.

6.2. *Higgs FCNC*

In this subsection, we discuss the interaction between Higgs and quarks in our model. From Eq. (3.90), we extract the interactions between $h_L - h_R$ and quarks, given by

$$\begin{aligned}
\mathcal{L}_q \supset \mathcal{L}_{hH} = &-\frac{1}{v_L}\sum_{k,i=3}^{4}\Bigg[\left(\mathcal{Z}_{T_L} m_t^{\text{diag}}\right)^{ki}\overline{\left(\hat{u}_L^m\right)^k}\left(\hat{u}_R^m\right)^i + \left(m_t^{\text{diag}}\mathcal{Z}_{T_L}\right)^{ki}\overline{\left(\hat{u}_R^m\right)^k}\left(\hat{u}_L^m\right)^i \\
&+ \left(\mathcal{Z}_{B_L} m_b^{\text{diag}}\right)^{ki}\overline{\left(\hat{d}_L^m\right)^k}\left(\hat{d}_R^m\right)^i + \left(m_b^{\text{diag}}\mathcal{Z}_{B_L}\right)^{ki}\overline{\left(\hat{d}_R^m\right)^k}\left(\hat{d}_L^m\right)^i\Bigg] h_L \\
&- \frac{1}{v_R}\sum_{k,i=3}^{4}\Bigg[\left((1-\mathcal{Z}_{T_L}) m_t^{\text{diag}}\mathcal{Z}_{T_R}\right)^{ki}\overline{\left(\hat{u}_L^m\right)^k}\left(\hat{u}_R^m\right)^i \\
&+ \left(\mathcal{Z}_{T_R} m_t^{\text{diag}}(1-\mathcal{Z}_{T_L})\right)^{ki}\overline{\left(\hat{u}_R^m\right)^k}\left(\hat{u}_L^m\right)^i + \left((1-\mathcal{Z}_{B_L}) m_b^{\text{diag}}\mathcal{Z}_{B_R}\right)^{ki}\overline{\left(\hat{d}_L^m\right)^k}\left(\hat{d}_R^m\right)^i \\
&+ \left(\mathcal{Z}_{B_R} m_b^{\text{diag}}(1-\mathcal{Z}_{B_L})\right)^{ki}\overline{\left(\hat{d}_R^m\right)^k}\left(\hat{d}_L^m\right)^i\Bigg] h_R,
\end{aligned} \tag{6.13}$$

where $\mathcal{Z}_{T_L}$, $\mathcal{Z}_{B_L}$, $\mathcal{Z}_{T_R}$, $\mathcal{Z}_{B_R}$, m_t^{diag}, and m_b^{diag} are given in Eqs. (3.84), (3.85), (3.88), (3.89), (3.68), and (3.69), respectively. By transforming the $h_L - h_R$ basis into the $h - H$ mass eigenstate with Eq. (4.23), the Lagrangian in Eq. (6.13) transforms into

$$\begin{aligned}
\mathcal{L}_{hH} = &-\Bigg\{\frac{\cos\phi}{v_L}\sum_{k,i=3}^{4}\Bigg[\left(\mathcal{Z}_{T_L} m_t^{\text{diag}}\right)^{ki}\overline{\left(\hat{u}_L^m\right)^k}\left(\hat{u}_R^m\right)^i + \left(m_t^{\text{diag}}\mathcal{Z}_{T_L}\right)^{ki}\overline{\left(\hat{u}_R^m\right)^k}\left(\hat{u}_L^m\right)^i \\
&+ \left(\mathcal{Z}_{B_L} m_b^{\text{diag}}\right)^{ki}\overline{\left(\hat{d}_L^m\right)^k}\left(\hat{d}_R^m\right)^i + \left(m_b^{\text{diag}}\mathcal{Z}_{B_L}\right)^{ki}\overline{\left(\hat{d}_R^m\right)^k}\left(\hat{d}_L^m\right)^i\Bigg] \\
&- \frac{\sin\phi}{v_R}\sum_{k,i=3}^{4}\Bigg[\left((1-\mathcal{Z}_{T_L}) m_t^{\text{diag}}\mathcal{Z}_{T_R}\right)^{ki}\overline{\left(\hat{u}_L^m\right)^k}\left(\hat{u}_R^m\right)^i + \left(\mathcal{Z}_{T_R} m_t^{\text{diag}}(1-\mathcal{Z}_{T_L})\right)^{ki}\overline{\left(\hat{u}_R^m\right)^k}\left(\hat{u}_L^m\right)^i \\
&+ \left((1-\mathcal{Z}_{B_L}) m_b^{\text{diag}}\mathcal{Z}_{B_R}\right)^{ki}\overline{\left(\hat{d}_L^m\right)^k}\left(\hat{d}_R^m\right)^i + \left(\mathcal{Z}_{B_R} m_b^{\text{diag}}(1-\mathcal{Z}_{B_L})\right)^{ki}\overline{\left(\hat{d}_R^m\right)^k}\left(\hat{d}_L^m\right)^i\Bigg]\Bigg\} h \\
&- \Bigg\{\frac{\sin\phi}{v_L}\sum_{k,i=3}^{4}\Bigg[\left(\mathcal{Z}_{T_L} m_t^{\text{diag}}\right)^{ki}\overline{\left(\hat{u}_L^m\right)^k}\left(\hat{u}_R^m\right)^i + \left(m_t^{\text{diag}}\mathcal{Z}_{T_L}\right)^{ki}\overline{\left(\hat{u}_R^m\right)^k}\left(\hat{u}_L^m\right)^i \\
&+ \left(\mathcal{Z}_{B_L} m_b^{\text{diag}}\right)^{ki}\overline{\left(\hat{d}_L^m\right)^k}\left(\hat{d}_R^m\right)^i + \left(m_b^{\text{diag}}\mathcal{Z}_{B_L}\right)^{ki}\overline{\left(\hat{d}_R^m\right)^k}\left(\hat{d}_L^m\right)^i\Bigg] \\
&+ \frac{\cos\phi}{v_R}\sum_{k,i=3}^{4}\Bigg[\left((1-\mathcal{Z}_{T_L}) m_t^{\text{diag}}\mathcal{Z}_{T_R}\right)^{ki}\overline{\left(\hat{u}_L^m\right)^k}\left(\hat{u}_R^m\right)^i + \left(\mathcal{Z}_{T_R} m_t^{\text{diag}}(1-\mathcal{Z}_{T_L})\right)^{ki}\overline{\left(\hat{u}_R^m\right)^k}\left(\hat{u}_L^m\right)^i \\
&+ \left((1-\mathcal{Z}_{B_L}) m_b^{\text{diag}}\mathcal{Z}_{B_R}\right)^{ki}\overline{\left(\hat{d}_L^m\right)^k}\left(\hat{d}_R^m\right)^i + \left(\mathcal{Z}_{B_R} m_b^{\text{diag}}(1-\mathcal{Z}_{B_L})\right)^{ki}\overline{\left(\hat{d}_R^m\right)^k}\left(\hat{d}_L^m\right)^i\Bigg]\Bigg\} H,
\end{aligned} \tag{6.14}$$

where h and H denote the Higgs and the heavy Higgs bosons, respectively. In this discussion, we will focus on the interaction of the Higgs boson with the quarks in our model.

6.2.1. *Top sector.* We collect the interaction terms between the Higgs boson and the top quark (t) and heavy top quark (t') from Eq. (6.14):

$$\begin{aligned}\mathcal{L}_{hH} \supset \mathcal{L}_{ht} = &-\left[\frac{\cos\phi}{v_L}\cos^2\phi_{T_L} m_t - \frac{\sin\phi}{v_R}\Big(\sin^2\phi_{T_L}\cos^2\beta_{T_R} m_t\right.\\ &\left.-\sin\phi_{T_L}\cos\phi_{T_L}\sin\beta_{T_R}\cos\beta_{T_R} m_{t'}\Big)\right]\bar{t}th\\ &+\left[\frac{\cos\phi}{v_L}\sin\phi_{T_L}\cos\phi_{T_L} m_{t'} + \frac{\sin\phi}{v_R}\Big(\sin\phi_{T_L}\cos\phi_{T_L}\sin^2\beta_{T_R} m_{t'}\right.\\ &\left.-\sin^2\phi_{T_L}\sin\beta_{T_R}\cos\beta_{T_R} m_t\Big)\right]\left(\bar{t}_L t'_R + \bar{t}'_R t_L\right)h\\ &+\left[\frac{\cos\phi}{v_L}\sin\phi_{T_L}\cos\phi_{T_L} m_t + \frac{\sin\phi}{v_R}\Big(\sin\phi_{T_L}\cos\phi_{T_L}\cos^2\beta_{T_R} m_t\right.\\ &\left.-\cos^2\phi_{T_L}\sin\beta_{T_R}\cos\beta_{T_R} m_{t'}\Big)\right]\left(\bar{t}'_L t_R + \bar{t}_R t'_L\right)h\\ &-\left[\frac{\cos\phi}{v_L}\sin^2\phi_{T_L} m_{t'} - \frac{\sin\phi}{v_R}\Big(\cos^2\phi_{T_L}\sin^2\beta_{T_R} m_{t'}\right.\\ &\left.-\sin\phi_{T_L}\cos\phi_{T_L}\sin\beta_{T_R}\cos\beta_{T_R} m_t\Big)\right]\bar{t}'t'h,\end{aligned} \tag{6.15}$$

