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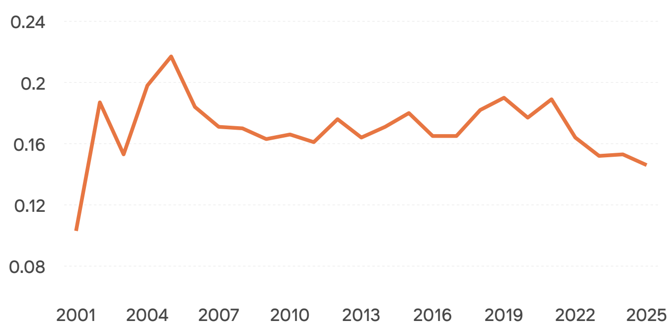

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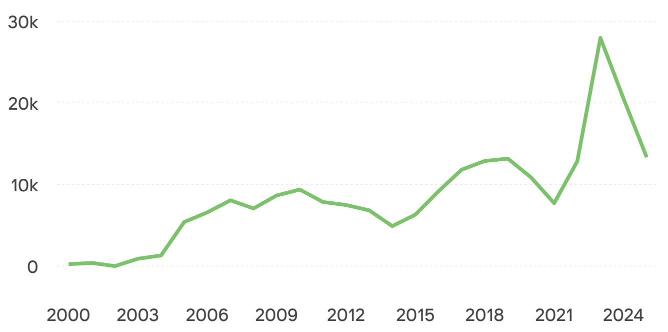

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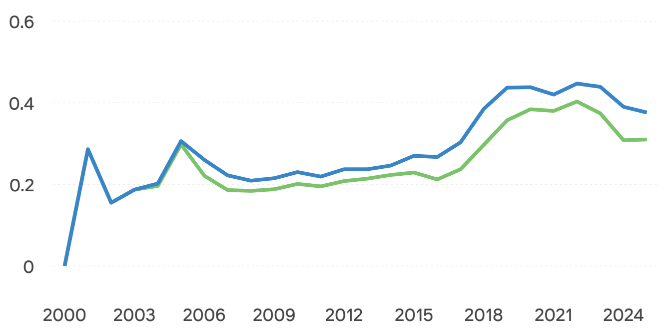

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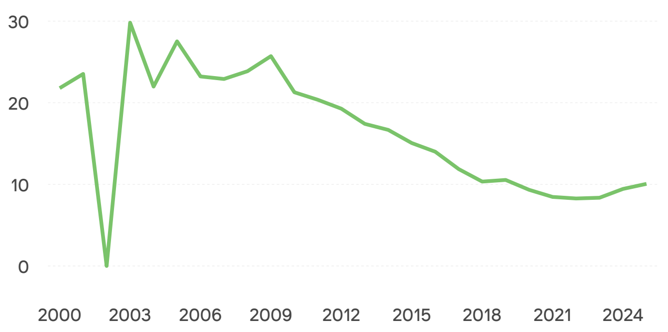

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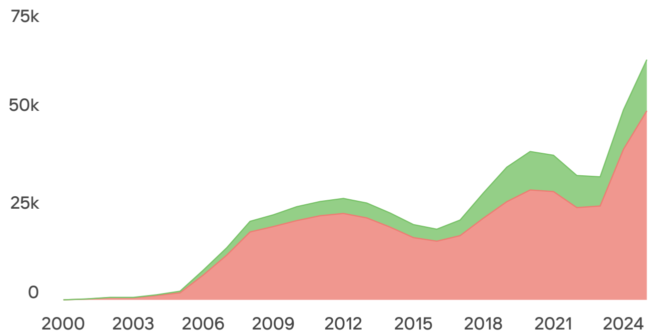

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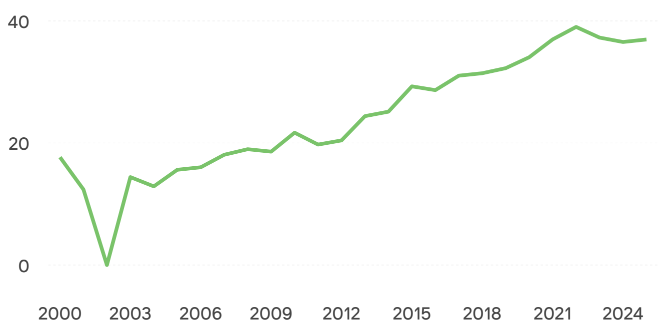

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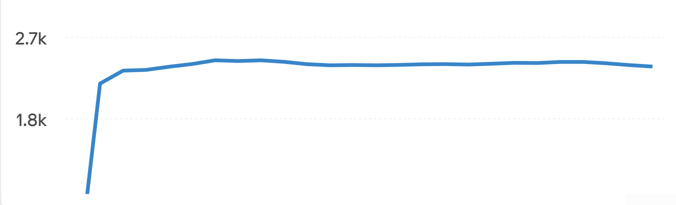

