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About the conference

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PDF (125 KB)

Students in Practice Learning Mathematics and Science Using STEAM Approach
01001
Wayan Maharani and Hongki Julie
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101001
Abstract | PDF (283.6 KB) | References

Open Access
Implementation of Ignatian Pedagogy Paradigm on The Topic of Probability at Seminary Mertoyudan Senior High School Magelang 01002
Maria Agustina Reforma Putri and Eko Budi Santoso
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101002
Abstract | PDF (809.6 KB) | References

Open Access
The Relationship Between Mathematical Thinking and Resilience in Number Sequence Lesson Through Ethnomathematics Among Pre-service Primary School Teachers 01003
Christiyanti Aprinastuti and Maria Agustina Amelia
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101003
Abstract | PDF (363.8 KB) | References

Open Access
Implementation of Mathematics Teaching Module Based on Reflective Pedagogy Paradigm (RPP) at Taruna Nusantara High School 01004
Rizky Anwari, Angelin Ica Pramesti and Eko Budi Santoso
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101004
Abstract | PDF (534.6 KB) | References

Open Access
Ethnomatematics Study on Ikat Woven Fabric of The *Kwatek* Lamalera 01005
Elisabeth Gunu Lyany and Hongki Julie
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101005
Abstract | PDF (692.4 KB) | References

Visual mathematics: An implementation to students in an indigenous community and a sub urban area 01006
Jalina Widjaja, Oki Neswan and Yudi Soeharyadi
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101006
Abstract | PDF (966.4 KB) | References

Open Access
Implementing The Ignatian Pedagogy Paradigm in Mathematics Learning: Annuity Material at The Vocational School Level 01007
Yuliastuti Dwi Lestari and Eko Budi Santoso
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101007
Abstract | PDF (646.6 KB) | References

Open Access
Validity Analysis of Digital Puzzle Game Media with a Realistic Mathematics Education Approach on Arithmetic Operations Material for Early Childhood 01008
Eem Kurniasih and Pukky Tetralian Bantining Ngastiti
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101008
Abstract | PDF (545.8 KB) | References

Open Access
The Mathematics of Finance: Pricing Volatility derivatives 01009
Parkpoom Phetpradap and Natkamon Sripanitan
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101009
Abstract | PDF (545.7 KB) | References

Open Access
Numerical Investigation of β-Amyloid Aggregation Process In Alzheimer's Disease 01010
Zefanya Putri Rida Wibowo and Lusia Krismiyati Budiasih
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101010
Abstract | PDF (714.8 KB) | References

Occupation Times of Fractional Brownian Motion as White Noise Distributions 01011
Herry Pribawanto Suryawan
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101011
Abstract | PDF (475.5 KB) | References

Open Access
Counting The Number of Arrangements of Tatami Mats in a Rectangular Room of Vertical Length 2, 3 and 4 01012
Yoshiaki Ueno
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101012
Abstract | PDF (481.7 KB) | References

Open Access
Decision-Level Fusion on Healthcare 01013
Astri Ayu Nastiti, Laurentinus Anindito Wisnu Susanto, Desi Natalia Muskananfola and Gusti Ayu Dwi Yanti
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101013
Abstract | PDF (861.3 KB) | References

Open Access
Development of an Epidemiological Model with Transmission Matrix to Understand the Dynamics of Tuberculosis Spread 01014
Meliana Pasaribu, Fransiskus Fran, Helmi, Angela Nadya Putri Ditya, Alexander and Tegar Rama Priyatna
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101014
Abstract | PDF (584.1 KB) | References

Open Access
Application of the XGBoost Algorithm for Predicting the Target Effective Temperature in Closed Broiler Chicken Cage 01015
Hartono
Published online: 06 February 2025
DOI: https://doi.org/10.1051/itmconf/20257101015
Abstract | PDF (677.7 KB) | References

Long Short-Term Memory and Bidirectional Long Short-Term Memory Algorithms for Sentiment Analysis of Skintific Product Reviews 01016

