frontiers in Education

TYPE Mini Review
PUBLISHED 21 July 2026
DOI 10.3389/feduc.2026.1897747

OPEN ACCESS

EDITED BY
Jennifer Cutri,
Swinburne University of Technology, Australia

REVIEWED BY
Manwai Yuen,
The Education University of Hong Kong, Hong Kong SAR, China

*CORRESPONDENCE
Margaretha Madha Melissa
✉ d250866@hiroshima-u.ac.jp

RECEIVED 02 June 2026
REVISED 26 June 2026
ACCEPTED 30 June 2026
PUBLISHED 21 July 2026

CITATION
Melissa MM, Anuniwat D and Pratini HS (2026) A review of mathematics teachers' noticing of students' algebraic thinking.
Front. Educ. 11:1897747.
doi: 10.3389/feduc.2026.1897747

A review of mathematics teachers' noticing of students' algebraic thinking

Margaretha Madha Melissa[1]*, Dhanachat Anuniwat[1] and Haniek Sri Pratini[2]

[1]International Education Development Program, Hiroshima University, Hiroshima, Japan, [2]Mathematics Education Department, Universitas Sanata Dharma, Yogyakarta, Indonesia

Teacher noticing is central to responsive teaching. However, how teachers notice students' algebraic thinking has not yet been synthesized. This review consolidates emerging work and examines the methods these studies use and the findings that can be drawn across them. Following a systematic search and screening, 13 original studies were identified from the three databases and appraised for quality. The results reveal that the studies are mostly small qualitative designs that examine preservice teachers through written or recorded videos and concentrate on pattern generalization. Across them, teachers attend well to the surface features of students' work, however interpret it less reliably and respond least reliably, while a procedural view of algebra narrows what they notice. These findings imply that teacher education should provide content-specific lens and sustained support to develop interpretation and response.

KEYWORDS
algebraic thinking, mathematics teacher education, pattern generalization, professional vision, teacher noticing

1 Introduction

Teaching that is complex and responsive to students depends on what teachers can observe and hear (Choy, 2025). The construct of teacher noticing captures this professional competency, defined by Jacobs et al. (2010) as three interrelated skills: attending to students' strategies, interpreting their mathematical understanding, and deciding how to respond. A parallel tradition frames noticing as a professional vision, selective attention, and knowledge-based reasoning through which teachers make sense of classroom events (van Es and Sherin, 2021). Mason (2002) described it as a discipline of intentional attention that can be deliberately developed in the competency-as-continuum model, noticing is understood as a situation-specific skill that mediates between teachers' underlying dispositions and performance (Kaiser et al., 2025b). These perspectives are complementary (cf. Kaiser et al., 2025a). The competency-as-continuum model situates noticing within a chain linking teachers' dispositions to their performance, and its situation-specific skills of perception, interpretation, and decision-making align with the skills of attendance, interpretation, and response (Jacobs et al., 2010), which provide the analytic lens for this review. Across these, noticing is treated not as a general skill but as a situation-specific and content-specific skill; what a teacher attends to depends on the mathematical domain at hand (Copur-Gencturk and Rodrigues, 2021).

Algebra is a demanding domain for such noticing. Following Kaput (2008) and Kieran (2022), this study consider algebraic thinking as generalization and reasoning about relationships, structure, and variation, rather than as symbol manipulation alone,

realized through generalized arithmetic, functional thinking, pattern generalization, and early algebraic reasoning. Young children can reason algebraically through these strands before formal instruction (Blanton and Kaput, 2005; Carraher et al., 2006; Sibgatullin et al., 2022). Nonetheless, teaching algebra in this conceptual way is difficult, partly because teachers often hold a narrow, manipulation-focused view of the subject (Walkoe, 2015). Therefore, helping teachers recognize a wide range of students' algebraic thinking is a recognized challenge in mathematics teacher education.

