



About Journal

Absis is a research journal managed by the Mathematics Education study program at Pasir Pengaraian University. Absis is taken from a mathematical term which is the distance of a point from the vertical axis. Absis contains articles on research results or literature studies in the field of mathematics and mathematics education. The Absis Journal is published three times a year, in April, August, and December. Absis is expected to be a place for researchers in mathematics or mathematics education, from various universities, and teachers to publish scientific articles.



ISSN: Electronic: 2654-8739, Print: 2655-4518

Frequency: 3 per year

Distribution: Prefix 10.30606/absis by Crossref.

Accreditation: SINTA 3.

Editor in chief: Ratri Isharyadi

Publisher: Program Studi Pendidikan Matematika, FKIP, Universitas Pasir Pengaraian.

[Author Guidelines](#)

[Submit Paper Now!](#)

Current Issue

Vol. 8 No. 1 (2025): Jurnal Absis



Published: 2025-04-30

Articles

Development of experiential learning-based tools to enhance students' creative thinking in quadrilaterals and triangles

Sri Intan Retno Dewi, Elfis Suanto, Titi Solfitri

1-14

 PDF

DOI : [10.30606/absis.v8i1.2658](https://doi.org/10.30606/absis.v8i1.2658)

Abstract View : 4

PDF downloads: 8

Development of teaching materials based on realistic mathematics education to facilitate students' mathematical critical thinking skills

Ranty Andika Abinaya Zameza, Maimunah, Nahor Murani Hutapea

15-28



PDF

DOI : 10.30606/absis.v8i1.2683

Abstract View : 3

PDF downloads: 0

Max-plus algebra modeling for outpatient service queues in Yogyakarta private hospitals

Helena Noventyas Pradnyamita Budiarta, Brigitta Kierra Purwa Adhiananta, Marcellinus Andy Rudhito

29-42



PDF

DOI : 10.30606/absis.v8i1.2785

Abstract View : 1

PDF downloads: 0

Development of test instruments for the metacognition ability of linear program materials

Umroh, Yuyu Yuhana

43-54



PDF

DOI : 10.30606/absis.v8i1.2662

Abstract View : 1

PDF downloads: 1

Validity of integral calculus textbook with indonesian cultural and religious values

Marhayati, Arini Mayan Fa'ani, Siti Faridah

55-69



PDF

DOI : 10.30606/absis.v8i1.2991

Abstract View : 2

PDF downloads: 0

Development of ispring-assisted interactive learning media to support students' self-regulated learning on flat-sided geometric solids

Susy Irawaty Nainggolan, Susda Heleni, Elfis Suanto

 70-82 PDF

DOI : 10.30606/absis.v8i1.2661

Abstract View : 1

PDF downloads: 0

No gender differences in mathematical problem-solving ability: A study with mathematics anxiety control

Ahmad Dzulfikar, Turmudi, Tatang Herman, Kusnandi

 83-93 PDF

DOI : 10.30606/absis.v8i1.3171

Abstract View : 1

PDF downloads: 1

Development of a test instrument to measure mathematical communication skills on the topics of three-variable linear equations and two-variable linear inequalities

Aprillia Anggita Simanullang, Kartini, Titi Solfitri

 94-104 PDF

DOI : 10.30606/absis.v8i1.2745

Abstract View : 2

PDF downloads: 0

[Author Guidelines](#)[View All Issues >](#)[Peer Review](#)[Reviewers](#)[Focus and Scope](#)[Publication Ethics](#)

[Article Template](#)[Plagiarism Policy](#)[Open Access Policy](#)[Copyright and License](#)[Archiving](#)[Commitment Letter](#)[Article Processing Fees](#)[Journal Indexing](#)[Journal Accreditation](#)

Our Editorial Team



Ratri Isharyadi
Editor in Chief
Universitas Pasir
Pengaraian, Indonesia
5984123



Marfi Ario
Editors
Universitas Pasir
Pengaraian, Indonesia
6124676



Hesty Marwani
Siregar
Editors
Universitas Riau,
Indonesia
6706820

[Read More](#)

Sinta Certificate



Article Template



Statistic Counter





[View Absis Stats](#)



Information

[For Readers](#)

[For Authors](#)

[For Librarians](#)

Address:

Kampus Universitas Pasir Pengaraian Jl. Tuanku Tambusai, Desa Kumu, Kecamatan Rambah Hilir, Kabupaten Rokan Hulu, Riau

Contact Info:

Tel: [+62852 7809 9389](tel:+6285278099389)

Email: jurnalabsis@gmail.com

Web: journal.upp.ac.id/index.php/absis

Information :

[Authors](#)

[Terms of Use](#)

[Privacy Policy](#)

© 2018, All rights reserved. This is an open-access article distributed under license [Creative Commons Attribution-NonCommercial 4.0 International License \(CC BY-NC 4.0\)](#)



[Home](#) / [Archives](#) / [Vol. 8 No. 1 \(2025\): Jurnal Absis](#) / [Articles](#)

Max-plus algebra modeling for outpatient service queues in Yogyakarta private hospitals

<https://doi.org/10.30606/absis.v8i1.2785>

 **Helena Noventyas Pradnyamita Budiarta**

 Universitas Sanata Dharma, Indonesia

 **Brigitta Kierra Purwa Adhiananta**

 Universitas Sanata Dharma, Indonesia

 **Marcellinus Andy Rudhito**

 Universitas Sanata Dharma, Indonesia

Keywords:

Aljabar Max-Plus

Antrian

Rawat Jalan

Max-Plus algebra

Petri net

outpatient queue

simulation

mathematical modeling

Abstract

This study aims to model the outpatient service queue system in a private hospital in Yogyakarta using the Max-Plus algebra approach

and to represent the model using a Petri net. The research was motivated by the complexity and inefficiency observed in outpatient queues, which often lead to long and unpredictable waiting times. Data were collected through direct observation of the outpatient process on the fourth floor of a private hospital. The sequence of services was first illustrated using a flowchart to map each stage experienced by patients. The time data for each process were then used to construct a Max-Plus algebra matrix, providing a mathematical model of the system. This model was further expressed in the form of a Petri net to illustrate the discrete and sequential nature of the service flow. The simulation was performed using Scilab software to analyze the system's dynamics. Results from the simulation revealed that the service system is non-periodic, indicated by the absence of an eigenvalue. This suggests that the total service time may increase as the number of patients grows. The findings provide insight into the structure of the outpatient queue system and offer a mathematical framework for future system analysis or optimization.