Laurensia Simanihuruk and Hari Suparwito

Published online: 06 February 2025

DOI: https://doi.org/10.1051/itmconf/20257101016

Abstract | PDF (491.1 KB) | References

Open Access

On Independent [1, 2]-sets in Hypercubes 01017

Eko Budi Santoso, Reginaldo M. Marcelo and Mari-Jo P. Ruiz

Published online: 06 February 2025

DOI: https://doi.org/10.1051/itmconf/20257101017

Abstract | PDF (839.5 KB) | References

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Preface of the Conference Proceeding of the 2nd International Conference on Mathematics, its Applications, and Mathematics Education (ICMAME) 2024

We are pleased to present the proceedings of the 2nd International Conference on Mathematics, its Applications, and Mathematics Education (ICMAME) 2024. This esteemed event was a collaborative effort organized by the Department of Mathematics, Faculty of Science and Technology, and the Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Sanata Dharma, Yogyakarta, Indonesia.

The conference was held on September 21, 2024, at Universitas Sanata Dharma, Indonesia, under the theme *"21st Century Mathematics and Mathematics Education."* This theme reflects the urgent need for innovative approaches to mathematics and mathematics education, aligned with the demands and opportunities of the 21st century. ICMAME 2024 brought together mathematicians, educators, and researchers to discuss emerging trends and share insights in these critical areas.

We were privileged to have distinguished keynote and invited speakers who enriched the conference with their expertise and perspectives. Our heartfelt thanks go to:

- **Assoc. Prof. Dr. Wanty Widjaya** (Deakin University, Australia)
- **Dr. rer. nat. Wolfgang Bock** (Linnaeus University, Sweden)
- **Dr. Martianus Frederic Ezerman** (Nanyang Technological University, Singapore)
- **Asst. Prof. Dr. Parkpoom Phetpradap** (Chiang Mai University, Thailand)
- **Veronica Fitri Rianasari, M.Sc., Ph.D.** (Universitas Sanata Dharma, Indonesia)

We extend our deepest gratitude to the organizing committee for their dedication and hard work, to the authors for their invaluable contributions, and to the reviewers for their diligent evaluations and constructive feedback. Without their combined efforts, this conference would not have been possible.

We hope this collection of proceedings, featuring contributions from authors across various countries (Indonesia, Philippines, Thailand, Japan, etc.), serves as a significant resource for researchers, educators, and practitioners. May it inspire future innovations, foster collaboration, and contribute to the advancement of mathematics and mathematics education for the betterment of our global community.

Thank you for being a part of the 2nd ICMAME 2024. We look forward to continuing this journey together and to exploring new horizons in the fascinating realms of mathematics and mathematics education.

Yogyakarta, 15 January 2025
Eko Budi Santoso
Chair, The 2nd ICMAME 2024

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In submitting conference proceedings to ITM Web of Conferences, I certify to the Publisher that I adhere to the **Policy on Publishing Integrity** of the journal in order to safeguard good scientific practice in publishing.

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Occupation Times of Fractional Brownian Motion as White Noise Distributions

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Abstract. Occupation times of a stochastic process models the amount of time the process spends inside a spatial interval during a certain finite time horizon. It appears in the fiber lay-down process in nonwoven production industry. The occupation time can be interpreted as the mass of fiber material deposited inside some region. From application point of view, it is important to know the average mass per unit area of the final fleece. In this paper we use white noise theory to prove the existence of the occupation times of one-dimensional fractional Brownian motion and provide an expression for the expected value of the occupation times.

1 Introduction

Technical textiles have attracted great attention to diverse branches of industry over the last decades due to their comparatively cheap manufacturing. By overlapping thousands of individual slender fibers, random fiber webs emerge yielding nonwoven materials that find applications e.g. in textile, building and hygiene industry as integral components of baby diapers, closing textiles, filters and medical devices, to name but a few. They are produced in melt-spinning operations: hundreds of individual endless fibers are obtained by the continuous extrusion of a molten polymer through narrow nozzles that are densely and equidistantly placed in a row at a spinning beam. The viscous or viscoelastic fibers are streched and spun until they solidify due to cooling air streams. Before the elastic fibers lay down on a moving conveyor belt to form a web, they become entangled and form loops due to the highly turbulent air flows. The homogeneity and load capacity of the fiber web are the most important textile properties for quality assesment of industrial nonwoven fabrics. The optimization and control of the fleece quality require modeling and simulation of fiber dynamics and lay-down. Available data to judge the quality, at least on the industrial scale, are usually the mass per unit area of the fleece.