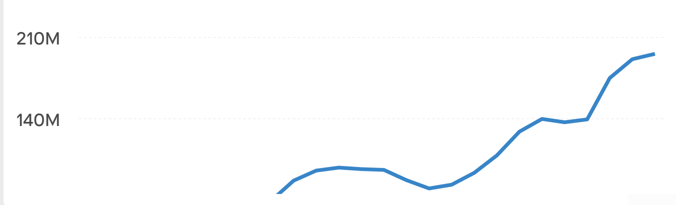

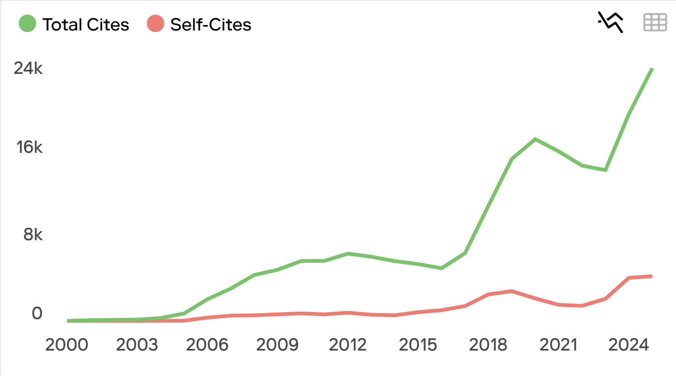

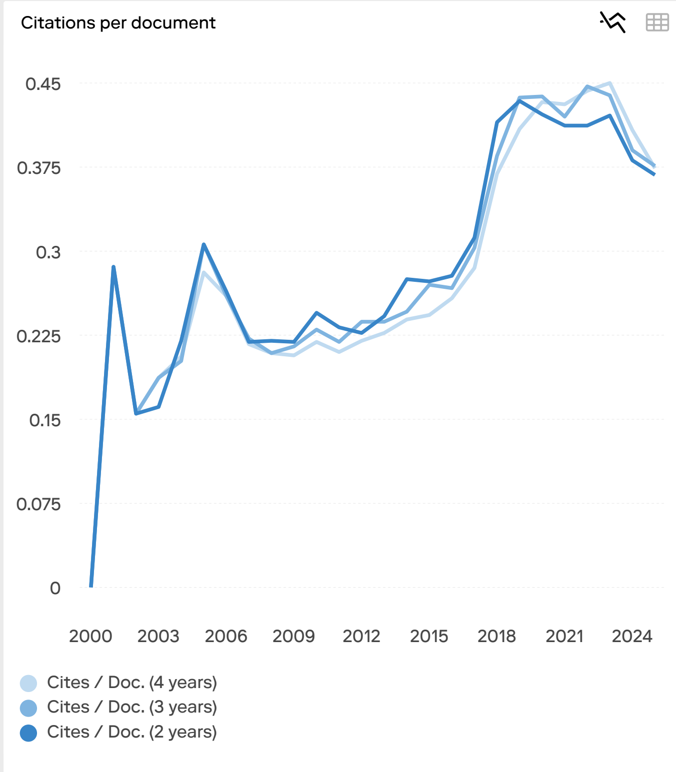

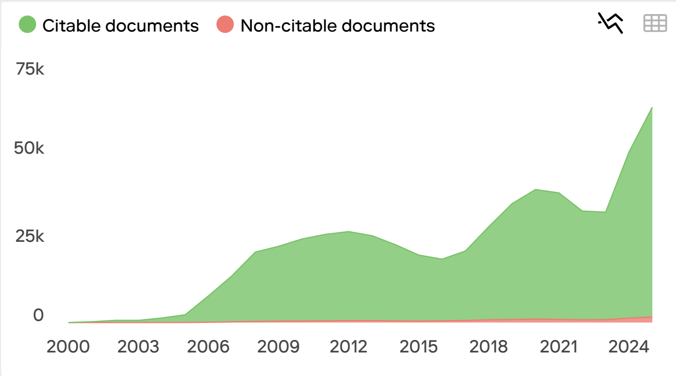

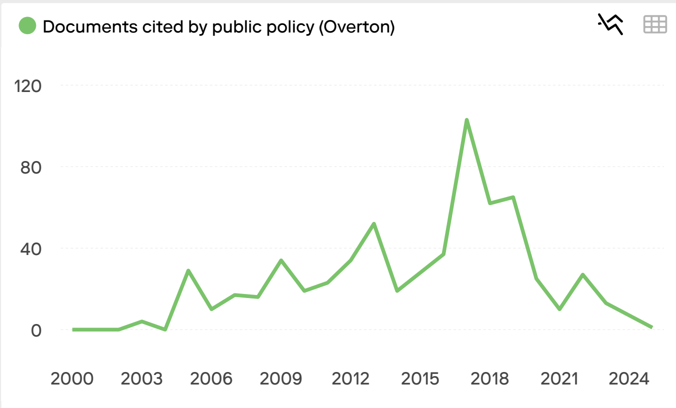

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International Conference of Mathematics and Mathematics Education (ICMME) 2022

Surakarta, Indonesia • 24–26 July 2022

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RESEARCH ARTICLE | JUNE 10 2026

Preface: International Conference of Mathematics and Mathematics Education (ICMME) 2022

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AIP Conf. Proc. 3464, 010001 (2026)
https://doi.org/10.1063/12.0042963

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Preface: International Conference of Mathematics and Mathematics Education (ICMME) 2022

It is with great pleasure and anticipation that we present the proceedings of the International Conference on Mathematics and Mathematics Education (ICMME) 2022. This esteemed conference, held under the theme "Improving Knowledge through Sustainable Innovation in the field of Education, Science, Technology, Engineering, and Mathematics," marks a significant milestone in the global pursuit of academic excellence and collaboration.

This theme highlights the importance of sustainable innovation in strengthening the fields of education, science, technology, engineering, and mathematics. In the context of a rapidly evolving global era, sustainable innovative approaches become key in improving educational practices and expanding our understanding of the underlying concepts of science and technology. Through interdisciplinary and international collaboration, this seminar aims to stimulate creative thinking, encourage the development of sustainable solutions, and promote enriching idea exchange to support sustainable progress in education and science.

We extend our sincere gratitude to the keynote speakers whose expertise and insights have enriched our discussions:

1. Dr. Ahmad Ridwan Tresna Nugraha, Head of Research Center for Quantum Physics, Indonesia
2. Dr. Michiel Veldhuis, from IPABO University of Applied Science and Utrecth University, the Netherlands
3. Assoc. Prof. Dr. Masitah Shahrill, Graduate School of Education, Universiti Brunei Darussalam, Brunei Darussalam
4. Assoc. Prof. Dr. Nor Zila binti Abd Hamid, Universiti Pendidikan Sultan Idris, Malaysia

This conference would not have been possible without the dedication and collaboration between the Department of Mathematics Education at Universitas Sebelas Maret Surakarta and the Department of Mathematics Education at Universiti Pendidikan Sultan Idris. Your commitment to fostering academic exchange and advancement in the fields of mathematics and education is truly commendable.

Held in a hybrid format from July 24th to 26th, 2022, at the Auditorium FKIP UNS, this conference has brought together scholars, researchers, and practitioners from Malaysia, the Philippines, Japan, Brunei Darussalam, and Indonesia. Through a blend of physical and virtual participation, we have transcended geographical boundaries to facilitate meaningful dialogue and collaboration.

The diverse perspectives and innovative ideas presented during this conference have undoubtedly contributed to the advancement of knowledge and practice in mathematics and mathematics education. We are confident that the papers included in these proceedings will serve as valuable resources for scholars and practitioners alike, inspiring further research and development in the pursuit of excellence.

We express our heartfelt appreciation to all participants, presenters, reviewers, and organizers for their invaluable contributions to the success of ICMME 2022. May the insights gained and connections forged during this conference continue to resonate and drive positive change in our respective fields.

Thank you for your participation and support.

