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Publisher
American Institute of Physics

SJR 2025
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H-Index
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Publication type
Conferences and Proceedings

ISSN
0094243X, 15517616

Coverage
1972-2026

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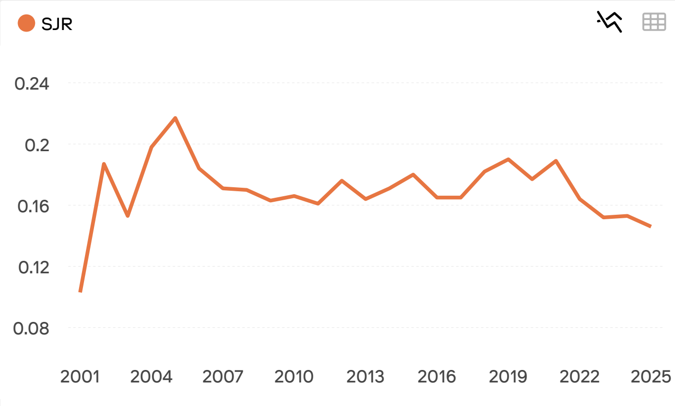

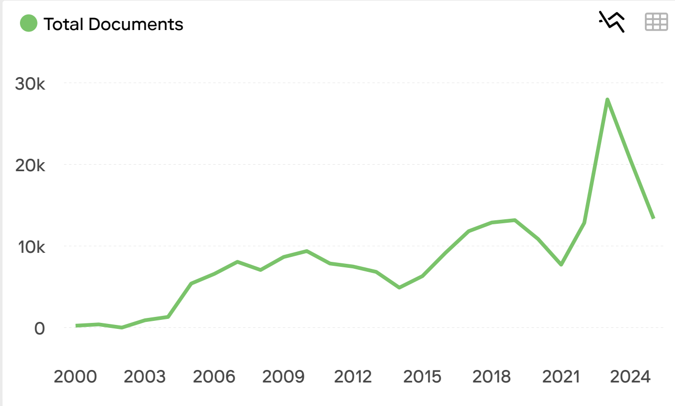

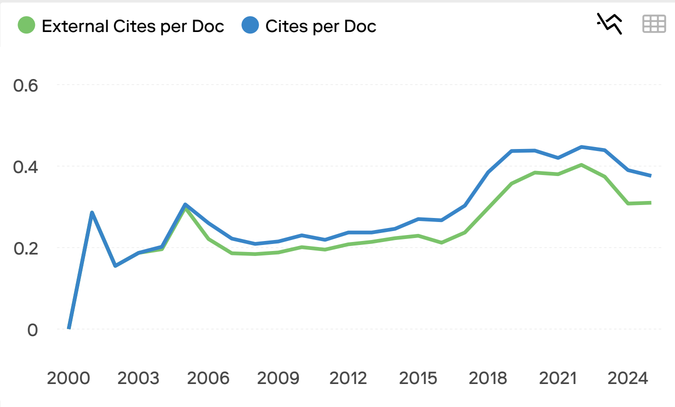

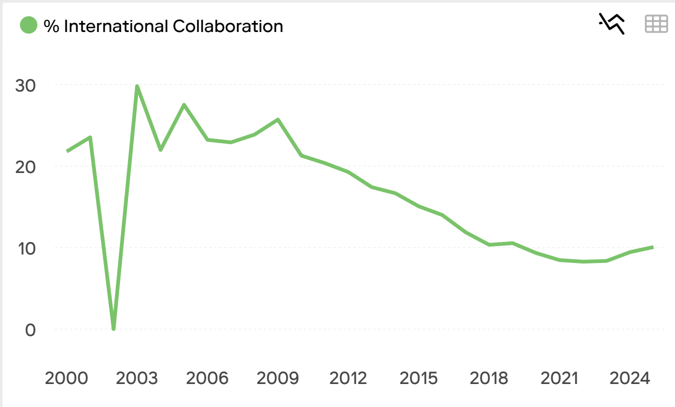

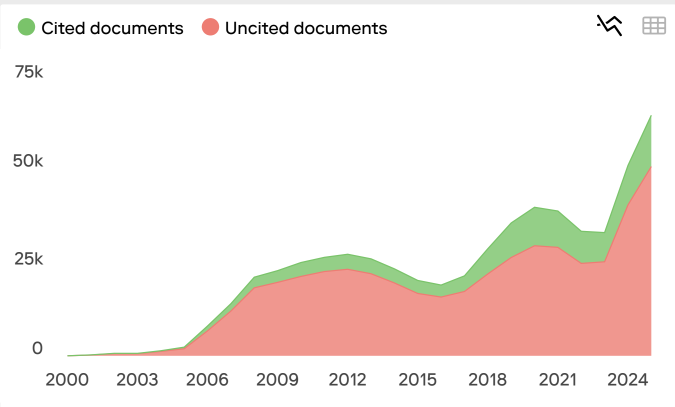

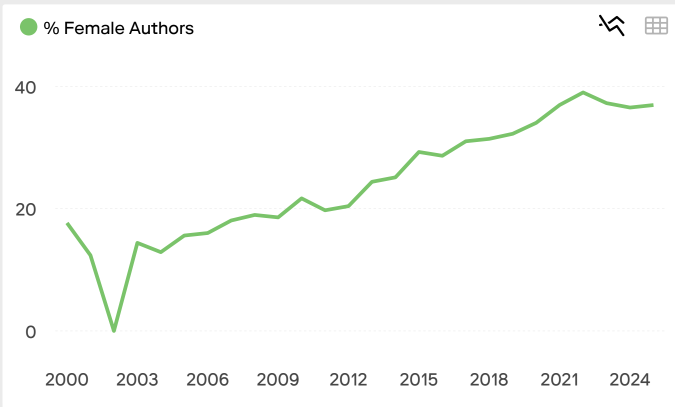

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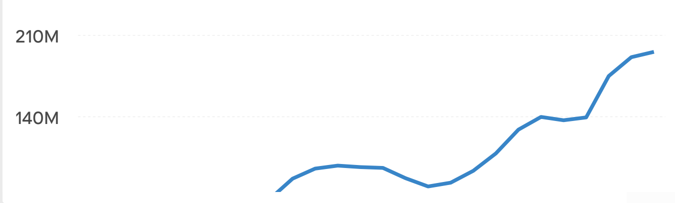