where we substitute the elements of $\mathcal{Z}_{T_L}$ and $\mathcal{Z}_{T_R}$ in Eqs. (3.84) and (3.88), respectively. Then, we take the approximations for the mixing angles in Eqs. (C.26) and (D.13). In addition, using the hierarchy in the top sector, i.e. $v_L < M_T < v_R$, and the approximation of mixing angle ϕ in Eq. (4.31), we obtain the interaction between the Higgs and top-sector quarks as follows:

$$\begin{aligned}\mathcal{L}_{ht} \simeq &-\cos\phi\frac{m_t}{v_L}\left(1-\frac{\lambda_{LR}}{\lambda_R}\frac{M_T^2}{m_{u_R}^2}\frac{v_L^2}{v_R^2}\right)\bar{t}th - \cos\phi\frac{M_T}{m_{u_R}}\left(1+\frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)\left(\bar{t}_L t'_R + \bar{t}'_R t_L\right)h\\ &-\cos\phi\frac{M_T}{m_{u_R}}\frac{v_L}{v_R}\left(1+\frac{\lambda_{LR}}{\lambda_R}\right)\left(\bar{t}'_L t_R + \bar{t}_R t'_L\right)h - \cos\phi\frac{m_{t'}}{v_R}\frac{v_L}{v_R}\left(\frac{M_T^2}{m_{u_R}^2}-\frac{\lambda_{LR}}{\lambda_R}\right)\bar{t}'t'h.\end{aligned} \tag{6.16}$$

In this expression, we also assume that $Y^3_{u_L} \simeq Y^3_{u_R} \simeq 1$. From Eq. (6.16) we extract some useful information regarding our model: the Higgs and top quark pairs coupling receives a small correction, while the Higgs and heavy top quark pairs coupling receives an overall suppression of $\mathcal{O}(v_L/v_R)$. Another important point is the tree-level FCNC interaction is suppressed. The Higgs FCNC of $\bar{t}'_L t_R$ and $\bar{t}_R t'_L$ type is more suppressed by a factor $\mathcal{O}(v_L/v_R)$ compared to the $\bar{t}_L t'_R$ and $\bar{t}'_R t_L$ type.

6.2.2. *Bottom sector.* In the same way, from Eq. (6.14) we collect the interactions between the Higgs boson and the bottom quark (b) and heavy bottom quark (b'). By expressing $\mathcal{Z}_{B_L}$ and $\mathcal{Z}_{B_R}$ in terms of their elements, we obtain

$$\begin{aligned}\mathcal{L}_{hH} \supset \mathcal{L}_{hb} = &-\Bigg[\frac{\cos\phi}{v_L}\cos^2\phi_{B_L} m_b - \frac{\sin\phi}{v_R}\Big(\sin^2\phi_{B_L}\cos^2\beta_{B_R} m_b \\ &- \sin\phi_{B_L}\cos\phi_{B_L}\sin\beta_{B_R}\cos\beta_{B_R} m_{b'}\Big)\Bigg]\bar{b}bh \\ &+\Bigg[\frac{\cos\phi}{v_L}\sin\phi_{B_L}\cos\phi_{B_L} m_{b'} + \frac{\sin\phi}{v_R}\Big(\sin\phi_{B_L}\cos\phi_{B_L}\sin^2\beta_{B_R} m_{b'} \\ &- \sin^2\phi_{B_L}\sin\beta_{B_R}\cos\beta_{B_R} m_b\Big)\Bigg]\Big(\bar{b}_L b'_R + \text{h.c.}\Big)h \\ &+\Bigg[\frac{\cos\phi}{v_L}\sin\phi_{B_L}\cos\phi_{B_L} m_b + \frac{\sin\phi}{v_R}\Big(\sin\phi_{B_L}\cos\phi_{B_L}\cos^2\beta_{B_R} m_b \\ &- \cos^2\phi_{B_L}\sin\beta_{B_R}\cos\beta_{B_R} m_{b'}\Big)\Bigg]\Big(\bar{b}'_L b_R + \text{h.c.}\Big)h \\ &-\Bigg[\frac{\cos\phi}{v_L}\sin^2\phi_{B_L} m_{b'} - \frac{\sin\phi}{v_R}\Big(\cos^2\phi_{B_L}\sin^2\beta_{B_R} m_{b'} \\ &- \sin\phi_{B_L}\cos\phi_{B_L}\sin\beta_{B_R}\cos\beta_{B_R} m_b\Big)\Bigg]\bar{b}'b'h. \end{aligned} \tag{6.17}$$

We use the approximations for the mixing angles in Eqs. (C.26), (D.13), and (4.31). In addition, by using the hierarchy in the bottom sector $v_L < v_R \ll M_B$, we obtain the interaction between the Higgs and bottom-sector quarks as follows:

$$\begin{aligned}\mathcal{L}_{hb} \simeq &-\cos\phi\frac{m_b}{v_L}\left(1 - \frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{v_R^2}\right)\bar{b}bh - \cos\phi\frac{m_b m_{b'}}{m_{d_L} m_{d_R}}\left(1 + \frac{\lambda_{LR}}{\lambda_R}\frac{v_L^2}{M_B^2}\right)\Big(\bar{b}_L b'_R + \bar{b}'_R b_L\Big)h \\ &- \frac{v_L}{v_R}\left(\frac{\lambda_{LR}}{\lambda_R} + \frac{v_R^2}{M_B^2}\right)\Big(\bar{b}'_L b_R + \bar{b}_R b'_L\Big)h - \cos\phi\frac{m_{d_L}}{m_{b'}}\left(1 - \frac{\lambda_{LR}}{\lambda_R}\right)\bar{b}'b'h.\end{aligned} \tag{6.18}$$

Similarly to the top sector, the interaction between the Higgs and the bottom quark pairs receives a small correction compared to the SM. The interaction between the Higgs and the heavy bottom quark pairs is suppressed by a factor $\mathcal{O}(v_L/M_B)$. The Higgs FCNC of $\bar{b}'_L b_R$ and $\bar{b}_R b'_L$ type is suppressed by a factor $\mathcal{O}(v_L/v_R)$. On the other hand, the Higgs FCNC of $\bar{b}_L b'_R$ and $\bar{b}'_R b_L$ type is not suppressed. This is because we assume $Y^3_{d_L} \simeq 1$.

6.3. *Z FCNC*

In this subsection we discuss interaction between the Z-boson and quarks. We begin by extracting the interaction terms between $Z_L - Z_R$ and quarks from Eq. (3.90), which reads as

$$\begin{aligned}\mathcal{L}_q \supset \mathcal{L}_{ZZ'} = &-\left[\frac{g_L}{2\cos\theta_W}\left(j^\mu_{3L}\right) - e\tan\theta_W\left(j^\mu_{\text{em}}\right)\right]Z_{L\mu} \\ &-\left[\frac{g_R}{2\cos\theta_R}\left(j^\mu_{3R}\right) - g'\tan\theta_R\left(j^\mu_{\text{em}} - \frac{1}{2}\left(j^\mu_{3L}\right)\right)\right]Z_{R\mu}.\end{aligned} \tag{6.19}$$

