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CONCEPTUAL CHANGE IN PROBABILITY AND RANDOMNESS OF HIGH SCHOOL STUDENTS USING COMPUTER SIMULATIONS

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(Order no.)

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ABSTRACT

This study investigated high school students' conceptual change on probability and randomness after the students used computer simulation programs. More specifically the research examined: (1) whether students' knowledge of probability and randomness improved; (2) what students' concepts were changed; (3) why students changed their concepts; (4) how students changed their concepts; and (5) what students thought and experienced about computer simulation programs. Question (1) was a quantitative study. It was investigated through pretest, posttest, and students' concept maps. A statistical analysis - one tailed *t*-test - was used. Questions (2) to (5) applied qualitative methodologies. They were investigated through multiple data sources such as: students' concept maps, learning logs, students' reasoning in pretest and posttest, fieldnotes, and evaluation on teaching-learning. The study is based on constructivist theory, conceptual change theory, and schema theory.

The computer simulation programs were developed by the Boston University Center for Polymer Studies. These programs were titled: Coin Flipping, 1-D Random Walk, Pascal's Triangle, Many Walkers, Diffusion, Coastline Dimension, and Fractal Dimension.

Thirty six grade-11 mathematics students from a suburban high school were used as the sample. They finished a pretest before the program, did the computer simulation programs, and 40 days after the experience they took a posttest. The students worked during regular classroom time.

The research found that students improved their knowledge of probability and randomness. Students changed some ideas of probability and randomness by changing their concept maps and their reasoning in their posttest. Students developed their ideas by explaining more exactly, explicitly, and clearly using statistical terms. Students liked the computer simulation programs and found that the class was fun and interesting.

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dimension, a plane two dimensions, and a cube three dimensions; but fractals have non-integer dimensions (between 1 and 2) (Stanley, Taylor, & Trunfio, 1994; Crownover, 1995).

There are two kinds of fractals: mathematical fractals and natural fractals. Mathematical fractals are described mathematically such that for a given formula, the resulting structure is always identical. Natural fractals never repeat themselves (Stanley et al., 1994). Bunde and Havlin (1994) distinguished fractals into two categories: deterministic and random fractals. Deterministic fractals are generated iteratively in an exact way, while random fractals are generated using a stochastic process. Deterministic fractals are the same as mathematical fractals. According to Bunde and Havlin, all fractal structures in nature are random fractals. The Koch curve with fractal dimension log 4 / log 3, the Cantor set with log 2 / log 3, and the Sierpinski Gasket with log 3 / log 2 are examples of mathematical fractals. Lightening bolts, coastlines, nerve cells, termite tunnels, root systems, broccoli, lungs, and kidneys are some examples of object that display natural fractal patterns (Stanley et al., 1994). According to Stanley et al., fractals have spidery patterns.

Some natural fractals are formed by molecules or atoms that move randomly. Investigating how molecules move is difficult because of their small size. Random events that mimic the way molecules move will help to understand the molecules. Random events are used as a model that simplifies the natural behavior of molecules or atoms (Stanley et al., 1994). According to Bunde and Havlin (1994), random fractals are generated using random and probabilistic process, and random fractals mimic the fractal system in nature. That is why some studies on natural fractals use random event approaches.

According to Shaughnessy (1992), there has been much research on probability and statistics in elementary schools and universities, but very little with secondary school students. Part of the reason is due to the lack of probability and statistics instruction at the secondary school level in the United States. Very few secondary schools offer a separate course in probability and statistics. Many secondary schools that include probability and statistics in their curriculum treat it as a 6-9 week unit within another course. Many students do not take it and teachers are tempted to skip it (Shaughnessy, 1992). Shaughnessy classified two types of studies on probability. First, studies described how students think. These studies investigated primitive conceptions, naive conceptions, fallacies of probability, and so forth. The studies were carried on mostly by psychologists. Second, studies concerned influencing students' beliefs or conceptions. They were conducted mostly by mathematics or statistics educators (Shaughnessy, 1992).

Because of the strong influences of cognitive psychology, research conducted by mathematics educators fell primarily into three categories: (1) feasibility studies to determine whether topics in probability and statistics could be taught to elementary or middle school students; (2) experimental or correlational studies to look for the effects of teaching probability to students' understanding of probability; and (3) studies that compared the effects of several approaches to teaching a unit on probability (Shaughnessy, 1992).

In 1994, the Boston University Center for Polymer Studies and the Science and Mathematics Education Center, with funding from the National Science Foundation, conducted a two week workshop on Patterns in Nature for high school science teachers. The teachers explored and analyzed natural fractals as patterns in nature using hands-on activities and computer simulation programs developed by the Center for Polymer Studies, Boston University. In the workshop, fractals were approached using probability and random event models.

The computer simulation programs are titled: Lottery, Coin Flipping, 1-D Random Walk (One Dimension Random Walk), Pascal's Triangle, 2-D Random Walk (Two Dimension Random Walk), Many Walkers, Deer, Diffusion, Coastline Dimension, and Fractal Dimension. Some programs emphasize such concepts of random events and probability as independence, chance, and distribution. Some programs emphasize fractal concepts and how to measure fractal dimension. The programs are simulations in which students actively interact with the computer programs. Students are allowed to predict, manipulate variables, collect data, analyze data, discuss in groups, and draw conclusions.

Some high schools in Massachusetts have used the computer simulation programs to teach about random events, probability, and fractals. The schools used the same programs that were used in this workshop. Students in the schools investigated the fractal dimension and how fractals were formed using the random event model. They made connections between random events and fractals. Random events were considered as a model to understand fractal formation. The use of the programs needed a systematic study to investigate its effectiveness with students in those high schools. Whether the programs help students understand the concepts of randomness, probability, and fractals needed to be evaluated.

Constructivism, conceptual change, and misconception

Over the last decade, many studies have investigated conceptual change (Basili & Sanford, 1991; Chinn, 1993; Dykstra, Boyle, & Monarch, 1992; Posner, Strike, Hewson, & Gertzog, 1982; Woods & Thorley, 1993). Conceptual change model of learning has evolved from the constructivist philosophy. According to constructivism, students construct their own knowledge. Meaning is constructed from what students see, hear, discuss, learn, experience, and so forth (Bettencourt, 1989; Bodner, 1986; Watts & Pope, 1989). Learning, according to constructivists, is in essence, a form of conceptual change. Students' understanding progresses, regresses, or remains unchanged. Changes in understanding occur when the learner's ideas are engaged, challenged and extended through direct experiences, as the learners interact with others to resolve conceptual conflict and as the learners reflect on their own ideas (Shymansky et al., 1993). Constructivists are most concerned about the process of student learning rather than the final outcomes.

The constructivist philosophy of learning provides a means for understanding students' misconceptions. Sometimes students construct correct knowledge, but sometimes they construct a "wrong" knowledge through their learning. Misconception is an idea that is incompatible with currently accepted scientific knowledge (Brown, 1989). Novak (1984) explained misconception as an unaccepted interpretation of concepts in a statement. Feldsine (1987) found misconception to be an error and incorrect relationship between two concepts. Fowler (1987) viewed misconception as an inaccurate understanding of concepts, misuse of a concept name, wrong classification of concept examples, confusion between different concepts, or improper hierarchical relationship. Some researchers use alternative conception rather than misconception. The alternative conception refers to experienced-based explanations constructed by a learner. It confers intellectual respect on the learner who has the idea. Sometimes alternative conceptions are contextually valid and help the learner to understand some phenomena (Wandersee, Mintzes, & Novak, 1994).

Many researchers find that students have misconceptions or alternative conceptions about probability and randomness before they receive formal instruction in classrooms (Baturo, 1993; Garfield & Ahlgren, 1988; Hawkins & Kapadia, 1984; Konold, 1988a, 1988b; Shaughnessy, 1983). For example, in tossing coins, some students think that the sequence HTHTTH is more likely to occur than the sequence HHHHTT (Tversky & Kahneman, 1982a). Some students consider the frequency distribution wrong because they think only of the samples they remember or are familiar with (Tversky & Kahneman, 1982a, 1982d). Many students believe that successive outcomes of a random process are not independent ("the gambler's fallacy") (Konold, 1988a). Students have different interpretations about the meanings of terms such as probable, impossible, and certain (Fischbein, Nello, & Marino, 1991). In daily experiences students hear many terms or words about probability such as "probable, likelihood, never, uncertainty, equally-likely, and maybe." Sometimes the words have different meanings from the statisticians' meaning, thereby influence students' misconception (Ernest, 1984; Konold, 1988b).

Shaughnessy listed some misconceptions on probability such as: judgmental heuristics (representativeness and availability), conjunction fallacy, and troubles with conditional and independent probability (Shaughnessy, 1992; Shaughnessy & Bergman, 1993). According to the representativeness heuristic, students estimate the likelihood of events based on how well an outcome represents some aspect of its parent populations. People believe that samples should either reflect the distribution of the parent population or mirror the process by which random events are generated (Kahneman & Tversky, 1982a, 1982b; Shaughnessy, 1992). When students base their estimate of the likelihood of events on how easy it is for them to call to mind particular instances of the event, they employ the availability heuristic (Kahneman & Tversky, 1982a, 1982b; Shaughnessy, 1992).

Conjunction fallacy happens when students do not follow the property of probability of conjunction. According to probability, the chance of two distinct events occurring simultaneously is less than the chance of either one occurring separately. Research has discovered that many students think differently. For example, students think that the percentage of people who were 55 years old and had a heart attack higher than the percentage of people who only had a heart attack (Kahneman and Tversky, in Shaughnessy & Bergman, 1993). According to Falk, many students have difficulties with conditional probabilities. The difficulties arise primarily when a conditioning event occurs after the event that it conditions. Students have difficulties in finding the probability of W2 if W1 is given and the probability of W1 if W2 is given (Shaughnessy & Bergman, 1993).

Computer simulations and misconceptions

Garfield and Ahlgren (1988) suggested some methods to overcome students' misconceptions or preconceptions of probability and randomness. They suggested to examine students' misconceptions, to ask whether their misconceptions can be changed, and to ask how they can be changed. After studying students' misconceptions, they proposed possible strategies in teaching probability such as: teaching students to use experiments and simulations, helping students to feel that probability is related to reality, and using methods that allow students to explore data. They also proposed that teachers point out and confront students' common misconceptions and errors (Garfield & Ahlgren, 1988; Hawkins & Kapadia, 1984).

To eliminate students' misconceptions, Shaughnessy (1983) emphasized the following instructional methods: activities and experiments; carrying out clinical interviews with students to overcome reasoning difficulties on probability problems; and giving students challenging problems that require them to gather, simulate data, and make decisions.

Ganiel and Idar (1985) maintained that computer simulations can help avoid misconceptions in science (1) if students have frequent opportunities to probe their understanding, to answer diagnostic questions, and to test the validity of predictions based on their own interpretations; and (2) if students receive immediate corrective comments concerning the accuracy of their answers and predictions. These comments should point out and clarify inconsistencies between students' answers and predictions. According to Ganiel and Idar, simulations are useful when students interact with computers through altering values of variables, initiating processes, probing conditions and observing the results of these actions. Simulations can create discrepancies with students' expectations. They can challenge students' notions and in doing so stimulate conceptual change (1985).

Some studies of computer simulations have found that such programs help students to learn (Choi & Gennaro, 1987; Collis, 1982; Ponte, 1991; Walton, 1990). Bork (1987) explained that computer simulations have many advantages in classroom instruction. Computer simulations are, by definition, interactive. They allow students to play an active role in their learning of how a model of a real-world system operates. Computer simulations have an ability to individualize the learning experience to the needs of learners. Students are different, unique, and they learn differently. With a computer the students can move at their own paces. They allow students to engage in Socratic dialogue where students get input directly from the computers. Students are evaluated directly. Students can learn and repeat the programs without the whole class and without the teachers (Bork, 1987). Computers can provide much more experience with data manipulation and data representation than would ever be possible by hand in the same time frame (Biehler et al., in Shaughnessy, 1992).

Shaughnessy (1992) indicated that there are few studies on the effects of computers or computer simulations on students' learning and understanding of probability and statistics. Literature on the use of computers in probability and statistics is long on didactical suggestion, but short on research suggestion. There are many new computer software that have not been evaluated or researched. Based on the lack of studies on the secondary school level, he suggested that the next studies on probability should emphasize questions such as: (1) What are secondary students' conceptions and misconceptions? (2) What are teachers' conceptions of probability and statistics? (3) What are the effects of computer software? (4) How can we use computers to help students change their beliefs and misconceptions?

PILOT STUDY

In a pilot study conducted in a suburban high school in Massachusetts, computer simulations have been shown to have effects on students' conceptual development about randomness and probability. In that study, one class of physics students was used as the sample population. The class consisted of 12 students. There were six Macintosh computers and one scanner. Over a two week period students used at least nine programs (Lottery, Coin Flipping, 1-D Random Walk, Pascal's Triangle, 2-D Random Walk, Many Walkers, Deer, Coastline Dimension, and Fractal Dimension). Students were engaged in groups of two or three. The students conducted experiments manually, worked on computer simulations, manipulated variables, collected data in computers, created graphics, discussed in groups, and made conclusions. Students wrote electronic learning logs - computerized diaries or notes - which described what they were doing and learning.

Students constructed concept maps about probability, randomness, and fractals. Concept maps are "two dimensional diagrams that illustrate relationships between ideas in a content area. These maps are organized hierarchically, with the broadest, most inclusive concept at the top of the page and subordinate, detailed concepts lower on the page. Concept words are linked by lines that are labeled to identify the type of relationship between the concepts" (Jonassen, Beissner, & Yacci, 1993, p.155). Their teacher acted as a tutor, mentor, friend, and judge, and applied a cognitive apprenticeship instructional model (Collins, Brown, & Duguid, in Audet, 1993). The computer simulation programs were developed using the apprenticeship model (Shore, in Audet, 1993). The process of instruction was: the teacher gave an introduction, and after that students did experiments and computer simulations, discussed within groups, and reported their results in their learning-logs.

Based on the students' work such as concept maps and learning logs, it was found that students experienced conceptual changes in randomness and probability.

a) Most students changed their preconceptions about the <u>third category</u> of randomness. The first category is random. The second category is not random. The third category is "not very random." It is a category between random and not random. In their pretest, most students believed that there was a third category. They gave some examples of the third category such as weather, the people who come up to you in the hall, a piano falls on you from a sixteen story building. In the last learning logs most students wrote that there was no such category. They wrote:

There is <u>no such category as "not very random</u>." If something is random, then all the possible outcomes have equal probability for every trial. If the outcomes do not have equal probabilities, they are not random. There is no in-between.

b) Most of the students changed their concept maps by either adding new concepts, new propositions, new examples, or by changing propositions and old concepts.

c) From learning logs, most students understood better about the meaning of probability and how to calculate probability, such as: the probability of coin flipping is 1/2; if the trial is infinite, the result will be closer to 1/2; the bell curve will be smoother if the number of trials increases.