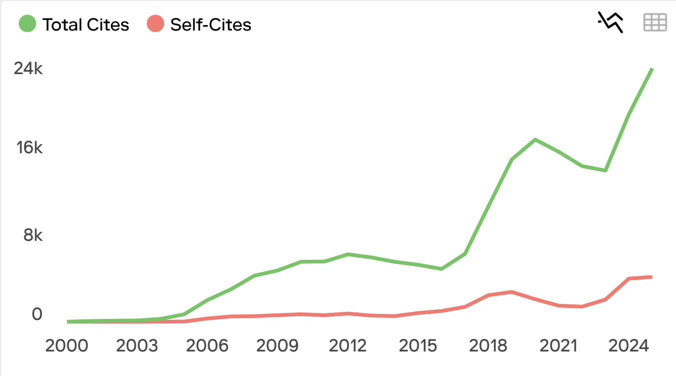

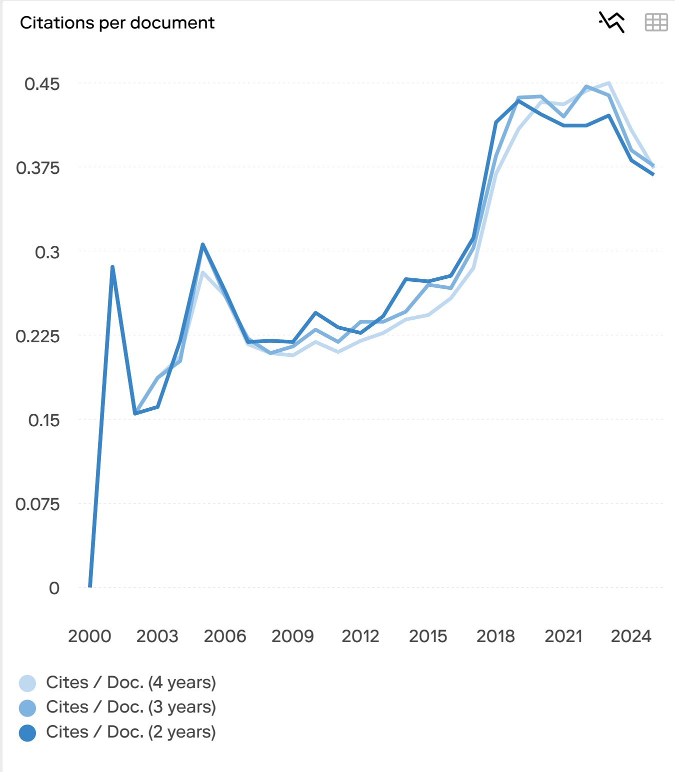

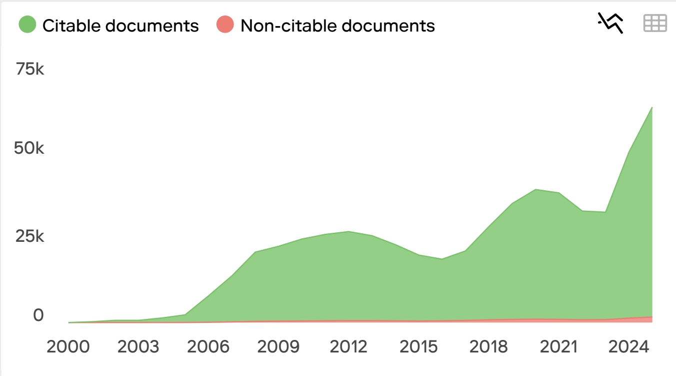

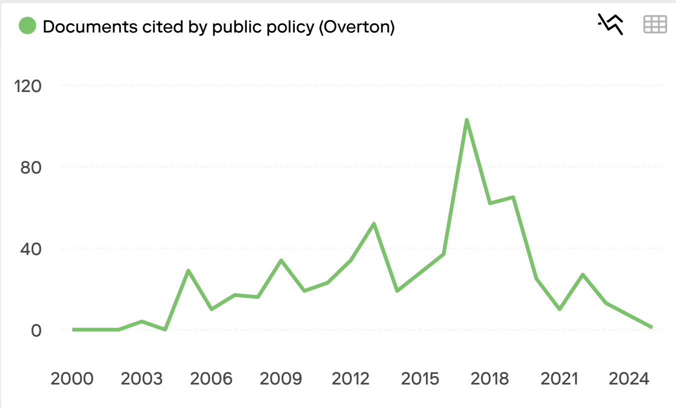

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International Conference of Mathematics and Mathematics Education (ICMME) 2022

Surakarta, Indonesia • 24–26 July 2022

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Preface: International Conference of Mathematics and Mathematics Education (ICMME) 2022

It is with great pleasure and anticipation that we present the proceedings of the International Conference on Mathematics and Mathematics Education (ICMME) 2022. This esteemed conference, held under the theme "Improving Knowledge through Sustainable Innovation in the field of Education, Science, Technology, Engineering, and Mathematics," marks a significant milestone in the global pursuit of academic excellence and collaboration.

This theme highlights the importance of sustainable innovation in strengthening the fields of education, science, technology, engineering, and mathematics. In the context of a rapidly evolving global era, sustainable innovative approaches become key in improving educational practices and expanding our understanding of the underlying concepts of science and technology. Through interdisciplinary and international collaboration, this seminar aims to stimulate creative thinking, encourage the development of sustainable solutions, and promote enriching idea exchange to support sustainable progress in education and science.

We extend our sincere gratitude to the keynote speakers whose expertise and insights have enriched our discussions:

1. Dr. Ahmad Ridwan Tresna Nugraha, Head of Research Center for Quantum Physics, Indonesia
2. Dr. Michiel Veldhuis, from IPABO University of Applied Science and Utrecth University, the Netherlands
3. Assoc. Prof. Dr. Masitah Shahrill, Graduate School of Education, Universiti Brunei Darussalam, Brunei Darussalam
4. Assoc. Prof. Dr. Nor Zila binti Abd Hamid, Universiti Pendidikan Sultan Idris, Malaysia

This conference would not have been possible without the dedication and collaboration between the Department of Mathematics Education at Universitas Sebelas Maret Surakarta and the Department of Mathematics Education at Universiti Pendidikan Sultan Idris. Your commitment to fostering academic exchange and advancement in the fields of mathematics and education is truly commendable.

Held in a hybrid format from July 24th to 26th, 2022, at the Auditorium FKIP UNS, this conference has brought together scholars, researchers, and practitioners from Malaysia, the Philippines, Japan, Brunei Darussalam, and Indonesia. Through a blend of physical and virtual participation, we have transcended geographical boundaries to facilitate meaningful dialogue and collaboration.

The diverse perspectives and innovative ideas presented during this conference have undoubtedly contributed to the advancement of knowledge and practice in mathematics and mathematics education. We are confident that the papers included in these proceedings will serve as valuable resources for scholars and practitioners alike, inspiring further research and development in the pursuit of excellence.