Here j_{3L}^{μ}, j_{3R}^{μ}, and $j_{\rm em}^{\mu}$ are defined in Eqs. (3.92–3.94), respectively. Next, we change the basis from the $Z_L - Z_R$ basis to the $Z - Z'$ basis using Eq. (4.14), and it leads to

$$\begin{aligned}\mathcal{L}_{ZZ'} = &-\left[\frac{1}{2\cos\theta_W}(g_L\cos\theta - e\tan\theta_R\sin\theta)j_{3L}^{\mu} - \frac{g_R\sin\theta}{2\cos\theta_R}j_{3R}^{\mu}\right.\\ &\left.-\frac{e}{\cos\theta_W}(\sin\theta_W\cos\theta - \tan\theta_R\sin\theta)j_{\rm em}^{\mu}\right]Z_\mu\\ &-\left[\frac{1}{2\cos\theta_W}(g_L\sin\theta + e\tan\theta_R\cos\theta)j_{3L}^{\mu} + \frac{g_R\cos\theta}{2\cos\theta_R}j_{3R}^{\mu}\right.\\ &\left.-\frac{e}{\cos\theta_W}(\sin\theta_W\sin\theta + \tan\theta_R\cos\theta)j_{\rm em}^{\mu}\right]Z'_\mu. \end{aligned} \tag{6.20}$$

In this discussion, we will focus on the interaction between the SM Z-boson and quarks. We expressed the Z-boson interaction in terms of vector and axial-vector couplings as follows:

$$\begin{aligned}\mathcal{L}_{ZZ'} \supset \mathcal{L}_{\bar{q}q}^{Z} = &-\frac{g_L}{2\cos\theta_W}\sum_{\alpha,\beta=1}^{4}\overline{(\hat{u}^m)^\alpha}\gamma^\mu\left[(g_V)_u^{\alpha\beta} - (g_A)_u^{\alpha\beta}\gamma^5\right](\hat{u}^m)^\beta Z_\mu\\ &-\frac{g_L}{2\cos\theta_W}\sum_{\alpha,\beta=1}^{4}\overline{(\hat{d}^m)^\alpha}\gamma^\mu\left[(g_V)_d^{\alpha\beta} - (g_A)_d^{\alpha\beta}\gamma^5\right](\hat{d}^m)^\beta Z_\mu, \end{aligned} \tag{6.21}$$

where

$$(g_V)_u^{\alpha\beta} = \frac{1}{2}\left((\kappa_{T_L})^{\alpha\beta} - (\kappa_{T_R})^{\alpha\beta}\right) - 2\kappa Q_u\delta^{\alpha\beta}, \tag{6.22}$$

$$(g_A)_u^{\alpha\beta} = \frac{1}{2}\left((\kappa_{T_L})^{\alpha\beta} + (\kappa_{T_R})^{\alpha\beta}\right), \tag{6.23}$$

$$(g_V)_d^{\alpha\beta} = -\frac{1}{2}\left((\kappa_{B_L})^{\alpha\beta} - (\kappa_{B_R})^{\alpha\beta}\right) - 2\kappa Q_d\delta^{\alpha\beta}, \tag{6.24}$$

$$(g_A)_d^{\alpha\beta} = -\frac{1}{2}\left((\kappa_{B_L})^{\alpha\beta} + (\kappa_{B_R})^{\alpha\beta}\right), \tag{6.25}$$

$$(\kappa_{T_L})^{\alpha\beta} = (\cos\theta - \sin\theta_W\tan\theta_R\sin\theta)\left(\mathcal{Z}_{T_L}^{\rm all}\right)^{\alpha\beta}, \tag{6.26}$$

$$(\kappa_{T_R})^{\alpha\beta} = \frac{\sin\theta_W\sin\theta}{\sin\theta_R\cos\theta_R}\left(\mathcal{Z}_{T_R}^{\rm all}\right)^{\alpha\beta}, \tag{6.27}$$

$$(\kappa_{B_L})^{\alpha\beta} = (\cos\theta - \sin\theta_W\tan\theta_R\sin\theta)\left(\mathcal{Z}_{B_L}^{\rm all}\right)^{\alpha\beta}, \tag{6.28}$$

$$(\kappa_{B_R})^{\alpha\beta} = \frac{\sin\theta_W\sin\theta}{\sin\theta_R\cos\theta_R}\left(\mathcal{Z}_{B_R}^{\rm all}\right)^{\alpha\beta}, \tag{6.29}$$

$$\kappa = \sin^2\theta_W\cos\theta - \sin\theta_W\tan\theta_R\sin\theta. \tag{6.30}$$

The matrix forms of 4×4 unitary matrices $\mathcal{Z}_{T_L}^{\text{all}}$, $\mathcal{Z}_{B_L}^{\text{all}}$, $\mathcal{Z}_{T_R}^{\text{all}}$, and $\mathcal{Z}_{B_R}^{\text{all}}$ are given as follows:

$$\mathcal{Z}_{T_L}^{\text{all}} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & \mathcal{Z}_{T_L} \end{pmatrix}, \quad \mathcal{Z}_{T_R}^{\text{all}} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & \mathcal{Z}_{T_R} \end{pmatrix},$$
$$\mathcal{Z}_{B_L}^{\text{all}} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & \mathcal{Z}_{B_L} \end{pmatrix}, \quad \mathcal{Z}_{B_R}^{\text{all}} = \begin{pmatrix} I_2 & 0_2 \\ 0_2 & \mathcal{Z}_{B_R} \end{pmatrix}, \tag{6.31}$$

where I_2 and 0_2 are the 2×2 unit matrix and zero matrix, respectively. The 2×2 submatrices $\mathcal{Z}_{T_L}$, $\mathcal{Z}_{B_L}$, $\mathcal{Z}_{T_R}$, and $\mathcal{Z}_{B_R}$ are given in Eqs. (3.84), (3.85), (3.88), and (3.89), respectively. $Q_u = 2/3$, $Q_d = -1/3$ are the electric charge of up-type and down-type quarks, respectively.

6.3.1. *Up sector.* In this part, we analyze the interaction between the Z-boson and the up sector in our model. From Eq. (6.21), it reads as

$$\begin{aligned} \mathcal{L}_{\bar{q}q}^{Z} \supset \mathcal{L}_{t}^{Z} = -\frac{g_L}{2\cos\theta_W} \Big\{ & \overline{(\hat{u}^m)^1}\gamma^\mu\Big[(g_V)_u^{11} - (g_A)_u^{11}\gamma^5\Big](\hat{u}^m)^1 \\ & + \overline{(\hat{u}^m)^2}\gamma^\mu\Big[(g_V)_u^{22} - (g_A)_u^{22}\gamma^5\Big](\hat{u}^m)^2 + \bar{t}\gamma^\mu\Big[(g_V)_u^{33} - (g_A)_u^{33}\gamma^5\Big]t \\ & + \bar{t}\gamma^\mu\Big[(g_V)_u^{34} - (g_A)_u^{34}\gamma^5\Big]t' + \overline{t'}\gamma^\mu\Big[(g_V)_u^{43} - (g_A)_u^{43}\gamma^5\Big]t \\ & + \overline{t'}\gamma^\mu\Big[(g_V)_u^{44} - (g_A)_u^{44}\gamma^5\Big]t'\Big\} Z_\mu, \end{aligned} \tag{6.32}$$

where the vector coupling $(g_V)_u$ and axial-vector coupling $(g_A)_u$ are defined in Eqs. (6.22) and (6.23), respectively. By using the definitions of κ_{T_L}, κ_{T_R}, and κ which are written in Eqs. (6.26), (6.27), and (6.30), we obtain

$$(\kappa_{T_L})^{11} = (\kappa_{T_L})^{22} = \cos\theta\left(1 - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right), \tag{6.33}$$

$$(\kappa_{T_R})^{11} = (\kappa_{T_R})^{22} = \frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right), \tag{6.34}$$

$$(\kappa_{T_L})^{33} = \cos\theta\left(1 - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right), \tag{6.35}$$

$$(\kappa_{T_R})^{33} = \frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\frac{M_T^2}{m_{u_R}^2}, \tag{6.36}$$

$$(\kappa_{T_L})^{34} = (\kappa_{T_L})^{43} = \cos\theta\left(1 - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right)\frac{m_{u_L} M_T}{m_{u_R}^2}, \tag{6.37}$$

$$(\kappa_{T_R})^{34} = (\kappa_{T_R})^{43} = -\frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\frac{M_T}{m_{u_R}}, \tag{6.38}$$

$$(\kappa_{T_L})^{44} = \cos\theta\left(1 - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right)\frac{m_{u_L}^2 M_T^2}{m_{u_R}^4}, \tag{6.39}$$

$$(\kappa_{T_R})^{44} = \frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right), \tag{6.40}$$

$$\kappa = \cos\theta\left(\sin^2\theta_W - \sin\theta_W \tan\theta_R \mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right). \tag{6.41}$$

Here we write the suppression coming from the small mixing angle θ as $\mathcal{O}(v_L^2/v_R^2)$. The exact form of the mixing angle θ is given in Eq. (4.19). From Eqs. (6.37) and (6.38), the κ_{T_L} and κ_{T_R} terms related to the Z-boson FCNC process with the top and heavy top quarks are suppressed by $\mathcal{O}(v_L M_T/v_R^2)$ and $\mathcal{O}(v_L^2 M_T/v_R^3)$, respectively. This indicates that the Z-mediated FCNC process in the up sector is suppressed within our model. In addition, the interaction between the Z-boson and heavy top quark is also suppressed. Moreover, the deviation of the SM-like terms in $(\kappa_{T_L})^{ii}$ and κ, with $i \in \{1, 2, 3\}$ are suppressed by a factor $\mathcal{O}(v_L^2/v_R^2)$.