Two lines of literature indirectly address this challenge. Teacher noticing has been broadly synthesized, including its conceptualizations and development (König et al., 2022; Scheiner et al., 2026), and algebraic thinking has been reviewed as a content-specific area (Sibgatullin et al., 2022). To our knowledge, the specific intersection of how teachers notice students' algebraic thinking has not yet been synthesized. This review consolidates emerging studies and clarifies the current position of this field. The review was guided by two questions:

1. What research methodologies (designs, data collection methods, participants and sample sizes, contexts, and countries) have been employed in studies on mathematics teacher noticing of students' algebraic thinking?
2. What findings can be synthesized from this body of literature?

2 Methodology

This review followed a structured search and screening procedure based on the PRISMA guidelines, adapted to the scope of a focused review. Three databases were searched in May 2026: Scopus, Web of Science Core Collection, and ERIC (via EBSCOhost). The same Boolean query was applied to the title, abstract, and keyword fields of each database and adapted to each database's field tags and truncation syntax. The query combined a teacher-noticing block ("teacher noticing" OR "professional noticing" OR "professional vision" OR "noticing of student*") with an algebraic-thinking block, as mentioned in the Introduction, ("algebraic thinking" OR "algebraic reasoning" OR "early algebra*" OR "functional thinking" OR "pattern generaliz*" OR "generalized arithmetic"). The search returned 33 records (Scopus=14, Web of Science=9, and ERIC=10).

Studies were included if they (1) were published in a peer-reviewed journal, (2) were written in English, (3) explicitly addressed teacher noticing or professional vision, (4) were relevant to the discourse on teacher noticing in mathematics education, and (5) explicitly addressed algebraic thinking. Records that did not fulfill all five criteria were excluded. The selection process is summarized in Figure 1.

As depicted in Figure 1, 15 duplicate records were excluded from the 33 identified records, leaving 18 records for title and abstract screening. Three records were excluded at this stage because they were not peer-reviewed journal articles, comprising one book chapter, one dissertation, and one conference paper. Thereafter, the full texts of the remaining 15 reports were then assessed for eligibility, and two were excluded because they were not in English, but in Spanish. This process yielded 13 studies included in the review.

Each study was appraised for methodological quality using the Mixed Methods Appraisal Tool (Hong et al., 2018), suitable for reviews combining qualitative, quantitative, and mixed-methods designs. Thereafter, a standardized matrix was used to extract

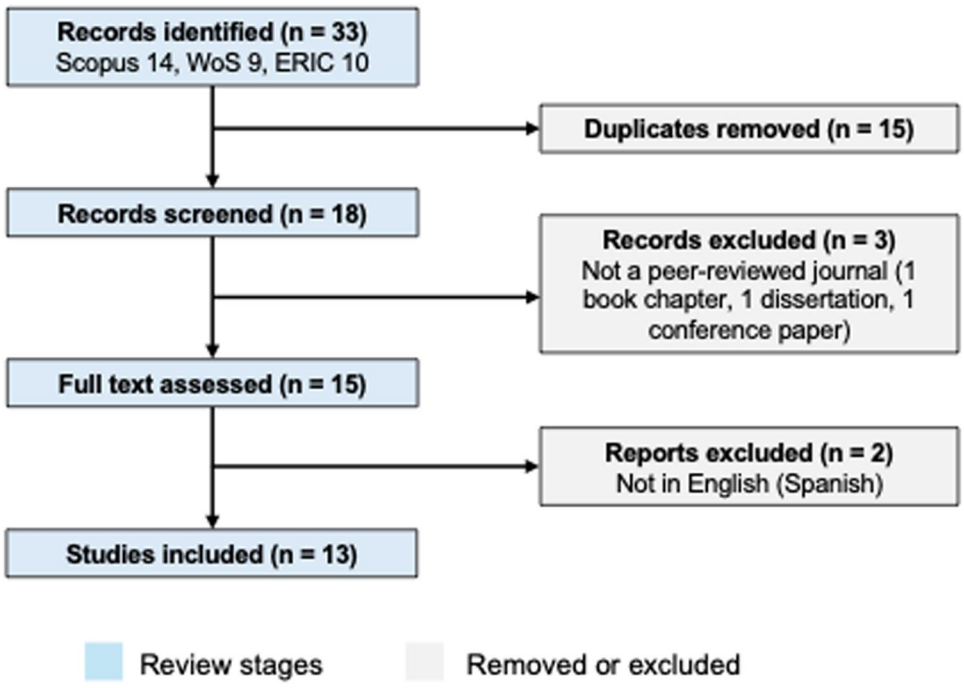

FIGURE 1
Articles selection process.

the elements addressed by the research questions from each study. The results are presented in the following section.