We express our heartfelt appreciation to all participants, presenters, reviewers, and organizers for their invaluable contributions to the success of ICMME 2022. May the insights gained and connections forged during this conference continue to resonate and drive positive change in our respective fields.

Thank you for your participation and support.

Sincerely,
Editors
Dr. Budi Usodo, M.Pd.
Dr. Imam Sujadi, M.Si.
Dr. Laila Fitriana, M.Pd.
Mr. Agus Hendriyanto, M.Pd.
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Dinda U. R. Yani; Kartika Fithriasari; Dedy D. Prastyo

AIP Conf. Proc. 3464, 080017 (2026) https://doi.org/10.1063/5.0330810

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SCIENCE

Measurement of vapor-liquid equilibrium for binary systems limonene and citronellal in kaffir lime essential oil as antimicrobial substance

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RESEARCH ARTICLE | JUNE 10 2026

Mathematical model of the spread of African swine fever disease and its numerical solutions based on the fourth-order Runge-Kutta and the Milne methods

Maria Y. Bhoki ; Sudi Mungkasi

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AIP Conf. Proc. 3464, 070001 (2026)

https://doi.org/10.1063/5.0330907

In this paper, we describe the dynamics of the spread of African swine fever (ASF) between pigs and ticks. A mathematical model for the problem is recalled in this paper. The mathematical model can be used to find strategies to reduce the number of pigs and ticks infected with African swine fever disease. To be specific, the Susceptible-Infected-Recovered-Susceptible-Infected (SIR-SI) mathematical model for the ASF disease will be solved numerically using the fourth-order Runge-Kutta and the Milne methods. From simulation results, both methods perform well. The solution patterns produced by the two methods are the same as the pattern of the reference solution generated by the ODE45 algorithm in MATLAB software. We use the ODE45 algorithm as the reference, because the exact analytical solution of the SIR-SI model has not yet been found. In theory, the order of accuracy of the fourth-order Runge-Kutta method is equal to that of the Milne method. This is confirmed by simulations in this paper. From our simulation results, the fourth-order Runge-Kutta method is more effective in solving the SIR-SI model of the ASF disease spread in comparison with the Milne method. This is supported by our research findings based on smaller error values, but the same order of accuracy of the fourth-order Runge-Kutta method in comparison with those of the Milne method.

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Mathematical model of the spread of African swine fever disease and its numerical solutions based on the fourth-order Runge-Kutta and the Milne methods

Maria Y. Bhoki ; Sudi Mungkasi

AIP Conf. Proc. 3464, 070001 (2026)
https://doi.org/10.1063/5.0330907

Mathematical Model of the Spread of African Swine Fever Disease and Its Numerical Solutions Based on the Fourth-Order Runge-Kutta and the Milne Methods

Maria Y. Bhoki[1, a)] and Sudi Mungkasi[2]

[1]*Department of Mathematics Education, FKIP, Sanata Dharma University, Yogyakarta, Indonesia.*
[2]*Department of Mathematics, FST, Sanata Dharma University, Yogyakarta, Indonesia.*

[a)] Corresponding author: kudomrs8@gmail.com

Abstract. In this paper, we describe the dynamics of the spread of African swine fever (ASF) between pigs and ticks. A mathematical model for the problem is recalled in this paper. The mathematical model can be used to find strategies to reduce the number of pigs and ticks infected with African swine fever disease. To be specific, the Susceptible-Infected-Recovered-Susceptible-Infected (SIR-SI) mathematical model for the ASF disease will be solved numerically using the fourth-order Runge-Kutta and the Milne methods. From simulation results, both methods perform well. The solution patterns produced by the two methods are the same as the pattern of the reference solution generated by the ODE45 algorithm in MATLAB software. We use the ODE45 algorithm as the reference, because the exact analytical solution of the SIR-SI model has not yet been found. In theory, the order of accuracy of the fourth-order Runge-Kutta method is equal to that of the Milne method. This is confirmed by simulations in this paper. From our simulation results, the fourth-order Runge-Kutta method is more effective in solving the SIR-SI model of the ASF disease spread in comparison with the Milne method. This is supported by our research findings based on smaller error values, but the same order of accuracy of the fourth-order Runge-Kutta method in comparison with those of the Milne method.

INTRODUCTION

African swine fever or commonly called ASF is a disease caused by a kind of virus that affects domesticated pigs or wild boars. ASF is a disease in the livestock of pigs that is quite dangerous and accompanied by a very high mortality rate. This infectious disease is caused by the ASF virus which can infect members of the suidae [1]. Suidae is a family of mammals commonly called pigs. This disease can spread quickly and the mortality rate it causes reaches more than 90% so that it can result in huge economic losses for some areas that are indeed one of the country's incomes in the form of pig livestock or pork. When a pig is infected with ASF, it will develop high fever and bleeding of the internal organs, redness of the chest and abdomen, bloody diarrhea, anorexia, atasia, paresis, convulsions, sometimes vomiting, diarrhea, constipation, and cyanosis skin bleeding [2]. Usually pigs become depressed, on their backs, have difficulty beating, and do not want to eat. Approximately within a week pigs infected with ASF will experience these things. The disease is transmitted by contact with pigs infected with blood, body fluids, or corpses. The virus can remain in the body from month to year, but it cannot infect human.

ASF virus is a double-stranded DNA virus of the genus Asfivirus. To date the ASF virus has only one serotype although there are 23 genotypes with varying virulence. ASF is caused by a DNA virus with double strands of the genus Asfivirus and the family Asfarviridae [3], [4]. The ASF virus is highly resistant to environmental influences, and is stable at a pH of 4-13, as well as being able to withstand life in the blood (4℃) for 18 months, in cold meat for 15 weeks, in frozen meat for several years, in ham for 6 months and in pigsty for 1 month [5].

Sources of transmission such as blood, body fluids, and tissues of infected pigs are very risky because they contain high concentrations of the virus. Contagion can occur in case of direct contact with a sick pig. Transmission can also

International Conference of Mathematics and Mathematics Education (ICMME) 2022
AIP Conf. Proc. 3464, 070001-1–070001-9; https://doi.org/10.1063/5.0330907

occur through equipment, feed, and beverages that are already contaminated by the virus. In addition, transmission can also occur through ticks that act as vectors of the ASF virus [6].

Until now, the government has not found a vaccine that can be used to prevent the spread of ASF disease. Therefore, the most important strategic step in the process to prevent the spread of ASF is through the implementation of biosecurity, good pig farm management, and strict and intensive supervision for areas at high risk of spreading ASF disease.