6.3.2. *Down sector.* In this part, we analyze the interaction between the Z-boson and the down sector in our model. From Eq. (6.21), we extract

$$\begin{aligned}\mathcal{L}^Z_{\bar{q}q} \supset \mathcal{L}^Z_b = -\frac{g_L}{2\cos\theta_W}\Bigg\{&\overline{(\hat{d}^m)^1}\gamma^\mu\Big[(g_V)^{11}_d - (g_A)^{11}_d\gamma^5\Big](\hat{d}^m)^1 \\ &+ \overline{(\hat{d}^m)^2}\gamma^\mu\Big[(g_V)^{22}_d - (g_A)^{22}_d\gamma^5\Big](\hat{d}^m)^2 + \bar{b}\gamma^\mu\Big[(g_V)^{33}_d - (g_A)^{33}_d\gamma^5\Big]b \\ &+ \bar{b}\gamma^\mu\Big[(g_V)^{34}_d - (g_A)^{34}_d\gamma^5\Big]b' + \overline{b'}\gamma^\mu\Big[(g_V)^{43}_d - (g_A)^{43}_d\gamma^5\Big]b \\ &+ \overline{b'}\gamma^\mu\Big[(g_V)^{44}_d - (g_A)^{44}_d\gamma^5\Big]b'\Bigg\}Z_\mu, \end{aligned} \tag{6.42}$$

where the vector coupling $(g_V)_d$ and axial-vector coupling $(g_A)_d$ are defined in Eqs. (6.24) and (6.25), respectively. By using the definitions of κ_{B_L}, κ_{B_R}, and κ written in Eqs. (6.28), (6.29), and (6.30) respectively, we obtain

$$(\kappa_{B_L})^{11} = (\kappa_{B_L})^{22} = \cos\theta\left(1 - \sin\theta_W\tan\theta_R\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right), \tag{6.43}$$

$$(\kappa_{B_R})^{11} = (\kappa_{B_R})^{22} = \frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right), \tag{6.44}$$

$$(\kappa_{B_L})^{33} = \cos\theta\left(1 - \sin\theta_W\tan\theta_R\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right), \tag{6.45}$$

$$(\kappa_{B_R})^{33} = \frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right), \tag{6.46}$$

$$(\kappa_{B_L})^{34} = (\kappa_{B_L})^{43} = \cos\theta\left(1 - \sin\theta_W\tan\theta_R\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right)\frac{m_{d_L}}{M_B}, \tag{6.47}$$

$$(\kappa_{B_R})^{34} = (\kappa_{B_R})^{43} = -\frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\frac{m_{d_R}}{M_B}, \tag{6.48}$$

$$(\kappa_{B_L})^{44} = \cos\theta\left(1 - \sin\theta_W\tan\theta_R\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right)\frac{m^2_{d_L}}{M^2_B}, \tag{6.49}$$

$$(\kappa_{B_R})^{44} = \frac{\sin\theta_W\cos\theta}{\sin\theta_R\cos\theta_R}\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\frac{m^2_{d_R}}{M^2_B}, \tag{6.50}$$

$$\kappa = \cos\theta\left(\sin^2\theta_W - \sin\theta_W\tan\theta_R\mathcal{O}\left(\frac{v_L^2}{v_R^2}\right)\right). \tag{6.51}$$

The FCNC process in the down sector is suppressed, similarly to the up sector. As shown in Eqs. (6.47) and (6.48), the κ_{B_L} and κ_{B_R} terms are suppressed by a factor $\mathcal{O}(v_L/M_B)$ and $\mathcal{O}(v_L^2/v_R M_B)$, respectively. In addition, the interaction between the Z-boson and heavy bottom quark is also suppressed. Furthermore, the deviation of the SM-like terms in $(\kappa_{B_L})^{ii}$ and κ, with $i \in \{1, 2, 3\}$ is suppressed by a factor $\mathcal{O}(v_L^2/v_R^2)$.

7. Conclusion

We have presented a systematic analysis of the quark sector in the universal seesaw model. We derived the Lagrangian of the model, including the quark sector, Higgs sector, and kinetic terms of the gauge fields. We start by writing the Lagrangian which is invariant under $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_{Y'}}$. After the $\mathrm{SU(2)_R}$ Higgs doublet acquires nonzero vev, we obtain the Lagrangian, which is invariant under SM gauge symmetry. Furthermore, the SM gauge group is broken into $\mathrm{U(1)_{em}}$ after the $\mathrm{SU(2)_L}$ Higgs doublet acquires nonzero vev. In the gauge interactions sector, we classify the terms based on the number of fields, such as linear, quadratic, cubic, and quartic interactions. In addition, we found that the massless Nambu–Goldstone bosons are mixed to become new states χ_Z and $\chi_{Z'}$. We have shown clearly that χ_Z and $\chi_{Z'}$ become the longitudinal components of the massive Z- and Z'-bosons, respectively.

Our model focuses on the third family of the quark sector. Within this framework we explain the hierarchy between the top and bottom quark masses by mixing with the heavy VLQs. We use the direct measurement of the top quark mass and the running mass of the bottom quark. Additionally, the lower bounds on the heavy top and heavy bottom quark masses also serve as constraints. The lower mass limit of the Z'-boson, linked to the W_R-boson mass, also imposes a stringent constraint on v_R. By setting g_R and the Yukawa couplings equal to 1, the lower limit of v_R is 10 TeV in this model. We obtained that the heavy top quark mass is in the order of v_R ($m_{t'} = 7.13$ TeV) and the heavy bottom mass is in the order of M_B ($m_{b'} = 293.82$ TeV). We confirmed that the hierarchy of VLQ's mass parameters, v_L, and v_R in our model is $v_L < M_T < v_R \ll M_B$.

Moreover, the presence of VLQs in the model induces the FCNC at the tree level. In the SM, the FCNC process is highly suppressed and only occurs at the loop level due to the Glashow–Iliopoulos–Maiani (GIM) mechanism. In our model, we have shown that the Z-boson-mediated FCNC process is suppressed for both (up and down) sectors. The deviation from the SM values is suppressed by $\mathcal{O}(v_L^2/v_R^2)$, which comes from the small mixture in the lighter mass eigenstate Z from Z_R. On the other hand, Higgs-mediated FCNCs of $\bar{b}_L b'_R$ and $\bar{b}'_R b_L$ types are not suppressed when $Y_{d_L}^3 \simeq 1$.

Acknowledgements

We would like to thank Apriadi Salim Adam, Yuta Kawamura, Yusuke Shimizu, Hironori Takei, and Kei Yamamoto for the useful comment and discussion.