Sincerely,
Editors
Dr. Budi Usodo, M.Pd.
Dr. Imam Sujadi, M.Si.
Dr. Laila Fitriana, M.Pd.
Mr. Agus Hendriyanto, M.Pd.
Mr. Sani Sahara, M.Pd.
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Blended learning as an alternative learning model post Covid-19: A pre-experimental design of the teaching mathematics based on the character building of elementary school students

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Development of STEM-C module integrating project-based learning in integration topic

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Edi Supriyadi; Jarnawi A. Dahlan

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A systematic literature review on mathematics mobile applications in education

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EDUCATION AND TEACHER PROFESSIONAL DEVELOPMENT

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The implementation of academic supervision by the principal in improving teacher competence

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Determining the presence of chaos based on variation data of water level in Lebir River, Kelantan, Malaysia

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Identifying the presence of chaos based on variation sediment transport data in Sungai Selangor, Malaysia

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SCIENCE

RESEARCH ARTICLE | JUNE 10 2026

Performance comparison between the fourth-order Runge-Kutta method and the Milne method in solving a mathematical model for the dengue fever disease spread

Maria R. Lipat ; Sudi Mungkasi

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AIP Conf. Proc. 3464, 070003 (2026)

https://doi.org/10.1063/5.0330908

In this paper, we consider a Susceptible-Infected-Recovered-Susceptible (SIRS) mathematical model for the spread of the Dengue fever disease. This disease involves human and mosquitoes in the system. The problem is formed into a system of nonlinear ordinary differential equations. The goal of this paper is to compare the performance of two numerical methods in solving the problem. The two numerical methods are the fourth-order Runge-Kutta and the Milne methods. These two methods are considered, because of their popularity. Based on our research results, the fourth-order Runge-Kutta method is able to solve the problem using various values of time steps. However, the Milne method is unable to solve the problem if the time step is relatively large. This is related to the numerical stability of the methods. That is, the fourth-order Runge-Kutta method is more stable than the Milne method for relatively large values of time steps.

REFERENCES

1. D. K. Ningsih, *J. Sains dan Seni ITS*, 5, A7–A13, (2016).
Google Scholar

2. Y. M. Rangkuti and S. Side, *J Conserv Dent.* 2013, 16, 2013, (2013).
Google Scholar

3. H. S. Rodrigues, M. T. T. Monteiro, and D. F. M. Torres, *Conf. Pap. Math.*, 1, 1–7, (2013).
Google Scholar

4. P. Y. Shu *et al*, *Clin. Diagn. Lab. Immunol.*, 10, 622–630, (2003).
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5. Y. Pusparisa, *Katadata Media Network*, 2022.
Google Scholar

6. Widyawati, 2022.

7. S. A. Londah, C. E. Mongi, and C. E. J. C. Montolalu, *d'CARTESIAN*, 8, 114, (2019).
https://doi.org/10.35799/dc.8.2.2019.24066
Google Scholar Crossref

8. H. Purwanto, E. Noviani, and M. N. Mara, *Bimaster*, 03, 153–162, (2014).
Google Scholar

9. W. Sanusi, M. I. Pratama, M. Rifandi, S. Sidjara, Irwan, and S. Side, *J. Phys. Conf. Ser.*, 1752, 1–11, (2021).
https://doi.org/10.1088/1742-6596/1752/1/012004
Google Scholar Crossref

10. R. U. Hurit and Sudi Mungkasi, *Mathline J. Mat. dan Pendidik. Mat.*, 6, 140–153, (2021).
Google Scholar

11. D. V. Griffiths and I. M. Smith, *Numerical Methods for Engineers*. McGraw-Hill Science/Engineering/Math, 2006.
Google Scholar Crossref

12. E. R. Wulan and F. Hasanudin, *J. Mat. Integr.*, 9, 131, (2013).
Google Scholar

13. S. L. Ross, *Differential equations 3rd edition Shepley L. Ross.pdf*. John Wiley and Sons, New York, 1984.
Google Scholar

14. Stillman, Gloria, Brown, and Jill, *Vinculum*, 48, 20, (2011).
Google Scholar

15. S. Side, Alimuddin, and A. Bani, *J. Math. Comput. Stat.*, 1, 169–182, (2018).

https://doi.org/10.35580/jmathcos.v1i2.9240
Google Scholar Crossref

16. W. Sanusi *et al*, *J. Appl. Math.*, 1, 1–8, (2021).
https://doi.org/10.1155/2021/2918080
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17. N. Hidayat, Suhariningsih, A. Suryanto, and S. Mungkasi, *Appl. Math. Sci.*, 8, 1411–1420, (2014).
Google Scholar

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Performance Comparison between the Fourth-Order Runge-Kutta Method and the Milne Method in Solving a Mathematical Model for the Dengue Fever Disease Spread

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Abstract. In this paper, we consider a Susceptible-Infected-Recovered-Susceptible (SIRS) mathematical model for the spread of the Dengue fever disease. This disease involves human and mosquitoes in the system. The problem is formed into a system of nonlinear ordinary differential equations. The goal of this paper is to compare the performance of two numerical methods in solving the problem. The two numerical methods are the fourth-order Runge-Kutta and the Milne methods. These two methods are considered, because of their popularity. Based on our research results, the fourth-order Runge-Kutta method is able to solve the problem using various values of time steps. However, the Milne method is unable to solve the problem if the time step is relatively large. This is related to the numerical stability of the methods. That is, the fourth-order Runge-Kutta method is more stable than the Milne method for relatively large values of time steps.

INTRODUCTION

Infectious diseases are still major problems for human living. Regarding this, infectious disease spread is important to research. Disease can be caused by bacteria, microbes, or viruses. Viruses are unique organisms that are genetically somewhere between life and death. This happens when outside the host's body is a capsid and attaches to the host [1]. This virus will infect the host and reproduce new viruses. Viruses are unique in that they cannot die but can only be crystallized. This fact makes viruses a dangerous organism. One of these viruses is the Dengue virus. Dengue virus can cause a disease known as Dengue fever [2]. According to Rodrigues [3] Dengue fever is a dangerous disease that can cause death. This disease is only transmitted through mosquito bites, so this disease is classified as *Anthropod Borne* disease. The Dengue virus is carried by a vector, namely the *Aedes aegypti* mosquito and the *Aedes albopictus* mosquito. According to Shu [4] the main vector for carrying the Dengue virus is the *Aedes aegypti* mosquito.

Several countries are prone to Dengue fever disease. Indonesia is the second country after Thailand which is infected with Dengue fever. Even until September 2020 there were 84,734 people infected with Dengue disease throughout Indonesia [5]. In Indonesia, the death rate reached 584 people. This is known in the Ministry of Health data on September 15, 2020. Then a number of Dengue cases in Sikka district, East Nusa Tenggara reached 1,816 people. Total of 16 people died and 1740 people recovered, while 60 people were still being treated at the hospital [6].