Students also changed their ideas about the computer. In the beginning they considered a computer only as a tool for typing papers and for playing games, but these activities convinced them that a computer could be used as a tool for learning science. The computer was thought of as a useful tool for learning because it helped students to visualize and clarify ideas about science. It generated simple models by which to understand phenomena about the growth of patterns in nature. Students also had more respect for the computer as a tool for learning. For example, one group said that computers helped them to understand population growth more than any amount of reading could have accomplished.

This study in a small class of twelve students indicated that the computer simulation programs helped students to understand the concepts of randomness and probability. The students experienced conceptual change and improved their understanding of probability and randomness.

RATIONALE

The research of Garfield and Ahlgren (1988) and Shaughnessy (1983) suggest that misconceptions on randomness and probability might be overcome by using activities that allowed students to explore data. Computer simulation programs have many activities by which students could manipulate variables, collect data, analyze data, and make conclusions. It is proposed that computer simulations can eliminate misconceptions or improve students' ability to experience conceptual change in randomness and probability.

This study proposes to examine students' conceptions of probability and randomness in secondary school. It includes the effects of computer simulations on students' conceptual changes. It investigates the effects of computer simulations on students' understanding of probability. The study is designed to implement Shaughnessy's suggestion for future research in probability and statistics at the secondary level.

Most research in probability and randomness focuses more mathematically in which probability was treated as part of mathematics or statistics. The special value of the Boston University computer simulation programs is that the probability and randomness are connected to the sciences: chemistry, physics, and biology. The programs are more interdisciplinary, linking mathematics and sciences. The programs emphasize concepts and intuitive probability more than mathematical calculation. In addition, the probability and randomness are connected to fractals, a new topic of study in the secondary schools. The programs could be used as a means of introducing an interdisciplinary study in science learning as suggested by the Association for the Advancement of Science (AAAS) in Project 2061: Science for all Americans (AAAS, 1989).

PURPOSE OF THE STUDY

The purpose of this study is to investigate changes in high school students' concepts of probability and randomness after the students use computer simulation programs. Specifically, this research examines: (1) whether students' understanding of probability and randomness improved, (2) what conceptual changes students experienced, (3) why students changed their concepts, (4) how such a process of conceptual change might have happened, and

(5) what students thought and experienced about computer simulation programs.

The students' conceptual change about probability and randomness are identified through multiple data sources such as: the improvement of students' understanding about randomness and probability through a pretest and a posttest; the changes of concept maps either by adding new concepts, new links, and examples, or by changing the old concepts, links, and examples; and changes in the way students explain their answers on why-questions in a pretest and a posttest. In addition, the questionnaire data on computers gives answers on how useful the computer programs are for learning probability and randomness.

HYPOTHESIS

It was hypothesized that students would experience conceptual change after using computer simulation programs, that students' understanding of probability and randomness would improve, and that the computer programs would enrich students' understanding of randomness and probability.

Specifically, this study would answer the question: "Did students' knowledge of probability and randomness improve?" It was hypothesized that students would improve their knowledge after they used computer simulations. There would be a significant difference between students' scores on the pre and posttest.

This study also investigated the questions "What conceptual changes did students experience and how did students change their ideas of randomness and probability?" It was hypothesized that students would experience some conceptual changes. It was also hypothesized that students would change their ideas either by adding new concepts, new propositions, and new examples in their concept map, or by changing their old concepts, links, and examples. Students also would change their ideas by replacing their reasons on posttest.

The study also investigates the question "Why did students change their ideas?" It was hypothesized that the reasons were because students had created more concepts, links, and examples that related to probability and randomness. Students also changed their ideas because they thought that their old concepts and reasoning were not appropriate to their new understanding.

Lastly, the study also seeks to answer several questions about computer simulation programs: are the programs helpful for students in understanding the concepts of probability and randomness? Do students like the programs? It was hypothesized that the computer programs were helpful for students to understand probability and randomness. Students would like the programs and in the process they would change their ideas about computers.

THEORETICAL BACKGROUND

The study is based on constructivist theory, conceptual change theory, and schema theory. According to constructivist theory, knowledge is constructed by the learners either individually or socially. The constructivist perceptive has influenced science education by stressing the importance of students' activities in constructing knowledge. According to constructivist theory, learning is a form of conceptual change. Conceptual change can be classified as weak and strong. A strong change happens if there is a reconceptualization. In the strong restructuring, students change their existing concepts. A weak change happens if students develop their old ideas (Carey, in Dykstra et al., 1992). The strong change happens when there is dissatisfaction in students' minds. The dissatisfaction occurs as a result of a conflict between students' old ideas and the new situation (Posner et al., 1982).

The study is also based on schema theory (Rumelhart, 1980, in Jonassen et al., 1993), "knowledge is stored in information packets or schema that comprise our mental constructs of ideas" (p.6). In learning process, students restructure their knowledge by adding schema or developing new conceptualization from the old schema.

The schema are arranged in networks of interrelated concepts (semantic network). Because the semantic network describes structural knowledge, meaning does not exit until some structures are achieved. According to Ausubel, meaningful learning happens if students link the new information to the existing knowledge structure. Because a concept map is used as a means to represent a structural knowledge, the meaningful learning occurs if students reformulate their concept maps (Jonassen et al., 1993).

SUMMARY

This chapter has introduced some studies on fractals as patterns in nature. Most of the models for explaining the growth of fractals as patterns of nature is approached by probability and random events that mimic the natural fractals. Computer simulation programs are used in learning random events and probability. It is believed that the use of computer simulation programs can eliminate misconceptions of randomness and probability and computer simulations can facilitate students' conceptual changes. This study proposes to investigate whether the computer programs really stimulate students' conceptual changes.

Chapter two reviews the literature related to this study including constructivism, conceptual change, misconception, and computer simulations.

CHAPTER 2
REVIEW OF LITERATURE

INTRODUCTION

In the last two decades constructivism has influenced science education, especially in science learning and teaching. Constructivists claim that learning is an active process in which students construct their own knowledge. Constructivists believe teachers should provide activities through which students build their own knowledge. For constructivists, conceptual changes are a measure of how students change and develop their preconceptions into new concepts that are more similar to the experts' concepts. During knowledge construction sometimes students construct misconceptions. However, according to constructivism, students tend to reconstruct their old concepts over time to improve their conceptual framework. In this process, students' misconceptions can be overcome. There are many types of misconceptions in science education, but in this study, the researcher limited himself to two conceptual areas: probability and randomness.

Computer simulations are consistent with constructivist perspectives because they encourage students to actively construct their own knowledge by manipulating variables, collecting data, analyzing data, discussing results, and making conclusions about how the system behaves. Thus, the computer simulations can create the condition for students to develop their conceptual framework.

This chapter examines studies relating to constructivism, conceptual changes, misconceptions, and computer simulations. The first part discusses constructivist epistemology, Piagetian constructivism, two kinds of constructivism, and some implications of constructivist theory to teaching and learning. The second part describes the process of conceptual change and how to trace students' conceptual change. The third part examines some studies of misconceptions, especially misconceptions of probability and randomness, and the last part discusses studies about computers and computer simulations in science and mathematics education.

CONSTRUCTIVISM

Constructivist epistemology

Constructivism is an epistemology that deals with questions about knowledge and knowing. The main questions of epistemology are: what is knowledge, how do we get knowledge, how do we know what we know? Epistemologists also ask about the true nature of knowing by asking what is truth? (Bodner, 1986; Ryan & Cooper, 1992). Constructivist epistemology states that all the knowledge we have is of our own construction (Bettencourt, 1989). Bettencourt maintains that knowledge is never a simple copy of reality. In fact, it is always the result of a construction of reality through the subjects' activities. The subjects construct the cognitive schema, categories, concepts, and structures that are necessary for knowledge (Bettencourt, 1989). Constructivists assert that the only first tools available to a knower are the senses. Thus, an individual interacts with the environment only through seeing, hearing, touching, smelling, and tasting. From these messages individuals build a picture of the world. Therefore, constructivists believe that knowledge resides in individuals. Knowledge cannot be transferred from the teacher's head to the students' heads. Students themselves have to make sense of what is taught by trying to fit it with their experiences (Lorsbach & Tobin, 1992). For a constructivist, knowledge is not deterministic, but a process of becoming.

According to von Glasersfeld (1988), the roots of constructivism lie in Vico's epistemology. In 1711, Vico expressed his epistemology by saying: "God is the artificer of nature, man the god of artifacts." Vico explained that "to know" means "to know how to make it." It means that someone knows a thing only if one can explain of what components it consists. That is why only God alone can know the real world, because only he knows how to make it and from what he created it. However, people can know only what they have constructed (von Glasersfeld, 1988). For Vico, knowledge refers to conceptual structures (construction). It differs from empiricism which claims that knowledge should refer to an external reality.

For von Glasersfeld, radical constructivism is different from traditional constructivism because the former states that knowledge is not always a true representation of a world as it exists prior to being experienced (Bettencourt, 1989). In other words, radical constructivism has abandoned the relationship between knowledge and "reality" as a criterion of truth. Knowledge does not reflect an "objective" ontological reality, but is rather exclusively an ordering and organization of a world constituted by an individual's experience (Bettencourt, 1989).

Staver (1986) tried to understand constructivism through the major debate in the history of epistemology. The main questions of the debate are: "In what realm does the structure of knowledge lie? What is the truth of knowledge?" Thinkers throughout history have maintained that reality consists of an external and internal dimension. External refers to the objective dimension; internal is the subjective dimension. Rationalism states that our knowledge conforms to objects and that truth is the result of logical deduction. Rene Descartes, a prototypical rationalist, said "Cogito ergo sum" ("I think, therefore I am") (Staver, 1986). Empiricists also assume that our knowledge conforms to objects, but they use inductive reasoning from evidence based on experience. Empiricism assumes that all reality is perceived through the senses, and their criterion of truth is verification by experience (Staver, 1986). Thus rationalists emphasize ratio, logical, deductive knowledge; empiricists emphasize experience and inductive knowledge. According to Staver, constructivism is a synthesis of rationalist and empiricist viewpoints, and it indicates the interaction between subjects and objects, between the external and the internal dimension of reality.

According to Matthews (1994), the two major traditions of constructivism are psychological constructivism and sociological constructivism. The psychological constructivist starts with the works of Piaget regarding how a child constructs his/her cognitive knowledge. The psychological constructivist bifurcates into two directions: (a) a more personal, individual, and subjective constructivism such as Piaget's and that of his followers; and (b) the social constructivism of Vygotsky. Piaget stressed the individual activities in construction of knowledge; Vygotsky, the importance of language communities for the cognitive constructions of individuals. Sociological constructivism claims that scientific knowledge is socially constructed. It emphasizes the circumstances and dynamics of science's construction. Sociological constructivists focus upon the extra individual social circumstances that determine individuals' beliefs (Matthews, 1994).

Osborne (1993) and Matthews (1994) explained that constructivism contains a danger of leading to empiricism and relativism, especially in science education. Some constructivists in science education emphasize experience and sensation which could lead to empiricism, where all concepts should be based on an external reality. Some constructivists emphasize abstraction or the construction which could lead to relativism where all concepts are equally valid because any idea which is derived by abstraction must be considered as legitimate. Constructivist epistemology has no basis for judging the merits of competing abstractions (Matthews, 1994; Osborne, 1993).

Piagetian constructivism

Piaget is one of the first psychologists who applied constructivist epistemology to the learning process. For Piaget, knowing was an intellectual adaptation process in which new experiences and ideas interact with what learners already know to produce new cognitive structures (Shymansky, 1992; von Glasersfeld, 1988). According to Piaget, there are prior knowledge structures in people's minds (schemata). Each schema acts as a filter and a facilitator of new ideas and experiences. Schemata order, coordinate and synthesize principles. The consciousness of self and its differentiation from the external dimension rests upon constructive activity by the knower. For Piaget, reality becomes the phenomena we experience through construction. Through contact with new experiences, the schema can be changed or developed. So, learning involves conceptual change. According to Piaget, schema develops during intellectual growth, especially during the formal operational stage. So, for Piaget, reality is neither external and ready-made, nor innate and predetermined, but is acquired through the constructive activity of an individual who produces new mental schemata (Staver, 1986).

In the theory of cognitive development, Piaget used assimilation and accommodation processes to explain how concept or schemata change (Wadsworth, 1989). Wadsworth described Piaget's assimilation and accommodation as:

> Assimilation is the cognitive process by which a person integrates new perceptual, motor, or conceptual matter into existing schemata or patterns of behavior... Assimilation theoretically does not result in change of schemata, but it does affect the growth of schemata and is thus a part of development (Wadsworth, 1989, p.13).

According to Piaget, sometimes students would create a new schema or modify existing schema. The process of schemata change is called accommodation (Wadsworth, 1989).

According to Piaget, "equilibrium is a state of balance between assimilation and accommodation. Disequilibrium is a state of imbalance between assimilation and accommodation. Equilibration is the process of moving from disequilibrium to equilibrium. It is a self-regulatory process whose tools are assimilation and accommodation" (Wadsworth, 1989, p.16). Equilibration allows external experience to be incorporated into internal structures (schemata). If disequilibrium occurs, it provides motivation for a child to seek equilibrium through further assimilating or accommodating. In assimilation, the organism fits stimuli into existing schemata. In accommodation, the organism changes schemata to fits the stimulus. Equilibration is the internal mechanism that regulates those processes (Wadsworth, 1989).

According to Matthews (1994), Piaget's constructivism is more personal and individual. It stresses how an individual constructs his or her knowledge by activity in the world. O'Loughlin (1992) criticized Piaget's constructivism as being too subjective, as not sufficiently sociocultural. Von Glasersfeld said that in the Piagetian definition of knowledge, the subject's experience always includes social interaction with other knowing subjects, something that is especially important in education (von Glasersfeld, 1988). Piaget (1981) in The Psychology of Intelligence described social factors in students' intellectual development. Before the level of concrete operation the social environment is not essentially distinct from the physical environment, but at the stage of concrete operation and particularly of formal operation, there is a role of the social environment for students' intelligence.