Since the mathematical treatment of the whole process at a stroke is not possible due to its complexity, a hierarchy of models that adequately describe partial aspects of the process chain has been developed in research during the last years. A stochastic model for the fiber deposition in the nonwoven production was proposed and analyzed in [1-4]. The model is based on stochastic differential equations describing the resulting position of the fiber on the belt under the influence of turbulent air flows. In [5] parameter estimation of the Ornstein-

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Uhlenbeck process from available mass per unit area data, the occupation time in mathematical terms, was done.

Definition 1.

Let $X = (X_t)_{t\in[0,T]}$, $T > 0$, be a stochastic process and consider an interval $[a, b] \subseteq \mathbb{R}$. The occupation time $M_{T,[a,b]}(X)$ is defined as

$$M_{T,[a,b]}(X) \coloneqq \int_0^T 1_{[a,b]}(X_t)\, dt = \int_0^T \int_a^b \delta(X_t - x)\, dx\ dt. \tag{1}$$

Here, $1_{[a,b]}$ denoted the indicator function of the interval $[a, b]$ and δ is the Dirac-delta distribution.

Formally, occupation times models the time the stochastic process spends inside the spatial interval $[a, b]$ during the time interval $[0, T]$. In terms of our physical model for the nonwoven production, the occupation time can be interpreted as the mass of fiber material deposited inside the interval $[a, b]$, i.e. the mass per unit area of the final fleece.

Motivated by the above mentioned problem, in [6-7] the occupation times of one-dimensional Brownian motion were studied. In particular, it has been proved that occupation times of one-dimensional Brownian motion is a white noise distribution in the sense of Hida. In the present paper, we extend these results to occupation time of one-dimensional fractional Brownian motion. Although it is possible to study the problem by classical probabilistic method, we use a white noise approach to generalize the results also to higher dimensions in later research. In the next section we provide neccesary background on the white noise theory. The main result together with its proof are given afterward.

2 White Noise Theory

In this section we give background on the white noise theory used throughout this paper. For a more comprehensive discussions including various applications of white noise theory we refer to [8-10] and references therein. We start with the Gelfand triple

$$S(\mathbb{R}) \hookrightarrow L^2(\mathbb{R}) \hookrightarrow S'(\mathbb{R}) \tag{2}$$

where $S(\mathbb{R})$ is the space of real-valued Schwartz test function, $S'(\mathbb{R})$ is the space of real-valued tempered distributions, and $L^2(\mathbb{R})$ is the real Hilbert space of all real-valued Lebesgue square-integrable functions. Next, we construct a probability space $(S'(R), B, \mu)$ where B is the Borel σ-algebra generated by cylinder sets on $S'(\mathbb{R})$ and the unique probability measure μ is established through the Bochner-Minlos theorem by fixing the characteristic function

$$C(f) \coloneqq \int_{S'(\mathbb{R})} \exp(i\langle\omega, f\rangle)\, d\mu(\omega) = \exp\left(-\tfrac{1}{2}|f|_0^2\right) \tag{3}$$

for all $f \in S(\mathbb{R})$. Here $|\cdot|_0^2$ denotes the usual norm in the $L^2(\mathbb{R})$ and $\langle\cdot,\cdot\rangle$ denotes the dual pairing between $S'(\mathbb{R})$ and $S(\mathbb{R})$. The dual pairing is considered as the bilinear extension of the inner product on $L^2(\mathbb{R})$, i.e.

$$\langle g, f\rangle = \int_{\mathbb{R}} g(x) f(x) dx \tag{4}$$

for all $g \in L^2(\mathbb{R})$ and $f \in S(\mathbb{R})$. This probability space is known as the real-valued white noise space since it contains the sample paths of the one-dimensional Gaussian white noise.