Seeing the problems that occur due to the spread of Dengue disease, mathematics provides an alternative way to solve the spread of the disease. Mathematical models of Dengue fever disease spreads can be formed using various approaches. A number of researchers used the SIR model (Suspected-Infected-Recovered) [7] and the SEIR model (Suspected-Exposed-Infected-Recovered) [8], assuming that recovered individuals do not return to being susceptible individuals. This is the basis for modifying the SIR model into SIRS because recovered individuals will likely become susceptible individuals for the case of Dengue fever disease [9].

In this paper, we will model the spread of disease in a mathematical form using the Susceptible-Infected-Recovered-Susceptible (SIRS) model type on the spread of Dengue fever disease. This model will be solved using the fourth-order Runge-Kutta method and the Milne method, then we compare the results of these two methods with those

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of the ODE45 method to see which method is better in solving the model. All computations and simulations are conducted using the MATLAB software.

RESEARCH METHOD

The research method that the author uses is literature study and numerical simulations, that is, by studying papers and references that support the solving of the equations and method used. The solving steps are the SIRS model development, model discretization, computer programming, numerical simulation, and observation of simulation results. The methods used are the fourth-order Runge-Kutta method and the Milne method with the Susceptible-Infected-Recovered-Susceptible (SIRS) epidemic model. Research starts from studying and understanding the process of making SIRS schemes and models for the spread of Dengue fever disease. After the model is obtained, we solve the SIRS model using the fourth-order Runge-Kutta method and the Milne method. Following it, we analyze the solution graphs obtained from the simulations using the MATLAB program. Finally, we conclude the research based on those results.

Fourth-Order Runge-Kutta Method

The description about the fourth-order Runge-Kutta method is as follows. Consider the following Initial Value Problem (IVP):

$$\frac{dy}{dx} = f(x, y),$$
$$y(x_0) = y_0.$$

We will use this generic form of IVP for the mathematical model of disease spread problem.

The fourth-order Runge-Kutta method is one of methods used to solve differential equations with initial conditions. The fourth-order Runge-Kutta method has higher level of accuracy compared to the Euler, Heun, and third-order methods. The accuracy of these results makes the fourth-order Runge-Kutta method widely used in solving ordinary differential equations with initial conditions [10].

The fourth-order Runge-Kutta scheme for the aforementioned IVP is as follows [11]:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (1)$$
$$k_1 = hf(x_i, y_i) \quad (2)$$
$$k_2 = hf\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1\right) \quad (3)$$
$$k_3 = hf\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2\right) \quad (4)$$
$$k_4 = hf(x_i + h, y_i + k_3) \quad (5)$$

Milne Method

Solving ordinary differential equations using the Milne method is the process of finding the value of the function $y(x)$ at a certain point x from the known ordinary differential equation $f(x, y)$ by making predictions with the predictor formula and making corrections with the corrector formula [12].

According to [13] the numerical scheme of the Milne method for numerical solutions to the aforementioned IVP is as follows:

Predictor:

$$y_{n+1} = y_{n-3} + \frac{4h}{3}(2f_n - f_{n-1} + 2f_{n-2}) \quad (6)$$

Corrector:

$$y_{n+1} = y_{n-1} + \frac{h}{3}(f_{n+1} + 4f_n + f_{n-1}) \quad (7)$$

RESULTS AND DISCUSSION

In this section, we recall the SIRS mathematical model and provide the numerical schemes of the fourth-order Runge-Kutta and the Milne methods.

SIRS Model and Numerical Schemes

Mathematical models [14] are of important study. According to [15], the Susceptible-Infected-Recovered (SIR) model was used to simulate the spread the Dengue fever disease in South Sulawesi. The SIR model assumes that recovered individuals are not infected again. However, each individual has the opportunity to be susceptible to disease, so the SIRS model will be used in the transmission of Dengue fever disease in the present paper. The assumptions used are: the birth and death rates are considered constants, the total populations of humans and mosquitoes are considered constants, the SIRS model of the human population N^h is divided into three groups of individuals, namely Susceptible (S^h) representing the number of individuals who are susceptible to disease, Infected (I^h) denoting the number of individuals who have suffered from the disease and actively transmit Dengue fever disease, and Recovered (R^h) representing the number of individuals who have recovered from the disease. Individuals who have recovered have the possibility to return to the Susceptible subpopulation [16].

The mosquito population is divided into two subpopulations, namely, Susceptible (S^v) which is a population of susceptible mosquitoes and Infected (I^v) which is a population of infected mosquitoes. Schematically, the pattern of the spread of the Dengue Fever Disease can be described as in Figure 1.

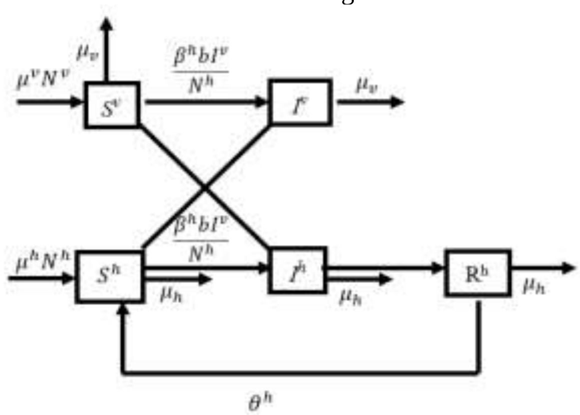

FIGURE 1. Human and vector populations in the SIRS model diagram

From Figure 1, the system of differential equations is obtained as follows. The system of differential equations for human population (host):

$$\frac{dS^h}{dt} = \mu^h N^h - \frac{\beta^h b}{N^h} I^v S^h - \mu^h S^h + \theta^h R^h \tag{8}$$
$$\frac{dI^h}{dt} = \frac{\beta^h b}{N^h} I^v S^h - (\mu^h + \gamma^h) I^h$$
$$\frac{dR^h}{dt} = \gamma^h I^h - (\mu^h + \theta^h) R^h$$

The system of differential equations for mosquito population (vector):

$$\frac{dS^v}{dt} = \mu^v N^v - \frac{\beta^v b}{N^h} I^h S^v - \mu^v S^v \tag{9}$$
$$\frac{dI^v}{dt} = \frac{\beta^v b}{N^h} I^h S^v - \mu^v I^v$$