> At the stage at which grouping of concrete operations and particularly when those of formal operations are constructed, ...the problem of the respective roles of social interaction and individual structures in the development of thought arises in all its acuteness... Considered from this angle, logic requires common rules or norms; it is morality of thinking imposed and sanctioned by others. Thus, <u>the obligation not to contradict oneself is not simply a conditional necessity</u>... for anybody who accepts the exigencies of operational activity; <u>It is also a moral "categorial" imperative</u>, <u>inasmuch as it is indispensable for intellectual interaction and co-operation</u>. Indeed, the child first seeks to avoid contradicting himself when he is in the presence of others. In the same way, objectively, the need for verification, the need for words and ideas to keep their meaning constant, etc. are <u>as much social obligations</u> as conditions of operational thought (Piaget, 1981, p.162-163) [italics added].

Socioculturalism and sociological constructivism

Inspired by the work of Vygotsky, socioculturalism pays attention to cultural practices in the learners' milieu and emphasizes the socially and culturally situated nature of activity (Bereiter, 1994). According to socioculturalism, individual activity is profoundly influenced by participation in encompassing cultural practices -- such as completing worksheets in school, shopping in supermarkets, selling candy on the street, and the like (Cobb, 1994). Minick, in Cobb (1994) questioned whether mathematical learning is primarily a process of active cognitive reorganization or a process of inculturation into a community of practice. To Minick, socioculturalism examines the individual in his or her social action as its unit of analysis. Socioculturalists use the individual's participation in culturally organized practices and face-to-face interaction as a primary explanatory construct. Sociocultural theorists view classroom interaction as an example of the culturally organized practices of schooling (Cobb, 1994).

Cobern (1991) stated that constructivism is contextual. Students always construct their knowledge in a special situation and context. If the context is different, students will understand the concepts differently. In discussing mathematical development, Cobb suggested that constructivism should be combined with sociocultural perspectives. For Cobb, the two perspectives are complementary, that is, mathematical learning should be viewed as both a process of active individual construction and a process of inculturation into the mathematical practices of wider society (Cobb, 1994).

Sociological constructivism is different from the constructivism of Piaget and the socioculturalism of Vygotsky. Sociological constructivists maintain that scientific knowledge is socially constructed and vindicated. The circumstances and dynamic of science's construction are very important. Sociological constructivism ignores the individual psychological mechanism of belief construction. Sociologists focus upon the social circumstances which determine individual beliefs. "Extreme sociological constructivism claims that science is nothing but a form of human cognitive construction comparable to artistic or literary construction, and having no particular claim to truth" (Matthews, 1994, p.138).

Implication of constructivist theory on education

- **On learning**

It is clear that for constructivists learning is an active process, by which meaning is constructed. It is a process of fitting new ideas and experience into an existing conceptual framework

(Bettencourt, 1989; Shymansky, 1992; Watts & Pope, 1989). According to constructivists, learners are responsible for their own learning. They bring their prior conceptions to learning situations. They make sense of what is learned by negotiating meaning, comparing what is known to the new experience, and resolving discrepancies between what is known and what seems to be implied by new experience. Individuals or groups can negotiate meaning. Because knowledge is personally and socially constructed, group study should be encouraged (Shymansky, 1992; Watts & Pope, 1989). According to Bodner (1986), individuals' knowledge must be viable and must be tested against reality. If it fails the reality test, it must be revised.

For Driver et al. (1994), social constructivism recognizes that learning involves being introduced to a symbolic world. Knowledge and understanding are constructed when an individual engages socially in dialogue and activity about shared experiments or tasks. Thus, meaning-making is a dialog process involving persons in conversation. Learning is seen as the process by which individuals are introduced to a culture by more skilled members. In this sense, learners need access not only to physical experiences but also to the concepts and models of conventional science. Here the teacher's role is important, for the teacher provides appropriate experiential opportunities and makes the cultural tools and conventions of the scientific community available to students. So, learning science involves both personal and social processes (Driver et al., 1994).

• **On teaching**

From the constructivist perspective, teaching is not the transmission of knowledge, but an activity that enables students to construct their knowledge. Thus lecturing should not be the primary practice of teachers, but rather they should provide learning experiences that allow students to take responsibility for the design, process, and outcomes of the investigation. Teachers should evaluate students' hypotheses and conclusions, generate students' discussion, and encourage students to express their ideas. Teachers can assign activities that stimulate curiosity among students, give them opportunities for developing basic science concepts, and help them to work together and communicate their scientific ideas (Watts & Pope, 1989).

According to Driver and Oldham in Matthews (1994), constructivist teaching is characterized by a number of stages:

1. Orientation, in which students are given the opportunity to develop a sense of purpose and motivation for learning the topic;
2. Elicitation, during which students make their current ideas on the topic clear. It is achieved by group discussions, writing, posters, etc.;

3. Restructuring of ideas: (a) clarification and exchange of ideas by contrasting them with the ideas of others, (b) construction of new ideas in the light of the above discussion and demonstration, and (c) evaluation of new ideas experimentally;

4. Application of ideas, with opportunity to use and develop ideas in a variety of situations;

5. Review: how the ideas have changed.

CONCEPTUAL CHANGE

Conceptual change process

Kuhn, in The Structure of Scientific Revolutions, stated that science was more characterized by scientists' paradigms than by their methods of inquiry. A paradigm is a conceptual scheme through which scientists in a given discipline view problems in their field. A researchable problem and methods which will be used to solve the problem are primarily determined by scientists' relevant paradigm. Historically, the paradigms used by scientists have changed. According to Kuhn, the change is comparatively abrupt. He described two kinds of scientific activities: puzzle solving and discovering new paradigms. In puzzle solving, scientists conduct experiments and make observations. When the paradigm fails to apply to an important group of problems, or there is conflicting result, a new paradigm must be invented. New paradigms are necessary for any revolutionary ideas in the sciences (Kuhn, 1970; Novak, 1977).

Toulmin described human understanding through the concept of evolution. The most important part of human understanding is the evolutionary development of concepts, not fixed concepts. In the development of concepts, individuals change their ideas. He wrote:

> A man demonstrates his rationality, not by a commitment to fixed ideas, stereotyped procedures, or immutable concepts, but by the manner in which, and the occasions on which, he changes those ideas, procedures, and concepts (Novak, 1977, p.47).

As did Kuhn, so Posner, Strike, Hewson, and Gertzog (1982) explained two distinguishable phases of conceptual change in the philosophy of science: (1) central commitments, and (2) the central commitments in need of modification. In the central commitments, scientists define problems, strategies to deal with the problems, and specify criteria for solutions. In the second phase, scientists need to modify the central commitments, which they must do if they face a challenge to their basic assumptions.

According to Posner et al. (1982), in learning there are analogous processes of conceptual change. The first phase of conceptual change is called assimilation, the second phase, accommodation. In assimilation, students use existing concepts to deal with new phenomena; in accommodation, students have to replace and change their central concepts. Here, Posner et al. used Piaget's terms but did not commit themselves to Piaget's theory.

Accommodation (the radical conceptual change) needs some conditions in order to take place (Posner et al., 1982):

1. There must be dissatisfaction with the existing conception. Students change their concepts if they believe that their existing concepts no longer work in dealing with new situation, experiences, or phenomena.

2. The new concept must be intelligible. Students are able to understand how the new experiences can be structured by the new concepts.

3. The new concept must be plausible, i.e., have the capacity to solve problems generated by its predecessors, and be consistent with other theories or knowledge and with the past experience.

4. The new concept should be fruitful, useful for research programs, and have the potential to extend and to open new inquiry.

According to Posner et al. (1982), the major source of dissatisfaction with the existing conception is the anomaly. It happens when students cannot assimilate something or cannot make sense of something. When students experience an anomaly, they can revise or change their concepts to avoid conflict.

Many science educators have used anomalous data to promote a theory of change (Chinn, 1993). They provide experiments or experience that give contrary data from students' prediction or understanding. For example, for students who think that all solid things have higher density than liquid, science teachers provide some solids having lower density than liquid. Anomalous data have also played a central role in conceptual change in the history of science (Kuhn, in Chinn, 1993). However, anomalous data frequently fail to promote conceptual change, because scientists and students often find ways to discount the data (Chinn, 1993).

According to Chinn (1993), people may respond to anomalous data by ignoring or rejecting, or by excluding the data from the existing theory, or by reinterpreting the data, reinterpreting the data with peripheral changes on the existing theory, or by accepting the data and changing the theory. The response to the anomalous data is determined by the individual's current beliefs, the characteristics of the alternative theory, the characteristics of the anomalous data, and the individual's processing strategies. There should be a plausible alternative theory which is able to explain new anomalous data. People are likely to reject the alternative theory if there does not appear to be a plausible mechanism. The quality of the alternative theory should be better than the current theory. The alternative theory should explain a wider scope of evidence, be more accurate than the current theory, and be internally consistent. According to Chinn, the anomalous data that influences conceptual change should be credible. Replicating the data, allowing people to observe the data directly, using the data that people already know, will make the data more credible.

Carey described two kinds of knowledge restructuring: weak and strong. The weak restructuring does not change the concept, but the strong restructuring does (Dykstra et al., 1992). For example, changing from the concept "probability has no relation to randomness" to the concept "probability can be used to predict some random phenomena" is a strong conceptual change. Changing concept from "randomness is unpredictable" to "randomness is unpredictable and has no precision" is an example of weak conceptual change. Some researchers emphasize the instructional strategies which may induce conceptual change. They encourage students to question their beliefs and to use conceptual maps in instruction. They provide strategy that causes disequilibrium in the students' minds. Disequilibrium can happen if students experience conflict between their own beliefs and the new experiences (Dykstra et al., 1992).

Dykstra, Boyle, and Monarch (1992) organized conceptual change into a non-hierarchical taxonomy. The categories in this taxonomy are characterized by unique changes in the representation from preconception to post conception. According to Dykstra et al., there are three types of conceptual change: differentiation, class extension, and reconceptualization. Differentiation happens when new concepts emerge from existing concepts which are more general. For example, velocity and acceleration emerge from general ideas of motion. Class extension occurs when the existing concepts considered different are found to be cases of one subsuming concept. For example, rest and constant velocity are viewed as equivalent from the Newtonian point-of-view. And reconceptualization takes place when a significant change in the nature and relationship between concepts occurs. For example, "Force implies motion" becomes "Force implies acceleration" (Dykstra et al., 1992).

Howe (1993) described a model of conceptual change which has influenced science education. The model indicates that students come to school with misconceptions. These misconceptions need to be elicited and challenged by explaining or demonstrating contrary examples, and corrected by providing more general concepts which students will accept and assimilate. This model creates conditions for the misconceptions to be refuted. According to Vygotsky, students have spontaneous concepts and scientific concepts which continuously interrelate. Spontaneous concepts are achieved through everyday life, and the scientific concepts are achieved through school instructions. What students are learning in schools influences the development of concepts acquired through everyday experiences, and vice versa. The crucial difference between the two categories of concepts is the presence and absence of a system. Spontaneous concepts are based on particular instances and are not part of a coherent system of thought, but scientific concepts are presented as a part of a system of relationships (Howe, 1993; Newman & Holzman, 1993; van der Veer & Valsiner, 1991). Using Vygotsky's theory, Howe suggested that teachers or educators might rethink the labeling of students' spontaneous concepts as misconceptions. They should not refute the spontaneous concepts, but should help students to integrate their preconceptions into more systematic framework. According to Howe, it is not a misconception that the earth is flat because within the child's experience the earth is flat. Concept development cannot happen in weeks or even months. It takes a long time to develop.

How to trace conceptual change

How can we trace and measure conceptual change? Many researchers used conceptual maps to investigate the conceptual change of students (Dykstra et al., 1992; Markham et al., 1994; Novak, 1990; Novak & Gowin, 1984; Pendley & Bretz, 1994; Schreiber & Abegg, 1991; Wallace & Mintzes, 1990). According to Novak and Gowin (1984) in Learning How to Learn, a concept map is a schematic device for representing a set of conceptual meanings embedded in a framework of propositions. Conceptual maps explicitly illustrate students' concepts. They represent meaningful relationships between concepts, and consist of concepts and propositions. A proposition consists of two or more concept-labels linked by words in a semantic unit. It provides a visual road map showing some of the pathways to concept-meaning. A concept map is constructed hierarchically from general concepts to specific concepts. By using concept maps, researchers can identify misconceptions such as concepts that miss links or miss ideas. Figure 1. is an example of a student concept map on fractals. "Fractals", "natural", "mathematical", "randomness", "probability", "fractal dimension", "self-similarity", "iterations", "finite", "diffusion", etc. are concepts or concepts' labels. "Is", "is example of", "are made by", "can be", "cause", "show", "have", etc., are links which connect a concept with another concept. Links are also indicated by arrows (--->). "Fractals can be natural", "fractals show self-similarity", "mathematical fractals are not finite", etc. are propositions. "Natural fractals" and "mathematical fractals" are specific concepts and "fractals" is a general concept. So, "fractals" is constructed in the upper level, and "natural" and "mathematical" are in the lower level.