In this setting a one-dimensional Brownian motion can be represented by a continuous modification of the stochastic process $B = (B_t)_{t\geq 0}$ with

$$B_t = \langle \cdot, 1_{[0,t]} \rangle \quad (5)$$

In the sequel we will use the Gel'fand triple

$$(S) \hookrightarrow L^2(\mu) \hookrightarrow (S)' \quad (6)$$

where (S) is the space of white noise test functions obtained by taking the intersection of a family of Hilbert subspaces of $L^2(\mu)$. The space of white noise distributions $(S)'$ is defined as the topological dual space of (S). Elements of $(S)$and $(S)'$ are known as Hida test functions and Hida distributions, respectively. Within this framework white noise can be considered as the time derivative of Brownian motion with respect to the topology of $(S)'$. An important tool in white noise analysis is the S-transform which can be considered as the Laplace transform with respect to the infinite dimensional Gaussian measure. The S-transform of $\Phi \in (S)'$ is defined as

$$S\Phi(\varphi) := \langle\langle \Phi, :\exp(\langle \cdot, \varphi \rangle): \rangle\rangle, \quad \varphi \in S(\mathbb{R}) \quad (7)$$

where

$$:\exp(\langle \cdot, \varphi \rangle): := C(\varphi) \exp(\langle \cdot, \varphi \rangle)$$

is the so-called Wick exponential and $\langle\langle \cdot,\cdot \rangle\rangle$denotes the dual pairing between $(S)'$ and (S). We define this dual pairing as the bilinear extension of the sesquilinear inner product on $L^2(\mu)$. The S-transform provides a convenient way to identify a Hida distribution $\Phi \in (S)'$, in particular, when it is hard to find the explicit form for the Wiener-Ito chaos decomposition of Φ.

The following theorem is the main tool to our main result.

Theorem 2. [11]

Let (Ω, A, ν) be a measure space and $\lambda \mapsto \Phi_\lambda$ be a mapping from Ω to $(S)'$. If

1. the mapping $\lambda \mapsto S\Phi_\lambda(\varphi)$ is measurable for all $\varphi \in S(\mathbb{R})$, and
2. there exist $C_1(\lambda) \in L^1(\Omega, A, \nu)$, $C_2(\lambda) \in L^\infty(\Omega, A, \nu)$, and a continuous seminorm $\|\cdot\|$ on $S(\mathbb{R})$ such that for all $z \in \mathbb{C}$, $\varphi \in S(\mathbb{R})$

$$|S\Phi_\lambda(z\varphi)| \leq C_1(\lambda) \exp(C_2(\lambda)|z|^2\|\varphi\|^2) \quad (8)$$

then Φ_λ is Bochner integrable with respect to some Hilbertian norm which topologizing $(S)'$. Hence $\int_\Omega \ \Phi_\lambda \, d\nu(\lambda) \in (S)'$, and furthermore

$$S\big(\int_\Omega \Phi_\lambda \, d\nu(\lambda)\big) = \int_\Omega S\Phi_\lambda \, d\nu(\lambda) \quad (9)$$

Let $0 < T < \infty$. Recall that fractional Brownian motion with Hurst parameter $H \in (0,1)$ is a centered Gaussian process $B^H = (B_t^H)_{t\in[0,T]}$ defined on some probability space $(\Omega, F, \mathbb{P})$ taking values in $\mathbb{R}$ with covariance function

$$\mathbb{E}(B_t^H B_s^H) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}), \quad s,t \geq 0 \quad (10)$$

Fractional Brownian motion is Gaussian extension of the standard Brownian motion which is not Markov process nor semimartingale. It possesses Holder continuous trajectories and long/short-range dependence depending on the value of the Hurst parameter. Within the white noise analysis framework fractional Brownian motion can be represented by

$$B_t^H = \langle \cdot, M_-^H 1_{[0,t]} \rangle \tag{11}$$

where M_-^H is the Weyl fractional integral operator for $H \in \left(\frac{1}{2}, 1\right)$ and is the Marchaud fractional derivative operator for $H \in \left(0, \frac{1}{2}\right)$, see [12] for details. Note that $M_-^{1/2}$ is the identity operator and fractional Brownian motion reduces to the standar Brownian motion. The corresponding Donsker's delta distribution is given by

$$\delta(B_t^H - x) = \delta\left(\langle \cdot, M_-^H 1_{[0,t]} \rangle\right) = \frac{1}{2\pi} \int_{\mathbb{R}} \exp\left(i\lambda\left(\langle \cdot, M_-^H 1_{[0,t]} \rangle - x\right)\right) d\lambda \tag{12}$$

