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This study explores how hands-on activities, specifically paper folding, reveal the number sense performance of preservice primary school teachers. By engaging in tasks introducing and exploring number patterns, participants demonstrated their ability to transition from the real world of quantities to numerical expressions, recognise relationships among numbers, and intuitively identify sequence patterns. The result discloses varying ability levels to translate between number representations, understand the additive and multiplicative structures of numbers, and grasp the interaction between this translation process and the conceptual understanding of operations.

Keywords: Number sense, paper folding, preservice primary school teachers.

Introduction and theoretical background

Primary school teachers in Indonesia must develop a comprehensive mathematical foundation to effectively convey mathematical concepts to young learners, with number sense being a fundamental component, providing the critical foundation for further learning and cognitive development. While there is still no consensus on the definition of number sense (Pitta-Pantazi et al., 2014), the significance of fostering number sense among students has gained increased attention from mathematics educators globally. Studies by Yang et al. (2009) emphasise the critical role of number sense in children's mathematical growth and advocate for its inclusion in mathematics curricula across primary and secondary education. Therefore, assessing the number sense of preservice teachers is critical (Aktaş & Özdemir, 2017).

The study introduced a hands-on activity for preservice primary school teachers (PSTs) to introduce number sequence, allowing them to explore patterns, transition from tangible to abstract, and intuitively recognize sequences. This approach supports educational practices that link numerical patterns to physical or visual representations, which benefit both students and preservice teachers (Rivera & Becker, 2005; Whitin & Whitin, 2014). PSTs should thoroughly understand number sequence as it underpins essential arithmetic skills, pattern identification, and cognitive development (Griffin, 2009), all critical in early mathematics instruction.

The importance of number sense as a foundational idea in mathematics (NCTM, 2000) necessitates research on how preservice teachers demonstrate this skill. While much research has been conducted on number sense in students, there is relatively limited literature on the number sense of PSTs. This study aims to contribute to the literature by investigating how paper folding may reveal the number sense of PSTs. Specifically, this study explores PSTs' abilities across selected dimensions of number sense, as defined by Sowder (1992), which are elaborated further in the following section.

There has yet to be a universal consensus on the exact definition of number sense (Pitta-Pantazi et al., 2014). Some studies indicate that the number sense has two dimensions (Berch, 2005; Jordan et al., 2006). The first involves basic skills like counting, number recognition, pattern understanding, and nonverbal calculations. The second pertains to arithmetic, including problem-solving and number combinations. Additionally, number sense is the ability to see numbers as meaningful, involving skills like using different representations, recognizing magnitudes, using benchmarks, composing and decomposing numbers, understanding operations, estimating, doing mental math, and assessing result accuracy (Pitta-Pantazi et al., 2014).

Building on the existing definitions and characteristics of number sense, Sowder (1992) has provided a comprehensive list of behaviours that demonstrate the presence of number sense. This list, while not without some overlap, is a valuable resource in understanding the complex and multifaceted nature of number sense.

> (1) Ability to compose and decompose numbers, to move flexibly among different representations, to recognize when one representation is more useful than another; (2) Ability to recognize the relative magnitude of numbers; (3) Ability to deal with the absolute magnitude of numbers; (4) Ability to use benchmarks; (5) Ability to link numeration, operation, and relation symbols in meaningful ways; (6) Understanding the effects of operations on numbers; (7) The ability to perform mental computation through "invented" strategies that take advantage of numerical and operational properties; (8) Being able to use numbers flexibly to estimate numerical answers to computations, and to recognize when an estimate is appropriate; and (9) A disposition toward making sense of numbers. (Sowder, 1992, pp. 4–6)

The central question guiding this study: how do PSTs perform number sense across Sowder's dimensions 1, 5, 6, and 9? We interpret having good number sense in the context of our study as the ability to precisely translate between different number representations, grasp the structure of addition and multiplication, comprehend how these conversion processes interact, and have a conceptual understanding of operations.