From the fractal concept map (Figure 1) we are able to identify what fractals are and how a student constructs his/her cognitive framework about fractals. According to the student, fractals consist of natural fractals and mathematical fractals. Julia set, Mandelbrot

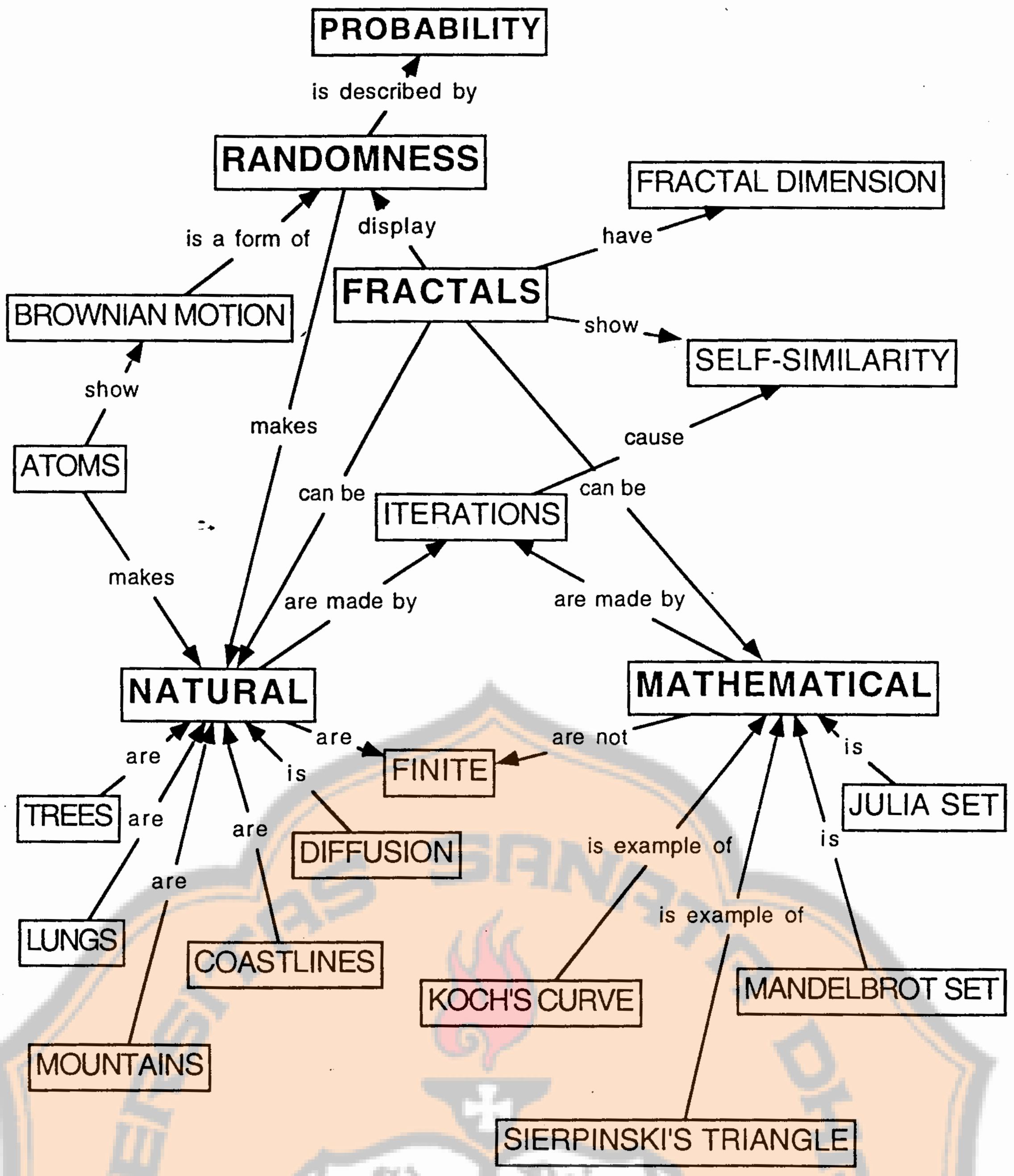

Figure 1. A high school student's concept map on fractals.

set, Koch's curve, and Sierpinski's triangle are examples of mathematical fractals; and tress, lungs, coastlines, diffusion, and mountains are examples of natural fractals. Fractals display randomness and fractals have self-similarity characteristics. Natural fractals are finite, but mathematical fractals are not. Both natural and mathematical fractals are made by iterations.

Researchers compare the concept maps of students before and after instruction or students' concept maps during instructional activities to look for differences, development, and changes in their ideas. Then, they analyze the concept maps quantitatively or qualitatively. Wallace and Mintzes in their study, prepared a set of concept labels and asked students to develop a pre-instruction concept map for that topic. They used the same questions for the pretest and posttest. Wallace and Mintzes scored the concept map according to Novak's suggestions and used statistical methods for analysis. They also interviewed students based on their concept maps (Wallace & Mintzes, 1990).

Shymansky et al. (1993) used concept maps and interviews in studying changes of students' understanding of force, motion, work, and energy. The sample was students enrolled in a ten week level-10 physics course at a public school in Western Australia. They collected four concept maps from students: one at the start of instruction, two concept maps during the instruction, and one in one month after formal instruction. Students were allowed to use the course outline as a resource in constructing their initial concept map. At a new collection point, students were given the original of their previous map and encouraged to use the map as a reference point for making a revised map or an entirely new one. Prior to the interviews, copies of student maps were distributed to interviewers to facilitate planning. An outline of target concepts based on the course outline was drawn by interviewers. The outline served as a guide to allow students to explain and expand their maps. They found that students experienced conceptual growth in kinematics and energy, but not in Newton's laws (Shymansky et al., 1993).

According to Jonassen, Beissner, and Yacci (1993), concept maps stem from the work of Ausubel. According to Ausubel's assimilation theory, learning becomes meaningful only when it occurs in the context of the learner's prior knowledge. Meaningful learning happens when learners try to link new information into their existing knowledge structures. It is accomplished through the learning concepts, propositions, and reformulation of their previous concepts. Ausubel believed that knowledge structures are organized hierarchically with more inclusive concepts subsuming more detailed concepts. The concepts are defined through propositions, which identify relationships between concepts. The interrelated networks of concepts and propositions are essential elements of human learning. Concept maps are an explicit representation of these integrated knowledge networks (Jonassen et al., 1993).

A concept map is a means for conveying structural knowledge. Structural knowledge is a knowledge of how concepts within a domain are interrelated (Diekhoff, 1983). According to Jonassen et al. (1993), the structural knowledge mediates the translation of the declarative into procedural knowledge. According to Ryle, declarative knowledge represents a cognition or awareness of some objects, events, or ideas. It is a knowing that knowledge. Learners are able to describe what they know, but they do not apply it. Procedural knowledge describes how learners use their declarative knowledge. It is a knowing how knowledge. It is not enough to "know that" in order to "know how." We have to know why. This is the role of structural knowledge. It provides the conceptual bases for why by describing how the declarative knowledge is interconnected (Jonassen et al., 1993).

Jonassen et al. (1993) explained that structural knowledge is based on schema theory. According to the schema theory, knowledge is stored in information packets, or schemata, that comprise our mental construct for ideas. "A schema for an object, event, or ideas is comprised of a set of attributes or slots that describe and therefore help us to recognize that object or event" (Rumelhart, 1980, in Jonassen et al., 1993, p.6). These slots contain relationships to other schemata. It is the relationship between schemata that gives meaning to whole ideas. Learners fill slots by encoding their experiences as attributes of schemata. The process can also result in the addition of new schemata. When new experiences cannot be described by existing schemata, the schemata are restructured. Jonassen et al. (1993) described that a basic premise of schema theory is that human memory is analyzed semantically. Schemata are arranged in networks of interrelated concepts. These networks are known as our semantic networks (Quillian, 1968, in Jonassen et al., 1993). These networks describe what a learner knows and provide the foundation for learning new ideas, and altering and expanding the existing semantic networks.

Structural knowledge theory has limitations. Clancy (1992) said that the semantic network theory is based on traditional conceptions of memory. It assumes that human memory consists of stored facts and procedures. Semantic network theorists see nets as isomorphic representations of what is stored in memory. Memory structures exist as representations in the mind. According to the theory, the semantic storing of information in humans can be cognitively mapped. Clancy said that is not the case. Memory is not static but changes according to the context in which the events occurred (Jonassen et al., 1993).

Jonassen et al. (1993) explained that structural knowledge is a hypothetical construct that is important in understanding human information processing and describing how individuals construct and store knowledge. The structural tools such as concept maps are technically used for representing the current state of the learners' knowledge. But those states are dynamic. They change with different contexts imposed on their creation. As tools for reflecting on current knowledge structures, they are constructive. They generate representations of semantic networks. But the net is not encoded in memory at all (Jonassen et al., 1993). It is clear that these tools and their resulting nets are contextually driven. Most of the tools will result in different products each time they are used in different social contexts.

Concept maps used as a means of representing structural knowledge can serve as the basis for assessing differences between learners' and experts' structural knowledge. Thus, concept maps can be used to look for conceptual changes in structural knowledge. Concept maps can be used as teaching tools as pre and post instructional summaries of the content area. Most studies of concept maps find that they are helpful as learning and teaching tools. As research tools, concept maps have disadvantages. Generating good maps is a time-consuming process. It is sometimes difficult to interpret concept maps because they have multiple lines and labels (Jonassen et al., 1993). <u>C-map computer programs</u> built by Cornell

University can reduce the time-consuming problem. C-map is a computer program for constructing concept maps. The program is easy to use by students. Using C-map students can construct clear and neat concept maps faster than manually.

MISCONCEPTIONS/ALTERNATIVE CONCEPTIONS

Probability and randomness concepts

Random events are occurrences that are unconnected. They do not influence one another, and are independent and unpredictable (Compagner, 1991); they are not guided in a special direction, and are not effected by any causal laws (McShane, 1970; Svozil, 1993). The principal characteristic of these random events is randomness. For example, every number in rolling a die has the same chance, and they do not influence each other; they are random.

Historically, Maistrov (1974) described the development of probability theory as consisting of five stages: prehistory, the origins of probability as a science, the appearance of James Bernoulli, the Russian school, and the modern period (Maistrov, 1974). It was not clear when the probability notion began. But even before Christ, the elements of probability such as calculation, prediction for uncertain experiments like gambling have been found.

This stage was concluded by the work of Cardano in his book of games Liber de Ludo Aleae, The Book on Games of Chance. Cardano defined the concept of probability of an event in repeated throws of an ideal die as the ratio of the favorable outcomes to the possible ones (Maistrov, 1974; Rao, 1987).

During the second period (17-18 century), the concept of mathematical expectation was developed and addition and multiplication theorems of probability were established. For the first time, probability theory was used in demography, in the insurance business, and in the estimation of observation errors. The third stage was dominated by the work of Bernoulli in his 1713 book Ars Conjectandi, The Art of Conjecturing, in which he proved a limit theorem pertaining to the law of larger numbers. He also discussed permutation, combinations, and variations of repetitions. He was famous for his Binomial distribution theorem that applied probability theory to statistics. In this period the first applications of probability to various areas of natural sciences were initiated.

In the Russian school period, the extensions of the laws of larger numbers and central limit theorem were used in various classes of random variables. The probability theorem was used in dependent random variables. So, probability theory could be applied to many branches of natural sciences. In this time statistical physics developed. Axiomatization was the principle change

CHAPTER 1
INTRODUCTION

BACKGROUND OF THE PROBLEM

In the last decade there have been many scientific studies on fractals as patterns in nature (Fujikawa & Matsushita, 1990; Matsushita, Sano, Hayakawa, Honjo, & Sawada, 1984; May & Maher, 1989). Fujikawa and Matsushita (1990) studied bacterial fractal growth in the concentration field of nutrient; Matsushita et al. (1984) investigated the fractal structures of zinc metal grown by electrodeposition; and May and Maher (1989) measured the fractal dimension of a radial fingering pattern in a smooth Hele-Shaw cell. Fractals are self-similar objects which appear unchanged after increasing or shrinking their size (Schrouder, 1991). Devaney (1990) described a fractal as a geometric shape that has two special properties: self-similarity and fractional dimension. Self-similarity means that if "we take a part of a fractal and magnify it by the same magnification factor in all directions, the magnified picture cannot be distinguished from the original". (Bunde & Havlin, 1994, p.4). Crownover (1995) explained a repetition pattern in nature as an example of self-similarity: "A tree has branches. These branches have smaller branches, and so on. Theoretically the pattern of branches is repeated infinitely many times at ever smaller scales" (p.2). Many objects have integer dimensions such as a line having one initiated during the modern period. The axiomatization made probability theory more abstract and became deductive mathematical discipline close to the set theory and other branches of mathematics. The axiomatization is to make an orderly structure for basic notions of probability, because probability is used in many disciplines such as physics, biology, chemistry, engineering, etc. (Maistrov, 1974).

Probability can be interpreted from three different perspectives: classical interpretation, frequentist interpretation, and subjective interpretation (Konold, 1988b). According to the classical interpretation, the probability of an event is equal to the ratio of the number of alternatives favorable to that event to the total number of equally likely alternatives. For example, the probability of rolling a #2 with a die is 1/6. It is sometimes called theoretical probability. John Venn defined the probability in terms of long-term frequencies because if we roll the die as long as possible, the its probability will be 1/6 (Garnham & Oakhill, 1994). This notion of probability was first systematically described by G. Cardano in the sixteenth century in his book Liber de Ludo Aleau (The Book of Games of Chance) (Maistrov, 1974).

According to frequentist interpretation, probability is defined as the relative frequency of occurrence of an event in an infinite or near infinite number of trials. For example, the probability of rolling a #3 with dice is the ratio of trials resulting in a #3 to the total number of trials. The frequency interpretation sometimes is called empirical probability. John Maynard Keynes called the probability a comparison with all other possibilities (Garnham & Oakhill, 1994). According to subjectivist interpretation, probability is a measure of the belief in the truth of a proposition. Different people could validly assign different values to the probability of rolling a #3 with a particular die (Konold, 1988b). The subjectivist interpretation can be traced to early work of Pierre de Laplace: Bayesian statistics. "Bayesians take a judgment about the probability of an event to represent the degree of belief of an 'ideal' person, whose beliefs are perfectly consistent, that the event will occur. The degree of belief may, indeed, be completely subjective, as long as they obey certain rules" (Garnham & Oakhill, 1994, p.155).

The three views of probability agree on the fundamental mathematical properties of probabilities, such as (Garnham & Oakhill, 1994):

1. A probability is a number in the range of 0 to 1. Zero is impossible and 1 is certain.

2. The sum of the probabilities of all the possible events in a given situation is 1.

3. The probability of one or other events in mutually exclusive events in the same situation is the sum of the probabilities of the events. For example in a fair die; the probability of a five or a six is

equal to the probability of a five plus the probability of a six. It is equal to (1/6 + 1/6) = 1/3.

4. The probability of two independent events is equal to the product of each probability.

5. The probability of an event A given B (conditional probability), is probability of both A and B happening divided by the probability of B happening. Mathematically it is formulated as

$p(A|B) = p(A \text{ and } B) / p(B)$.

Misconceptions in sciences and mathematics

Many researchers prefer to use a term "alternative conception" than "misconception" because: (1) "alternative conception" refers to experience-based explanations constructed by students; (2) the term gives intellectual respect to students who have those ideas; (3) sometimes alternative conceptions are contextually rational and work to explain some phenomena. Many researchers prefer to use a term "misconception" because (1) the term already has a meaning to lay persons; (2) in science education the term readily carries the concept of an idea at variance with current scientific thought; (3) the term is easily understood by both practicing teachers and the general public (Wandersee, Mintzes, and Novak, 1994).

Many researchers find that students have misconceptions before they receive instructions in formal settings. Misconceptions are found in all areas of sciences including physics (Brown, 1987; Clement, 1987; Gilbert et al., 1982; Mohapatra, 1988), biology (Marek et al., 1994), chemistry (Pendley & Bretz, 1994), and astronomy (Comins, 1993).