Downloads

Download data is not yet available.

References

- Alamsyah, A., & Sari, R. F. (2023). Optimisasi Multi-Objektif Pada Pelayanan Kesehatan Di Ruang Rawat Inap Rumah Sakit . Jurnal Absis: Jurnal Pendidikan Matematika Dan Matematika, 6(1), 826–838. <https://doi.org/10.30606/absis.v6i1.2200>
- Hardiyanti, S. A., Yuniwati, I., & Yustita, A. D. (2017). Bentuk Petri Net dan model aljabar Max-Plus pada sistem pelayanan pasien rawat jalan Rumah Sakit Al Huda Genteng, Banyuwangi. Jurnal UJMC, 3(2), 1–

8.

Hariputra, R. P., Defit, S., & Sumijan. (2022). Analisis sistem antrian

[Read More](#)





Published

2025-04-29

How to Cite

Budiarta, H. N. P., Adhiananta, B. K. P., & Rudhito, M. A. (2025). Max-plus algebra modeling for outpatient service queues in Yogyakarta private hospitals. *Jurnal Absis: Jurnal Pendidikan Matematika Dan Matematika*, 8(1), 29–42. <https://doi.org/10.30606/absis.v8i1.2785>

More Citation Formats



Issue

[Vol. 8 No. 1 \(2025\): Jurnal Absis](#)

Section

Articles

License

Copyright (c) 2025 Helena Noventyas Pradnyamita Budiarta, Brigitta Kierra Purwa Adhiananta, Marcellinus Andy Rudhito



This work is licensed under a [Creative Commons Attribution-NonCommercial 4.0 International License](https://creativecommons.org/licenses/by-nc/4.0/).

Similar Articles

- Hendrika Bete, Cecilia Novianti Salsinha, Maria Naimnule, [Mathematical connections: An analysis of students' ability in solving mathematical problems based on learning styles](#) , [Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika: Vol. 7 No. 2 \(2024\): Jurnal Absis](#)
- Musfira Musfira, Rafiq Badjeber, Nursupiamin, [Kesulitan Peserta Didik dalam Menyelesaikan Soal Higher Order Thinking Skill \(HOTS\) Ditinjau dari Kemampuan Komunikasi Matematis](#) , [Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika: Vol. 6 No. 1 \(2023\): Jurnal Absis](#)
- Taufik Rahman, Fauzia Nisa, [Pengaruh Model Problem Based Learning Dengan Strategi Everyone Is A Teacher Here Terhadap Kemampuan Komunikasi Matematis Dan Self-Confidence Siswa SMP](#) , [Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika: Vol. 2 No. 2 \(2020\): Jurnal Absis](#)
- Devi Utami Putri, Sirwanti, Aspikal, [Improving students' mathematical reasoning through the learning cycle instructional model](#) , [Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika: Vol. 7 No. 2 \(2024\): Jurnal Absis](#)
- Dini Julia Putri, Armis, Sakur, [Pengembangan Instrumen Asesmen Kemampuan Penalaran Matematis Konten Perbandingan untuk Siswa Fase D](#) , [Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika: Vol. 6 No. 2 \(2024\): Jurnal Absis](#)
- Helfi Safrita, Syarifah Nur Siregar, Atma Murni, [Improving students' mathematical representation ability through problem-based learning model](#) , [Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika: Vol. 7 No. 2 \(2024\): Jurnal Absis](#)
- Fazrina Saumi, Rizki Amalia, Amelia, Nurviana, [Analisis Sistem Antrian Pada Pelayanan Poli Jantung RSUD Kota Langsa](#) , [Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika: Vol. 3 No. 2 \(2021\): Jurnal Absis](#)
- Saniah Afiqoh Kholisah, Rini Dian Anggraini, Kartini, [Implementation of the problem-based learning model to improve students' mathematical problem-solving skills](#) , [Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika: Vol. 7 No. 2 \(2024\): Jurnal Absis](#)
- Hermanto, Dadang Juandi, Uly Hidayati, [A Systematic Literature Review: The Impact of ICT on Students' Mathematical Literacy](#) , [Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika: Vol. 5 No. 1 \(2022\): Jurnal Absis](#)

- Arfatin Nurrahmah, Lasia Agustina, [Deskripsi Kemampuan Berpikir Kreatif Matematis Mahasiswa Ditinjau dari Kemampuan Awal Matematika](#) , [Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika: Vol. 4 No. 1 \(2021\): Jurnal Absis](#)

[1](#) [2](#) [3](#) [4](#) [5](#) [6](#) [7](#) [>](#) [>>](#)

You may also [start an advanced similarity search](#) for this article.

Address:

Kampus Universitas Pasir Pengaraian Jl. Tuanku Tambusai, Desa Kumu, Kecamatan Rambah Hilir, Kabupaten Rokan Hulu, Riau

Contact Info:

Tel: [+62852 7809 9389](tel:+6285278099389)

Email: jurnalabsis@gmail.com

Web: journal.upp.ac.id/index.php/absis

Information :

[Authors](#)

[Terms of Use](#)

[Privacy Policy](#)

© 2018, All rights reserved. This is an open-access article distributed under license [Creative Commons Attribution-NonCommercial 4.0 International License \(CC BY-NC 4.0\)](#)

MAX-PLUS ALGEBRA MODELING FOR OUTPATIENT SERVICE QUEUES IN YOGYAKARTA PRIVATE HOSPITALS

Helena Noventyas Pradnyamita Budiarta¹, Brigitta Kierra Purwa Adhiananta²,
*Marcellinus Andy Rudhito³

^{1,2,3} Universitas Sananta Dharma, Indonesia

rudhito@usd.ac.id

ABSTRACT This study aims to model the outpatient service queue system in a private hospital in Yogyakarta using the Max-Plus algebra approach and to represent the model using a Petri net. The research was motivated by the complexity and inefficiency observed in outpatient queues, which often lead to long and unpredictable waiting times. Data were collected through direct observation of the outpatient process on the fourth floor of a private hospital. The sequence of services was first illustrated using a flowchart to map each stage experienced by patients. The time data for each process were then used to construct a Max-Plus algebra matrix, providing a mathematical model of the system. This model was further expressed in the form of a Petri net to illustrate the discrete and sequential nature of the service flow. The simulation was performed using Scilab software to analyze the system's dynamics. Results from the simulation revealed that the service system is non-periodic, indicated by the absence of an eigenvalue. This suggests that the total service time may increase as the number of patients grows. The findings provide insight into the structure of the outpatient queue system and offer a mathematical framework for future system analysis or optimization.