3 Result and discussion

3.1 Research methodologies

All five dimensions, design, data collection, participants, sample size, and country, mentioned in the first research question are reported for each of the 13 studies in Table 1.

In addition to the methodological dimensions listed in Table 1, three other characteristics were scrutinized. Most studies use the attending, interpreting, and deciding model of Jacobs et al. (2010), often with its robustness, limitations, and lack of evidence coding (Fisher et al., 2019; Jong et al., 2021; LaRochelle et al., 2019; Özel et al., 2025; Tekin-Sitrava et al., 2025), whereas a few use the professional vision framework (Rodrigues et al., 2022; Walkoe, 2015; Walkoe et al., 2026). Some have extended this base to include knowledge for teaching and affect (Jong et al., 2021), task performance (Zhang and Li, 2026), learning trajectories (Lee and Lee, 2023; Zapatera Llinares, 2022), or multimodal communication (Walkoe et al., 2026). This shared use of one model provides researchers a consensus but overlooks what lies beyond the noticing skills aspect.

The content base was narrow. Pattern generalization, mostly figural and linear, dominated (e.g., Callejo and Zapatera, 2017; LaRochelle et al., 2019 Lee and Lee, 2023;; Zapatera Llinares, 2022), whereas functional thinking appeared once (Zhang and Li, 2026) and justification only occasionally (Fisher et al., 2019; Magiera and Zambak, 2021). Core topics, such as equal signs and generalized arithmetic, were largely missing.

The student thinking under study sits mostly in the primary to lower secondary level, matching the pattern tasks (e.g., Özel et al., 2025; Rodrigues et al., 2022; Zapatera Llinares, 2022), with secondary thinking rarely the focus (LaRochelle et al., 2019; Walkoe, 2015; Zhang and Li, 2026) as this level often differs from teachers' own, the grade band of noticed thinking is a more useful descriptor.

3.2 Synthesis of findings

A consistent gradient was observed across all three skills. The clearest pattern is that teachers attend more readily than they interpret and interpret more readily than they decide how to respond. LaRochelle et al. (2019) reported that three-quarters of secondary in-service teachers attended to the details of students' strategies, but only half demonstrated limited (and non-robust) evidence of interpreting, and only a quarter demonstrated limited evidence of deciding. Intervention studies echo this asymmetry; attending and interpreting improved significantly, while deciding remained a persistent challenge (Fisher et al., 2019; Jong et al., 2021). When responses were examined closely, teachers often posed repetitive or procedural questions rather than questions that extended student thinking (Özel et al., 2025; Tekin-Sitrava et al., 2025). This phenomenon must be critically observed. Across the corpus, responding was elicited almost entirely through written artifacts in non-interactive settings,

TABLE 1 Methodological characteristics of the thirteen studies.