For example, some students misconceive Newton's third law. They consider force as an innate property of objects rather than what arises from an interaction between objects (Brown, 1989). Thus, students believe that heavier objects will fall faster than lighter ones and that a heavier object has stronger force than a lighter one. Some students view force as a push or pull that must be exerted by muscular action. For them, if objects do not move, they have no force acting upon them (Arons, 1981). Some individuals believe that uniform motion only occurs if the acceleration is constant (Ivowi, 1984). Mohapatra (1988) found that many students in India have misconceptions regarding the second law of light reflection. They think that the equality between the angle of incidence and reflection happens only on a plane mirror (Mohapatra, 1988). Nussbaum and Novak in 1976 found that some second-grade students understood that the earth is flat. Stavy and Wax in 1989 found that only about half of 11-12 old Israeli students considered plants to be alive and reproduce. Anderson found that some 12-16 years old Swedish students thought that atoms have different shapes: e.g., square or rectangular (Wandersee, Mintzes, and Novak, 1994).

Misconceptions or preconceptions also have been found to exist in mathematics and statistics (Baturo, 1993; Green, 1983; Hirsch & O'Donnell, 1993; Konold, 1988a, 1988b; Shaughnessy, 1983). Research in mathematics and psychology indicates that students have already established beliefs about chance before they get formal instruction on statistics and probability (Green, 1983; Shaughnéssy, 1983, 1994). Some students make heuristic judgments that develop misconceptions on probability (Green, 1983; Shaughnessy, 1983; Tversky & Kahneman, 1982a). According to Konold, students commonly believe that the successive outcomes of a random process are not independent (Konold, 1988a). For example, after getting heads (H) four times in flipping a coin, students believe that the next outcome will be tails (T). Some students conclude falsely that many people smoke, because only people whom they have met come to mind. Terms such as, possible, impossible, probable, and certain mean different things to different students (Fischbein, Nello, & Marino, 1991; Shaughnessy, 1983).

Green (1983) investigated the intuitions of chance and the concepts of probability among 11-16 year old English school pupils. He also established patterns of development of probability concepts and related these to other mathematical concepts. He used the Probability Concepts Test and the Test of General Reasoning Ability. The probability concept test consisted of questions with multiple choices and some "why" questions. His tasks included tree diagrams, visual representations of randomness, spinners with area models of probability, and questions on the language of probability (Shaughnessy, 1994). Green found that: (1) ratio concepts are very poor; (2) items involving an appreciation of randomness, the stability of frequencies, and inference are particularly poorly done; (3) pupils' verbal ability is often inadequate for accurately describing probabilistic situations. Very often students describe "certainty" and "high probability" as the same and "impossibility" as the same as "low probability" (Green, 1983). To improve students' conceptions of chance, randomness, and probability, Green emphasized experimental activities and explicit classroom discussion (Shaughnessy, 1994).

Smock and Belovicz (1968) studied junior high school students for their understanding of probability theory concepts. They specifically looked for variation and consistency in usage of quantitative language terms, the ability to generate probable combinations and permutations, and the recognition and utilization of the concepts of independence. They concluded that students have some kind of strategies to guess but generally students gave many indications of not comprehending the nature of the task or the principles involved.

Baturo (1993) analyzed misconceptions of probability among students of grade-12 in Australia. The purpose of her study was to:

(1) explore students' cognitive functioning in probability; (2) examine students' intuitive, concrete, computational and conceptual knowledge of probability; and (3) identify relationships between these knowledge types and students' knowledge in the domain of fractions and ratios. She used clinical interviews as methodology. Baturo found that students had a variety of cognitive schemata in doing probability problems such as fractions, ratios, and comparisons. Students who used a fraction schema (part/whole) had better results compared to students who used "part/part" or "whole/whole" schemata. There was a link between probability competence and fraction competence. Students did not see the purpose in learning probability, did not really enjoy or understand the probability lesson they had. Some students had misconceptions such as "P=2 was legitimate probability."

The reasons for misconceptions

Misconceptions originate from many sources. Misconceptions in physics come from everyday language which has different meanings from the language of physics (Arons, 1981; Gilbert et al., 1982). For example, in daily life, motion and rest are different, but in physics, motion and rest can be equivalent, e.g., the concept of inertia. Gilbert et al. (1982) explained that students sometimes consider objects anthropocentrically. If they see a picture of a man sitting still on a steadily moving bicycle, they think there is no force, because the man is not pedaling. Brown (1987) discovered that student misconceptions of Newton's Third Law come from their naive view (non scientific view). For example, students thought that force is a property of a single object. Stavy (1991) indicated that misconceptions often come from students' experiences that are inappropriate or contradict the concepts of physics.

Some researchers discovered that some misconceptions come from textbooks (Iona, 1987; Ivowi, 1984; Renner, 1990). Iona and Ivowi found that many physics textbooks teach misconceptions, contain errors, and have internal inconsistencies. Iona gave an example from a high school book that describes kinetic energy as a negative energy.

> In analyzing the motion of a falling object the authors find, due to erroneous interpretation of the conditions, that the object "<u>has gained an amount of kinetic energy equal to -1/2 mv</u>^{2}," which does not disturb them. They explain: the negative sign indicates that the motion of the object is in the downward direction (Iona, 1987, p.303).

The use of analogies during teaching aimed at overcoming some obstacles seems to provide a source for new misconceptions (Dupin et al., 1987). For example, Dupin noted that teachers compared electricity to a flowing fluid. This good explanation can eliminate some problems, but it misrepresents voltage. Some researches also found that some misconceptions originate in the teachers themselves (Arons, 1981; Iona, 1987; Ivowi, 1987). Teachers with misconceptions also influence students' misconceptions and make misconceptions difficult to eliminate.

Some researchers in mathematics and psychology find that some misconceptions on probability arise from heuristic thinking (Baturo, 1993; Konold, 1988a, 1988b; Shaughnessy, 1983; Tversky & Kahneman, 1982a, 1982c, 1982d). Tversky and Kahneman described three heuristic judgments that influence people who predict probabilities: representativeness, availability, and adjustment on anchoring.

Representativeness

What is the probability that event A originates from B? In answering this question many people rely on the representativeness thinking. According to representativeness thinking, the probability of A is evaluated by how well A represents B, or how well A resembles B (Tversky & Kahneman, 1982a, 1982c). If A highly represents B, then the probability that A is thought to originate from B is high. Students tend to estimate likelihood based on how similar an event is to the distribution from which it is drawn or how similar the event is to the process by which the outcomes are generated (Shaughnessy, 1983). According to Tversky and Kahneman, representativeness is a directional relation. A sample is more or less representative of a particular population and an act is representative of a person. A relation of representativeness can be defined for (1) a value and a distribution, (2) an instance and a category, (3) a sample and a population, and (4) an effect and a cause (Tversky & Kahneman, 1982a, 1982c). Representativeness is used because it is readily accessible.

Even though representativeness is useful for deciding probability, reliance on representativeness will lead to predictable errors of judgment because the logic of representativeness differs from the logic of probability. Prior happenings or base rate frequency of the outcomes should affect probability, but have no effect on representativeness. Similarity does not depend on the size of the sample, but probability depends on the size of the sample (Tversky & Kahneman, 1982a, 1982c).

One example of representativeness occurs in tossing a coin for heads (H) or tails (T). The sequence HTHTTH is regarded by many students as more likely than sequence HHHHTT, which does not appear to represent the fairness of the coin and randomness. People expect that the essential characteristic of the process will be represented not only globally in the entire sequence, but locally in each of its parts (Tversky & Kahneman, 1982a, 1982c).

Another example happens in the problem below:

Probability of having a baby boy is 1/2. Which of these sequences is more likely occur for having six children?

a. BGGGGB b. BGBGBG c. About the same chance

Some students choose (b) because it seems to be true (Shaughnessy,1983).

Availability

Many people assess the frequency of a class or the probability of an event by the ease with which instances or occurrences can be brought to mind. For example, one may assess the risk of heart attack among middle-aged people by recalling such occurrences among one's acquaintances (Tversky & Kahneman, 1982a, 1982d). Another example, a student is asked to estimate the percentage of people who smoke. It will be a much higher estimation if he or she meets frequently with smokers. This judgmental heuristic is a useful clue for assessing frequency or probability, because instances of large classes are usually reached better and faster than instances of less frequent classes. Availability is affected by factors other than frequency and probability, so reliance on availability could lead to biases. The biases are due to the retrievability of instances, effectiveness of a search set, imaginability, and illusory correlation. Famous instances are more easily retrieved than the other, and are thus interpreted as having higher frequency and probability (Tversky & Kahneman, 1982a, 1982d).

Adjustment on anchoring

In a phenomena called anchoring, people make estimations by starting from an initial value that is adjusted to yield the final answer. Different starting points yield different estimates, which are biased toward the initial values. It is usually employed in numerical prediction when a relevant value is available.

For example:

We have number (1). 8x7x6x5x4x3x2x1 and

(2). 1x2x3x4x5x6x7x8

Ask students to predict which number is higher. Most students choose the first, when in fact they are the same. Adjustments are typically insufficient and usually lead to underestimation (Tversky & Kahneman, 1982a).

Shaughnessy added that some students err because of their deterministic or belief system. Students who do not believe in probability or chance do not care about probability when in schools. They make decisions depending on their beliefs. Many students who do not believe in chance or probability might be influenced by the overemphasis on deterministic models in our schools, such as Newton's laws and deductive axiomatic approach to geometry (Shaughnessy, 1983).

Konold (1988a) studied student beliefs with respect to equally-likely vs. unequally-likely events. One common belief is that successive outcomes of a random process are not independent, the so called "gamblers' fallacies." For example, students believe that after a long run at heads, the next flip will be a tail. In his study of high school students attending a summer mathematics program at Mount Holyoke College, Konold found that most students do not commit the gamblers' fallacy even without instruction. Most students have a correct intuition that all sequences are equally-likely. Erring students believed that a series of random events is governed by a self-correcting process (balancing process). They also believed in the law of larger number; that is, as the sample size increases the number of heads and tails become closer.

In his study on students' beliefs about probability, Konold indicated that before they received formal instructions on probability, students had heard the words "probable", "random", "independent", "chance", "fair", etc.. Statistically, however, these words have different meanings that could lead to misconceptions. Konold (1988b) also discussed how <u>outcome approaches</u> could develop misconceptions. For many people, a question about probability is interpreted from a normative perspective, not a probabilistic one. Thus, the primary goal is to successfully predict the outcome of a single trial. The outcome approach often ignores frequency information.

Fischbein et al. (1991) discussed the origins of misconceptions in terms of linguistic difficulties, lack of logical abilities, and the difficulty of extracting the mathematical structure from practice. If students cannot define some probabilistic terms correctly, they will find it difficult to understand the concept of probability. In this study Fischbein et al. found that many students did not have clear definitions of the terms "probable", "impossible", and "certain." Hawkins and Kapadia (1984) also found that language difficulties cause misconceptions.

Garfield and Ahlgren (1988) identified three reasons why students have difficulties developing correct intuitions about probability. Many students have difficulties with rational number concepts and proportional reasoning which are used in calculating, reporting, and interpreting probabilities. Results reported by the National Assessment of Educational Progress (NAEP) from the second and third mathematics assessments indicated that students were generally weak in rational number concepts and had difficulties with basic concepts involving fractions, decimals, and percents (Carpenter et al., 1981, 1983 in Garfield and Ahlgren, 1988). Probability ideas often appear to conflict with students' experiences and how they view the world (Kapadia, 1985 in Garfield & Ahlgren, 1988). Many students have already developed a distaste for probability through having been exposed to its study in a highly abstract and formal way.

Piaget and Inhelder (1975) found that the concepts of permutation, combination, chance, and probability have not been fully developed before the level of formal operations. Because some high school students are still in the concrete operation level and not yet in the formal operation level, they cannot grasp probability. To Piaget and Inhelder, the concept of probability develops spontaneously at an appropriate stage of development.

Piaget and Inhelder (1975) described three stages of the development of the idea of chance. The <u>first stage</u> occurs before seven or eight years old. It is characterized by the absence of operations, a type of reversible composition. The reasoning at this stage is not logical, but is ruled only by an intuitive regulation. The child does not distinguish the possible from the necessary. There is neither chance nor probability, because there is no system of reference based on deductive operations. The <u>second stage</u> is from seven to eleven or twelve years old. The stage is characterized by the construction of operative groupings by a logical order or by numeral sets, but still related to concrete objects. It is the beginning development of the idea of chance. The <u>third stage</u> occurs after eleven years of age (adolescence). Adolescents think beyond the present and form theories about everything. They think reflexively and become capable of reasoning in a hypothetical-deductive manner. They reason not necessarily based on reality or their beliefs, but on the validity of an inference (Piaget, 1981). Judgment of probability becomes organized in its general aspects by a kind of operations (Piaget and Inhelder, 1975).

Fischbein (1975) disagreed with Piaget. Fischbein's study found that children have an intuition for chance and probability. Pre-school children can distinguish the random in the sense of the unpredictable. In the period of concrete operation, children become able to distinguish between the random and the deducible. The process is not completed during this period, since thinking is still largely tied to the concrete level. The intuition of chance, relative frequency, and probability develop with age as a function of intellectual development and the day to day experiences of the child. The synthesis between the random and the deducible is not realized spontaneously and completely at the level of formal operation. Teaching and experience influence the development of probabilistic intuition. For Fischbein, probabilistic intuition does not develop spontaneously, except within narrow limits. "The understanding, interpretation, evaluation, and prediction of probabilistic phenomena cannot be entrusted to primary intuition which has been neglected and abandoned in a rudimentary state of development. But it must be trained" (Fischbein, 1975, p.131). The cognition regarding probability develops slowly in children. Social mediation and instruction help to develop this cognition (Fischbein, 1975; Hawkins & Kapadia, 1984).

Overcoming misconceptions about probability

Konold stressed encouraging students to recognize conflicts between normative concepts and erroneous intuition and argued that students should be encouraged to evaluate their intuitions by questioning whether their beliefs agree with the beliefs of others, whether their beliefs are internally consistent, and whether their beliefs are compatible with empirical observation (Konold, 1988b). Schroeder (1983) investigated the effect of using microcomputer games to help children in grades 4 to 6 develop and apply concepts of probability. The object of the games is to capture and to hold at the end of the games more places than the opponent. To achieve this goal, students choose moves (captures) so that the probability of recapture by an opponent is minimized. He investigated whether students' strategies improved with experience. The results indicated that some subjects had little difficulty applying probability concepts and explaining their strategies. He concluded that the games are suitable activities for children.

Hirsch & O'Donnell (1993) investigated the effectiveness of several instructional interventions to eliminate student's misconception of representativeness. They suggest that instruction

Interventions should have cognitive dissonance, the experience of contradictory concepts or events; interaction among students; experience conflicts; interaction with materials; and classroom activity. Interventions with the above characteristics worked well in eliminating misconceptions.