MATHEMATICAL MODEL

In addition to the application of biosecurity and animal husbandry management, the development of science in the field of mathematics also has an important role to be able to control the spread of the ASF disease, namely by applying mathematical modeling. In this study, it will be discussed about mathematical modeling of the spread of ASF disease with the SIR-SI model.

One example of a mathematical model that can be applied in the dynamics of the spread of ASF disease is the SIR-SI (Susceptible-Infected-Recovered-Susceptible-Infected) epidemic model. This model has been widely used to predict patterns of disease behavior that spread in pig and tick livestock over time in a given environment.

The mathematical model used consists of S_P, I_P, R_P, S_T, I_T which describes the dynamics of the spread of ASF disease. The population is divided into several parts, namely, pigs susceptible to disease (S_P), infected pigs (I_P), recovered pigs (R_P), ticks that are susceptible to disease (S_T), and infected ticks (I_T). A graphical representation of the proposed model is shown in Figure 1.

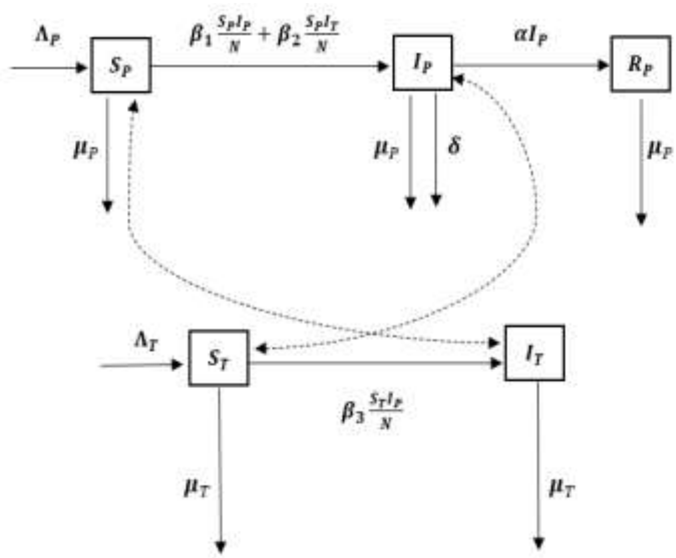

FIGURE 1. ASF disease spread model diagram.

Therefore, the mathematical model of the dynamics of the spread of ASF disease is regulated by a system of differential equations as follows:

$$\left.\begin{aligned}
\frac{dS_P}{dt} &= \Lambda_P - \mu_P S_P - \beta_1 \frac{S_P I_P}{N} - \beta_2 \frac{S_P I_T}{N} \\
\frac{dI_P}{dt} &= \beta_1 \frac{S_P I_P}{N} + \beta_2 \frac{S_P I_T}{N} - (\mu_P + \alpha + \delta) I_P \\
\frac{dR_P}{dt} &= \alpha I_P - \mu_P R_P \\
\frac{dS_T}{dt} &= \Lambda_T - \mu_T S_T - \beta_3 \frac{S_T I_P}{N} \\
\frac{dI_T}{dt} &= \beta_3 \frac{S_T I_P}{N} - \mu_T I_T
\end{aligned}\right\} \quad (1)$$

where $S_P(0) \geq 0$, $I_P(0) \geq 0$, $R_P(0) \geq 0$, $S_T(0) \geq 0$, and $I_T(0) \geq 0$ is the initial state.

Descriptions of variables and parameters are given in Table 1.

TABLE 1. Parameters used in the SIR-SI model.

Notation	Description
Λ_P	The rate of increase in susceptible pigs
Λ_T	The rate of increase susceptible ticks
μ_P	The natural mortality rate of pigs
μ_T	The natural mortality rate of ticks
β_1	The rate of infection of the disease in pigs through contact with infected pigs
β_2	The rate of infection of the disease in pigs through contact with infected ticks
β_3	The rate of infection of the disease in ticks through contact with infected pigs
α	The rate of recovery of pigs recovering from the disease
δ	Mortality rate from disease

NUMERICAL METHODS

Based on the above mathematical model, the fourth-order Runge-Kutta (RK4) method and the Milne method are used to find solutions to the system of equations.

Fourth-Order Runge Kutta Method

The fourth-order Runge-Kutta method has been widely used because it has a high degree of accuracy. Suppose that we are given the following system of equations:

$$\begin{cases} \dfrac{dy}{dx} = f(x,y) \\ y(x_0) = y_0 \end{cases} \tag{2}$$

Equation system (2) can be solved using the fourth-order Runge-Kutta method as follows:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{3}$$

with:

$$k_1 = hf(x_i, y_i) \tag{4}$$

$$k_2 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1) \tag{5}$$

$$k_3 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2) \tag{6}$$

$$k_4 = hf(x_i + h, y_i + k_3) \tag{7}$$

where $i = 0, 1, 2, \ldots, M-1$ and $h = x_{i+1} - x_i$. Here M is the number of steps in the free variable, and x_i denotes the discretized free variable.

Milne Method

Milne method is a multi-step method, consisting of a predictor and corrector to find solutions to a given initial value problem. The Milne numerical scheme for solving equation system (2) has the following form [7]:

Predictor:

$$y_{n+1}^* = y_{n-3} + \frac{4h}{3}(2y_n' - y_{n-1}' + 2y_{n-2}') \tag{8}$$

Corrector:

$$y_{n+1} = y_{n-1} + \frac{h}{3}(y_{n+1}^* + 4y_n' + y_{n-1}') \tag{9}$$

with $h = x_{i+1} - x_i$.

NUMERICAL RESULTS

In this section, the simulation results will be described based on numerical calculations from the fourth-order Runge-Kutta method and the Milne method. Numerical programming and simulation is performed using MATLAB software. For all numerical simulations, the initial values and parameter values are presented in Table 2. The unit of time used is day.