Study	Design	Data Collection	Participants and Sample size	Country
Callejo and Zapatera (2017)	Qualitative	Written responses to pattern tasks, questionnaire	38 preservice teachers	Spain
Fisher et al. (2019)	Quasi-experimental	Instructional module, classroom video, pre and post	296 preservice elementary	USA
Jong et al. (2021)	Quasi-experimental	Module with noticing, MKT, and affect measures, pre and post	296 preservice elementary	USA
LaRochelle et al. (2019)	Quantitative	Written responses to student-work artifacts	16 secondary teachers	USA
Lee and Lee (2023)	Qualitative	Written task with hypothetical student examples	154 preservice teachers	USA
Magiera and Zambak (2021)	Mixed Methods	Written artifacts and video records	15 preservice teachers	USA
Özel et al. (2025)	Qualitative	Open-ended questionnaire and semi-structured interviews	32 preservice teachers	Türkiye
Rodrigues et al. (2022)	Qualitative	Online multimedia case, whole-class reflective discussion	15 preservice teachers	Portugal
Tekin-Sitrava et al. (2025)	Qualitative	Two-year online professional development, pre and post	31 in-service teachers	Türkiye
Walkoe (2015)	Design Experiment	Eight-week video club, individual tasks, algebraic thinking framework	7 preservice teachers	USA
Walkoe et al. (2026)	Design-Based Research	Video club with a pin-and-draw tool	Two cohorts (9 and 11 preservice teachers)	USA
Zapatera Llinares (2022)	Qualitative	Teaching module built on a learning trajectory	18 preservice teachers	Spain
Zhang and Li (2026)	Mixed Methods	Questionnaire and written artifacts	86 in-service teachers	China

Sources: Authors' own elaboration.

where teachers described a hypothetical next step rather than acting on student thinking as it unfolded. This is a methodological weakness of the present study. Such tasks cannot reproduce the immediate, contingent, and interactive nature of classroom responding. They capture how teachers reason about the next step in writing, not how they respond to students in real time.

However, this gradient is not universally applicable. Zhang and Li (2026) determined that novice in-service Chinese teachers demonstrated strong attending and responding, but less robust interpreting, and that teachers with medium task performance attended and responded more strongly than their high-performing peers. These results suggest that the relative difficulty of the three skills is mediated by curriculum, teacher preparation, and cultural context.

Regarding content-specific quality of attending, several studies have revealed that teachers tend to foreground surface features, that is correctness, efficiency, and producing a formula or rule, rather than the relational and structural reasoning that define algebraic thinking (Lee and Lee, 2023; Walkoe, 2015). This narrow, manipulation-oriented view of algebra limits the depth of interpretation, particularly for figural generalization strategies. This finding aligns with long-standing concerns that algebra is often treated as symbol manipulation rather than as generalization and reasoning about structures.

Noticing is learnable with appropriate support. A consistent and optimistic finding is that targeted interventions develop noticing. Video clubs (Walkoe, 2015; Walkoe et al., 2026), online professional development enriched by collaborative discussion (Tekin-Sitrava et al., 2025), online multimedia cases (Rodrigues et al., 2022), and modules built on learning trajectories (Lee and Lee, 2023; Zapatera Llinares, 2022) all strengthen teachers' attending and interpreting. Design features are significant. A learning-trajectory framework supports interpretation, collaborative discussion sustains growth, and specific tools shape what teachers observe. Walkoe et al. (2026) determined that preservice teachers noticed students' gestures only when prompted to use a dedicated drawing tool, highlighting that students express algebraic thinking multimodally through gestures, gaze, and movement, and that written records alone miss this dimension.

Regarding connections to knowledge and affect, noticing is linked to the broader characteristics of teachers. Jong et al. (2021) reported statistically significant but complex connections among professional noticing, mathematical knowledge for teaching, and affect, while Zhang and Li (2026) demonstrated that the relationship between task performance and noticing is non-linear.

4 Conclusion

This review of 13 studies on teachers' noticing of students' algebraic thinking is an emerging area that has not yet been synthesized. Methodologically, the field is dominated by qualitative designs, concentrated in the USA, and focuses on preservice teachers analyzing student artifacts. The findings reveal a promising landscape with challenges: teachers can develop noticing through video clubs, multimedia cases, and learning trajectory modules. However, addressing what the students reveal remains the most challenging component to develop.

This indicates a method that the corpus has rarely examined. The reviewed interventions mostly asked teachers to notice student thinking after it had occurred. However, interpreting and responding were the most challenging. A promising alternative is to anticipate student thinking before a lesson or while watching a video by predicting the strategies and errors that a task is likely to elicit. Anticipation engages interpreting and responding directly because predicting how students will reason requires interpreting the mathematics of the task and rehearsing the decisions that teachers found most difficult.