Shaughnessy (1977, 1992) designed an experimental activity-based course to overcome misconceptions of college students on judgment heuristics. He developed a series of nine activities in probability, game theory, expected value, and elementary statistics. In the experimental activity-based course, students meet frequently and work in group. The instructional model of activities is: (1) Students make a guess for the outcomes. (2) Students carry on the task such as gathering and organizing data. (3) Students answer questions based on the data only. (4) Students compare the result with their first guessing to confront their misconceptions with experimental evidence. (5) Students build a theoretical probability model that accounts for the experimental data. (6) Students compare the three results: first guessing, data result, and last model.

Students keep a daily journal of experiments, home work problems, and what they learn and feel. Shaughnessy found that many students experienced a conceptual change in their judgment heuristics. According to Shaughnessy, the results suggest that the manner in which students learn probability makes a difference in their ability to overcome misconceptions. Shaughnessy also found that several students did not change their misconceptions. Thus, he concluded that misconceptions are slow to change (Shaughnessy, 1977, 1992).

COMPUTERS AND COMPUTER SIMULATIONS IN SCIENCE

Computers in science & mathematics instruction

From 1980-85 the number of computers used for instruction in American schools increased from one or two thousand to over one million (Becker, in Baird, 1989). The NSTA reported that about one-fourth of all secondary science teachers (about 95,000) used computers in 1985-86. The number of biology, chemistry, and physics teachers who reported using computers was approximately the same. They used computers for drill, tutorials, programming, and word processing. Drill or practice dominated application for which the computers were utilized. During drill, computers provide rehearsal of factual knowledge. In tutorials, they present new knowledge and skills. In both drill and tutorial, students are passive. In programming, however, students play a more active role (Baird, 1989).

According to Baird (1989), there were at least three additional ways in which science teachers used computers. These are Microcomputer-Based Laboratories (MBL), management and teacher utilities, and telecommunications. In microcomputer-based laboratories, science becomes truly interactive. The computer collects data in real time as students conduct an experiment. The students can monitor variables in the same way researchers observe experiments. Science becomes more realistic. Teachers use microcomputers as management tools as in spreadsheets and databases to maintain inventories of equipments and supplies. Word processors are used to maintaining and updating correspondence, tests, handouts, etc. The most interesting new application of computers is in telecommunication. It expands the traditional boundaries of the classroom. Teachers and students can get information from outside schools and learn, take, and share data from other states and nations (Baird, 1989).

Taylor (1987) compared computer-use in physics education through tutorials, demonstrations, modeling tool kits, laboratory aids, and programming. Tutorials guide students through segments of materials to be learned and problems to be solved. In demonstration programs students change parameters and constants. A modeling tool kit is a utility program that mimics various aspects of nature that students use to solve problems. Working with this model allows students to develop an intuition for certain aspects of nature.

Computers are used as laboratory aids to collect and analyze data. Here computers are used as in a microcomputer based laboratory (MBL) setting (Taylor, 1987)

In mathematics curricula, most computers are used for drilling, practices, problem solving, and for playing games (Ediger, 1989). In drills, students repeat what have been taught. In practice, students have opportunities to use the new skills they have acquired. In problem solving, students identify a new problem, gather data, and make and test hypotheses. Games stimulate students' interest in mathematics. Based on those uses, Ediger suggested that computer software should focus on attracting students' interest, provide purpose or reasons to learn, supply sequential content for learners, and give feedback and relevant content for students (Ediger, 1989).

The drill-use of computer in mathematics according to Ediger (1988) is appropriate to the reinforcement theory of Skinner. According to Skinner, a behaviorist, giving reinforcement to students who have finished their job correctly will motivate them in doing their next job. A correct response by students to a program Item increases the possibility of students responding correctly in the future. Computers provide reinforcement when students are doing drill and practice correctly (Ediger, 1988).

The Educational Technology Center of Harvard University (1988) produced a position paper on the future role of technology in science, mathematics, and computer education. They examined the impact of technology-enhanced teaching units in five Massachusetts high schools during 1987. The goal of the units was to help students learn how knowledge is constructed in each subject area. Their approach was to (1) use computers as a tool, not the main focus of attention, (2) choose targets of difficulties for future learning, and (3) involve active teachers in all phases of design and implementation. They found that computers can serve as a stimulus to learning when teachers use them to present dynamic visual images of ideas in a variety of symbolic representations. Effective teachers are still critical to the success of each unit (Baird, 1989).

The office of Technology Assessment (1982) predicted that by 1994 schools will have over four million microcomputers. These machines offer a major potential for changing the way science is taught and learned. For this change to occur, they recommended that teachers should be trained well how to use computers; schools should provide support for the purchase of quality courseware and peripheral devices; and research on the benefit of computers in science learning should be conducted. Full software libraries must be available to institutions that train preservice and inservice teachers (Baird, 1989).

Computer Simulations In Science and Mathematics Education

Teachers' use of computers in probability classes usually follows this format: (1) teachers read the problems; (2) students perform real life trials of experiments, record the data, and draw conclusions; (3) students run computer simulations for a larger number of experiments and compare their computer result with their life trial result; and (4) teachers present the mathematical solution through the discovery approach (Collis, 1982; Walton, 1986; Walton, 1990).

Akbari-Zarin and Gray (1990) found that computers can improve the critical thinking of students. Using five questions that characterize critical thinking, they found that students who used computers scored higher on questions requiring more analysis. Sterling and Gray (1991) discovered that computer simulations affect students' attitudes and understanding in introductory statistics. Students using computer simulations in introductory statistics scored better on examinations and believed that computers were useful for learning statistics.

Zietsman and Hewson (1986) studied the effect of instruction using microcomputer simulations and conceptual change strategies on science learning. The computer program was designed according to a model of conceptual change learning to diagnose and remedy an alternative conception of velocity. They found that microcomputer simulations provide credible representations of reality. There was no difference in the concept of velocity identified using the relative motion of real objects or a microcomputer simulation. The remedial part of the program produced a significant conceptual change in students who previously held the alternative conception.

Borghi, De Ambrosis, Mascheretti, and Massara (1987) used computer simulations and laboratory work in teaching mechanics. They built a strategy for teaching mechanics that included four phases. First, students observed simple experiments that introduced the topic. Second, students watched a film that illustrated the operation and results of more complex experiments which cannot be conducted in a laboratory. Third, students ran a computer simulation to simulate the experiments of the first phase and are allowed to vary parameters. Fourth, students become involved in a laboratory activity. Borghi et al. found that the strategy works for students in a school for teachers. They learned that interactive graphic computers can be very effective in teaching mechanics; and are a particularly effective remedy for some misconceptions on motion. They suggested that computer simulations be combined with experimental works (Borghi et al., 1987).

Choi and Gennaro (1987) observed that computer-simulated experiences are as effective as hands-on experiences. They conclude that computers can be used for simulation, that they are effective, that they could be used for students who are absent, that they could replace expensive experiments, and that they require less instruction time. In addition, teachers do not need to re-explain if students want to rerun computer programs. Zietsman and Hewson (1986) found that microcomputers have at least two capabilities: (1) to simulate the real world and experiences that are difficult to replicate, and (2) to give tests to diagnose student conception. Using computers, students receive feedback about their understanding. Schroeder (1983) used computer games to help children to develop the concept of probability. He concluded that games are suitable activities and that discussion of them is valuable. His results indicate that students develop the concept of probability.

SIGNIFICANCE OF THE LITERATURE REVIEW TO THIS RESEARCH

Constructivist theory provides the underlying theoretical framework for this study. Like Driver et al. (1994), Cobern (1991), and Cobb (1994), the researcher believes that constructivism is personal and social. It means that students construct their knowledge individually but they are also influenced by their situation, their groups, and their environments. Constructivist perspective influenced the research setting and design. Especially in the learning and teaching process, students were allowed to be active learners. They were encouraged to complete hands on activities and run computer simulations. Their teacher acted as a tutor and helped students to find their own understanding. Students worked in small groups within which they discussed their problems and experiments. In the research design, the constructivist theory Influenced how the researcher chose test instruments. Concept maps and "why questions" in pretest and posttest were chosen because the researcher believed they gave students more freedom to reveal their knowledge framework through the concept maps and open ended questions.

The conceptual change ideas such as: (1) students learn if they experience cognitive dissatisfaction, (2) students respond differently to anomalous data, and (3) there are strong and weak changes, are used in the study especially in the data analysis. Those theories help to explain why some students change their concepts and other students do not. These theoretical understandings help to analyze why students change their ideas and how their concepts change.

Novak's suggestion for using concept maps to trace the development of students' conceptual change is applied in the research design. Schema theory supports the use of concept maps for a study such as this one. According to the schema theory, knowledge is stored in schema and a concept map is a representing of these schema. In a meaningful learning, students change or restructure their schema. The changing concept maps indicate the changing schema or concepts. The researcher proposes to analyze students' concept maps qualitatively and quantitatively to determine if students change their concepts and improve their knowledge of probability and randomness.

The findings of misconception studies, especially as they relate to the topic of probability, were used in the data analysis. In this study, multiple choice questions are adapted from the studies by Kahneman and Tversky (1982), Green (1983), Konold (1988), Hirsch (1993), Baturo (1993), Schroeder (1983), Shaughnessy (1983), and Fischbein and Gazit (1984). The researcher selected questions that represented the basic concepts of probability and randomness.

Some studies of computer simulations and statistics find that in certain condition, students improve their scores in examinations and that computer simulations are useful to study statistics. In this research, the researcher proposes that the computer simulations would not only improve students' scores, but also enhance students' conceptual change with respect to probability and randomness.

SUMMARY

The chapter reviewed constructivism, conceptual change model of learning, misconceptions, and the use of computer simulations in science. Constructivist philosophy maintains that meaning is constructed by the learners, either individually or in a social context. The constructivist perceptive has influenced science education by stressing the importance of students' activities in constructing knowledge.

There are two kinds of conceptual change: strong and weak. The strong conceptual change happens when students change their ideas through accommodation. The weak conceptual change happens when students do not change their old ideas, but develop their old ideas through assimilation. Students change their ideas if they experience anomalous situations or if they dissatisfy with their old concepts. To trace students' conceptual change some researchers use concept maps.

Students have misconceptions before they get formal instructions. In changing preconceptions or misconceptions, students come to the new concepts that are closer to experts' concepts. Computer simulations that rely upon constructivist principles and conceptual change perspectives, allow students to predict, manipulate variables, collect data, discuss, and draw conclusions.

They are intended to enhance conceptual development. In this study, the researcher examined how understanding principles of probability and randomness were affected by the use of computer simulations.

CHAPTER 3
METHODOLOGY

INTRODUCTION

This chapter explains the research methodology. It includes the research questions, hypotheses, treatment, setting, sample strategy, design, instrumentation, and procedure. The hypothesis that students experienced conceptual changes in probability and randomness as a result of learning using computer simulation programs was studied quantitatively and qualitatively. The quantitative study measured student improvement in understanding probability and randomness. The qualitative study investigated which students' concepts were changed, why they changed their ideas, how they experienced conceptual change, and what they thought about the computer simulation programs. The instruments included content-pretests and posttests, concept maps, learning logs, fieldnotes, and examples of students' work. Content analysis and statistical methods were used in examining the data. Students from a suburban Massachusetts high school provided the sample population.

RESEARCH QUESTIONS AND HYPOTHESES

Research questions

The purpose of this study was to investigate how high school students' concepts of probability and randomness changed after they used computer simulation programs on the topics of randomness and fractals developed by the Center for Polymer Studies, Boston University. This study sought to determine whether students experienced conceptual change regarding probability and randomness and whether the computer simulation programs helped students to better understand the concepts of probability and randomness. Specifically, the research examined:

1. whether students' understanding of probability and randomness improved;
2. what students' concepts of probability and randomness were changed;
3. why students changed their concepts;
4. how the process of conceptual change happened; and
5. what students thought and experienced the computer simulation programs.

Hypotheses

It was hypothesized that students would experience conceptual change as a result of using the computer simulation programs and that students' understanding of probability and randomness would improve. A significant difference between students' preconceptions and students' final concepts on probability and randomness at the end of the programs was expected. Using a one tailed *t*-test with a significance level $\alpha = .05$ or $p \leq .05$ the difference between students' scores on the pretest and the posttest of probability and randomness was tested.

TREATMENT

Computer simulation programs

The computer simulation programs were created by the Center for Polymer Studies, Boston University. The programs that were used in the study consisted of seven units: *Coin Flipping, 1-D Random Walk, Pascal's Triangle, Many Walkers, Diffusion, Fractal Coastline, and Fractal Dimension.* Coin Flipping, 1-D Random Walk, Pascal's Triangle, Many Walkers, and Diffusion have a basis in randomness and emphasize such concepts of probability as independence, chance, distribution, normal distribution (bell curve). Fractal Coastline and Fractal Dimension emphasize fractals and how to calculate fractal dimension. The programs are simulations in which students interact actively with the computer programs. The basic theoretical model underlying the programs is that random events mimic fractal formation in nature so that fractals can be understood through random events.

The computer simulation programs use scientific processes. In the methods, students have to predict what will happen with their experiment. They make a hypothesis about their experiment. Then, they do hands-on activities and run computer simulations to collect data. Students analyze data and make a conclusion. Students compare their conclusion with their prediction. If their prediction and conclusion are different, students should change their prediction or run more experiments to get more data which will influence a new result. They discuss their result with friends or teacher.

The Coin Flipping program focuses on the concept of randomness, chance, and probability distribution. It asks students to flip coins with a different number of trials and steps. Students count the number of heads and construct histograms. By looking at the histograms, students will be able to discover the distribution pattern of coin flippings and probability of coin flipping, especially if the experiments are done several times with a larger number of coins. The computers allow students to change the number of coins, the number of experiments, and the speed of the simulations. Students can view more than one experiment at the same time in the window so that it is easier for them to see the patterns and make generalizations about the pattern (Stanley et al., 1994).

1-D Random Walk (one dimension random walk) operates the same principle as coin flipping. It is a different means of representing coin-flipping. How can someone predict the position of something random? In the program, a walker is standing in the middle of a horizontal line. When students press a flip button, the walker will move randomly one step to the right or left. In the beginning, students run the random walk one step at a time, so that they can see what happens with the walker. After that, students can change the step and do the experiment several times. Students have to see the pattern of the bar graphs which are built as a result of the random walker in several trials. Students are asked to explain the connection between the random walk and the coin flipping program. According to Stanley et al., the random walker is very important because it mimics the way a molecule moves.