Keywords: Max-Plus algebra, Petri net, outpatient queue, simulation, mathematical modeling

ABSTRAK Penelitian ini bertujuan untuk memodelkan sistem antrian layanan rawat jalan di rumah sakit swasta di Yogyakarta menggunakan pendekatan aljabar Max-Plus, serta merepresentasikan model tersebut dalam bentuk jaringan Petri. Penelitian ini dilatarbelakangi oleh kompleksitas dan ketidakefisienan dalam sistem antrian pasien rawat jalan, yang sering kali menyebabkan waktu tunggu yang panjang dan tidak terprediksi. Data diperoleh melalui observasi langsung terhadap proses layanan di lantai 4 salah satu rumah sakit swasta. Urutan pelayanan digambarkan terlebih dahulu dalam bentuk diagram alir berdasarkan tahapan-tahapan yang dialami pasien. Selanjutnya, data waktu dari setiap proses digunakan untuk membentuk model matematis berupa matriks Max-Plus. Model

tersebut kemudian dinyatakan dalam bentuk jaringan Petri untuk menggambarkan sifat sistem layanan yang diskrit dan berurutan. Simulasi dilakukan menggunakan perangkat lunak Scilab untuk menganalisis dinamika sistem. Hasil simulasi menunjukkan bahwa sistem layanan bersifat non-periodik, yang ditunjukkan dengan tidak ditemukannya nilai eigen. Hal ini mengindikasikan bahwa waktu layanan total dapat meningkat seiring bertambahnya jumlah pasien. Temuan ini memberikan pemahaman tentang struktur sistem antrian rawat jalan dan dapat menjadi dasar untuk analisis dan evaluasi sistem lebih lanjut secara matematis.

Kata-kata kunci: aljabar Max-Plus, jaringan Petri, antrian rawat jalan, simulasi, pemodelan matematis

INTRODUCTION

Queueing systems are a significant subject of study in queueing theory and discrete system modeling, as they represent real-world situations where the demand for services exceeds the available service capacity. This phenomenon is commonly found in various sectors of everyday life, including transportation, banking, and healthcare. When service capacity fails to accommodate high demand, users are forced to wait, which can lead to several negative consequences. Long queues not only reduce efficiency and productivity but also harm the service provider's reputation and profitability (Hariputra, Defit, & Sumijan, 2022; Paulina Maure et al., 2021).

One of the most critical types of queueing systems can be found in healthcare facilities, particularly in hospitals. According to the Ministry of Health, hospitals are healthcare service facilities that provide comprehensive medical services, including outpatient, inpatient, and emergency care. In Yogyakarta, there is a private hospital known for its high-quality service, making it a primary referral center for a wide range of medical needs. The high volume of patients, especially during peak hours, often leads to long queues in outpatient services.

To address queue-related problems and analyze service systems systematically, mathematical modeling is essential. One effective approach is the use of Max-Plus algebra. Max-Plus algebra is a mathematical structure defined on the extended set of real numbers $\mathbb{R} \cup \{-\infty\}$, using the maximum operator ($\oplus$) for addition and regular addition ($\otimes$) for multiplication (Rudhito, 2016). This algebra has several applications in modeling dynamic systems, such as transportation scheduling, production systems, shortest path analysis (Subiono, 2015), and queueing networks (Rudhito, 2016). Employing Max-Plus algebra allows for efficient and structured modeling of discrete service system dynamics in a mathematical framework.

Max-Plus algebra is a mathematical structure defined over the set $\mathbb{R} \cup \{-\infty\}$, equipped with two operations: the maximum ($\oplus$) and the usual addition ($\otimes$). According to Rudhito (2016), for $\alpha \in \mathbb{R}_{\max}$ and matrices $A, B \in \mathbb{R}_{\max}^{m \times n}$, the scalar-matrix product $\alpha \otimes A$ results in a matrix where each element is computed as $(\alpha \otimes A)_{ij} = \alpha \otimes A_{ij}$ for $i = 1, \dots, m$ and $j = 1, \dots, n$. Meanwhile, the matrix product $A \otimes B$ is defined by $(A \otimes B)_{ij} = \oplus_{k=1}^p A_{ik} \otimes B_{kj}$ for the same indices.

Some properties of Max-Plus algebra, as stated in Theorem 2.2.4 (Rudhito, 2016), include:

Associativity: $(A \oplus B) \oplus C = A \oplus (B \oplus C)$,

Commutativity: $A \oplus B = B \oplus A$,

Distributivity: $A \otimes (B \oplus C) = (A \otimes B) \oplus (A \otimes C)$,

Idempotency: $A \oplus A = A$,

Distributive with respect to right multiplication: $(A \oplus B) \otimes C = (A \otimes C) \oplus (B \otimes C)$.

To complement the algebraic model, this study also uses Petri nets, which are widely applied to represent discrete event systems (Subiono, 2015). In Petri nets, the relationship between events is described using transitions and places. A transition occurs when all input places are marked, representing a change of state. A Petri net is formally defined as a 4-tuple (P, T, A, ω) , where:

- P is a finite set of places $\{p_1, p_2, \dots, p_n\}$,
- T is a finite set of transitions $\{t_1, t_2, \dots, t_m\}$,
- A is a set of directed arcs such that $A \subseteq (P \times T) \cup (T \times P)$,
- ω is a weight function mapping each arc to a positive integer.