Funding

Open Access funding: SCOAP[3]

Appendix A. Weak-basis of Yukawa interaction

In this appendix, we show how to obtain the Yukawa interaction that is written in Eq. (2.3). We start from the general Yukawa interaction terms,

$$\begin{aligned}\mathcal{L}_{\rm YM} = &-\overline{q^i_L} y^i_{u_L} \tilde{\phi}_L T_R - \overline{T_L} y^{i*}_{u_R} \tilde{\phi}^\dagger_R q^i_R - \overline{T_L} M_T T_R - \text{h.c.} \\ &- \overline{q^i_L} y^i_{d_L} \phi_L B_R - \overline{B_L} y^{i*}_{d_R} \phi^\dagger_R q^i_R - \overline{B_L} M_B B_R - \text{h.c.}.\end{aligned} \tag{A.1}$$

The Yukawa couplings are general complex vectors in $\mathbb{C}^3$ with the following parameterization:

$$y^i_{u_{L(R)}} = \mathbf{y}_{u_{L(R)}} = \begin{pmatrix} \sin\theta^u_{L(R)} \sin\phi^u_{L(R)} e^{i\alpha^1_{u_{L(R)}}} \\ \sin\theta^u_{L(R)} \cos\phi^u_{L(R)} e^{i\alpha^2_{u_{L(R)}}} \\ \cos\theta^u_{L(R)} e^{i\alpha^3_{u_{L(R)}}} \end{pmatrix} Y^3_{u_{L(R)}}, \tag{A.2}$$

$$y^i_{d_{L(R)}} = \mathbf{y}_{d_{L(R)}} = \begin{pmatrix} \sin\theta^d_{L(R)} \sin\phi^d_{L(R)} e^{i\alpha^1_{d_{L(R)}}} \\ \sin\theta^d_{L(R)} \cos\phi^d_{L(R)} e^{i\alpha^2_{d_{L(R)}}} \\ \cos\theta^d_{L(R)} e^{i\alpha^3_{d_{L(R)}}} \end{pmatrix} Y^3_{d_{L(R)}}, \tag{A.3}$$

where $Y^3_{u_{L(R)}}$ and $Y^3_{d_{L(R)}}$ are real positive numbers. Define the following weak-basis transformations (WBTs) as follows:

$$(q'_L)^i = e^{-i\alpha^i_{u_L}} q^i_L, \tag{A.4}$$

$$(q'_R)^i = e^{-i\alpha^i_{u_R}} q^i_R. \tag{A.5}$$

Applying this WBT into Eq. (A.1), we obtain

$$\begin{aligned}\mathcal{L}_{\rm YM} = &-\overline{(q'_L)}^i \left(y'_{u_L}\right)^i \tilde{\phi}_L T_R - \overline{T_L} \left(y'_{u_R}\right)^{i*} \tilde{\phi}^\dagger_R \left(q'_R\right)^i - \overline{T_L} M_T T_R - \text{h.c.} \\ &- \overline{(q'_L)}^i y^i_{d_L} \phi_L B_R - \overline{B_L} y^{i*}_{d_R} \phi^\dagger_R \left(q'_R\right)^i - \overline{B_L} M_B B_R - \text{h.c.},\end{aligned} \tag{A.6}$$

where

$$\left(y'_{u_L}\right)^i = y^i_{u_L} e^{-i\alpha^i_{u_L}}, \tag{A.7}$$

$$\left(y'_{u_R}\right)^i = y^i_{u_R} e^{-i\alpha^i_{u_R}} \tag{A.8}$$

are real vectors. On the other hand, $y^i_{d_L}$ and $y^i_{d_R}$ remain complex vectors with the redefined phases.

Next we write the $(y'_{u_L})^i$ Yukawa coupling explained above as,

$$\begin{aligned}(y'_{u_L})^i &= \begin{pmatrix} \sin\theta^u_L \sin\phi^u_L \\ \sin\theta^u_L \cos\phi^u_L \\ \cos\theta^u_L \end{pmatrix} Y^3_{u_L} \\ &= \mathbf{e}^u_{L_3} Y^3_{u_L}\end{aligned} \tag{A.9}$$

and define another WBT,

$$\left(q'_L\right)^i = (V_{u_L})^{ij} \left(q''_L\right)^j, \tag{A.10}$$

where in general V_{u_L} is a 3×3 unitary matrix formed by three orthonormal vectors with the third column chosen as $\mathbf{e}^u_{L_3}$ in Eq. (A.9),

$$V_{u_L} = \left(\mathbf{e}^u_{L_1}\ \mathbf{e}^u_{L_2}\ \mathbf{e}^u_{L_3}\right), \tag{A.11}$$

which leads to the product $(V^\dagger_{u_L})^{ji}(y'_{u_L})^i = \delta^{j3} Y^3_{u_L}$.

For the $(y'_{u_R})^i$ Yukawa coupling, it can be derived similarly by changing $L \to R$ in Eqs. (A.9–A.11). For the down sector, the product of Eq. (A.11) and the down-type Yukawa coupling yields down-type Yukawa coupling on another basis. For example, $(V^\dagger_{u_L})^{ji}(y_{d_L})^i = (y''_{d_L})^j$. Therefore, the Lagrangian in Eq. (A.6) becomes

$$\begin{aligned}\mathcal{L}_{\rm YM} = &-Y^3_{u_L}\overline{(q''_L)^3}\tilde{\phi}_L T_R - Y^3_{u_R}\overline{T_L}\tilde{\phi}^\dagger_R (q''_R)^3 - \overline{T_L}M_T T_R - \text{h.c.}\\ &-\overline{(q''_L)^i}\,(y''_{d_L})^i\,\phi_L B_R - \overline{B_L}\,(y''_{d_R})^{i*}\,\phi^\dagger_R\,(q''_R)^i - \overline{B_L}M_B B_R - \text{h.c.},\end{aligned} \tag{A.12}$$

and it has the form such that the Yukawa couplings of the up-type quark doublet ($Y^3_{u_L}$ and $Y^3_{u_R}$) are given by real positive numbers while the Yukawa couplings of the down-type quark are general complex vectors as written in Eq. (2.3).

Appendix B. Parameterization of V_{d_R} and V_{d_L}

In this appendix, we explain in more detail how to parameterize and remove the unphysical phases of V_{d_R} and V_{d_L}. Both V_{d_R} and V_{d_L} have the following form:

$$V = \begin{pmatrix}\mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3\end{pmatrix}, \tag{B.1}$$

where the third column is related to either y_{d_R} or y_{d_L} and is parameterized by,

$$\mathbf{v}_3 = \begin{pmatrix}\sin\theta\sin\phi e^{i\alpha_1}\\ \sin\theta\cos\phi e^{i\alpha_2}\\ \cos\theta e^{i\alpha_3}\end{pmatrix}. \tag{B.2}$$

Since V is a unitary matrix, the column vectors satisfy $\mathbf{v}^\dagger_\mathbf{i}\cdot\mathbf{v}_\mathbf{j} = \delta_{ij}$ and V has a matrix form as follows:

$$V = (\alpha_1,\alpha_2,\alpha_3)R_{12}(\phi)R_{23}(\theta)(0,\delta,0)R_{12}(\alpha)(\rho,\sigma,0), \tag{B.3}$$

where $(\alpha_1,\alpha_2,\alpha_3) = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$; $(0,\delta,0) = \text{diag}(1, e^{i\delta}, 1)$;$(\rho,\sigma,0) = \text{diag}(e^{i\rho}, e^{i\sigma}, 1)$; and

$$R_{12}(\phi) = \begin{pmatrix}\cos\phi & \sin\phi & 0\\ -\sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{pmatrix},\quad R_{23}(\theta) = \begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{pmatrix},$$
$$R_{12}(\alpha) = \begin{pmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}. \tag{B.4}$$

We have the freedom to rotate V by U(2) transformations from both sides. As shown in Eqs. (3.18) and (3.57), we can remove the unphysical phases and angles in Eq. (B.3) by following

$$\widetilde{V} = \widetilde{U}^\dagger V \widetilde{W}, \tag{B.5}$$

where $\widetilde{U}$ and $\widetilde{W}$ are 3×3 unitary matrices which have the following expressions:

$$\begin{aligned}\widetilde{U}^\dagger &= \left(0, \frac{\alpha_3}{2}, 0\right) R^{-1}_{12}(\phi)(-\alpha_1, -\alpha_2, 0),\\ \widetilde{W} &= (-\rho, -\sigma, 0)R^{-1}_{12}(\alpha)(0, -\delta, 0)\left(0, -\frac{\alpha_3}{2}, 0\right).\end{aligned} \tag{B.6}$$

Thus, we obtain,

$$\widetilde{V} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta e^{i\frac{\alpha_3}{2}} \\ 0 & -\sin\theta e^{i\frac{\alpha_3}{2}} & \cos\theta e^{i\alpha_3} \end{pmatrix}. \tag{B.7}$$

Appendix C. Diagonalization of quark mass matrix

In this appendix, we derive the exact mass eigenvalues of the top-bottom SM quarks and the heavy VLQ partners, as well as the matrices used for the diagonalization procedure. We will show the diagonalization procedure for the top sector. The bottom sector can be done similarly because the form of $\mathbb{M}_b$ is the same as $\mathbb{M}_t$. We start from Eq. (3.66), explicitly writing the $(W_{T_R})^{43}$ and $(W_{T_R})^{44}$ values,

$$\mathbb{M}_t \equiv \begin{pmatrix} -\frac{Y^3_{u_L} Y^3_{u_R} v_L v_R}{2m_{u_4}} & Y^3_{u_L} \frac{v_L}{\sqrt{2}} \frac{M_T}{m_{u_4}} \\ 0 & m_{u_4} \end{pmatrix} = \begin{pmatrix} -m_{t_1} & m_{t_2} \\ 0 & m_{u_4} \end{pmatrix}, \tag{C.1}$$

where m_{t_1} and m_{t_2} in Eq. (C.1) are defined as follows:

$$m_{t_1} = \frac{Y^3_{u_L} Y^3_{u_R} v_L v_R}{2m_{u_4}}, \qquad m_{t_2} = Y^3_{u_L} \frac{v_L}{\sqrt{2}} \frac{M_T}{m_{u_4}}. \tag{C.2}$$