It has been proved that $\delta(B_t^H - x) \in (S)'$. Furthermore, its S-transform is given by

$$S\delta(B_t^H - x)(\varphi) = \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{1}{2t^{2H}}\left(\langle M_+^H \varphi, 1_{[0,t\,]} \rangle - x\right)^2\right) \tag{13}$$

for any $\varphi \in S(\mathbb{R})$. Here M_+^H is the dual operator of M_-^H and for any $\varphi \in S(\mathbb{R})$ the function $M_+^H \varphi$ is continuous. For details and proofs we refer to [12-13]. The Donsker delta distribution is an important research object in the Gaussian analysis. For example, it can be used to study local times, self-intersection local times, stochastic current and Feynman integrals, see e.g. [14-20]. The derivatives of Donsker's delta distribution has been also studied in [21]. In [22] Donsker's delta distribution is analyzed in the context of stochastic processes with memory.

3 Main Result

Now we are ready to prove the main finding of the paper.
Theorem 3.
Let $B^H = (B_t^H)_{t \in [0,T]}$ be a one-dimensional fractional Brownian motion and consider an interval $[a, b] \subset \mathbb{R}$The occupation time

$$M_{T,[a,b]}(B^H) = \int_0^T \int_a^b \delta(B_t^H - x)\, dx\, dt \tag{14}$$

is a Hida distribution.

Proof. It is apparent, at least formally, that occupation times can be obtained by integrating Donsker's delta distribution with respect to the product measure on $[a, b] \times [0, t]$. In this regard we will use Kondratiev-Streit integration theorem (Theorem 2) to prove the statement. Observe that for any $\varphi \in S(\mathbb{R})$ $S\delta(B_t^H - x)(\varphi)$ is a measurable function with respect to the product measure on $[a, b] \times [0, t]$. Now for any $z \in \mathbb{C}$ and $\varphi \in S(\mathbb{R})$ we have
$|S\delta(B\_t\text{^}H - x)(z\varphi)|$

$$= \left|\frac{1}{\sqrt{2\pi t^{2H}}} \exp(-\frac{1}{2t^{2H}}\left(\langle M_+^H\, z\varphi, 1_{[0,t\,]} \rangle - x\right)^2\ )\ \right|$$

$$= \frac{1}{\sqrt{2\pi t^{2H}}} \left|\exp(-\frac{1}{2t^{2H}}\left(\langle zM_+^H \varphi, 1_{[0,t\,]} \rangle - x\right)^2\ )\ \right|$$

$$= \frac{1}{\sqrt{2\pi t^{2H}}} \left|\exp(-\frac{1}{2t^{2H}}\left(\langle zM_+^H \varphi, 1_{[0,t\,]} \rangle^2 - 2x\langle zM_+^H \varphi, 1_{[0,t\,]} \rangle + x^2\right)\ )\ \right|$$

$$= \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{x^2}{2t^{2H}}\right) \left|\exp\left(-\frac{1}{2t^{2H}} \langle z M_+^H \varphi, 1_{[0,t\,]}\rangle^2\right) \exp\left(\frac{x}{t^{2H}} \langle z M_+^H \varphi, 1_{[0,t\,]}\rangle\right)\right|$$
$$\leq \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{x^2}{2t^{2H}}\right) \exp\left(\frac{1}{2t^{2H}} \left|\langle z M_+^H \varphi, 1_{[0,t\,]}\rangle\right|^2\right) \exp\left(\left|\frac{x}{t^{2H}} \langle z M_+^H \varphi, 1_{[0,t\,]}\rangle\right|\right)$$
$$= \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{x^2}{2t^{2H}}\right) \exp\left(\frac{1}{2t^{2H}} |z|^2 \left|\langle M_+^H \varphi, 1_{[0,t\,]}\rangle\right|^2\right) \exp\left(\frac{|x|}{\sqrt{t^{2H}}} \frac{|z|}{\sqrt{t^{2H}}} \langle z M_+^H \varphi, 1_{[0,t\,]}\rangle\right)$$
$$\leq \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{x^2}{2t^{2H}}\right) \exp\left(\frac{1}{2t^{2H}} |z|^2 \left|\langle \varphi, M_-^H 1_{[0,t\,]}\rangle\right|^2\right) \exp\left(\frac{x^2}{2t^{2H}}\right)$$
$$\exp\left(\frac{1}{2t^{2H}} |z|^2 \left|\langle \varphi, M_-^H 1_{[0,t\,]}\rangle\right|^2\right)$$
$$= \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(\frac{1}{t^{2H}} |z|^2 \left|\langle \varphi, M_-^H 1_{[0,t\,]}\rangle\right|^2\right)$$
$$\leq \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(\frac{|z|^2}{t^{2H}}\ |\varphi|^2 \left|M_-^H 1_{[0,t\,]}\right|^2\right)$$
$$= \frac{1}{\sqrt{2\pi t^{2H}}} \exp(\,|z|^2\ |\varphi|^2)$$