Although P and T are typically finite, they can also be countably infinite depending on the complexity of the system. In most practical cases, finite models are sufficient to represent systems such as outpatient service queues in hospitals.

According to Sari and Asih (2015), patients often wait for more than one hour to receive internal medicine services, highlighting the need for mathematical approaches capable of evaluating such systems more efficiently. Unlike traditional models that rely on probability or simulation, Max-Plus algebra offers a deterministic method to analyze discrete dynamic systems. When integrated with Petri nets, this approach allows for a clearer and more detailed visualization of service processes, making it easier to identify and improve inefficiencies.

Several previous studies have applied Max-Plus algebra to various real-world problems. These include outpatient service modeling with Petri net representations (Hardiyanti, Yuniwati, & Yustita, 2017; Tutupary & Lesnussa, 2013; Paulina Maure et al., 2021), scheduling in craft production (Yahya, Nurwan, & Resmawan, 2022), and optimization of tofu production processes (Nawar, Rahakbauw, & Patty, 2023). A related study by Saumi and Amalia (2021) modeled queues in a hospital's cardiology department using a single-channel, single-phase FIFO system. Another by Alamsyah and Sari (2023) utilized the simplex method via POM software to optimize resource allocation and healthcare quality.

The novelty of this study lies in the specific outpatient workflow and service structure observed, starting from patient registration to prescription retrieval at the pharmacy in a private hospital in Yogyakarta. Therefore, the aim of this study is to develop a Max-Plus algebra and Petri net-based model for outpatient service queues

in a hospital, providing an analytical framework to better understand and evaluate service efficiency.

METHODS

This study is a qualitative descriptive research aimed at modeling the outpatient service queue system in a private hospital in Yogyakarta using the Max-Plus algebra approach and Petri net representation. Data were collected through direct observation of the outpatient service process, covering the flow from patient registration to medication retrieval at the pharmacy. Observations involved recording the arrival times and service durations at each stage using a stopwatch and structured observation sheets.

The collected data were used to construct a service flowchart. Based on this flow, a mathematical model was developed in the form of a Max-Plus matrix, which was then visualized as a Petri net to represent the discrete transitions and conditions within the system.

Data analysis was conducted descriptively using Scilab software to simulate the system and examine its dynamics. The analysis focused on determining whether the system exhibits periodic behavior, which is assessed based on the presence or absence of an eigenvalue in the Max-Plus matrix model.

FINDING AND DISCUSSION

Steps or Algorithm Used

The initial step of this research involved collecting data through direct observation. The service times at each stage of the outpatient process were recorded using a stopwatch. Based on the recorded data, a flowchart was developed to visualize the sequence of services experienced by patients from entry to exit.

Before constructing the mathematical model, variables were defined to represent each service duration and the transition time between stages. Using these variables, Max-Plus algebraic expressions were formulated to represent the relationships between service stages. These expressions were then used to build the corresponding Max-Plus matrix model.

The resulting matrix was then analyzed using Scilab software to calculate the eigenvalue of the system. The result of this calculation served as the basis for concluding the system's periodicity, service efficiency, and identifying possible improvements.

Outpatient Service Flowchart

The following figure presents the flowchart of the outpatient service process on the fourth floor of the private hospital in Yogyakarta. It includes all stages from patient admission, administrative procedures, consultation in examination rooms, to prescription collection and patient discharge.