The top quark mass matrix in Eq. (C.1) can be diagonalized by bi-unitary transformation, which gives

$$K^\dagger_{T_L} \mathbb{M}_t K_{T_R} = \left(m_t^{\mathrm{diag}}\right) = \mathrm{diag}\left(m_t, m_{t'}\right). \tag{C.3}$$

Initially, we transform $\mathbb{M}_t$ into a real symmetric matrix by multiplying it on the left side with an orthogonal matrix S_t, which yields

$$\mathbb{M}'_t = S_t \mathbb{M}_t, \tag{C.4}$$

where

$$S_t = \begin{pmatrix} \cos\phi_{T_l} & -\sin\phi_{T_l} \\ \sin\phi_{T_l} & \cos\phi_{T_l} \end{pmatrix}. \tag{C.5}$$

$\mathbb{M}'_t$ becomes a real symmetric matrix with the following expression:

$$\mathbb{M}'_t = \begin{pmatrix} -m_{t_1}\cos\phi_{T_l} & -m_{t_1}\sin\phi_{T_l} \\ -m_{t_1}\sin\phi_{T_l} & m_{t_2}\sin\phi_{T_l} + m_{u_4}\cos\phi_{T_l} \end{pmatrix} \tag{C.6}$$

if the mixing angle satisfies the following condition:

$$\tan\phi_{T_l} = \frac{m_{t_2}}{m_{u_4} - m_{t_1}}. \tag{C.7}$$

Then, a real symmetric matrix can be diagonalized by multiplying from both sides another 2×2 orthogonal matrix R_t and its transpose,

$$R_t \mathbb{M}'_t R_t^T = \mathrm{diag}(-m_t, m_{t'}), \tag{C.8}$$

where

$$R_t = \begin{pmatrix} \cos\phi_{T_R} & \sin\phi_{T_R} \\ -\sin\phi_{T_R} & \cos\phi_{T_R} \end{pmatrix}. \tag{C.9}$$

The minus sign inside the diagonal matrix on the right-hand side of Eq. (C.8) arises because the determinant of the top quark mass matrix $\mathbb{M}_t$ is negative. Since m_t is lighter than $m_{t'}$, we assign the minus sign to m_t. However, we could eliminate the minus sign by multiplying Eq. (C.8) by

$-\tau_3$ on the right side, where τ_3 is the third component of the Pauli matrices. The mixing angle can then be obtained as

$$\tan 2\phi_{T_R} = \frac{2m_{t_1}m_{t_2}}{m_{u_4}^2 + m_{t_2}^2 - m_{t_1}^2}. \tag{C.10}$$

The eigenvalues of Eq. (C.8) can be computed using the following equation:

$$\lambda^2 - \left(\mathrm{tr}\mathbb{M}_t'\right)\lambda + \det\mathbb{M}_t' = 0. \tag{C.11}$$

After performing the calculations, we obtain

$$\lambda_1 = -m_t = \frac{\sqrt{m_{t_2}^2 + (m_{u_4} - m_{t_1})^2}}{2} - \frac{\sqrt{m_{t_2}^2 + (m_{u_4} + m_{t_1})^2}}{2}, \tag{C.12}$$

$$\lambda_2 = m_{t'} = \frac{\sqrt{m_{t_2}^2 + (m_{u_4} - m_{t_1})^2}}{2} + \frac{\sqrt{m_{t_2}^2 + (m_{u_4} + m_{t_1})^2}}{2}. \tag{C.13}$$

We can also equivalently express them with the parameters of the mass matrix as follows:

$$m_t = -\frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}, \tag{C.14}$$

$$m_{t'} = \frac{\sqrt{M_T^2 + (m_{u_R} - m_{u_L})^2}}{2} + \frac{\sqrt{M_T^2 + (m_{u_R} + m_{u_L})^2}}{2}, \tag{C.15}$$

where

$$m_{u_R} = Y_{u_R}^3 \frac{v_R}{\sqrt{2}}, \qquad m_{u_L} = Y_{u_L}^3 \frac{v_L}{\sqrt{2}}. \tag{C.16}$$

Finally, we can summarize all the matrix transformations explained above as

$$R_t S_t \mathbb{M}_t R_t^T(-\tau_3) = \mathrm{diag}(m_t, m_{t'}). \tag{C.17}$$

Additionally, the product of two orthogonal matrices is also an orthogonal matrix. Then we can define O_t as

$$O_t = R_t S_t = \begin{pmatrix} \cos\phi_{T_L} & \sin\phi_{T_L} \\ -\sin\phi_{T_L} & \cos\phi_{T_L} \end{pmatrix} \tag{C.18}$$

with $\phi_{T_L} = \phi_{T_R} - \phi_{T_l}$. Hence, by comparing Eq. (C.17) and Eq. (C.3) we obtain the expression for the mixing matrices as follows:

$$K_{T_L}^\dagger = \begin{pmatrix} \cos\phi_{T_L} & \sin\phi_{T_L} \\ -\sin\phi_{T_L} & \cos\phi_{T_L} \end{pmatrix}, \tag{C.19}$$

$$K_{T_R} = \begin{pmatrix} \cos\phi_{T_R} & -\sin\phi_{T_R} \\ \sin\phi_{T_R} & \cos\phi_{T_R} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\cos\phi_{T_R} & -\sin\phi_{T_R} \\ -\sin\phi_{T_R} & \cos\phi_{T_R} \end{pmatrix}. \tag{C.20}$$

For the bottom sector, we can derive the results similarly by replacing t with b, T with B, and u with d. Thus, we write the mass eigenvalues and the mixing matrices for the bottom sector as follows:

$$m_b = -\frac{\sqrt{M_B^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_B^2 + (m_{d_R} + m_{d_L})^2}}{2}, \tag{C.21}$$

$$m_{b'} = \frac{\sqrt{M_B^2 + (m_{d_R} - m_{d_L})^2}}{2} + \frac{\sqrt{M_B^2 + (m_{d_R} + m_{d_L})^2}}{2}, \tag{C.22}$$

where

$$m_{d_R} = Y^3_{d_R}\frac{v_R}{\sqrt{2}}, \quad m_{d_L} = Y^3_{d_L}\frac{v_L}{\sqrt{2}}, \tag{C.23}$$

$$K^\dagger_{B_L} = \begin{pmatrix} \cos\phi_{B_L} & \sin\phi_{B_L} \\ -\sin\phi_{B_L} & \cos\phi_{B_L} \end{pmatrix}, \tag{C.24}$$

$$K_{B_R} = \begin{pmatrix} \cos\phi_{B_R} & -\sin\phi_{B_R} \\ \sin\phi_{B_R} & \cos\phi_{B_R} \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\cos\phi_{B_R} & -\sin\phi_{B_R} \\ -\sin\phi_{B_R} & \cos\phi_{B_R} \end{pmatrix}. \tag{C.25}$$

For the approximate masses already written in Eqs. (6.9–6.12) and the approximate mixing angle this yields,

$$\sin\phi_{T_L} \simeq -\frac{m_{u_L}M_T}{M_T^2 + m_{u_R}^2}, \quad \cos\phi_{T_L} \simeq 1, \quad \sin\phi_{T_R} \simeq \frac{m_{u_L}^2 m_{u_R} M_T}{\left(M_T^2 + m_{u_R}^2\right)^2}, \quad \cos\phi_{T_R} \simeq 1$$

$$\sin\phi_{B_L} \simeq -\frac{m_{d_L}}{M_B}, \quad \cos\phi_{B_L} \simeq 1, \quad \sin\phi_{B_R} \simeq \frac{m_{d_L}^2 m_{d_R}}{M_B^3}, \quad \cos\phi_{B_R} \simeq 1, \tag{C.26}$$

or in the approximate matrix form as follows:

$$K^\dagger_{T_L} \simeq \begin{pmatrix} 1 & -\frac{m_{u_L}M_T}{M_T^2+m_{u_R}^2} \\ \frac{m_{u_L}M_T}{M_T^2+m_{u_R}^2} & 1 \end{pmatrix}, \quad K_{T_R} \simeq \begin{pmatrix} 1 & -\frac{m_{u_L}^2 m_{u_R} M_T}{(M_T^2+m_{u_R}^2)^2} \\ -\frac{m_{u_L}^2 m_{u_R} M_T}{(M_T^2+m_{u_R}^2)^2} & 1 \end{pmatrix} \tag{C.27}$$

$$K^\dagger_{B_L} \simeq \begin{pmatrix} 1 & -\frac{m_{d_L}}{M_B} \\ \frac{m_{d_L}}{M_B} & 1 \end{pmatrix}, \qquad K_{B_R} \simeq \begin{pmatrix} -1 & -\frac{m_{d_L}^2 m_{d_R}}{M_B^3} \\ -\frac{m_{d_L}^2 m_{d_R}}{M_B^3} & 1 \end{pmatrix}. \tag{C.28}$$