Let us consider system (8) and system (9). The numerical scheme for solving the SIRS model using the fourth-order Runge-Kutta method employs the following values:

$$k_{1S^h} = \Delta t\left(\mu^h N^h - \frac{\beta^h b}{N^h} {I_n}^v {S_n}^h - \mu^h {S_n}^h + \theta^h {R_n}^h\right) \tag{10}$$

$$k_{1I^h} = \Delta t\left(\frac{\beta^h b}{N^h} {I_n}^v {S_n}^h - (\mu^h + \gamma^h) {I_n}^h\right) \tag{11}$$

$$k_{1R^h} = \Delta t\left(\gamma^h {I_n}^h - (\mu^h + \theta^h) {R_n}^h\right) \tag{12}$$

$$k_{1S^v} = \Delta t\left(\mu^v N^v - \frac{\beta^v b}{N^h} {I_n}^h {S_n}^v - \mu^v {S_n}^v\right) \tag{13}$$

$$k_{1I^v} = \Delta t\left(\frac{\beta^v b}{N^h} {I_n}^h {S_n}^v - \mu^v {I_n}^v\right) \tag{14}$$

$$k_{2S^h} = \Delta t\left[\mu^h N^h - \frac{\beta^h b}{N^h}\left({I_n}^v + \frac{k_{1I^v}}{2}\right)\left({S_n}^h + \frac{k_{1S^h}}{2}\right) - \mu^h\left({S_n}^h + \frac{k_{1S^h}}{2}\right) + \theta^h\left({R_n}^h + \frac{k_{1R^h}}{2}\right)\right] \tag{15}$$

$$k_{2I^h} = \Delta t\left[\frac{\beta^h b}{N^h}\left({I_n}^v + \frac{k_{1I^v}}{2}\right)\left({S_n}^h + \frac{k_{1S^h}}{2}\right) - (\mu^h + \gamma^h)\left({I_n}^h + \frac{k_{1I^h}}{2}\right)\right] \tag{16}$$

$$k_{2R^h} = \Delta t\left[\gamma^h\left({I_n}^h + \frac{k_{1I^h}}{2}\right) - (\mu^h + \theta^h)\left({R_n}^h + \frac{k_{1R^h}}{2}\right)\right] \tag{17}$$

$$k_{2S^v} = \Delta t\left[\mu^v N^v - \frac{\beta^v b}{N^h}\left({I_n}^h + \frac{k_{1I^h}}{2}\right)\left({S_n}^v + \frac{k_{1S^v}}{2}\right) - \mu^v\left({S_n}^v + \frac{k_{1S^v}}{2}\right)\right] \tag{18}$$

$$k_{2I^v} = \Delta t\left[\frac{\beta^v b}{N^h}\left({I_n}^h + \frac{k_{1I^h}}{2}\right)\left({S_n}^v + \frac{k_{1S^v}}{2}\right) - \mu^v\left({I_n}^v + \frac{k_{1I^v}}{2}\right)\right] \tag{19}$$

$$k_{3S^h} = \Delta t\left[\mu^h N^h - \frac{\beta^h b}{N^h}\left({I_n}^v + \frac{k_{2I^v}}{2}\right)\left({S_n}^h + \frac{k_{2S^h}}{2}\right) - \mu^h\left({S_n}^h + \frac{k_{2S^h}}{2}\right) + \theta^h\left({R_n}^h + \frac{k_{2R^h}}{2}\right)\right] \tag{20}$$

$$k_{3I^h} = \Delta t\left[\frac{\beta^h b}{N^h}\left({I_n}^v + \frac{k_{2I^v}}{2}\right)\left({S_n}^h + \frac{k_{2S^h}}{2}\right) - (\mu^h + \gamma^h)\left({I_n}^h + \frac{k_{2I^h}}{2}\right)\right] \tag{21}$$

$$k_{3R^h} = \Delta t\left[\gamma^h\left({I_n}^h + \frac{k_{2I^h}}{2}\right) - (\mu^h + \theta^h)\left({R_n}^h + \frac{k_{2R^h}}{2}\right)\right] \tag{22}$$

$$k_{3S^v} = \Delta t\left[\mu^v N^v - \frac{\beta^v b}{N^h}\left({I_n}^h + \frac{k_{2I^h}}{2}\right)\left({S_n}^v + \frac{k_{2S^v}}{2}\right) - \mu^v\left({S_n}^v + \frac{k_{2S^v}}{2}\right)\right] \tag{23}$$

$$k_{3I^v} = \Delta t\left[\frac{\beta^v b}{N^h}\left({I_n}^h + \frac{k_{2I^h}}{2}\right)\left({S_n}^v + \frac{k_{2S^v}}{2}\right) - \mu^v\left({I_n}^v + \frac{k_{2I^v}}{2}\right)\right] \tag{24}$$

$$k_{4S^h} = \Delta t\left[\mu^h N^h - \frac{\beta^h b}{N^h}({I_n}^v + k_{3I^v})({S_n}^h + k_{3S^h}) - \mu^h({S_n}^h + k_{3S^h}) + \theta^h\left({R_n}^h + \frac{k_{3R^h}}{2}\right)\right] \tag{25}$$

$$k_{4I^h} = \Delta t\left[\frac{\beta^h b}{N^h}({I_n}^v + k_{3I^v})({S_n}^h + k_{3S^h}) - (\mu^h + \gamma^h)({I_n}^h + k_{3I^h})\right] \tag{26}$$

$$k_{4R^h} = \Delta t\left[\gamma^h({I_n}^h + k_{3I^h}) - (\mu^h + \theta^h)({R_n}^h + k_{3R^h})\right] \tag{27}$$

$$k_{4S^v} = \Delta t\left[\mu^v N^v - \frac{\beta^v b}{N^h}({I_n}^h + k_{3I^h})({S_n}^v + k_{3S^v}) - \mu^v({S_n}^v + k_{3S^v})\right] \tag{28}$$

$$k_{4I^v} = \Delta t\left[\frac{\beta^v b}{N^h}({I_n}^h + k_{3I^h})({S_n}^v + k_{3S^v}) - \mu^v({I_n}^v + k_{3I^v})\right] \tag{29}$$

The numerical integration using the fourth-order Runge-Kutta method for solving the SIRS model is stated as follows:

Numerical scheme for the system of differential equations for human population (host)

$${S_{n+1}}^h = {S_n}^h + {}^1\!/_6\left(k_{1S^h} + 2k_{2S^h} + 2k_{3S^h} + k_{4S^h}\right) \tag{30}$$

$${I_{n+1}}^h = {I_n}^h + {}^1\!/_6\left(k_{1I^h} + 2k_{2I^h} + 2k_{3I^h} + k_{4I^h}\right) \tag{31}$$

$${R_{n+1}}^h = {R_n}^h + {}^1\!/_6\left(k_{1R^h} + 2k_{2R^h} + 2k_{3R^h} + k_{4R^h}\right) \tag{32}$$

Numerical scheme for the system of differential equations for mosquito population (vector)

$${S_{n+1}}^v = {S_n}^v + {}^1\!/_6\left(k_{1S^v} + 2k_{2S^v} + 2k_{3S^v} + k_{4S^v}\right) \tag{33}$$

$${I_{n+1}}^v = {I_n}^v + {}^1\!/_6\left(k_{1I^v} + 2k_{2I^v} + 2k_{3I^v} + k_{4I^v}\right) \tag{34}$$