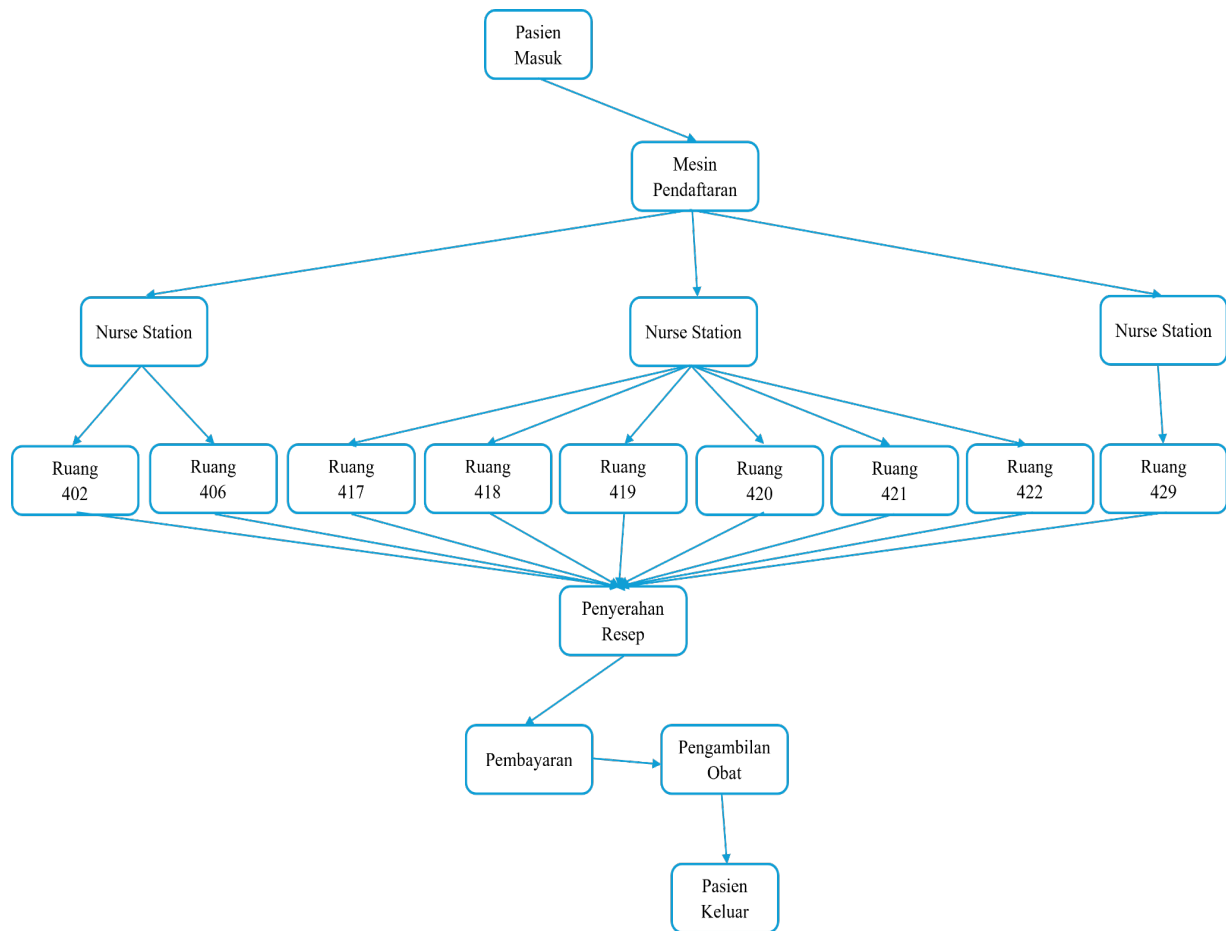


Figure 1. Outpatient Service Flowchart – 4th Floor

Max-Plus Algebra Model of the Outpatient Service Queue System in a Private Hospital in Yogyakarta

The variable descriptions used are as follows:

- a(i) = patient arrival time on the 4th floor at the i-th
- b(i) = time to start using the registration machine at the i-th
- c(i) = time to finish using the registration machine at the i-th
- d(i) = time to start service at nurse station 1 at the i-th
- e(i) = time to finish service at nurse station 1 at the i-th
- f(i) = time to start service at nurse station 2 at the i-th
- g(i) = time to finish service at nurse station 2 at the i-th
- h(i) = time to start service at nurse station 3 at the i-th
- i(i) = time to finish service at nurse station 3 at the i-th
- j(i) = time to start outpatient service in room 402 at the i-th

- $k(i)$ = time to finish outpatient service in room 402 at the i -th
 $l(i)$ = time to start outpatient service in room 406 at the i -th
 $m(i)$ = time to finish outpatient service in room 406 at the i -th
 $n(i)$ = time to start outpatient service in room 417 at the i -th
 $o(i)$ = time to finish outpatient service in room 417 at the i -th
 $p(i)$ = time to start outpatient service in room 418 at the i -th
 $q(i)$ = time to finish outpatient service in room 418 at the i -th
 $r(i)$ = time to start outpatient service in room 419 at the i -th
 $s(i)$ = time to finish outpatient service in room 419 at the i -th
 $t(i)$ = time to start outpatient service in room 420 at the i -th
 $u(i)$ = time to finish outpatient service in room 420 at the i -th
 $v(i)$ = time to start outpatient service in room 421 at the i -th
 $w(i)$ = time to finish outpatient service in room 421 at the i -th
 $x(i)$ = time to start outpatient service in room 422 at the i -th
 $y(i)$ = time to finish outpatient service in room 422 at the i -th
 $z(i)$ = time to start outpatient service in room 429 at the i -th
 $aa(i)$ = time to finish outpatient service in room 429 at the i -th
 $ab(i)$ = time to start prescription handover at the i -th
 $ac(i)$ = time to finish prescription handover at the i -th
 $ad(i)$ = time to start payment at the i -th
 $ae(i)$ = time to finish payment at the i -th
 $af(i)$ = time to start medication retrieval at the i -th
 $ag(i)$ = time to finish medication retrieval at the i -th

 V_a = average duration of registration machine service (in minutes)
 V_b = average walking time from registration to nurse station (in minutes)
 V_c = average nurse station service time (in minutes)
 V_d = average walking time to consultation room (in minutes)
 V_e = average consultation time with doctor (in minutes)
 V_f = average walking time from doctor to prescription
 V_g = average time for prescription handover (in minutes)
 V_h = average payment time (in minutes)
 V_i = average waiting time for medicine (in minutes)
 V_j = average medication retrieval time (in minutes)