Appendix D. CKM-like matrices

In this appendix, we will discuss CKM-like matrices in this model and the rephasing of the CKM-like matrices. The CKM-like matrix, which appears for the first time in Section 3, is an "intermediate" right-handed CKM-like matrix which has explicit form as follows:

$$V_R^{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_R^d} & s_{\theta_R^d} c_{\theta_{B_R}} e^{i\frac{\alpha^3_{d_R}}{2}} & s_{\theta_R^d} s_{\theta_{B_R}} e^{i\frac{\alpha^3_{d_R}}{2}} \\ 0 & -c_{\theta_{T_R}} s_{\theta_R^d} e^{i\frac{\alpha^3_{d_R}}{2}} & c_{\theta_{T_R}} c_{\theta_R^d} c_{\theta_{B_R}} e^{i\alpha^3_{d_R}} & c_{\theta_{T_R}} c_{\theta_R^d} s_{\theta_{B_R}} e^{i\alpha^3_{d_R}} \\ 0 & -s_{\theta_{T_R}} s_{\theta_R^d} e^{i\frac{\alpha^3_{d_R}}{2}} & s_{\theta_{T_R}} c_{\theta_R^d} c_{\theta_{B_R}} e^{i\alpha^3_{d_R}} & s_{\theta_{T_R}} c_{\theta_R^d} s_{\theta_{B_R}} e^{i\alpha^3_{d_R}} \end{pmatrix}, \tag{D.1}$$

where

$$c_{\theta_R^d} = \cos\theta_R^d, \quad s_{\theta_R^d} = \sin\theta_R^d, \quad c_{\theta_{T_R}} = \cos\theta_{T_R},$$
$$s_{\theta_{T_R}} = \sin\theta_{T_R}, \quad c_{\theta_{B_R}} = \cos\theta_{B_R}, \quad s_{\theta_{B_R}} = \sin\theta_{B_R}. \tag{D.2}$$

After Step 6 is done, we have the expressions of the left-handed CKM-like matrix and right-handed CKM-like matrix, which are defined in Eq. (3.73) and Eq. (3.74), respectively. The matrix forms of the left-handed CKM-like matrix and right-handed CKM-like matrix are as follows:

$$\mathcal{V}_L^{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_L^d} & s_{\theta_L^d} c_{\phi_{B_L}} e^{i\frac{\alpha^3_{d_L}}{2}} & -s_{\theta_L^d} s_{\phi_{B_L}} e^{i\frac{\alpha^3_{d_L}}{2}} \\ 0 & -c_{\phi_{T_L}} s_{\theta_L^d} e^{i\frac{\alpha^3_{d_L}}{2}} & c_{\phi_{T_L}} c_{\theta_L^d} c_{\phi_{B_L}} e^{i\alpha^3_{d_L}} & -c_{\phi_{T_L}} c_{\theta_L^d} s_{\phi_{B_L}} e^{i\alpha^3_{d_L}} \\ 0 & s_{\phi_{T_L}} s_{\theta_L^d} e^{i\frac{\alpha^3_{d_L}}{2}} & -s_{\phi_{T_L}} c_{\theta_L^d} c_{\phi_{B_L}} e^{i\alpha^3_{d_L}} & s_{\phi_{T_L}} c_{\theta_L^d} s_{\phi_{B_L}} e^{i\alpha^3_{d_L}} \end{pmatrix}, \tag{D.3}$$

where

$$\begin{aligned} &c_{\theta_L^d} = \cos\theta_L^d, \quad s_{\theta_L^d} = \sin\theta_L^d, \quad c_{\phi_{T_L}} = \cos\phi_{T_L}, \\ &s_{\phi_{T_L}} = \sin\phi_{T_L}, \quad c_{\phi_{B_L}} = \cos\phi_{B_L}, \quad s_{\phi_{B_L}} = \sin\phi_{B_L} \end{aligned} \tag{D.4}$$

and

$$\mathcal{V}_R^{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_R^d} & -s_{\theta_R^d} c_{\beta_{B_R}} e^{i\frac{\alpha^3_{d_R}}{2}} & s_{\theta_R^d} s_{\beta_{B_R}} e^{i\frac{\alpha^3_{d_R}}{2}} \\ 0 & c_{\beta_{T_R}} s_{\theta_R^d} e^{i\frac{\alpha^3_{d_R}}{2}} & c_{\beta_{T_R}} c_{\theta_R^d} c_{\beta_{B_R}} e^{i\alpha^3_{d_R}} & -c_{\beta_{T_R}} c_{\theta_R^d} s_{\beta_{B_R}} e^{i\alpha^3_{d_R}} \\ 0 & -s_{\beta_{T_R}} s_{\theta_R^d} e^{i\frac{\alpha^3_{d_R}}{2}} & -s_{\beta_{T_R}} c_{\theta_R^d} c_{\beta_{B_R}} e^{i\alpha^3_{d_R}} & s_{\beta_{T_R}} c_{\theta_R^d} s_{\beta_{B_R}} e^{i\alpha^3_{d_R}} \end{pmatrix}, \tag{D.5}$$

where

$$\begin{aligned} &c_{\theta_R^d} = \cos\theta_R^d, \quad s_{\theta_R^d} = \sin\theta_R^d, \quad c_{\beta_{T_R}} = \cos\beta_{T_R}, \\ &s_{\beta_{T_R}} = \sin\beta_{T_R}, \quad c_{\beta_{B_R}} = \cos\beta_{B_R}, \quad s_{\beta_{B_R}} = \sin\beta_{B_R}, \\ &\beta_{T_R} = \theta_{T_R} - \phi_{T_R}, \quad \beta_{B_R} = \theta_{B_R} - \phi_{B_R}. \end{aligned} \tag{D.6}$$

Recall the mass terms in the diagonal mass basis (including the massless two lightest quark fields) as follows:

$$\begin{aligned} \mathcal{L}_q \supset \mathcal{L}_{\rm mass} = &-\overline{\left(u_L^m\right)^\alpha} \left(m_t^{\rm diag}\right)^\alpha \left(u_R^m\right)^\alpha - {\rm h.c.} \\ &- \overline{\left(d_L^m\right)^\alpha} \left(m_b^{\rm diag}\right)^\alpha \left(d_R^m\right)^\alpha - {\rm h.c.}. \end{aligned} \tag{D.7}$$

We have the freedom to rephase the quark fields with the following transformations:

$$\left(u^m_{L(R)}\right)^\alpha = \left(\theta_{u_{L(R)}}\right)^\alpha \delta^{\alpha\beta} \left(\hat{u}^m_{L(R)}\right)^\beta, \tag{D.8}$$

$$\left(d^m_{L(R)}\right)^\alpha = \left(\theta_{d_{L(R)}}\right)^\alpha \delta^{\alpha\beta} \left(\hat{d}^m_{L(R)}\right)^\beta, \tag{D.9}$$

where $\theta_{u_{L(R)}} = {\rm diag}(e^{i\theta_{u_{L(R)1}}}, e^{i\theta_{u_{L(R)2}}}, e^{i\theta_{u_3}}, e^{i\theta_{u_4}})$ and $\theta_{d_{L(R)}} = {\rm diag}(e^{i\theta_{d_{L(R)1}}}, e^{i\theta_{d_{L(R)2}}}, e^{i\theta_{d_3}}, e^{i\theta_{d_4}})$. One can show that Eq. (D.7) is invariant under transformation in Eqs. (D.8–D.9).