The numerical scheme of the Milne method to solve systems (6) and (7) with $i = 3, 4, 5, \ldots$ is described in this subsection. Four initial values used in the Milne method will be taken from the fourth-order Runge-kutta method, which will be calculated first. The Milne method is a predictor-corrector method:

Predictor

$$Sh_{i+1}{}^{(0)} = Sh_{i-3} + \frac{4\Delta t}{3}\left[2\left(\mu^h N^h - \frac{\beta^h b}{N^h} I_i{}^v S_i{}^h - \mu^h S_i{}^h + \theta^h R_i{}^h\right) - \left(\mu^h N^h - \frac{\beta^h b}{N^h} I_{i-1}{}^v S_{i-1}{}^h - \mu^h S_{i-1}{}^h + \theta^h R_{i-1}{}^h\right) + 2\left(\mu^h N^h - \frac{\beta^h b}{N^h} I_{i-2}{}^v S_{i-2}{}^h - \mu^h S_{i-2}{}^h + \theta^h R_{i-2}{}^h\right)\right] \quad (35)$$

$$Ih_{i+1}{}^{(0)} = Ih_{i-3} + \frac{4\Delta t}{3}\left[2\left(\frac{\beta^h b}{N^h} I_i{}^v S_i{}^h - (\mu^h + \gamma^h) I_i{}^h\right) - \left(\frac{\beta^h b}{N^h} I_{i-1}{}^v S_{i-1}{}^h - (\mu^h + \gamma^h) I_{i-1}{}^h\right) + 2\left(\frac{\beta^h b}{N^h} I_{i-2}{}^v S_{i-2}{}^h - (\mu^h + \gamma^h) I_{i-2}{}^h\right)\right] \quad (36)$$

$$Rh_{i+1}{}^{(0)} = Rh_{i-3} + \frac{4\Delta t}{3}\left[2(\gamma^h I_i{}^h - (\mu^h + \theta^h) R_i{}^h) - (\gamma^h I_{i-1}{}^h - (\mu^h + \theta^h) R_{i-1}{}^h) + 2(\gamma^h I_{i-2}{}^h - (\mu^h + \theta^h) R_{i-2}{}^h)\right] \quad (37)$$

$$Sv_{i+1}{}^{(0)} = Sv_{i-3} + \frac{4\Delta t}{3}\left[2\left(\mu^v N^v - \frac{\beta^v b}{N^h} I_i{}^h S_i{}^v - \mu^v S_i{}^v\right) - \left(\mu^v N^v - \frac{\beta^v b}{N^h} I_{i-1}{}^h S_{i-1}{}^v - \mu^v S_{i-1}{}^v\right) + 2\left(\mu^v N^v - \frac{\beta^v b}{N^h} I_{i-2}{}^h S_{i-2}{}^v - \mu^v S_{i-2}{}^v\right)\right] \quad (38)$$

$$Iv_{i+1}{}^{(0)} = Iv_{i-3} + \frac{4\Delta t}{3}\left[2\left(\frac{\beta^v b}{N^h} I_i{}^h S_i{}^v - \mu^v I_i{}^v\right) - \left(\frac{\beta^v b}{N^h} I_{i-1}{}^h S_{i-1}{}^v - \mu^v I_{i-1}{}^v\right) + 2\left(\frac{\beta^v b}{N^h} I_{i-2}{}^h S_{i-2}{}^v - \mu^v I_{i-2}{}^v\right)\right] \quad (39)$$

Corrector

$$Sh_{i+1} = Sh_{i-1} + \frac{\Delta t}{3}\left[\left(\mu^h N^h - \frac{\beta^h b}{N^h} Iv_{i+1}{}^{(0)} Sh_{i+1}{}^{(0)} - \mu^h Sh_{i+1}{}^{(0)} + \theta^h Rh_{i+1}{}^{(0)}\right) + 4\left(\mu^h N^h - \frac{\beta^h b}{N^h} I_i{}^v S_i{}^h - \mu^h S_i{}^h + \theta^h R_i{}^h\right) + \left(\mu^h N^h - \frac{\beta^h b}{N^h} I_{i-1}{}^v S_{i-1}{}^h - \mu^h S_{i-1}{}^h + \theta^h R_{i-1}{}^h\right)\right] \quad (40)$$

$$Ih_{i+1} = Ih_{i-1} + \frac{\Delta t}{3}\left[\left(\frac{\beta^h b}{N^h} Iv_{i+1}{}^{(0)} Sh_{i+1}{}^{(0)} - (\mu^h + \gamma^h) Ih_{i+1}{}^{(0)}\right) + 4\left(\frac{\beta^h b}{N^h} I_i{}^v S_i{}^h - (\mu^h + \gamma^h) I_i{}^h\right) + \left(\frac{\beta^h b}{N^h} I_{i-1}{}^v S_{i-1}{}^h - (\mu^h + \gamma^h) I_{i-1}{}^h\right)\right] \quad (41)$$

$$Rh_{i+1} = Rh_{i-1} + \frac{\Delta t}{3}\left[(\gamma^h Ih_{i+1}{}^{(0)} - (\mu^h + \theta^h) Rh_{i+1}{}^{(0)}) + 4(\gamma^h I_i{}^h - (\mu^h + \theta^h) R_i{}^h) + (\gamma^h I_{i-1}{}^h - (\mu^h + \theta^h) R_{i-1}{}^h)\right] \quad (42)$$

$$Sv_{i+1} = Sv_{i-1} + \frac{\Delta t}{3}\left[\left(\mu^v N^v - \frac{\beta^v b}{N^h} Ih_{i+1}{}^{(0)} Sv_{i+1}{}^{(0)} - \mu^v Sv_{i+1}{}^{(0)}\right) + 4\left(\mu^v N^v - \frac{\beta^v b}{N^h} I_i{}^h S_i{}^v - \mu^v S_i{}^v\right) + \left(\mu^v N^v - \frac{\beta^v b}{N^h} I_{i-1}{}^h S_{i-1}{}^v - \mu^v S_{i-1}{}^v\right)\right] \quad (43)$$

$$Iv_{i+1} = Iv_{i-1} + \frac{\Delta t}{3}\left[\left(\frac{\beta^v b}{N^h} Ih_{i+1}{}^{(0)} Sv_{i+1}{}^{(0)} - \mu^v Iv_{i+1}{}^{(0)}\right) + 4\left(\frac{\beta^v b}{N^h} I_i{}^h S_i{}^v - \mu^v I_i{}^v\right) + \left(\frac{\beta^v b}{N^h} I_{i-1}{}^h S_{i-1}{}^v - \mu^v I_{i-1}{}^v\right)\right] \quad (44)$$

TABLE 1. Parameters and parameter descriptions.