Taking into account the initial values of each of the populations $S_P(0)$, $I_P(0)$, $R_P(0)$, $S_T(0)$, $I_T(0)$, the solution based on the fourth-order Runge-Kutta method is described using the numerical scheme as follows:

$$S_{P_{i+1}} = S_{P_i} + \frac{1}{6}\left(k_{1S_P} + 2k_{2S_P} + 2k_{3S_P} + k_{4S_P}\right) \tag{10}$$

$$I_{P_{i+1}} = I_{P_i} + \frac{1}{6}\left(k_{1I_P} + 2k_{2I_P} + 2k_{3I_P} + k_{4I_P}\right) \tag{11}$$

$$R_{P_{i+1}} = R_{P_i} + \frac{1}{6}\left(k_{1R_P} + 2k_{2R_P} + 2k_{3R_P} + k_{4R_P}\right) \tag{12}$$

$$S_{T_{i+1}} = S_{T_i} + \frac{1}{6}\left(k_{1S_T} + 2k_{2S_T} + 2k_{3S_T} + k_{4S_T}\right) \tag{13}$$

$$I_{T_{i+1}} = I_{T_i} + \frac{1}{6}\left(k_{1I_T} + 2k_{2I_T} + 2k_{3I_T} + k_{4I_T}\right) \tag{14}$$

with h is the time step, and i is the iteration index, as well as:

$$k_{1S_P} = h\left(\Lambda_P - \mu_P S_{P_i} - \beta_1 \frac{S_{P_i} I_{P_i}}{N} - \beta_2 \frac{S_{P_i} I_{T_i}}{N}\right) \tag{15}$$

$$k_{1I_P} = h\left(\beta_1 \frac{S_{P_i} I_{P_i}}{N} + \beta_2 \frac{S_{P_i} I_{T_i}}{N} - (\mu_P + \alpha + \delta) I_{P_i}\right) \tag{16}$$

$$k_{1R_P} = h\left(\alpha I_{P_i} - \mu_P R_{P_i}\right) \tag{17}$$

$$k_{1S_T} = h\left(\Lambda_T - \mu_T S_{T_i} - \beta_3 \frac{S_{T_i} I_{P_i}}{N}\right) \tag{18}$$

$$k_{1I_T} = h\left(\beta_3 \frac{S_{T_i} I_{P_i}}{N} - \mu_T I_{T_i}\right) \tag{19}$$

$$k_{2S_P} = h\left[\Lambda_P - \mu_P\left(S_{P_i} + \frac{1}{2}k_{1S_P}\right) - \beta_1 \frac{\left(S_{P_i} + \frac{1}{2}k_{1S_P}\right)\left(I_{P_i} + \frac{1}{2}k_{1I_P}\right)}{N} - \beta_2 \frac{\left(S_{P_i} + \frac{1}{2}k_{1S_P}\right)\left(I_{T_i} + \frac{1}{2}k_{1I_T}\right)}{N}\right] \tag{20}$$

$$k_{2I_P} = h\left[\beta_1 \frac{\left(S_{P_i} + \frac{1}{2}k_{1S_P}\right)\left(I_{P_i} + \frac{1}{2}k_{1I_P}\right)}{N} + \beta_2 \frac{\left(S_{P_i} + \frac{1}{2}k_{1S_P}\right)\left(I_{T_i} + \frac{1}{2}k_{1I_T}\right)}{N} - (\mu_P + \alpha + \delta)\left(I_{P_i} + \frac{1}{2}k_{1I_P}\right)\right] \tag{21}$$

$$k_{2R_P} = h\left[\alpha\left(I_{P_i} + \frac{1}{2}k_{1I_P}\right) - \mu_P\left(R_{P_i} + \frac{1}{2}k_{1R_P}\right)\right] \tag{22}$$

$$k_{2S_T} = h\left[\Lambda_T - \mu_T\left(S_{T_i} + \frac{1}{2}k_{1S_T}\right) - \beta_3 \frac{\left(S_{T_i} + \frac{1}{2}k_{1S_T}\right)\left(I_{P_i} + \frac{1}{2}k_{1I_P}\right)}{N}\right] \tag{23}$$

$$k_{2I_T} = h\left[\beta_3 \frac{\left(S_{T_i} + \frac{1}{2}k_{1S_T}\right)\left(I_{P_i} + \frac{1}{2}k_{1I_P}\right)}{N} - \mu_T\left(I_{T_i} + \frac{1}{2}k_{1I_T}\right)\right] \tag{24}$$

$$k_{3S_P} = h\left[\Lambda_P - \mu_P\left(S_{P_i} + \frac{1}{2}k_{2S_P}\right) - \beta_1 \frac{\left(S_{P_i} + \frac{1}{2}k_{2S_P}\right)\left(I_{P_i} + \frac{1}{2}k_{2I_P}\right)}{N} - \beta_2 \frac{\left(S_{P_i} + \frac{1}{2}k_{2S_P}\right)\left(I_{T_i} + \frac{1}{2}k_{2I_T}\right)}{N}\right] \tag{25}$$

$$k_{3I_P} = h\left[\beta_1 \frac{\left(S_{P_i} + \frac{1}{2}k_{2S_P}\right)\left(I_{P_i} + \frac{1}{2}k_{2I_P}\right)}{N} + \beta_2 \frac{\left(S_{P_i} + \frac{1}{2}k_{2S_P}\right)\left(I_{T_i} + \frac{1}{2}k_{2I_T}\right)}{N} - (\mu_P + \alpha + \delta)\left(I_{P_i} + \frac{1}{2}k_{2I_P}\right)\right] \tag{26}$$

$$k_{3R_P} = h\left[\alpha\left(I_{P_i} + \frac{1}{2}k_{2I_P}\right) - \mu_P\left(R_{P_i} + \frac{1}{2}k_{2R_P}\right)\right] \tag{27}$$

$$k_{3S_T} = h\left[\Lambda_T - \mu_T\left(S_{T_i} + \frac{1}{2}k_{2S_T}\right) - \beta_3\frac{\left(S_{T_i} + \frac{1}{2}k_{2S_T}\right)\left(I_{P_i} + \frac{1}{2}k_{2I_P}\right)}{N}\right] \tag{28}$$

$$k_{3I_T} = h\left[\beta_3\frac{\left(S_{T_i} + \frac{1}{2}k_{2S_T}\right)\left(I_{P_i} + \frac{1}{2}k_{2I_P}\right)}{N} - \mu_T\left(I_{T_i} + \frac{1}{2}k_{2I_T}\right)\right] \tag{29}$$

$$k_{4S_P} = h\left[\Lambda_P - \mu_P\left(S_{P_i} + k_{3S_P}\right) - \beta_1\frac{\left(S_{P_i} + k_{3S_P}\right)\left(I_{P_i} + k_{3I_P}\right)}{N} - \beta_2\frac{\left(S_{P_i} + k_{3S_P}\right)\left(I_{T_i} + k_{3I_T}\right)}{N}\right] \tag{30}$$

$$k_{4I_P} = h\left[\beta_1\frac{\left(S_{P_i} + k_{3S_P}\right)\left(I_{P_i} + k_{3I_P}\right)}{N} + \beta_2\frac{\left(S_{P_i} + k_{3S_P}\right)\left(I_{T_i} + k_{3I_T}\right)}{N} - (\mu_P + \alpha + \delta)\left(I_{P_i} + k_{3I_P}\right)\right] \tag{31}$$

$$k_{4R_P} = h\left[\alpha\left(I_{P_i} + k_{3I_P}\right) - \mu_P\left(R_{P_i} + k_{3R_P}\right)\right] \tag{32}$$

$$k_{4S_T} = h\left[\Lambda_T - \mu_T\left(S_{T_i} + k_{3S_T}\right) - \beta_3\frac{\left(S_{T_i} + k_{3S_T}\right)\left(I_{P_i} + k_{3I_P}\right)}{N}\right] \tag{33}$$

$$k_{4I_T} = h\left[\beta_3\frac{\left(S_{T_i} + k_{3S_T}\right)\left(I_{P_i} + k_{3I_P}\right)}{N} - \mu_T\left(I_{T_i} + k_{3I_T}\right)\right] \tag{34}$$