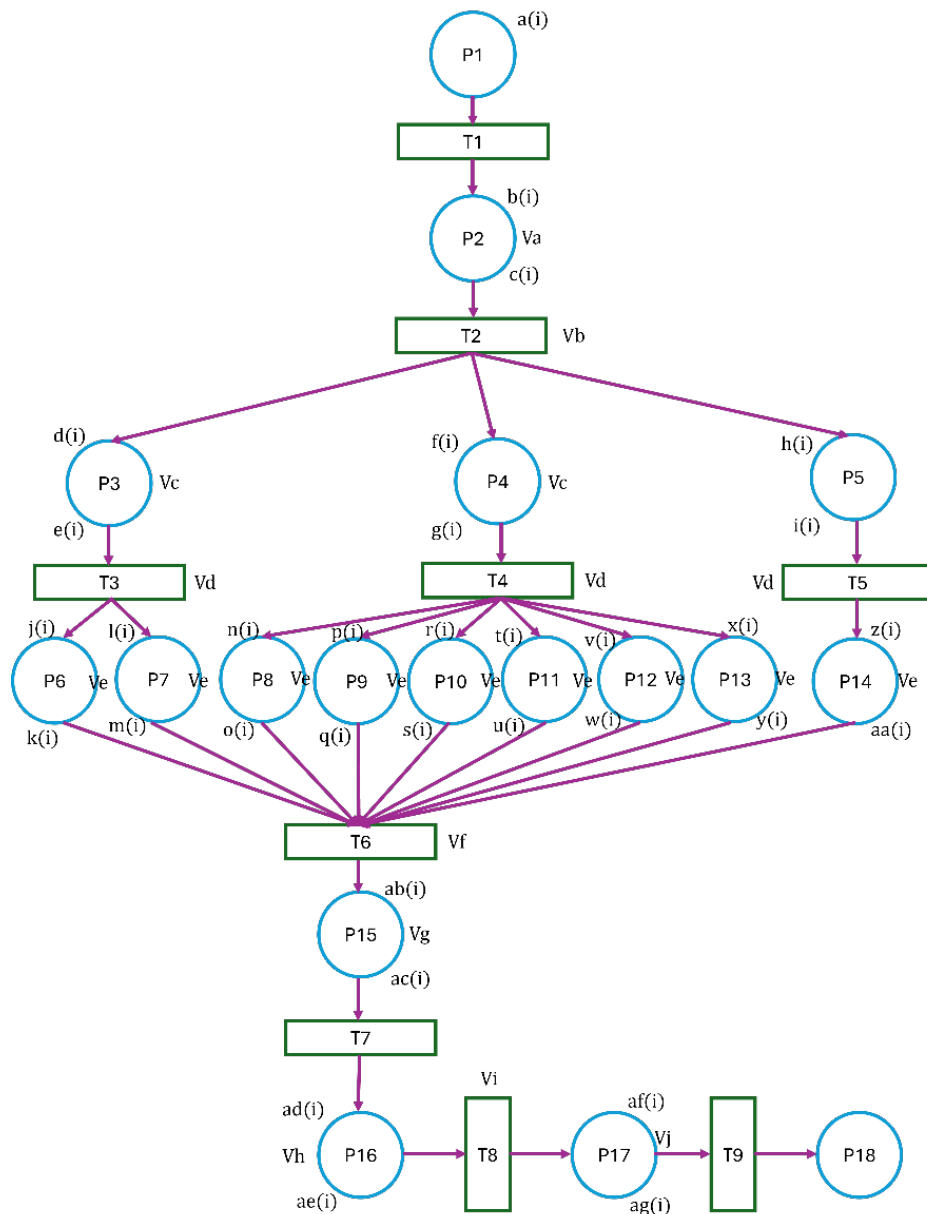


Figure 2. Petri Net of the Outpatient Queue System on the 4th Floor

Table 1. Average Service Time at Each Stage

Variable Code	Description of Service Time	Average (minutes)
Va	Registration machine service time	0.23
Vb	Walking time to nurse station	1.00
Vc	Nurse station service time	2.267
Vd	Walking time to doctor's room	1.00
Ve	Doctor consultation time	11.583

Variable Code	Description of Service Time	Average (minutes)
Vf	Walking time to prescription handover	1.00
Vg	Prescription handover time	0.80
Vh	Payment time	1.15
Vi	Waiting time for medication	28.083
Vj	Medication retrieval time	2.05

There are several assumptions used in this model, including: patients arrive at fixed intervals, each stage has a fixed average service time as listed in the table, all patients are served based on the order of arrival, and each service stage has a fixed number of staff.

$$a(i) = a(i)$$

$$b(i) = a(i) \oplus c(i - 1)$$

$$c(i) = b(i) \otimes Va$$

$$= (a(i) \otimes 0,23) \oplus (c(i - 1) \otimes 0,23)$$

$$d(i) = (c(i) \otimes Vb) \oplus d(i - 1)$$

$$= (a(i) \otimes 1,23) \oplus (c(i - 1) \otimes 1,23) \oplus d(i - 1)$$

$$e(i) = d(i) \otimes Vc$$

$$= (a(i) \otimes 3,497) \oplus (c(i - 1) \otimes 3,497) \oplus (d(i - 1) \otimes 2,267)$$

$$f(i) = (c(i) \otimes Vb) \oplus f(i - 1)$$

$$= (a(i) \otimes 1,23) \oplus (c(i - 1) \otimes 1,23) \oplus f(i - 1)$$

$$g(i) = f(i) \otimes Vc$$

$$= (a(i) \otimes 3,497) \oplus (c(i - 1) \otimes 3,497) \oplus (f(i - 1) \otimes 2,267)$$

$$h(i) = (c(i) \otimes Vb) \oplus h(i - 1)$$

$$= (a(i) \otimes 1,23) \oplus (c(i - 1) \otimes 1,23) \oplus h(i - 1)$$

$$i(i) = h(i) \otimes Vc$$

$$= (a(i) \otimes 3,497) \oplus (c(i - 1) \otimes 3,497) \oplus (h(i - 1) \otimes 2,267)$$

$$j(i) = (e(i) \otimes Vd) \oplus j(i - 1)$$

$$= (a(i) \otimes 4,497) \oplus (c(i - 1) \otimes 4,497) \oplus (d(i - 1) \otimes 3,267) \oplus j(i - 1)$$

$$k(i) = j(i) \otimes Ve$$

$$= (a(i) \otimes 16,08) \oplus (c(i - 1) \otimes 16,08) \oplus (d(i - 1) \otimes 14,85) \oplus (j(i - 1) \otimes 11,583)$$

$$l(i) = (e(i) \otimes Vd) \oplus l(i - 1)$$

$$= (a(i) \otimes 4,497) \oplus (c(i - 1) \otimes 4,497) \oplus (d(i - 1) \otimes 3,267) \oplus l(i - 1)$$

$$m(i) = l(i) \otimes Ve$$

$$= (a(i) \otimes 16,08) \oplus (c(i - 1) \otimes 16,08) \oplus (d(i - 1) \otimes 14,85) \oplus (l(i - 1) \otimes 11,583)$$

$$n(i) = (g(i) \otimes Vd) \oplus n(i - 1)$$

$$= (a(i) \otimes 4,497) \oplus (c(i-1) \otimes 4,497) \oplus (f(i-1) \otimes 3,267) \oplus n(i-1)$$

$$o(i) = n(i) \otimes Ve$$

$$= (a(i) \otimes 16,08) \oplus (c(i-1) \otimes 16,08) \oplus (f(i-1) \otimes 14,85) \oplus (n(i-1) \otimes 11,583)$$

$$p(i) = (g(i) \otimes Vd) \oplus p(i-1)$$

$$= (a(i) \otimes 4,497) \oplus (c(i-1) \otimes 4,497) \oplus (f(i-1) \otimes 3,267) \oplus p(i-1)$$

$$q(i) = p(i) \otimes Ve$$

$$= (a(i) \otimes 16,08) \oplus (c(i-1) \otimes 16,08) \oplus (f(i-1) \otimes 14,85) \oplus (p(i-1) \otimes 11,583)$$

$$r(i) = (g(i) \otimes Vd) \oplus r(i-1)$$

$$= (a(i) \otimes 4,497) \oplus (c(i-1) \otimes 4,497) \oplus (f(i-1) \otimes 3,267) \oplus r(i-1)$$