Pascal's Triangle stresses probability distribution. It is another means of representing coin flipping or random walks. A ball is randomly falling down one level to the right or left from the top of Pascal's triangle. The ball will continue falling down to other levels by randomly going to the left or right. By choosing many balls, students can see what kind of a bar graph is built as a result of the falling balls. Students are allowed to change the speed of the experiments, the number of trials, the number of the balls, and see the result on a window. Students learn that the number of balls or coins in flipping coins influences the smoothness of the curve (Stanley et al., 1994).

The Many Walkers is a random event program with some number of walkers. If the random walk mimics a moving molecule, then the many walkers mimic the way many molecules move. The program can be used as a model of studying molecular movement in real life such as in chemistry, biology, or physics. In the program, students can choose the number of walkers, the number of steps, the number of trials. The number of walkers in each position is shown in the bar graph at the bottom of the screen. Students are asked to decide whether the number of walkers results in a wider spread for the same number of steps and in a smoother graph. Students are asked to predict whether the average of square disposition increases with the number of walkers for a given number of steps. Students are encouraged to find the average displacement and the average squared displacement as a function of time. Diffusion has the same basis as Many Walkers. By calculating when the molecules of an aggregate react, students are able to understand that the molecules are moving randomly and not ballistically (Stanley et al.,1994).

The Fractal Coastline program helps students to find the dimension of a coastline. First, students draw the model of a coastline and second, they measure the dimension of the model. Students set the animation step-by-step, so that they can watch the construction of the coastline. Students will see a line in the computer window. If students click on the iteration button one, two, three, etc., they will see that the line comes to resemble the shape of a coastline. Students can measure the dimension of the coastline model using the ruler method either manually or automatically. Computers will analyze the data from the measurement. Students are able to measure the dimension of the coastline by grid method (box method or covering method) that is also a component in the computer programs. They can compare whether the two measurements have the same result. Students will be able to find that the dimension of the coastline is not integral. It is a fractal dimension (Stanley et al.,1994).

The Fractal Dimension program enables the user to find the dimension of several fractals in nature. The program has two methods: circles and covering methods. In the study, students built an experiment of electrochemical deposition (ECD). The ECD cell consists of a circular anode surrounding a central cathode. The space between anode and cathode is filled with an electrolytic solution of copper sulfate ($CuSO_4$) or zinc sulfate ($ZnSO_4$). In the experiment, atoms are reduced at the cathode surface. Copper ions gain an electron and precipitate as copper atoms, so that the cathode grows and builds a fractal. Students scan the fractal and use this picture to calculate the dimension of the fractal. Students are asked also to compare the fractal in nature with the random fractals in computers (Stanley et al.,1994).

Hands-on activities

Before using computer simulation programs students run experiments or hands-on activities. There are several activities that were accomplished:

Coin flipping activity. Students in a small group of three flipped ten coins at once. They counted the number of heads. They run the coin flipping 10 times. They collected their data in a histogram for all groups. They found how many times heads occur more often. The coin flipping activity was done before students did coin flipping, random walk, and Pascal's triangle programs in computers.

Many random game. The game was done outside a classroom. Students were standing in line. They recalled the first of the final four digits of their own phone number. When their teacher said "Move", students who had an odd number moved to the left one step and students who had even numbers moved to the right one step. The teacher said again "Move!" Students continued moving according to the second digit of their phone number. They did again and again with the third digit and the fourth digit of their phone numbers. In the end of the fourth movement, students could see the idea of many random walkers. The game was done before students began to run many random programs.

The length of Cape Cod. In a small group students measured the length of Cape Cod using their maps and calipers. Students placed the first end of the calipers at the beginning of the coastline to be measured and swung the calipers so that the second end rested on the coastline as close to the starting point as possible. They continued the process and stopped at the end of the coastline. They counted the number of steps. They calculated the distance of Cape Cod by multiplying the scale of calipers and the number of steps. Students did the same measurement by making the scale of calipers a half shorter. They made a conclusion about the length of the coastline (Stanley et al., 1994). The experiment was done before students ran the Fractal Coastline program.

Coastline model game. The game was done outside the classroom. Two students were standing at a certain distance, holding a long rope. A third student stood in the middle of the rope between the two students. The third student moved to the left or the right according to the coin flipping. If he/she got heads, he/she moved to the right. If not, he/she moved to the left. The student moved a number of steps according to the number of the die flipping. For example, if he/she got (**) he/she moved two steps. After that, another student stood between the three students. They did the same movement as the third student. In the end, students can see the model of coastline. The game was completed before students ran the Fractal Coastline program.

Diffusion experiment. Students who studied chemistry did a diffusion experiment. Students took a glass tube about 1.5 meters in length. They put NH_3 (ammonia) in one end of the tube and put HCl (hydrogen chloride) in the other end. The molecular weight of HCl is 36 amu and the molecular weight of NH_3 is 17 amu. The two gases meet and react forming a dusk of white dust NH_4Cl (ammonium chloride) part way along the tube. The question is where and when the reaction will take place? (Stanley et al., 1994). What model of molecular movement do you have: random or ballistic? Students ran the experiment before running the Diffusion program.

SETTING

The study was conducted in a suburban high school that was participating in an educational research project at the Boston University Center for Polymer Studies and the Science & Mathematics Education Center. Students were enrolled in mathematics courses. Some of them were also enrolled in chemistry classes. Their curriculum content included probability for a half semester. Students finished the curriculum in 40 days of class time over the period of three months. Students had two one-week vacations during this time.

Students used three rooms during the study: a mathematics classroom, a computer lab, and a chemistry lab. Most activities with computer simulation programs were run in the computer lab; the lecture/discussion was run in the mathematics classroom; and experiments on diffusion and fractals took place in the chemistry lab. There were 12 computers in the computer lab, but only 8 computers had the computer simulation programs used in the study. There were two laser printers and a scanner. In the study, each computer was used by groups of three students. In the chemistry lab there were two scanners and four computers which could be used. There was one computer and one overhead projector in the mathematics classroom. Students had no difficulties using computers because they had used them before.

The teacher was a mathematics teacher with extensive experience teaching mathematics including probability. This was the first occasion when he taught probability using the computer simulation programs. He was a dedicated teacher who the students liked. He had experience in using the computer simulation programs because he belonged to the Boston University Center for Polymer studies project. The chemistry teacher also worked with the program. He helped students do the experiments in diffusion and electrodeposition and he joined the students in class discussion. The curriculum content for the class activities is pictured in Table 1. below:

Table 1. CURRICULUM CONTENT

#	CONTENT	ACTIVITIES	PLACE	HOUR
1	Concept map	C-map program	lab	1
2	Random & probability	Coin flipping	lab	1
3	Independent prob.	Discussion	class	1
4	Randomness	Random walks	lab	1
5	Coastline	Cape Cod map	class	1
6	Dimension	Coastline Dimension	lab	1
7	Science model	Discussion	class	1
8	Randomness	Random walker &labs	lab	1
9	Random & probability	Discussion	class	1
10	Randomness	Diffusion	lab	1
11	Fractal dimension	Fractal Dimension	lab	1
12	Combination-permut.	Instruction	class	1
13	Free time	Learning log	lab	1
14	Fractal dimension	Gasket Triangle	lab	1
15	Fractal dimension	Discussion	class	2
16	Bayesian probability	Discussion	class	1
17	Fractal dimension	Fractal Dimension	lab	2
18	Fractal dimension	Electrodeposition	lab	1
19	Self-Affine	Hands on activity	lab	2
20	Rough surface	Discussion	lab	1
21	Rough surface	Computer program	lab	1

Notes:

1. Homework discussion, quiz, and tests are not included in the diagram.
2. Random Walk consists of three programs: Coin flipping, 1-D Random walk, and Pascal's triangle.
3. In the diagram only the main activity is written.
4. The curriculum was complete in 6 weeks, 4 days/week.

SAMPLE

The sample included 36 students from the grade 11 mathematics classes. One class had 21 students and the other 15 students. There were 6 females and 15 males in the first class and 5 females and 10 males in the second class. Students were grouped in threes, so there were 7 groups in the first class and 5 groups in the second class. Some students were also enrolled in chemistry. Those students guided other students to do diffusion and electrodeposition experiments.

RESEARCH DESIGN

The design of the study, as is pictured in the Table 2, combined qualitative and quantitative components. Indications of students' conceptual change can be determined by improvement of understanding as measured by the difference between the students' pretest and posttest scores, changes in students' concept maps, and the exchanges of student's reasoning in answering questions about probability and randomness. Some students might better express their changes in concept maps, some students in answering the why-questions in the pretest and posttest, and others in learning logs. Thus it was necessary to collect data in multiple ways. In collecting

Table 2. RESEARCH DESIGN MATRIX

RESEARCH QUESTIONS	TYPE OF STUDY	INSTRUMENTS	ANALYSIS
1. Did students' understanding improve?	quantitative	- pretest & posttest - concept maps	one tailed *t*-test
2. What concepts were changed?	qualitative	- concept maps - pretest & posttest	content analysis open coding
3. Why did students change their ideas?	qualitative	- questions about concept maps - learning logs - fieldnotes	content analysis open coding
4. How did students change their concepts?	qualitative	- concept maps - pretest & posttest - learning logs - fieldnotes	open coding
5. What did students experience and think about computer simulations?	qualitative	- questionnaires - evaluation of teaching-learning	open coding

data the researcher used data triangulation. Triangulation of methodology uses several methods to study the same object. It is the cross-validation among data sources, data collection strategies, and time periods. The triangulation is a form of replication that contributes greatly to researchers' confidence in research findings (Borg & Gall,1989; McMillan & Schumacher, 1989).

The research question 1 whether students' understanding of probability and randomness improved, was determined by examining the difference between the pretest and posttest mean scores, content exam, and the students' concept maps. Understanding was assumed to be correlated with higher scores. The pretest, posttest, first concept maps, and final concept maps were scored. The difference between pretest and posttest scores and the difference between first and final concept maps were analyzed using a *one tailed correlated t-test.* A Statview program was used for this analysis using a significance level of .05. The Null hypothesis was $\mu_1 = \mu_2$, where μ_1 was the mean of pretest, and μ_2 was the mean of posttest. If the probability was smaller than .05, the difference in the mean scores was significant. If the result was significantly different, the null hypothesis could be rejected.

The concept maps were analyzed quantitatively according to a method devised by Novak and Gowin (1984). They evaluated concept maps by examining several criteria: the valid relationship or propositions, valid levels of hierarchy, valid cross links, and correct examples. On the basis of the Novak's guidelines, the scoring of the concept maps in this study was completed according to the methods outlined in Table 3.

Table 3. Concept Map Scoring Rubric

CRITERIA	POOR	FAIR	GOOD
New concept (non example)	1	3	5
Relationship or link	1	3	5
Cross link	2	5	10
Example		1	2

A good concept has a full meaning and a very closed relationship to probability or randomness. A poor concept has meaning but it has very little relationship to probability or randomness. A fair concept has a full meaning and somewhat of a relationship to randomness or probability but one that is not very close. For example, coin flipping is a good concept because it has full meaning and has a very closed relationship to randomness and probability. A good relation or link connects two concepts correctly, clearly, and completely so that it builds an accurate proposition. A poor link connects wrongly two concepts. For example, randomness is not predictable. "Is not" is a poor relation because it builds an incorrect proposition. A fair relationship falls between the poor and good relation. Scoring of cross links is the same description as for links. A good example is a correct example. A fair example is one that is not totally correct. In this study, probability and randomness concepts were not scored because the two concepts were revered with students. An example of the scoring of a concept map is pictured in Figure 2.

After each score was obtained, the average score of students on the first and final concept maps was calculated. Using a one tailed correlated *t*-test, it could be determined whether the first and final concept maps were significantly different or not.

Research question 2 related to what students' concepts of probability and randomness were changed: information was drawn from concept maps, students' reasoning in pretest and posttest. Concept maps were transcribed in sentence forms. The first and final concept maps' transcriptions were compared. Through comparison, what concepts were changed, could be identified. For example, transcriptions of a first and final concept map were:

First concept map	Final concept map
- Nothing is random.	- Coin flipping is random.
- Random is unpredictable.	- Random is uncertain but predictable.

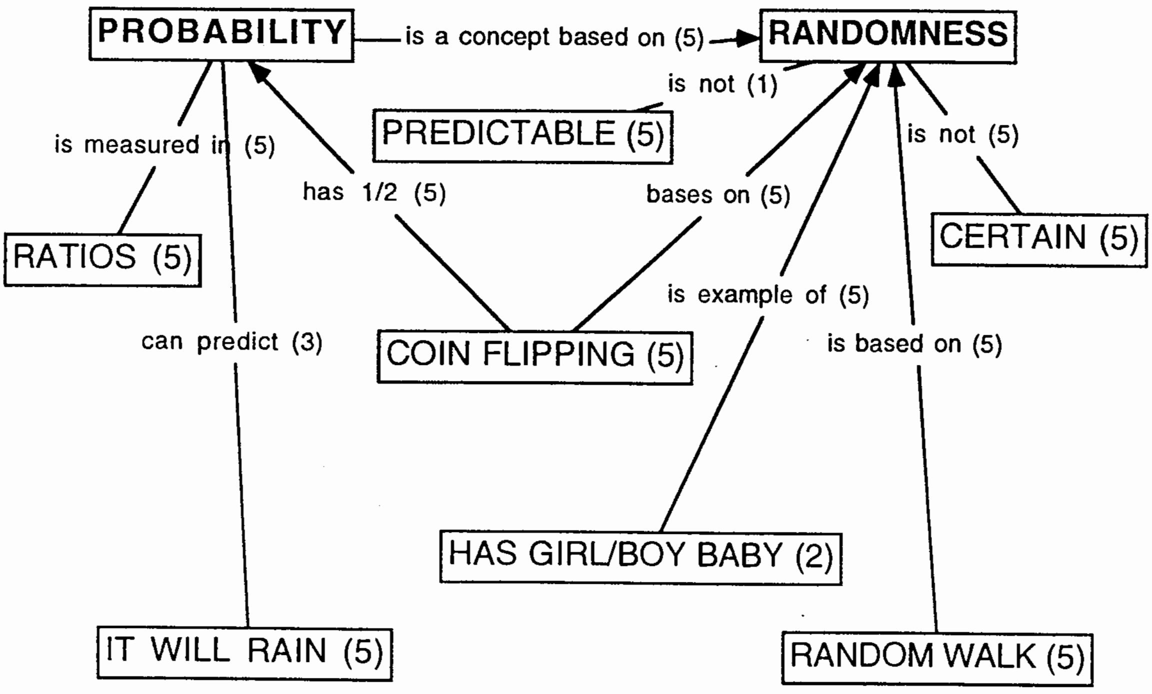

Figure 2. An example of concept map scoring.