While the Milne method is described as follows:

Predictor:

$$S^*_{P_{i+1}} = S_{P_{i-3}} + \frac{4h}{3}\left[2\left(\Lambda_P - \mu_P S_{P_{i-2}} - \beta_1\frac{S_{P_{i-2}}I_{P_{i-2}}}{N} - \beta_2\frac{S_{P_{i-2}}I_{T_{i-2}}}{N}\right) - \left(\Lambda_P - \mu_P S_{P_{i-1}} - \beta_1\frac{S_{P_{i-1}}I_{P_{i-1}}}{N} - \beta_2\frac{S_{P_{i-1}}I_{T_{i-1}}}{N}\right) + 2\left(\Lambda_P - \mu_P S_{P_i} - \beta_1\frac{S_{P_i}I_{P_i}}{N} - \beta_2\frac{S_{P_i}I_{T_i}}{N}\right)\right] \tag{35}$$

$$I^*_{P_{i+1}} = I_{P_{i-3}} + \frac{4h}{3}\left[2\left(\beta_1\frac{S_{P_{i-2}}I_{P_{i-2}}}{N} + \beta_2\frac{S_{P_{i-2}}I_{T_{i-2}}}{N} - (\mu_P + \alpha + \delta)I_{P_{i-2}}\right) - \left(\beta_1\frac{S_{P_{i-1}}I_{P_{i-1}}}{N} + \beta_2\frac{S_{P_{i-1}}I_{T_{i-1}}}{N} - (\mu_P + \alpha + \delta)I_{P_{i-1}}\right) + 2\left(\beta_1\frac{S_{P_i}I_{P_i}}{N} + \beta_2\frac{S_{P_i}I_{T_i}}{N} - (\mu_P + \alpha + \delta)I_{P_i}\right)\right] \tag{36}$$

$$R^*_{P_{i+1}} = R_{P_{i-3}} + \frac{4h}{3}\left[2\left(\alpha I_{P_{i-2}} - \mu_P R_{P_{i-2}}\right) - \left(\alpha I_{P_{i-1}} - \mu_P R_{P_{i-1}}\right) + 2\left(\alpha I_{P_i} - \mu_P R_{P_i}\right)\right] \tag{37}$$

$$S^*_{T_{i+1}} = S_{T_{i-3}} + \frac{4h}{3}\left[2\left(\Lambda_T - \mu_T S_{T_{i-2}} - \beta_3\frac{S_{T_{i-2}}I_{P_{i-2}}}{N}\right) - \left(\Lambda_T - \mu_T S_{T_{i-1}} - \beta_3\frac{S_{T_{i-1}}I_{P_{i-1}}}{N}\right) + 2\left(\Lambda_T - \mu_T S_{T_i} - \beta_3\frac{S_{T_i}I_{P_i}}{N}\right)\right] \tag{38}$$

$$I^*_{T_{i+1}} = I_{T_{i-3}} + \frac{4h}{3}\left[2\left(\beta_3\frac{S_{T_{i-2}}I_{P_{i-2}}}{N} - \mu_T I_{T_{i-2}}\right) - \left(\beta_3\frac{S_{T_{i-1}}I_{P_{i-1}}}{N} - \mu_T I_{T_{i-1}}\right) + 2\left(\beta_3\frac{S_{T_i}I_{P_i}}{N} - \mu_T I_{T_i}\right)\right] \tag{39}$$

Corrector:

$$S_{P_{i+1}} = S_{P_{i-1}} + \frac{h}{3}\left[\left(\Lambda_P - \mu_P S_{P_{i-1}} - \beta_1\frac{S_{P_{i-1}}I_{P_{i-1}}}{N} - \beta_2\frac{S_{P_{i-1}}I_{T_{i-1}}}{N}\right) + 4\left(\Lambda_P - \mu_P S_{P_i} - \beta_1\frac{S_{P_i}I_{P_i}}{N} - \beta_2\frac{S_{P_i}I_{T_i}}{N}\right) + \left(\Lambda_P - \mu_P S^*_{P_{i+1}} - \beta_1\frac{S^*_{P_{i+1}}I^*_{P_{i+1}}}{N} - \beta_2\frac{S^*_{P_{i+1}}I^*_{T_{i+1}}}{N}\right)\right] \tag{40}$$

$$I_{P_{i+1}} = I_{P_{i-1}} + \frac{h}{3}\left[\left(\beta_1\frac{S_{P_{i-1}}I_{P_{i-1}}}{N} + \beta_2\frac{S_{P_{i-1}}I_{T_{i-1}}}{N} - (\mu_P + \alpha + \delta)I_{P_{i-1}}\right) + 4\left(\beta_1\frac{S_{P_i}I_{P_i}}{N} + \beta_2\frac{S_{P_i}I_{T_i}}{N} - (\mu_P + \alpha + \delta)I_{P_i}\right) + \left(\beta_1\frac{S^*_{P_{i+1}}I^*_{P_{i+1}}}{N} + \beta_2\frac{S^*_{P_{i+1}}I^*_{T_{i+1}}}{N} - (\mu_P + \alpha + \delta)I^*_{P_{i+1}}\right)\right] \tag{41}$$

$$R_{P_{i+1}} = R_{P_{i-1}} + \frac{h}{3}\left[\left(\alpha I_{P_{i-1}} - \mu_P R_{P_{i-1}}\right) + 4\left(\alpha I_{P_i} - \mu_P R_{P_i}\right) + \left(\alpha I^*_{P_{i+1}} - \mu_P R^*_{P_{i+1}}\right)\right] \tag{42}$$

$$S_{T_{i+1}} = S_{T_{i-1}} + \frac{h}{3}\left[\left(\Lambda_T - \mu_T S_{T_{i-1}} - \beta_3\frac{S_{T_{i-1}}I_{P_{i-1}}}{N}\right) + 4\left(\Lambda_T - \mu_T S_{T_i} - \beta_3\frac{S_{T_i}I_{P_i}}{N}\right) + \left(\Lambda_T - \mu_T S^*_{T_{i+1}} - \beta_3\frac{S^*_{T_{i+1}}I^*_{P_{i+1}}}{N}\right)\right] \tag{43}$$

$$I_{T_{i+1}} = I_{T_{i-1}} + \frac{h}{3}\left[\left(\beta_3\frac{S_{T_{i-1}}I_{P_{i-1}}}{N} - \mu_T I_{T_{i-1}}\right) + 4\left(\beta_3\frac{S_{T_i}I_{P_i}}{N} - \mu_T I_{T_i}\right) + \left(\beta_3\frac{S^*_{T_{i+1}}I^*_{P_{i+1}}}{N} - \mu_T I^*_{T_{i+1}}\right)\right] \tag{44}$$

It is worth emphasizing that the initial values and parameter values in Table 2 are taken from Kouidere *et al.* [8]. The h value in Table 2 is specially selected considering that the simulation results will be quite accurate and the simulation process is relatively short.