$$s(i) = r(i) \otimes Ve$$

$$= (a(i) \otimes 16,08) \oplus (c(i-1) \otimes 16,08) \oplus (f(i-1) \otimes 14,85) \oplus (r(i-1) \otimes 11,583)$$

$$t(i) = (g(i) \otimes Vd) \oplus t(i-1)$$

$$= (a(i) \otimes 4,497) \oplus (c(i-1) \otimes 4,497) \oplus (f(i-1) \otimes 3,267) \oplus t(i-1)$$

$$u(i) = t(i) \otimes Ve$$

$$= (a(i) \otimes 16,08) \oplus (c(i-1) \otimes 16,08) \oplus (f(i-1) \otimes 14,85) \oplus (t(i-1) \otimes 11,583)$$

$$v(i) = (g(i) \otimes Vd) \oplus v(i-1)$$

$$= (a(i) \otimes 4,497) \oplus (c(i-1) \otimes 4,497) \oplus (f(i-1) \otimes 3,267) \oplus v(i-1)$$

$$w(i) = v(i) \otimes Ve$$

$$= (a(i) \otimes 16,08) \oplus (c(i-1) \otimes 16,08) \oplus (f(i-1) \otimes 14,85) \oplus (v(i-1) \otimes 11,583)$$

$$x(i) = (g(i) \otimes Vd) \oplus x(i-1)$$

$$= (a(i) \otimes 4,497) \oplus (c(i-1) \otimes 4,497) \oplus (f(i-1) \otimes 3,267) \oplus x(i-1)$$

$$y(i) = x(i) \otimes Ve$$

$$= (a(i) \otimes 16,08) \oplus (c(i-1) \otimes 16,08) \oplus (f(i-1) \otimes 14,85) \oplus (x(i-1) \otimes 11,583)$$

$$z(i) = (h(i) \otimes Vd) \oplus z(i-1)$$

$$= (a(i) \otimes 4,497) \oplus (c(i-1) \otimes 4,497) \oplus (h(i-1) \otimes 3,267) \oplus z(i-1)$$

$$aa(i) = z(i) \otimes Ve$$

$$= (a(i) \otimes 16,08) \oplus (c(i-1) \otimes 16,08) \oplus (h(i-1) \otimes 14,85) \oplus (z(i-1) \otimes 11,583)$$

$$ab(i) = ((k(i) \oplus m(i) \oplus o(i) \oplus q(i) \oplus s(i) \oplus u(i) \oplus w(i) \oplus y(i) \oplus aa(i)) \otimes Vf) \oplus ab(i-1)$$

$$\begin{aligned}
 &= (a(i) \otimes 17,08) \oplus (c(i-1) \otimes 17,08) \oplus (d(i-1) \otimes 15,85) \\
 &\quad \oplus (f(i-1) \otimes 15,85) \oplus (h(i-1) \otimes 15,85) \\
 &\quad \oplus (j(i-1) \otimes 12,582) \oplus (l(i-1) \otimes 12,582) \\
 &\quad \oplus (n(i-1) \otimes 12,582) \oplus (p(i-1) \otimes 12,582) \\
 &\quad \oplus (r(i-1) \otimes 12,582) \oplus (t(i-1) \otimes 12,582) \\
 &\quad \oplus (v(i-1) \otimes 12,582) \oplus (x(i-1) \otimes 12,582) \\
 &\quad \oplus (z(i-1) \otimes 12,582) \oplus ab(i-1)
 \end{aligned}$$

$$ac(i) = ab(i) \otimes Vg$$

$$\begin{aligned}
 &= (a(i) \otimes 17,88) \oplus (c(i-1) \otimes 17,88) \oplus (d(i-1) \otimes 16,65) \\
 &\quad \oplus (f(i-1) \otimes 16,65) \oplus (h(i-1) \otimes 16,65) \\
 &\quad \oplus (j(i-1) \otimes 13,382) \oplus (l(i-1) \otimes 13,382) \\
 &\quad \oplus (n(i-1) \otimes 13,382) \oplus (p(i-1) \otimes 13,382) \\
 &\quad \oplus (r(i-1) \otimes 13,382) \oplus (t(i-1) \otimes 13,382) \\
 &\quad \oplus (v(i-1) \otimes 13,382) \oplus (x(i-1) \otimes 13,382) \\
 &\quad \oplus (z(i-1) \otimes 13,382) \oplus (ab(i-1) \otimes 0,8)
 \end{aligned}$$

$$ad(i) = ac(i) \oplus ad(i-1)$$

$$\begin{aligned}
 &= (a(i) \otimes 17,88) \oplus (c(i-1) \otimes 17,88) \oplus (d(i-1) \otimes 16,65) \\
 &\quad \oplus (f(i-1) \otimes 16,65) \oplus (h(i-1) \otimes 16,65) \\
 &\quad \oplus (j(i-1) \otimes 13,382) \oplus (l(i-1) \otimes 13,382) \\
 &\quad \oplus (n(i-1) \otimes 13,382) \oplus (p(i-1) \otimes 13,382) \\
 &\quad \oplus (r(i-1) \otimes 13,382) \oplus (t(i-1) \otimes 13,382) \\
 &\quad \oplus (v(i-1) \otimes 13,382) \oplus (x(i-1) \otimes 13,382) \\
 &\quad \oplus (z(i-1) \otimes 13,382) \oplus (ab(i-1) \otimes 0,8) \oplus ad(i-1)
 \end{aligned}$$

$$ae(i) = ad(i) \otimes Vh$$

$$\begin{aligned}
 &= (a(i) \otimes 19,03) \oplus (c(i-1) \otimes 19,03) \oplus (d(i-1) \otimes 17,8) \\
 &\quad \oplus (f(i-1) \otimes 17,8) \oplus (h(i-1) \otimes 17,8) \oplus (j(i-1) \otimes 14,532) \\
 &\quad \oplus (l(i-1) \otimes 14,532) \oplus (n(i-1) \otimes 14,532) \\
 &\quad \oplus (p(i-1) \otimes 14,532) \oplus (r(i-1) \otimes 14,532) \\
 &\quad \oplus (t(i-1) \otimes 14,532) \oplus (v(i-1) \otimes 14,532) \\
 &\quad \oplus (x(i-1) \otimes 14,532) \oplus (z(i-1) \otimes 14,532) \oplus (ab(i-1) \\
 &\quad \otimes 1,95) \oplus (ad(i-1) \otimes 1,15)
 \end{aligned}$$