Methods

The study involved 90 PSTs enrolled in their third semester at a university in Yogyakarta, Indonesia. These participants had completed introductory mathematics courses on numbers, spatial reasoning, and symmetry. We conducted the study in the third month of the semester to ensure the participants had a thorough understanding of fundamental mathematics. In this research, participants were tasked with performing a folding activity, in which they received a piece of paper and were instructed to fold it in half multiple times (Mason et al., 2010) (see Figure 1 for clarification of the folding process). The questions were: (1) How many fold line segments are in the first, second, third, and seventh folds? (2) What is the general rule for the n^{th} fold? This paper focuses solely on the first question. The rationale for the seventh fold is the practical impossibility of folding paper seven times and more. The task prompts participants to explore what is happening in the paper folding, express the structure of how a sequence is generated, and turn to objects to manipulate, make, and justify conjectures (Mason, 2010). Consequently, the task aligns with and supports participants in engaging with the four overlapping number sense dimensions outlined by Sowder (1992). In addition, as noted by Sjaastad (2023), tasks that encourage inquiry activities like exploration, conjecture, and reasoning can enhance number sense.

The lesson was implemented in 90 minutes and divided into three phases: independent work, paired discussions, and a whole-class discussion. This study is part of a broader research project and focuses specifically on individual work to assess the participants' original performance in number sense. Several precautions were implemented during the individual work phase to ensure the validity of the result. Participants were seated individually and reminded to complete their tasks without collaborating. The instructor actively observed the room while a designated observer documented participant behaviour to monitor this process. Data were collected through written responses and researcher notes. The participants' solutions were coded at two levels, namely, (i) the correctness of the final solution and (ii) the correctness of the starting point.

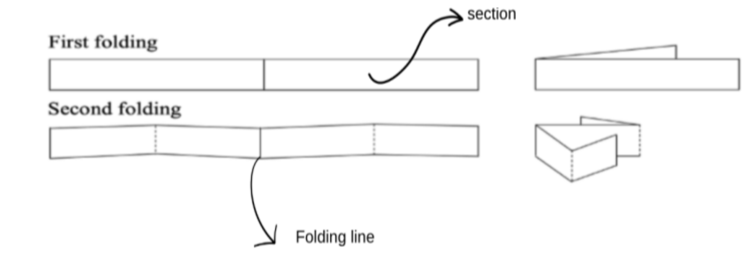


Figure 1: Illustration of the folding

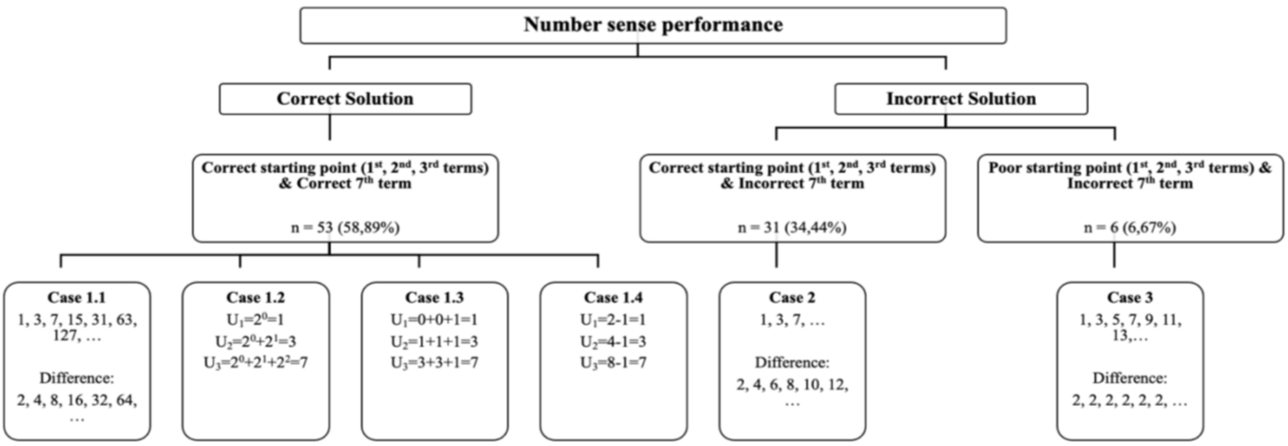


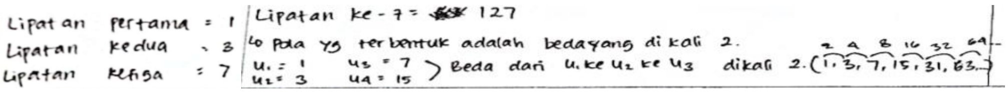
Figure 2: Distribution and details of performance

Results

The participants' performance was analysed by classifying the final results into correct and incorrect solutions, with Figure 2 highlighting the key steps leading to correct or incorrect outcomes. Based on the results, we selected the responses of several participants (HW, VH, AK, MA, NJ, and AB), which represent each case.