TABLE 2. Description, Initial Values, and Variable Parameter Values in Simulating the Spread of ASF Disease using the SIR-SI Model [1].

Notation	Description	Initial Value
S_P	The initial state of susceptible pigs	10000
I_P	The initial state of infected pigs	8000
R_P	The initial state of recovering pigs	3000
S_T	The initial state of susceptible ticks	30000

TABLE 2. Continued

Notation	Description	Initial Value
I_T	The initial state of infected ticks	10000
N	Population numbers	61000
Λ_P	The rate of increase in susceptible pigs	670
Λ_T	The rate of increase susceptible ticks	40
μ_P	The natural mortality rate of pigs	0.03
μ_T	The natural rate of death of ticks	0.01
β_1	The rate of infection of the disease in pigs through contact with infected pigs	0.02
β_2	The rate of infection of the disease in pigs through contact with infected ticks	0.01
β_3	The rate of infection of the disease in ticks through contact with infected pigs	0.01
α	The rate of recovery of pigs recovering from the disease	0.02
δ	Mortality rate from disease	0.02
h	Time steps for SIR-SI models	0.5

This section consists of three stages of solution where the first stage describes the solution using the fourth-order Runge-Kutta method, the second stage describes the solution using the Milne method, and the third stage describes the comparison of errors and the degree of accuracy between the fourth-order Runge-Kutta method and the Milne method, where the ODE45 algorithm solutions are the reference solutions.

Simulation Results of the fourth-order Runge-Kutta Method

The formulation of the fourth-order Runge-Kutta method for the solution of the SIR-SI model of the spread of ASF disease can be written as in equations (10), (11), (12), (13), and (14). Using the initial values and parameter values as shown in Table 2, a solution is obtained as shown in Figure 2. It is worth emphasizing that the exact analytical solution of the SIR-SI model has not yet been invented.

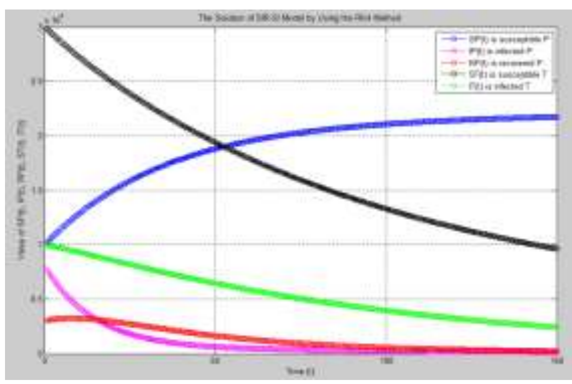

FIGURE 2. Solution to the SIR-SI model of ASF disease spread using the fourth-order Runge-Kutta method.

Figure 2 provides the behavior of the spread of ASF disease with the SIR-SI model using the fourth-order Runge-Kutta method. From the graph obtained, it can be seen that the magnitude of the S_P, that is, the population of pigs susceptible to the disease increases over time. The population of I_P, that is, the population of pigs infected with the disease has decreased. The population of R_P, that is, the population of pigs recovering from the disease increases and then decreases over time. The population of S_T, namely the population of ticks that are susceptible to disease, has decreased over time. The latter is the population of I_T, that is, the population of ticks infected with the disease decreases over time. Towards the end of the period, the population of S_P increased slowly corresponding to the populations of I_P, R_P, S_T, and I_T that fell asymptotically towards zero.

Milne Method Simulation Results

The formulation of Milne method for solving SIR-SI model of the dynamics of the spread of ASF disease can be written as in the equation (40), (41), (42), (43), and (44). Using the initial values and parameter values presented in Table 2, the solution obtained is shown as in Figure 3. It is worth emphasizing that the exact analytical solution of the SIR-SI model has not been invented to date.

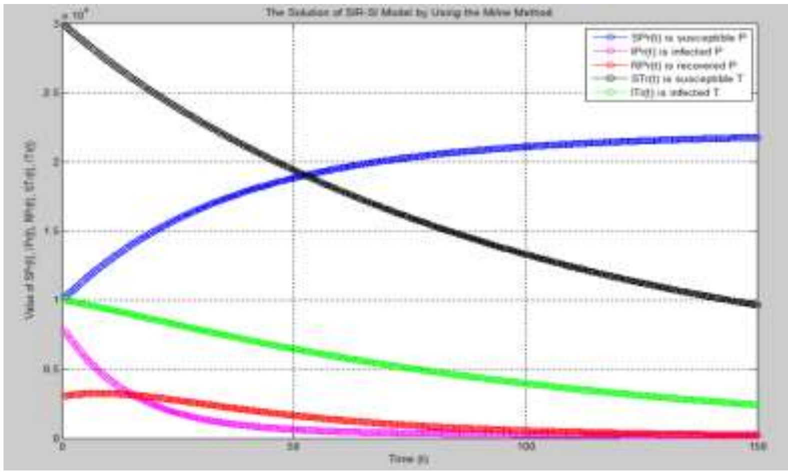

FIGURE 3. Solutions for the SIR-SI model of ASF deployment using the milne method

The results of simulating the spread of ASF disease with the SIR-SI model using the Milne method are shown in Figure 3. The acquisition of a solution graph using the Milne method is similar to the pattern of spreading the results of the fourth-order Runge-Kutta method. The behavior is the same.

Error Comparison between Fourth-Order Runge-Kutta and Milne Solutions

This section specifically describes the comparison of errors and the degree of accuracy between the solutions produced by the fourth-order Runge-Kutta method (Figure 2), the Milne method (Figure 3), and the reference solution shown in Figure 4. Figure 2-4 illustrates the same behavior of the solution for the SIR-SI model.