$$af(i) = (ae(i) \otimes Vi) \oplus (af(i-1) \otimes Vi)$$

$$\begin{aligned}
 &= (a(i) \otimes 47,113) \oplus (c(i-1) \otimes 47,113) \oplus (d(i-1) \otimes 45,883) \\
 &\quad \oplus (f(i-1) \otimes 45,883) \oplus (h(i-1) \otimes 45,883) \\
 &\quad \oplus (j(i-1) \otimes 42,615) \oplus (l(i-1) \otimes 42,615) \\
 &\quad \oplus (n(i-1) \otimes 42,615) \oplus (p(i-1) \otimes 42,615) \\
 &\quad \oplus (r(i-1) \otimes 42,615) \oplus (t(i-1) \otimes 42,615) \\
 &\quad \oplus (v(i-1) \otimes 42,615) \oplus (x(i-1) \otimes 42,615) \\
 &\quad \oplus (z(i-1) \otimes 42,615) \oplus (ab(i-1) \otimes 30,033) \oplus (ad(i-1) \\
 &\quad \otimes 29,233) \oplus (af(i-1) \otimes 28,083)
 \end{aligned}$$

$$ag(i) = af(i) \otimes Vj$$

The following is the result of the simulation using Scilab 5.5.2:

Figure 3. Queue System Simulation Results

The model developed in this study has significant potential to support hospital management in improving service efficiency. With this model, hospital administrators can monitor service bottlenecks and simulate outcomes before

implementing operational changes, such as increasing staff or optimizing workflow. Moreover, the model enables more precise resource planning based on daily patient volume. Thus, it functions not only as an analytical tool but also as a practical guide for strategic decision-making.

This model also holds potential for broader applications beyond outpatient services. For instance, it could be applied to radiology services, where long waits are common due to limited equipment and personnel. Similarly, the pharmacy's medication pickup queues could be optimized using this approach. Beyond healthcare, the model is relevant to queue systems in transportation (e.g., airport check-ins) and logistics (e.g., warehouse distribution). With its flexibility, the model can be adapted to various operational settings with similar queuing patterns.

CONCLUSIONS AND RECOMMENDATIONS

Based on the results and discussion, the outpatient queue system at a private hospital in Yogyakarta can be modeled using Max-Plus Algebra. The modeling process resulted in a Max-Plus algebra matrix that represents the flow of services mathematically. Prior to matrix construction, a Petri net was developed to visualize the service sequence and support the modeling process.

This study was limited to the formulation of the matrix model. The resulting Max-Plus matrix can be further utilized for analyzing the outpatient queue system in more depth, particularly in evaluating service performance and identifying potential delays. Follow-up research may focus on applying this model to conduct simulations, optimize the queue system, or support hospital management in improving operational efficiency.

REFERENCES

- Alamsyah, A., & Sari, R. F. (2023). Optimisasi Multi-Objektif Pada Pelayanan Kesehatan Di Ruang Rawat Inap Rumah Sakit . *Jurnal Absis: Jurnal Pendidikan Matematika Dan Matematika*, 6(1), 826–838.
<https://doi.org/10.30606/absis.v6i1.2200>
- Hardiyanti, S. A., Yuniwati, I., & Yustita, A. D. (2017). Bentuk Petri Net dan model aljabar Max-Plus pada sistem pelayanan pasien rawat jalan Rumah Sakit Al Huda Genteng, Banyuwangi. *Jurnal UJMC*, 3(2), 1–8.
- Hariputra, R. P., Defit, S., & Sumijan. (2022). Analisis sistem antrian dalam meningkatkan efektivitas pelayanan menggunakan metode accidental sampling. *Jurnal Sistem Informasi dan Teknologi*, 4(2), 70–75.
<https://doi.org/10.37034/jsisfotek.v4i2.127>
- Kementerian Kesehatan Republik Indonesia. (2020). *Peraturan Menteri Kesehatan Republik Indonesia Nomor 3 Tahun 2020 tentang Klasifikasi dan Perizinan Rumah Sakit*.

- Nawar, W., Rahakbauw, D. L., & Patty, D. (2023). Penjadwalan waktu proses produksi tahu menggunakan pendekatan aljabar Max-Plus (Studi kasus: Pabrik Sumber Rizki). *Tensor*, 4(2), 73–82. <https://doi.org/10.30598/tensorvol4iss2pp73-82>
- Maure, O. P., Ningsi, G. P., & Nay, F. A. (2021). Pemodelan sistem antrian pasien rawat jalan menggunakan Petri Net dan aljabar Max-Plus: Studi kasus RSU di Yogyakarta. *Leibniz: Jurnal Matematika*, 1(2), 1–11. <https://doi.org/10.59632/leibniz.v1i2.101>
- Rudhito, M. A. (2016). *Aljabar Max-Plus dan Penerapannya*. Sanata Dharma University Press.
- Saumi, F., Amalia, R., Amelia, & Nurviana. (2021). Analisis Sistem Antrian Pada Pelayanan Poli Jantung RSUD Kota Langsa. *Jurnal Absis: Jurnal Pendidikan Matematika Dan Matematika*, 3(2), 297–308. <https://doi.org/10.30606/absis.v3i2.668>
- Subiono. (2015). *Aljabar Min-Max Plus dan terapannya*.
- Tutupary, F. S., & Lesnussa, Y. A. (2013). Aplikasi Petri Net pada sistem pelayanan pasien rawat jalan peserta Askes di Rumah Sakit Umum Daerah Dr. Haulussy Ambon. *Gamatika*, 3(2), 147–154.
- Yahya, L., Nurwan, N., & Resmawan, R. (2022). Menentukan waktu optimal untuk pembuatan kerajinan sulaman Karawo menggunakan aljabar Max-Plus. *Vygotsky*, 4(1), 23. <https://doi.org/10.30736/voj.v4i1.442>