Parameter	Parameter and parameter descriptions
μ^h	The rate of birth and death of each individual
$\beta^h b$	The rate of interaction between I^v and S^h
θ^h	The rate of decline in individual immunity to disease.
γ^h	Each individual's rate of healing
μ^v	The birth and death rate of each of mosquito
$\beta^v b$	The rate of interaction between $I^h$and S^h

The values of the parameters in Table 1 are available in the References [5] and [16]. These values are listed in Table 2.

TABLE 2. Parameters and initial values.

Parameter	Initial values.
μ^h	0.000046
$\beta^h b$	0.75
θ^h	0.575
γ^h	0.328833
μ^v	0.0323
$\beta^v b$	0.357
$S(0)^h$	318,337
$I(0)^h$	1,816
$R(0)^h$	1,800
$S(0)^v$	94,400
$I(0)^v$	5,600
N^h	321,953
N^v	100,000

Numerical Results

Parameter values and initial values that are already known in Table 2 will be used to simulate the SIRS model using the fourth-order Runge-Kutta method and the Milne method. The simulation results can be seen in the following graphs and error values.

Simulation Using the MATLAB Software with Time Step is 2^{-4}

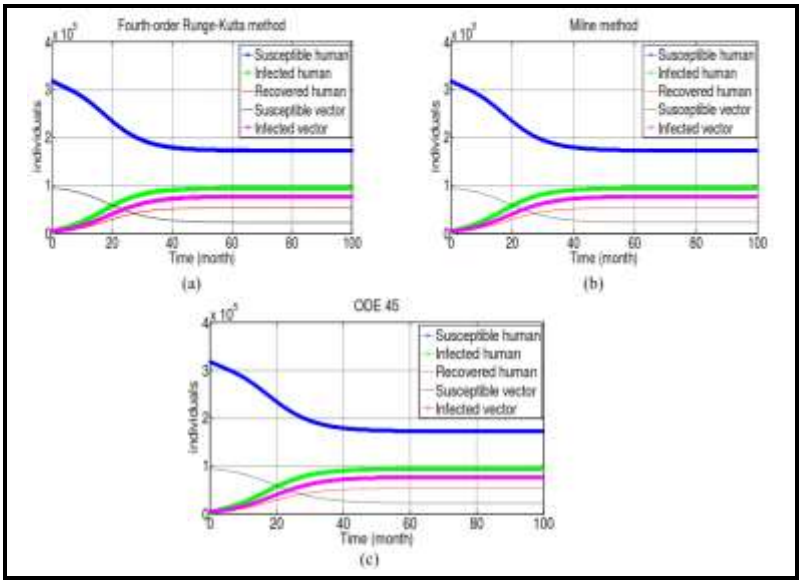

FIGURE 2. Simulation results of the SIRS model using time step equals to 2^{-4}: a) simulation results using the fourth-order Runge-Kutta method, b) simulation results using the Milne method, c) simulation results using ODE45 algorithm

Simulation Using the MATLAB Software with Time Step is 2^{0}

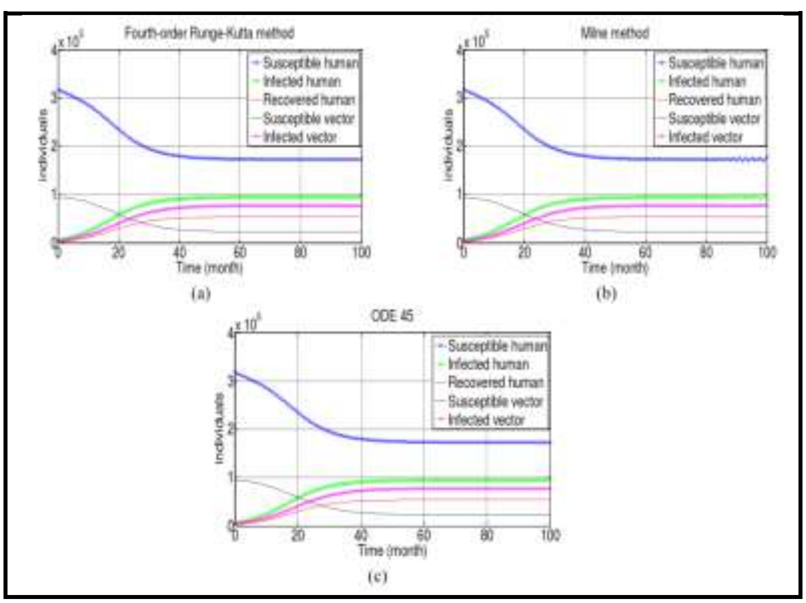

FIGURE 3. Simulation results of the SIRS model using time step equals to 2^{0}: a) simulation results using the fourth-order Runge-Kutta method, b) simulation results using the Milne method, c) simulation results using ODE45 algorithm.

Simulation results using the MATLAB software in Figure 2 and Figure 3 show that the rate of individuals who are susceptible to disease begins to decrease. This decline in this subpopulation is due to the birth rate and individuals losing immunity which is smaller than the infection rate which causes susceptible individuals to become infected. The infected population growth rate has increased; this is due to individuals who are susceptible to the disease will become infected individuals if bitten by mosquitoes that have been infected with the Dengue virus. The recovered population will also increase because after the infected individual experience treatment, they will recover from Dengue fever. Meanwhile, the number of mosquitoes that are susceptible to Dengue virus has decreased, and those infected have increased because from mosquitoes that are susceptible, they will become mosquitoes infected with Dengue virus which has the potential to spread Dengue fever in humans.

Both figures show graphs with different behaviour patterns. The first picture shows the graphic pattern that occurs when the time step taken is small, namely at the time, the time step is 2^{-4}, the graph of the fourth-order Runge-Kutta method and the Milne method run smoothly. While in the second picture, a larger time step is taken, when the time step is 1 it can be seen that the graph of the fourth-order Runge-Kutta method runs smoothly while the graph of the Milne method does not run smoothly. It can be seen at the end of the graph that the Milne method starts to oscillate. The Milne method has a good graph if the time steps taken are small. After looking at the graphs with different time steps, the following tables present the error values and accuracy levels for the fourth-order Runge-Kutta method and the Milne method.