Consequently, future studies should transcend pattern generalization to include functional and multimodal reasoning, in-service teachers and diverse geographical contexts, and study noticing in authentic classrooms. Along these lines, teacher noticing offers a promising lever for improving algebra education.

Author contributions

MM: Conceptualization, Funding acquisition, Investigation, Project administration, Supervision, Validation, Writing – original draft, Writing – review & editing. DA: Investigation, Validation, Writing – original draft, Writing – review & editing. HP: Validation, Writing – original draft, Writing – review & editing.

Funding

The author(s) declared that financial support was received for this work and/or its publication. This study was supported by JST SPRING (Grant Number JPMJSP2132).

Conflict of interest

The author(s) declared that this work was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Generative AI statement

The author(s) declared that generative AI was used in the creation of this manuscript. Claude.ai were used only for language refinement and limited conceptual clarification.

Any alternative text (alt text) provided alongside figures in this article has been generated by Frontiers with the support of artificial intelligence and reasonable efforts have been made to ensure accuracy, including review by the authors wherever possible. If you identify any issues, please contact us.

Publisher's note

References

Blanton, M. L., and Kaput, J. J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *J. Res. Math. Educ* 36, 412–446. doi: 10.2307/30034944

Callejo, M. L., and Zapatera, A. (2017). Prospective primary teachers' noticing of students' understanding of pattern generalization. *J. Math. Teach. Educ* 20, 309–333. doi: 10.1007/s10857-016-9343-1

Carraher, D. W., Schliemann, A. D., Brizuela, B. M., and Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *J. Res. Math. Educ* 37, 87–115. doi: 10.2307/30034843

Choy, B. H. (2025). Noticing to learn from lesson study: the role of critical events. *J. Math. Teach. Educ* 28, 1059–1076. doi: 10.1007/s10857-025-09680-8

Copur-Gencturk, Y., and Rodrigues, J. (2021). Content-specific noticing: a large-scale survey of mathematics teachers' noticing. *Teach. Teach. Educ* 101, 103320. doi: 10.1016/j.tate.2021.103320

Fisher, M. H., Thomas, J., Jong, C., Schack, E. O., and Dueber, D. (2019). Comparing preservice teachers' professional noticing skills in elementary mathematics classrooms. *Sch. Sci. Math* 119, 142–149. doi: 10.1111/ssm.12324

Hong, Q. N., Fàbregues, S., Bartlett, G., Boardman, F., Cargo, M., Dagenais, P., et al. (2018). The mixed methods appraisal tool (MMAT) version 2018 for information professionals and researchers. *Educ. Inf* 34, 285–291. doi: 10.3233/EFI-180221

Jacobs, V. R., Lamb, L. L. C., and Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *J. Res. Math. Educ* 41, 169–202. doi: 10.5951/jresematheduc.41.2.0169

Jong, C., Schack, E. O., Fisher, M. H., Thomas, J., and Dueber, D. (2021). What role does professional noticing play? Examining connections with affect and mathematical knowledge for teaching among preservice teachers. *ZDM Math. Educ* 53, 151–164. doi: 10.1007/s11858-020-01210-5

Kaiser, G., König, J., Krauss, S., Buchholtz, N., and Gruber, H. (2025a). Are teacher expertise and teacher competence complementary or contradictory research paradigms? Reflections on a possible integration and first conceptualizations. *ZDM Math. Educ* 57, 1077–1092. doi: 10.1007/s11858-025-01743-7

Kaiser, G., Yang, X., and König, J. (2025b). "Recent discourse in research on the competence of mathematics teachers," in *Proceedings of the 9th ICMI-East Asia Regional Conference on Mathematics Education, Vol. 1*, eds. O. Kwon, B. Kaur, J. Pang, J. Noh, S. Lee, S. Han, S. Yeo, and M. Lim (Siheung, Korea: EARCOME9), 169–188.