The last inequality is obtained by an application of the Cauchy-Schwarz inequality. We have used also the Ito isometry, that is $\left|M_-^H 1_{[0,t\,]}\right|^2 = var(B_t^H) = t^{2H}$. Note that

$$\int_0^T \int_a^b \frac{1}{\sqrt{2\pi t^{2H}}}\, dxdt = \frac{(b-a)T^{1-H}}{\sqrt{2\pi}(1-H)} < \infty$$

and $\exp(\,|z|^2\ |\varphi|^2)$ is a bounded function of t and x. Theorem 2 gives the desired result. ■

This is an immediate consequence of Theorem \ref{int} and Theorem \ref{main}.
Corollary 4.
The S-transform of the occupation times of fractional Brownian motion is given by

$$SM_{T,[a,b]}(B^H)(\varphi) = \int_0^T \frac{1}{\sqrt{2\pi t^{2H}}} \int_a^b \exp\left(-\frac{1}{2t^{2H}}\left(\int_0^t M_+^H\ \varphi\,(s)ds - x\right)^2\right)\, dx\ dt \quad (15)$$

for any $\varphi \in S(\mathbb{R})$.

Proof. Since $M_{T,[a,b]}(B^H) \in (S)'$ for every $\varphi \in S(\mathbb{R})$. Theorem 2 gives

$$SM_{T,[a,b]}(B^H)(\varphi)$$
$$= S\left(\int_0^T \int_a^b \delta(B_t^H - x)\, dx\, dt\right)(\varphi)$$
$$= \int_0^T \int_a^b S\delta(B_t^H - x)(\varphi)\, dx\, dt$$
$$= \int_0^T \int_a^b \frac{1}{\sqrt{2\pi t^{2H}}} \exp\left(-\frac{1}{2t^{2H}} \left(\langle M_+^H \varphi, 1_{[0,t\,]}\rangle - x\right)^2\right) dx\, dt$$
$$= \int_0^T \frac{1}{\sqrt{2\pi t^{2H}}} \int_a^b \exp\left(-\frac{1}{2t^{2H}}\left(\int_0^t M_+^H \varphi(s)ds - x\right)^2\right) dx\, dt$$

Corollary 5.
The expected value of the occupation times of fractional Brownian motion is given by

$$\mathbb{E}_\mu\left(M_{T,[a,b]}(B^H)\right) = \int_0^T \frac{1}{\sqrt{2\pi t^{2H}}} \int_a^b \exp\left(-\frac{x^2}{2t^{2H}}\right) dx\, dt \tag{16}$$

Proof. The expected value with respect to the white noise measure of the occupation times of fractional Brownian motion is obtained by taking the S-transform and evaluating the value at 0:

$$\begin{aligned}\mathbb{E}_\mu\left(M_{T,[a,b]}(B^H)\right) &= SM_{T,[a,b]}(B^H)(0) \\ &= \int_0^T \frac{1}{\sqrt{2\pi t^{2H}}} \int_a^b \exp\left(-\frac{1}{2t^{2H}}\left(\int_0^t 0\, ds - x\right)^2\right) dx\, dt \\ &= \int_0^T \frac{1}{\sqrt{2\pi t^{2H}}} \int_a^b \exp\left(-\frac{x^2}{2t^{2H}}\right) dx\, dt\,.\end{aligned}$$

4 Conclusion

We gave a mathematically rigorous meaning to the occupation times of a fractional Brownian motion as a Hida distribution. An expression for the expected value for the occupation times is also obtained. For the application point of view it is desirable to have a more explicit form for the expected value. This will be done in the future work. We would like also to mention that our present result is limited to one-dimensional setting. For further research we plan to generalize the result to higher spatial dimensions. Moreover, an extension to a more general class of processes (e.g. to the generalized grey Brownian motion) will be also considered.

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