TABLE 3. The average error value for the fourth-order Runge-Kutta method

h	S^h	I^h	R^h	S^v	I^v
1	4.89e-007	9.72e-006	6.67e-005	3.61e-007	2.76e-006
2^{-1}	2.35e-008	5.33e-007	3.29e-006	1.94e-008	1.42e-007
2^{-2}	1.29e-009	3.09e-008	1.80e-007	1.12e-009	7.95e-009
2^{-3}	7.57e-011	1.85e-009	1.05e-008	6.79e-011	4.71e-010
2^{-4}	4.59e-012	1.13e-010	6.33e-010	4.24e-012	2.87e-011
2^{-5}	2.88e-013	7.02e-012	3.89e-011	2.90e-013	1.78e-012
2^{-6}	2.46e-014	4.51e-013	2.43e-012	5.20e-014	1.21e-013

TABLE 4. The average error value for the Milne method

h	S^h	I^h	R^h	S^v	I^v
1	5.42e-003	4.91e-003	1.13e-002	7.71e-004	2.26e-004
2^{-1}	1.16e-003	1.16e-003	3.25e-003	1.01e-004	2.75e-005
2^{-2}	7.70e-004	5.42e-004	1.56e-003	4.28e-005	1.26e-005
2^{-3}	4.02e-004	1.40e-004	1.17e-004	3.90e-005	1.15e-005
2^{-4}	4.65e-005	6.08e-005	4.65e-005	1.10e-005	3.24e-006
2^{-5}	1.20e-006	3.61e-006	3.97e-006	8.88e-007	2.62e-007
2^{-6}	4.79e-008	1.35e-007	2.57e-007	3.91e-008	1.15e-008

TABLE 5. The level of accuracy of the fourth-order Runge-Kutta method

h	S^h	I^h	R^h	S^v	I^v
1	-	-	-	-	-
2^{-1}	4.38	4.19	4.34	4.22	4.29
2^{-2}	4.19	4.108	4.19	4.12	4.15
2^{-3}	4.09	4.06	4.10	4.04	4.08
2^{-4}	4.04	4.03	4.05	4.00	4.04
2^{-5}	4.00	4.01	4.03	3.87	4.01
2^{-6}	3.55	3.96	4.00	3.48	3.88

TABLE 6. The level of accuracy Milne method

h	S^h	I^h	R^h	S^v	I^v
1	-	-	-	-	-
2^{-1}	1.72	2.079	1.81	2.93	3.04
2^{-2}	1.09	1.10	1.06	1.24	1.11
2^{-3}	0.94	1.95	0.42	0.13	0.14
2^{-4}	3.11	1.20	4.65	1.83	1.83

TABLE 6. Continued

h	S^h	I^h	R^h	S^v	I^v
2^{-5}	5.29	4.07	3.55	3.63	3.63
2^{-6}	4.63	4.74	3.95	4.51	4.51

Table 3 and Table 4 show the error values for the fourth-order Runge-Kutta method and the Milne method**.** The time steps taken in calculating the average error value vary, which are 2^0, 2^{-1}, 2^{-2}, 2^{-3}, 2^{-4}, 2^{-5} and 2^{-6} for 5 compartments, namely the population susceptible to disease (S^h), the population infected with Dengue virus (I^h), the population that has recovered from disease (R^h), mosquitoes that are susceptible to the virus (S^v) and mosquitoes that have been infected with the Dengue virus (I^v). Tables 3 and 4 show that the average error values of the fourth-order Runge-Kutta method is smaller than the Milne method. This can be seen from the fourth-order Runge-Kutta method which has the smallest average error value of 2.46e-014, while the average error value of the Milne method is 1.15e-008.

Table 5 and Table 6 present the level (order) of accuracy. The level of accuracy of both numerical methods is calculated using the following formula [17].

$$R_i = \frac{\log\left(\frac{E_i}{E_{i+1}}\right)}{\log\left(\frac{h_i}{h_{i+1}}\right)} \tag{45}$$

where E_i is the numerical method error at the time step h_i.

Tables 5 and 6 show the level (order) of accuracy of the fourth-order Runge-Kutta method and the Milne method. The fourth-order Runge-Kutta method has a relatively stable level of accuracy to the value of four, whereas the level of accuracy of the Milne method is quite oscillatory around four.

CONCLUSION

The fourth-order Runge-Kutta method produces graphs that are smooth and do not oscillate as the time step increases, has a small error value, and the level of accuracy is relatively stable to the value of four. The Milne method has an oscillating graph when the step time increases, the error value is larger, and the accuracy level is relatively oscillatory around four. Thus, in simulating the spread of Dengue fever disease with the SIRS model, the fourth-order Runge-Kutta method is better than the Milne method.

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REFERENCES

1. D. K. Ningsih, J. Sains dan Seni ITS, **5**, A7–A13, (2016).
2. Y. M. Rangkuti and S. Side, J Conserv Dent. 2013, **16**, 2013, (2013).
3. H. S. Rodrigues, M. T. T. Monteiro, and D. F. M. Torres, Conf. Pap. Math., **1**, 1–7, (2013).
4. P. Y. Shu *et al.*, Clin. Diagn. Lab. Immunol., **10**, 622–630, (2003).
5. Y. Pusparisa, *Katadata Media Network*, 2022.
6. Widyawati, 2022.
7. S. A. Londah, C. E. Mongi, and C. E. J. C. Montolalu, d'CARTESIAN, **8**, 114, (2019).
8. H. Purwanto, E. Noviani, and M. N. Mara, Bimaster, **03**, 153–162, (2014).
9. W. Sanusi, M. I. Pratama, M. Rifandi, S. Sidjara, Irwan, and S. Side, J. Phys. Conf. Ser., **1752**, 1–11, (2021).
10. R. U. Hurit and Sudi Mungkasi, Mathline J. Mat. dan Pendidik. Mat., **6**, 140–153, (2021).
11. D. V. Griffiths and I. M. Smith, *Numerical Methods for Engineers*. McGraw-Hill Science/Engineering/Math, 2006.
12. E. R. Wulan and F. Hasanudin, J. Mat. Integr., **9**, 131, (2013).
13. S. L. Ross, *Differential equations 3rd edition Shepley L.Ross.pdf*. John Wiley and Sons, New York, 1984.

14. Stillman, Gloria, Brown, and Jill, Vinculum, **48**, 20, (2011).
15. S. Side, Alimuddin, and A. Bani, J. Math. Comput. Stat., **1**, 169–182, (2018).
16. W. Sanusi *et al.*, J. Appl. Math., **1**, 1–8, (2021).
17. N. Hidayat, Suhariningsih, A. Suryanto, and S. Mungkasi, Appl. Math. Sci., **8**, 1411–1420, (2014).

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Performance Comparison between the Fourth-Order Runge-Kutta Method and the Milne Method in Solving a Mathematical Model for the Dengue Fever Disease Spread

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