Kaput, J. J. (2008). "What is algebra? What is algebraic reasoning?," in *Algebra in the Early Grades*, eds. J. J. Kaput, D. W. Carraher, and M. L. Blanton (New York, NY: Routledge), 5–18. doi: 10.4324/9781315097435

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: background, overarching dimensions, and new directions. *ZDM Math. Educ* 54, 1131–1150. doi: 10.1007/s11858-022-01435-6

König, J., Santagata, R., Scheiner, T., Adleff, A.-K., Yang, X., and Kaiser, G. (2022). Teacher noticing: a systematic literature review of conceptualizations, research designs, and findings on learning to notice. *Educ. Res. Rev* 36, 100453. doi: 10.1016/j.edurev.2022.100453

LaRochelle, R., Nickerson, S. D., Lamb, L. C., Hawthorne, C., Philipp, R. A., and Ross, D. L. (2019). Secondary practising teachers' professional noticing of students' thinking about pattern generalisation. *Math. Teach. Educ. Dev* 21, 4–27.

Lee, M. Y., and Lee, J.-E. (2023). An analysis of elementary prospective teachers' noticing of student pattern generalization strategies in mathematics. *J. Math. Teach. Educ* 26, 155–177. doi: 10.1007/s10857-021-09520-5

Magiera, M. T., and Zambak, V. S. (2021). Prospective K-8 teachers' noticing of student justifications and generalizations in the context of analyzing written artifacts and video-records. *Int. J. STEM Educ* 8, 7. doi: 10.1186/s40594-020-00263-y

Mason, J. (2002). *Researching Your Own Practice: The Discipline of Noticing*. London: Routledge.

Özel, Z., Işıksal-Bostan, M., and Tekin-Sitrava, R. (2025). Prospective teachers' noticing of students' algebraic thinking: the case of pattern generalization. *Electron. J. Res. Sci. Math. Educ* 29, 59–84.

Rodrigues, R. V. R., Oliveira, H. M., and Cyrino, M. C. D. C. T. (2022). Promoting prospective mathematics teachers' professional vision on a whole-class reflective discussion: contributions of digital resources. *Int. J. Educ. Math. Sci. Technol* 10, 773–794. doi: 10.46328/ijemst.2181

Scheiner, T., Sherin, M. G., Van Es, E. A., and Kaiser, G. (2026). Towards theoretical synergies in mathematics teacher noticing. *ZDM Math. Educ* 58, 1–18. doi: 10.1007/s11858-026-01768-6

Sibgatullin, I. R., Korzhuev, A. V., Khairullina, E. R., Sadykova, A. R., Baturina, R. V., and Chauzova, V. (2022). A systematic review on algebraic thinking in education. *Eurasia J. Math. Sci. Technol. Educ* 18, em2065. doi: 10.29333/ejmste/11486

Tekin-Sitrava, R., Özel, Z., Işıksal-Bostan, M., and Yemen-Karpuzcu, S. (2025). How does online professional development program enriched with collaborative discussion develop teachers' noticing skills? *Int. J. Sci. Math. Educ* 23, 71–96. doi: 10.1007/s10763-024-10461-7

Van Es, E. A., and Sherin, M. G. (2021). Expanding on prior conceptualizations of teacher noticing. *ZDM Math. Educ* 53, 17–27. doi: 10.1007/s11858-020-01211-4

Walkoe, J. (2015). Exploring teacher noticing of student algebraic thinking in a video club. *J. Math. Teach. Educ* 18, 523–550. doi: 10.1007/s10857-014-9289-0

Walkoe, J., Walton, M., Elby, A., and Moon, P. (2026). Video tools to support multimodal teacher noticing. *ZDM Math. Educ* 58, 191–201. doi: 10.1007/s11858-025-01756-2

Zapatera Llinares, A. (2022). Prospective teachers' use of conceptual advances of learning trajectories to develop their teaching competence in the context of pattern generalization. *Math.* 10 (12), 1974. doi: 10.3390/math10121974

Zhang, Q., and Li, S. (2026). Examining novice secondary mathematics teachers' professional noticing of students' functional thinking in Chinese Mainland. *ZDM Math. Educ* 58, 203–215. doi: 10.1007/s11858-026-01769-5