Correct solution: Correct starting point and correct 7th term

Case 1.1

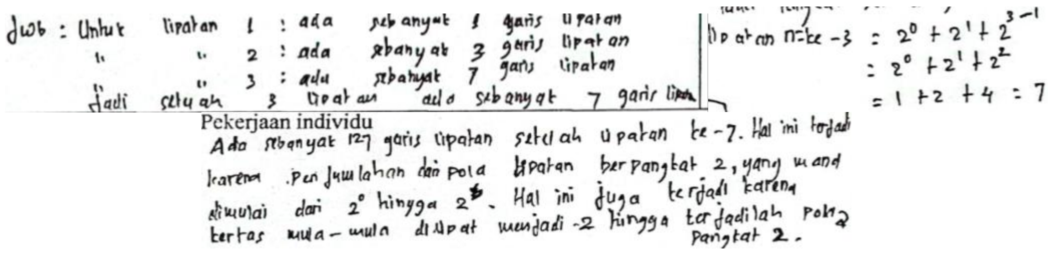


Translation: First fold = 1; second fold = 3; third fold = 7. Seventh fold = 127. The pattern formed is the difference multiplied by 2. $U_1 = 1$; $U_2 = 3$; $U_3 = 7, U_4 = 15$. The difference from U_1 to U_2 to U_3 is multiplied by 2.

Figure 3: Answer by HW

As evidenced by Figure 3, the PST correctly identified the sequence of fold line segments, suggesting she understands the basic idea of observing differences between successive terms, which increase in a structured way, i.e., kept getting multiplied by two. Each subsequent term can be determined by summing the previous term and the continuously doubled difference. Finding the seventh term using this rule remains a reliable method.

Case 1.2

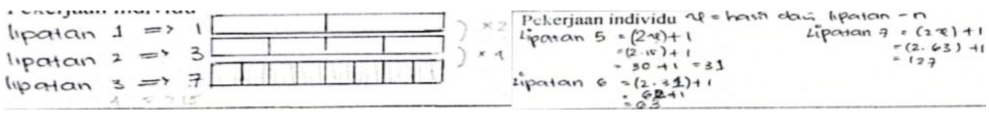


Translation: First fold, there is 1-fold line segment. Second fold, there are 3-fold line segments. Third fold, there are 7-fold line segments. Third fold = $2^0 + 2^1 + 2^{3-1} = 2^0 + 2^1 + 2^2 = 1 + 2 + 4 = 7$. There are 127-fold line segments on the seventh fold. This is due to the sum of the fold patterns in powers of 2, starting from 2^0 to 2^6. This is also because the paper is initially folded in 2 until a power of 2 pattern occurs.

Figure 4: Answer by VH

Based on Figure 4, the PST accurately recognized that the number of fold line segments follows an exponential pattern with base 2. She employed the formula $2^0 + 2^1 + \cdots + 2^{n-1}$ to articulate how the fold line segments accumulate, performing an advanced and accurate mathematical representation. By identifying the recursive relationship inherent in this exponential sequence, she demonstrated an understanding of how each fold builds upon the previous ones. Her statement, "the paper is initially folded in 2 until a power of 2 pattern occurs," indicated that she made a connection between the physical act of folding and the mathematical pattern.

Case 1.3

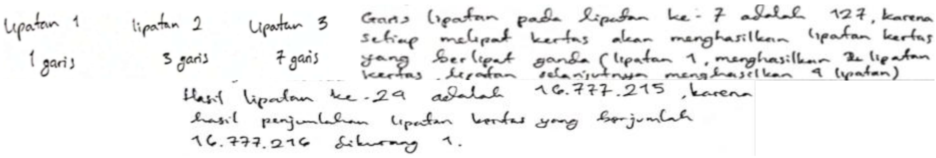


Translation: First fold: 1-line segment, second fold: 3-line segments, third fold: 7-line segments. x: result of the n^{th} – fold. Fifth fold $= (2x) + 1 = (2 \times 15) + 1 = 30 + 1 = 31$. Sixth fold $= (2 \times 31) + 1 = 62 + 1 = 63$. Seventh fold $= (2x) + 1 = (2 \times 63) + 1 = 127$. (Note: The sketch indicates that the number of sections doubles in each fold)

Figure 5: Answer by AK

Referring to Figure 5, the PST attempted to formulate a general rule for each subsequent fold using the expression $2x + 1$ to describe the relationship between the current and the previous terms. The appearance of this expression suggests that she fully captured the recursive nature of the sequence but struggled with fully articulating it, indicated by x being noted as a “result of the n^{th} fold” rather than “the number of the fold line segments on the previous fold”. Even though there is no verbal description regarding the number of sections or fold line segments, her sketch highlights that the number of sections doubles in each fold, which signals a connection to the folding activity.