In order to prove which method is the most accurate among the fourth-order Runge-Kutta method and the Milne method in the spread of ASF disease, the error and accuracy of these two methods must be measured. Therefore, the reference solution used is the ODE45 algorithm in MATLAB.

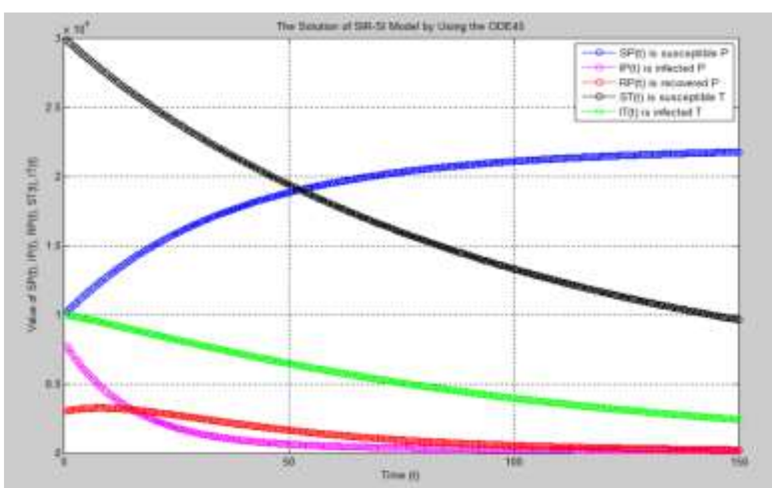

FIGURE 4. Solutions for the SIR-SI model of ASF disease spread using ODE45 algorithm.

In order to test the accuracy level of each method, we use timescales of 1, 0.5, 0.25, 0.125, and 0.0625. Next, it will be calculated the numerical errors and the accuracy (convergence) rate. To calculate the degree of accuracy (convergence) rate we use the formula based on Hidayat *et al.* [9]:

$$R_i = \frac{\log\left(\frac{E_i}{E_{i+1}}\right)}{\log\left(\frac{\Delta x_i}{\Delta x_{i+1}}\right)} \tag{45}$$

where E_i is the error corresponding to simulation using free variable step Δx_i and E_{i+1} is the error corresponding to simulation using free variable step Δx_{i+1}.

TABLE 3. Average error of the fourth-order Runge-Kutta method.

Δt	ES_P	EI_P	ER_P	ES_T	EI_T
1	4.69×10^{-9}	2.34×10^{-6}	7.91×10^{-7}	3.49×10^{-9}	1.16×10^{-8}
0.5	2.00×10^{-10}	7.99×10^{-8}	2.72×10^{-8}	1.39×10^{-10}	4.59×10^{-10}
0.25	9.41×10^{-12}	2.65×10^{-9}	8.86×10^{-10}	6.87×10^{-12}	2.21×10^{-11}
0.125	5.13×10^{-13}	9.14×10^{-11}	2.92×10^{-11}	4.21×10^{-13}	1.33×10^{-12}
0.0625	2.98×10^{-14}	3.47×10^{-12}	1.01×10^{-12}	2.70×10^{-14}	8.55×10^{-14}

TABLE 4. Accuracy rate of the fourth-order Runge-Kutta method.

Δt	CRS_P	CRI_P	CRR_P	CRS_T	CRI_T
1	–	–	–	–	–
0.5	4,55	4,88	4,87	4,65	4,67
0.25	4,41	4,91	4,94	4,34	4,38
0.125	4,20	4,86	4,93	4,03	4,05
0.0625	4,11	4,72	4,85	3,97	3,96

TABLE 5. Average error of the Milne method.

Δt	ES_P	EI_P	ER_P	ES_T	EI_T
1	3.27×10^{-9}	1.74×10^{-6}	5.68×10^{-7}	2.65×10^{-9}	8.92×10^{-9}
0.5	1.81×10^{-10}	6.98×10^{-8}	2.30×10^{-8}	1.33×10^{-10}	4.36×10^{-10}
0.25	9.86×10^{-12}	2.56×10^{-9}	8.35×10^{-10}	7.35×10^{-12}	2.36×10^{-11}
0.125	5.66×10^{-13}	9.66×10^{-11}	3.06×10^{-11}	4.53×10^{-13}	1.50×10^{-12}
0.0625	3.20×10^{-14}	3.55×10^{-12}	1.03×10^{-12}	2.82×10^{-14}	8.90×10^{-14}

TABLE 6. Accuracy rate of the Milne method.

Δt	CRS_P	CRI_P	CRR_P	CRS_T	CRI_T
1	–	–	–	–	–
0.5	4.18	4,64	4,63	4,31	4,36
0.25	4.21	4,78	4,79	4,18	4,22
0.125	4,14	4,79	4,84	4,02	4,04
0.0625	4,12	4,69	4,81	4,00	3,99

From Table 3 and Table 5, it can be seen that the average error value between compartments using the fourth-order Runge-Kutta method is smaller than the average error value between compartments using the Milne method. In addition, from Table 4 and Table 6 it can be seen that the accuracy of each compartment using the fourth-order Runge-Kutta method is generally higher when compared to the accuracy rate of each compartment using the Milne method. This fourth-order Runge-Kutta method can be applied to solve other models of disease spread, such as the problems discussed in [2], [3], [4], and [5]

CONCLUSION

The solution to the spread of ASF disease with the SIR-SI model in this paper is shown as in the numerically generated graph using the MATLAB program for the fourth-order Runge-Kutta method and the Milne method. The accuracy rate of the numerical solutions of these two methods is measured based on the reference solution of the ODE45 algorithm in MATLAB software.

In this study also applied the reference solution of the ODE45 algorithm because until now there is no proper analytical solution for the SIR-SI model. The solution graph for the SIR-SI model shows that the numerical solution generated by both methods has the same behavior as the reference solution used.

The accuracy of the fourth-order Runge-Kutta method is equal to as that of Milne method. This theory has been confirmed in this paper numerically by comparing the average error values and the degree of accuracy of each method. The calculation results show that the average error of the fourth-order Runge-Kutta method is smaller than that of the Milne method and the accuracy rate of the fourth-order Runge-Kutta method is equal to the accuracy level of the Milne method. Therefore, it can be concluded that the fourth-order Runge-Kutta method is more effective when compared with the Milne method in its application to solve the SIR-SI model of the spread of ASF disease. The use of large time steps is also very influential on the application of the Milne method. The greater the time step used, the resulting solution graph will undergo oscillations.

ACKNOWLEDGMENTS

Some of this work was a part of the first author's thesis written under the supervision of the second author. The funding support given by Sanata Dharma University in this research is gratefully acknowledged by the authors.

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Certificate number: 057/icmme/2/vii/2022

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