We apply this rephasing transformation into the $\mathcal{L}_q$. The left-handed and right-handed CKM-like matrices are rephased and become

$$\hat{\mathcal{V}}_L^{\rm CKM} = \theta^\dagger_{u_L} \mathcal{V}_L^{\rm CKM} \theta_{d_L}, \qquad \hat{\mathcal{V}}_R^{\rm CKM} = \theta^\dagger_{u_R} \mathcal{V}_R^{\rm CKM} \theta_{d_R}. \tag{D.10}$$

By choosing the proper phase and phase difference, we could rephase the left-handed and right-handed CKM-like matrices and they become the following matrix forms:

$$\hat{\mathcal{V}}_L^{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_L^d} & s_{\theta_L^d} c_{\phi_{B_L}} & -s_{\theta_L^d} s_{\phi_{B_L}} \\ 0 & -c_{\phi_{T_L}} s_{\theta_L^d} & c_{\phi_{T_L}} c_{\theta_L^d} c_{\phi_{B_L}} & -c_{\phi_{T_L}} c_{\theta_L^d} s_{\phi_{B_L}} \\ 0 & s_{\phi_{T_L}} s_{\theta_L^d} & -s_{\phi_{T_L}} c_{\theta_L^d} c_{\phi_{B_L}} & s_{\phi_{T_L}} c_{\theta_L^d} s_{\phi_{B_L}} \end{pmatrix}, \tag{D.11}$$

$$\hat{\mathcal{V}}_R^{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\theta_R^d} & -s_{\theta_R^d} c_{\beta_{B_R}} e^{i\frac{\delta}{2}} & s_{\theta_R^d} s_{\beta_{B_R}} e^{i\frac{\delta}{2}} \\ 0 & c_{\beta_{T_R}} s_{\theta_R^d} e^{i\frac{\delta}{2}} & c_{\beta_{T_R}} c_{\theta_R^d} c_{\beta_{B_R}} e^{i\delta} & -c_{\beta_{T_R}} c_{\theta_R^d} s_{\beta_{B_R}} e^{i\delta} \\ 0 & -s_{\beta_{T_R}} s_{\theta_R^d} e^{i\frac{\delta}{2}} & -s_{\beta_{T_R}} c_{\theta_R^d} c_{\beta_{B_R}} e^{i\delta} & s_{\beta_{T_R}} c_{\theta_R^d} s_{\beta_{B_R}} e^{i\delta} \end{pmatrix}, \tag{D.12}$$

where we redefine the phase difference as $\delta = \alpha_{d_R}^3 - \alpha_{d_L}^3$. Therefore, in this model, we have one CP-violating phase δ and in our choice, it is included in the right-handed CKM-like matrix as shown in Eq. (D.12).

Moreover, the mixing angles β_{T_R} and β_{B_R} can be expressed in the approximate form as,

$$\sin\beta_{T_R} \simeq \frac{m_{u_R}}{\sqrt{M_T^2 + m_{u_R}^2}}, \quad \cos\beta_{T_R} \simeq \frac{M_T}{\sqrt{M_T^2 + m_{u_R}^2}}, \quad \sin\beta_{B_R} \simeq \frac{m_{d_R}}{M_B}, \quad \cos\beta_{B_R} \simeq 1. \tag{D.13}$$

References

[1] P. Minkowski, Phys. Lett. B **67**, 421 (1977).
[2] T. Yanagida, Conf. Proc. C **7902131**, 95 (1979).
[3] M. Gell-Mann, P. Ramond, and R. Slansky, Conf. Proc. C **790927**, 315 (1979) [arXiv:1306.4669 [hep-th]] [Search inSPIRE].
[4] T. Yanagida, Prog. Theor. Phys. **64**, 1103 (1980).
[5] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[6] D. Wyler and L. Wolfenstein, Nucl. Phys. B **218**, 205 (1983).
[7] R. Foot, H. Lew, X.-G. He, and G. C. Joshi, Z. Phys. C **44**, 441 (1989).
[8] Z. G. Berezhiani, Phys. Lett. B **129**, 99 (1983).
[9] Z. G. Berezhiani, Phys. Lett. B **150**, 177 (1985).
[10] S. Rajpoot, Phys. Lett. B **191**, 122 (1987).
[11] D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **58**, 1600 (1987).
[12] A. Davidson and K. C. Wali, Phys. Rev. Lett. **59**, 393 (1987).
[13] Z. G. Berezhiani and R. Rattazi, Phys. Lett. B **279**, 124 (1992).
[14] Z. G. Berezhiani and R. Rattazi, Nucl. Phys. B **407**, 249 (1993) [arXiv:hep-ph/9212245] [Search in SPIRE].
[15] Y. Koide and H. Fusaoka, Z. Phys. C **71**, 459 (1996) [arXiv:hep-ph/9505201] [Search inSPIRE].
[16] T. Morozumi, T. Satou, M. N. Rebelo, and M. Tanimoto, Phys. Lett. B **410**, 233 (1997) [arXiv:hep-ph/9703249] [Search inSPIRE].
[17] Y. Kiyo, T. Morozumi, P. Parada, M. N. Rebelo, and M. Tanimoto, Prog. Theor. Phys. **101**, 3 (1999) [arXiv:hep-ph/9809333] [Search inSPIRE].
[18] R. Kawasaki, T. Morozumi, and H. Umeeda, Phys. Rev. D **88**, 033019 (2013) [arXiv:1306.5080 [hep-ph]] [Search inSPIRE].
[19] A. Patra and S. K. Rai, Phys. Rev. D **98**, 015033 (2018) [arXiv:1711.00627 [hep-ph]] [Search inSP IRE].
[20] A. E. C. Hernandez, J. M. Gonzalez, and U. J. Saldana-Salazar, Phys. Rev. D **100**, 035024 (2019) [arXiv:1904.09993 [hep-ph]] [Search inSPIRE].
[21] S.-P. Chen and P.-H. Gu, Nucl. Phys. B **986**, 116057 (2023) [arXiv:2211.01906 [hep-ph]] [Search in SPIRE].
[22] R. Dcruz and K. S. Babu, Phys. Rev. D **108**, 095011 (2023) [arXiv:2212.09697 [hep-ph]] [Search in SPIRE].
[23] R. Dcruz, Nucl. Phys. B **1001**, 116519 (2024) [arXiv:2301.10786 [hep-ph]] [Search inSPIRE].
[24] B. Belfatto, R. Beradze, and Z. Berezhiani, Eur. Phys. J. C **80**, 149 (2020) [arXiv:1906.02714 [hep-ph]] [Search inSPIRE].
[25] G. C. Branco, J. T. Penedo, P. M. F. Pereira, M. N. Rebelo, and J. I. Silva-Marcos, J. High Energy Phys. **07**, 099 (2021) [arXiv:2103.13409 [hep-ph]] [Search inSPIRE].
[26] S. Balaji, J. High Energy Phys. **05**, 015 (2022) [arXiv:2110.05473 [hep-ph]] [Search inSPIRE].

[27] A. K. Alok and S. Gangal, Phys. Rev. D **86**, 114009 (2012) [arXiv:1209.1987 [hep-ph]] [Search in SPIRE].

[28] F. Albergaria, G. C. Branco, J. F. Bastos, and J. I. Silva-Marcos, J. Phys. G Nucl. Part. Phys. **50**, 055001 (2023) [arXiv:2210.14248 [hep-ph]] [Search inSPIRE].

[29] A. H. Panuluh and T. Morozumi, J. Phys. Conf. Ser. **2446**, 012049 (2023) [arXiv:2211.02359 [hep-ph]] [Search inSPIRE].

[30] T. Morozumi, Y. Shimizu, S. Takahashi, and H. Umeeda, Prog. Theor. Exp. Phys. **2018**, 043B10 (2018) [arXiv:1801.05268 [hep-ph]] [Search inSPIRE].

[31] T. Morozumi, A. S. Adam, Y. Kawamura, A. H. Panuluh, Y. Shimizu, and K. Yamamoto, J. Phys. Conf. Ser. **2446**, 012046 (2023) [arXiv:2211.02360 [hep-ph]] [Search inSPIRE].

[32] J. M. Alves, G. C. Branco, A. L. Cherchiglia, C. C. Nishi, J. T. Penedo, P. M. F. Pereira, M. N. Rebelo, and J. I. Silva-Marcos, Phys. Rept. **1057**, 1 (2024) [arXiv:2304.10561 [hep-ph]] [Search in SPIRE].

[33] S. Navas et al. [Particle Data Group], Phys. Rev. D **110**, 030001 (2024).

Funded by SCOAP³