Case 1.4



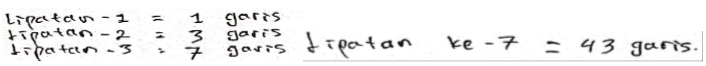
Translation: The number of the fold line segment at the 7^{th} fold is 127, because each fold will produce doubled sections (the first fold produces 2 sections, the next fold produces 4 sections). The result of the 24^{th} fold is 16.777.215 because the number of sections (16.777.216) is minus 1.

Figure 6: Answer by MA

As shown in Figure 6, the PST correctly identified that the number of sections doubles in each fold, reflecting an understanding of geometric progression. He grasped that the total number of fold line segments equals the number of sections minus one and correctly applied this reasoning to the 7^{th} and 24^{th} folds. His statement that “each fold will produce doubled sections” confirms a connection to the folding activity. This manifestation of the PST revisiting the folding activity indicates a deductive closure.

Incorrect solution: Correct starting point and incorrect 7^{th} term

Case 2



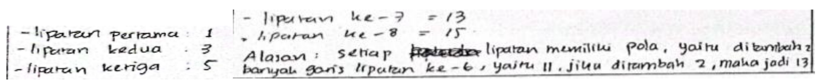
Translation: First fold = 1 line segment, second fold = 3 line segments, third fold = 7 line segments, seventh fold = 43 line segments.

Figure 7: Answer by NJ

As depicted in Figure 7, the identified sequence is $1, 3, 7, \dots, \dots, \dots, 43$, where the difference between two terms might be $2, 4, 6, 8, 10, 12$. While the fourth fold could have been physically performed, the PST could not accurately identify the fourth term, indicating disconnection with the folding activity. The incorrect sequence follows a pattern of adding successively increasing even numbers, reflecting a simple arithmetic progression. This point suggests that the PST overgeneralized his understanding of basic arithmetic sequences, applying a linear pattern where a non-linear pattern was required.

Incorrect solution: Poor starting point and incorrect 7^{th} term

Case 3



Translation: First fold: 1, second fold: 3, third fold: 5, seventh fold: 13, eighth fold: 15. Reason: Each fold has a pattern, which is plus 2. The number of lines on the sixth fold is 11, if it is plus 2 then it becomes 13.

Figure 8: Answer by AB

As presented in Figure 8, the identified sequence $1, 3, 5, 7, 9, 11, 13$ with a consistent difference of 2 between consecutive terms indicates that the PST incorrectly interpreted the pattern as a linear arithmetic progression. This reliance on a linear arithmetic pattern suggests an overgeneralization rooted in familiarity with basic sequences. The sequence that does not match the actual results reflects either a misunderstanding of the instruction or a disconnection with the folding activity. If not due to instructional misinterpretation, this case suggests a gap in his ability to translate concrete experiences into numbers as one of the mathematical representations.

Discussions

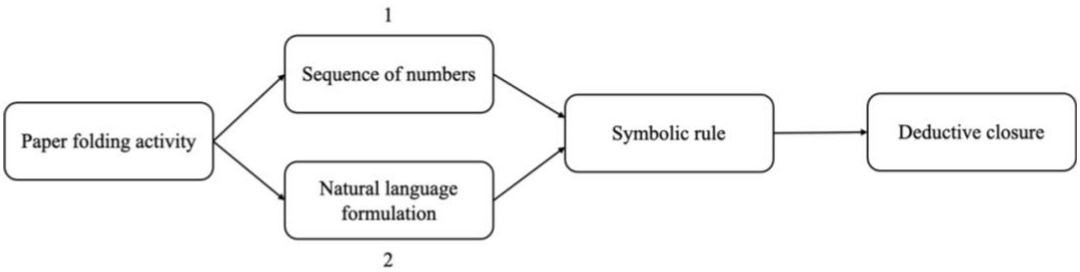


Figure 9: Translation process

Figure 9 depicts two possible approaches for translating the hands-on activity to reach the deductive closure as the optimal step. In this translation, certain visible features of the PSTs' responses are examined and connected to Sowder's (1992) dimensions of number sense. The folding activity could be translated into a mathematical object in two distinct ways:

First, if the fold line segments are simply enumerated, a sequence of numbers $(1, 3, 7, 15, ...)$ emerges without relational understanding from the translation process. This simplest manifestation of number sense aligns with dimension 1 (the ability to move flexibly among different representations), which is evident in how the manual activity produces a number at each step. If this counting is incorrect (Case 3), it could suggest a rudimentary number sense in this dimension or that the PST needed to recognize the need to count the fold line segments after each step. The error might stem from a tenuous grasp of numbers or misunderstanding the instruction. Meanwhile, Case 2 illustrates a specific circumstance where the PST accurately represented the number of line segments only for the first three folds, suggesting an inconsistency between representations.

Second, the translation process can also lead to formulating a rule based on natural numbers' additive and multiplicative structure. This rule could be established after the counting, either by abstracting the activity and identifying the pattern or directly linking the activity to a structured object. This process aligns with the manifestation of dimension 5 (the ability to link numeration, operation, and relation symbols in meaningful ways). Case 1.1 provides a concrete example of this dimension where the PST recognized the difference between consecutive terms. Case 1.3 gives another example, where the PST demonstrated a different approach to recursion, not by analysing the difference but by formulating a direct rule to describe the pattern.

In the latter approach, the relationship could be articulated in natural language: "In each fold, twice as many sections are formed as before, and the number of fold line segments is one less than the number of sections." This formulation necessitates an understanding of the operations' effects on numbers, reflecting dimension 6. Case 1.2 further exemplifies the presence of this dimension as the PST recognized that consecutive multiplication by two could be expressed as the sum of exponential numbers with base 2, suggesting the formulation of a general rule. This work contrasts with Case 1.1, where the progress stalled in identifying the difference between consecutive terms, and no general term was formulated. Thus, the PST's understanding was confined to the arithmetic level.

Optimally, recognizing the pattern in the number sequence concludes with a deductive reasoning process, connecting back to the folding activity. This manifestation reflects a disposition toward making sense of numbers, which characterizes dimension 9. The emergence of this dimension is evident in Case 1.4, where the PST explained the connection between the folding activity and the rule, providing a clear justification and argumentation for why the sequence is correct. Moreover, the sketch in Case 1.3 serves a visual explanation for the stated rule, simultaneously exemplifying the manifestation of dimension 9.

In summary, some PSTs accurately determine the seventh term, demonstrating their comprehension of the instructions. This performance implies their ability to smoothly transition between representations (dimension 1) from physical to mathematical forms. When PSTs demonstrate procedural knowledge, they often reflect their understanding of how operations affect numbers (dimension 6), as they can execute procedures accurately and make connections between their actions and numerical outcomes. Some PSTs demonstrate a conceptual understanding of exponential growth, recursive patterns, and their connection to the folding paper activity by linking numeration, operations, and relational symbols. It means they understand how the number of folds relates to exponential increases and can articulate these connections meaningfully using mathematical symbols and operations. On the other hand, some of the PSTs needed help to express the rule clearly and tend to overgeneralize arithmetic sequences (dimension 5). Finally, when PSTs connect the folding paper activity to mathematical representation, it shows a disposition toward making sense of numbers (dimension 9). They actively engage with the numerical patterns and relationships, demonstrating an inclination to understand and explore mathematical ideas. Overall, some participants performed well, and some others faced challenges in performing dimensions 1, 5, 6, and 9. In line with this result, another study by Aktaş & Özdemir (2017) highlighted the preservice elementary school mathematics teachers who have lower than expected number sense performance.

Conclusions and pedagogical implications

This study revealed varied performance of number sense among PSTs, particularly in recognising numerical patterns, recursion, and linking concrete actions to mathematical abstractions. To improve these areas, teacher educators should: (1) explicitly encourage PSTs to justify identified patterns, fostering deeper conceptual understanding rather than superficial recognition; (2) use activities like paper folding to strengthen the connections between tangible experiences and abstract mathematical concepts, encouraging PSTs to verbally and visually articulate mathematical relationships.

Acknowledgement

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