Notes:

1. Probability and randomness were not scored because the two concepts were given before.
2. Numbers inside ( ) are the scores of concepts, links, or examples.
3. The total scores of the concept map example above was 71.

- good concepts = 6,	score=	30
- good links = 7,	score=	35
- fair links =1,	score=	3
- poor link=1,	score=	1
- good example=1,	score=	2
	Total scores	71

There is a conceptual change on randomness from the first to the final concept maps' transcriptions. In the first concept map there is nothing which is random, but in the final concept map there is something which is random, i.e. coin flipping. In the first concept map, random is described being unpredictable, but in the final concept map, random is called predictable. Several categories were used by the author in grouping concepts such:

AC - adding concepts
AL - adding links
AE - adding examples
EC - erasing concepts
EL - erasing links
EE - erasing examples
CC - changing concepts
CL - changing links
CE - changing examples
CMR - changing the meaning or ideas of randomness
CMP - changing the meaning or ideas of probability
CMRP - changing the meaning or ideas of relation between randomness and probability.

The changing ideas or meaning have properties such as:

a. - more detail
b. - greater precision
c. - truer concepts

Students' reasonings in pretest and posttest were compared to find what other concepts were changed. Content analysis was used. "Content analysis is a research technique for the objective, systematic, and quantitative description of the manifest content of communication" (Berelson, in Borg & Gall, 1989, p.519). Most early content analysis studies examine the content of written materials by relying on simple frequency counts of objective variables. Recent studies consider not only content frequencies but also the interrelationships among several content variables (Borg & Gall, 1989). In the analysis the researcher made some classifications of grouping and examine how often each classification happened. The researcher also used open coding (Strauss & Corbin, 1990). "Open coding is a process of breaking down, examining, comparing, conceptualizing, and categorizing data" (p.61). The process of coding in the study was sentence coding in which sentences were labeled, conceptualized, and categorized. In the comparison of pretests and posttests, six classifications of grouping were used by the author:

N+ : reasoning not changed and answered correctly.

N- : reasoning not changed and answered incorrectly.

C+ : reasoning changed and answers changed to correct answers.

C- : reasoning changed and answers changed to incorrect answers.

CR+ : reasoning changed even though they had answered correctly

CR- : reasoning changed even though they still answered incorrectly.

If the number of students who did (N+ and N-) was smaller than the number of students who chose (C+, C-, CR+, CR-) in a problem, the test problem was classified as having conceptual change. If the number of students who chose C+ or CR+ individually was greater or the same as 9 in a test problem, then the problem was classified as showing a pattern of conceptual change.

After categorizing, each problem which showed a pattern of conceptual change was analyzed to find what kind of changes there were.

Question 3 of why students changed their ideas was studied by examining questionnaires about concept maps. There were two questions about concept maps: (1) Is your last concept map the same as your first concept map? (2) Why did you change your concept map? Open coding and content analysis were used in the study.

Data also was extracted from learning logs "vertically". Vertically means that all learning logs from the same activities were compared. For example, the logs of coin flipping activity from all students were compared. From the comparison, patterns of changes as a result of the coin flipping activity would be identified. Fieldnotes were examined whether there were data that indicated students' reason of changing concepts. Open coding was used to examine the patterns of changes.

Question 4 of how students changed their concepts on probability and randomness was studied using concept maps, students' reasoning on the pretest and posttest, learning logs, and fieldnotes that described how students were running the computer programs and learning in a classroom. By comparing old concept maps and new concept maps, by comparing students' reasoning on the pretest and posttest, some patterns of how students changed their ideas would be identified. From learning logs and researcher's fieldnotes patterns of how students changed their ideas would be analyzed using open coding.

Question 5 on students' thoughts and experiences regarding computer programs were studied using questionnaires and evaluation in learning logs. The questionnaires consisted of four questions about computer simulation programs. The evaluation consisted of how students experienced and thought about teaching-learning using computer programs. Open coding was used in this study.

To increase the validity of data, some steps were taken:

1. Students were chosen from the same class, the same grade, the same teacher, and had the same in-class time studying or using the computer simulation programs.

2. The computer programs were identical.
3. Students had the same time to complete the pretest and posttest, and construct concept maps.

INSTRUMENTS

Pretest

The pretest (student survey I) consisted of 20 multiple choice questions with an open-ended "Why" question (see Appendix C). It took 40 minutes to finish the pretest. Some multiple choice questions in the pretest are an adaptation from some researchers' questions such as Kahneman and Tversky (1982), Shaughnessy (1977), Konold (1988), Green (1983), Baturo (1993), and Hirsch and O'Donnell (1993), but they were only selected if the questions emphasized the concept of probability, chance, randomness, or frequency. The purpose of the "Why" questions is to give students an opportunity to explain their reasoning about why they chose a certain answer in each number. The "why" questions reveal what misconceptions students had. Analysis of students' answers and their reasoning gave the researcher a deeper understanding about students' misconceptions.

The questions that emphasize classical probability are similar to the one below:

You flip a coin once. What is the chance of getting a head?

A. 0

B. 1/2

C. 1

D. 2

Why? ....

Questions that illustrate the concept of randomness are like the one below:

Students flip one coin 5 times. Which sequence is more likely to occur in the five flippings? H = head, T = tail.

A. HHHTH

B. HTHTH

C. HTTTH

D. All the same

Why? ....

A question that emphasizes frequency probability or how often something happens is like the one below:

You flip 5 coins simultaneously. You do it 100 times. How many heads do you get most probably?

A. one heads

B. two heads

C. three heads

D. four heads

E. five heads

Why?..

Questions about the concept of independent probability are like the one below:

Which of the following statements has more chance of happening?

a. A man smokes and gets cancer.

b. A man gets cancer.

Why?..

A question on the meaning of probability terms is like the one below:

> There are two students in your school who have the same birthday. The event is
>
> a. uncertain
>
> b. definite
>
> c. probable
>
> d. impossible
>
> e. unexpected
>
> Why? ....

Concept maps

One class hour was required to train students to build concept maps using the C-map program. The teacher explained concept maps and how to construct a concept map with computers. Students built a concept map about familiar topics such as a sport, mathematics, their family, or a hobby. After they were familiar with doing a concept map on the computer, students were asked to construct a concept map on the topic of probability and randomness. The assignment for the first concept map was:

> Construct a concept map based on the concepts: probability and randomness. Add as needed other concepts, links between concepts, and examples which you have understood.

In the final concept map task, students were asked to compare their first and final concept maps and explain why they changed their concept maps. The assignments of the final concept maps were:

1. Construct a concept map based on the concepts: probability and randomness. Add as needed other concepts, links between concepts, and examples which you have understood.

2. Is your concept map the same as your first concept map? Why did you change your concept map? Explain.

Posttest

The posttest (student survey II) consisted of 20 multiple choice questions and four questions about the computer simulation programs (see Appendix D). It took 50 minutes to complete the posttest. The multiple choice questions were the same as on the pretest so that the researcher could make a pretest - posttest comparison. Four additional questions about computer simulation programs ask about students' thoughts and experiences regarding the programs and how they used the programs to understand probability and randomness. The questions are presented below:

1. Which of the following computer programs: Coin Flipping, 1-D Random Walk, Pascal's Triangle, Many Walkers, Diffusion,

Fractal Coastline, Fractal Dimension, were very useful in your learning about probability and randomness? Why?

2. Which of the above computer programs were not useful in your study of randomness and probability? Why?

3. What are your suggestions about improving those programs?

4. Suppose that you are asked to teach and explain probability and randomness to your friends in another school; which programs would you want to use? Why?

Learning logs

Learning logs were electronic journals maintained in computers in which students wrote what they were doing and learning. Students were free to write anything about the computer programs, problems in the study, questions, or inventions. Teachers responded in the journal to what students had written. Learning logs were analyzed to know how students developed their understanding, how they got new ideas, how they experienced difficulties, and what they thought about the content of the programs. Also investigated was how students changed their ideas through teachers' comments, and how students disagreed with their teacher on what they studied.

Students' work

Students' work was all of the students' products or activities connected with understanding randomness or probability. Students' work included homework and evaluations given by their teacher.

Evaluation of teaching-learning process

At the end of the program, students were asked to answer some questions on the teaching-learning process using computer simulation programs. Students were asked to describe how they experienced the simulation computer programs. The questions were:

a. How did you experience and think about the teaching-learning using computer simulation programs? Did the teaching-learning help you learn?

b. Do you prefer a lecture in classrooms or teaching using computer programs? Why?

c. What do you think about learning logs? Were they helpful for your study? If so, how did they help you?

d. How did your group work together? Did your group help you in this study?

e. How did you participate in your group? Did you feel satisfied with your group?

f. How many minutes did you usually run or use the computer programs in each lab?

Fieldnotes

During field observation in the classroom the researcher wrote fieldnotes on how the lesson was going, how students were working together, how students were running computer programs, how they were questioning, and how their teacher was teaching. The processes of learning and teaching were noted. The researcher was especially concerned about classroom activities that were thought to have an impact on students' conceptual changes.

The researcher always attended class activities: instructions and discussions in a mathematics classroom, computer simulations in a computer laboratory, hands-on activities in the mathematics classroom or outside the classroom, or chemistry experiments in a chemistry laboratory. He went to the school five days per week.

During class instructions the researcher sat down on a chair in the back of the classroom and observed what students and their teacher were doing. He wrote fieldnotes about what the teacher explained and students' reaction to the lecture. When students did hands-on activities, the researcher sometimes joined the students; but sometimes he walked around to observe what the students were doing.

During computer simulations the researcher did several things. First, he ran a computer that was located in the corner of the computer labs. He wrote notes on the computer about how many students came in the labs, the instruction of their teacher, and the task that students should do for the labs. Second, when students ran computer simulations in groups, the researcher observed each group. He made notes of what students were doing with the computers. Third, he sometimes helped a group who had difficulties to use the computer programs. If there were questions about the problem or the content of the topics, usually the researcher asked students to find their own answers by asking them to repeat the same experiment. Fourth, the researcher sometimes discussed computer programs with students after the formal classes were over.

DATA COLLECTION PROCEDURE

Before collected data, the researcher received approval from the school principal (see Appendix A) and parental consent (see Appendix B). Students received a pretest and were asked to build their first concept maps before the computer programs of randomness and fractals began.

During the randomness and fractal unit, students did hands-on activities, ran computer simulation programs, discussed their work, wrote learning logs, and did home work. The researcher took fieldnotes on the processes of learning-teaching. He collected students' learning logs and observed what students were doing, discussing, and concluding. The researcher focused more on those activities and patterns that had relevance to students' understanding and students' conceptual change regarding probability and randomness.

After students studied probability and randomness using the computer program simulations, they received a posttest and questionnaires about the computer programs and evaluation on the teaching-learning method. The posttest was done in 50 minutes. The students constructed their final concept maps as a last project.

LIMITATIONS OF THE STUDY

The study involved a small sample from only one school. Thus the interpretation of the results should be viewed from the perspective as the result of this limited sample.

The study had no control groups. It was difficult to find control groups who had the same level and situation from the same school. The grade 11 mathematics classes consisted of two classes. Both students in the two classes wanted to use computer simulations. Students wanted to use the same facilities and technologies because they paid the same tuition. If the researcher used students from other schools as control groups, they had different situation and different teachers that would influence the results. Many parameters needed to be considered. In the next study, if it is possible, control groups should be provided. The control groups will strengthen results.

SUMMARY

This chapter outlined the methodology of the study. Qualitative and quantitative methods were used to examine whether students experienced conceptual changes about probability and randomness after they used computer simulation programs. A one tailed *t*-test was used to find out whether the difference between students' pretest and posttest was statistically significant. Open coding: concept labeling, categorizing, and comparing categories were used as qualitative procedure to learn what students' concepts were changed, why students changed their ideas, how they changed the concepts, and what they thought about the computer programs.

The researcher used several instruments such as pretest, posttest, concept maps, learning logs, questionnaires about computer programs and teaching-learning method to collect data. Thirty six students from a suburban high school were used as the sample. They finished a pretest before the program, did the program, and after 40 days they completed the posttest. The school had good facilities including computer labs for students.

CHAPTER 4
RESULTS

INTRODUCTION

This chapter presents the results of the study of high school students' conceptual change on probability and randomness. The statistical analysis describes students' knowledge improvement on probability and randomness. The data which indicates what students' concepts were changed are presented. Students' misconceptions of randomness and probability found in the study are listed. The data which revealed students' reasons for changing their ideas and showed how students changed their concepts are described. Lastly, the data that indicates how students' thoughts and experiences regarding the usefulness of computer simulation programs are presented.

STUDENTS' KNOWLEDGE IMPROVEMENT

It was hypothesized that students would improve their understanding of probability and randomness after they used computer simulation programs and that there would be a significant difference between students' preconceptions and students' final concepts of probability and randomness.

A pretest consisting of multiple choice questions and concept map tasks was administered to 36 students in two classes. Forty days after the pretest, a posttest was given after students had used the computer programs. The actual time interval between the pretest and posttest was three months because students had two breaks of about three weeks. Thirty one students completed both pretest and posttests. Five students finished only their pretest or posttest, so their results could not be used in this analysis. Concept-mapping was administered separately from the multiple choice test as an individual project. Students completed their final concept maps one day before the posttest. During the posttest the researcher gave the first and final concept maps to students, so they could answer several questions based on their concept maps.

From the multiple choice test, there were complete data from the 31 students. The mean score of the pretest was 14.74, and the mean score of the posttest was 16.65. The standard deviation of the pretest was 2.05, and the standard deviation of the posttest was 1.56. The one tail *t*-test indicated that the difference of the pretest and the posttest was significant, t (30) = 4.12, p=.0002 (see Table 4).

From the concept map questions, data of 31 students were available for use in this analysis. The mean score of the pretest concept map was 44.13, and the mean score of the posttest concept map was 82.03. The standard deviation of the pretest concept map was 12.42, and the standard deviation of the posttest concept map was 29.69. The one tail *t*-test indicated that the difference of the pre and post concept maps was significant, $t(30) = 7.49$, $p = .0001$ (see Table 5).

Table 4

MULTIPLE CHOICE TEST: MEAN, STANDARD DEVIATION, AND ONE TAIL *t*-TEST OF PRETEST AND POSTTEST

Score Pretest	Score Posttest
Count = 31	Count = 31
Mean = 14.74	Mean = 16.65
SD = 2.05	SD = 1.56

Paired one tail *t*-test:

DF = 30

Paired *t* value = 4.12

Prob. (1-tail) = **.0002** (significant <0.05)

Table 5

CONCEPT MAP TEST: MEAN, STANDARD DEVIATION, AND ONE TAIL *t*-TEST OF PRETEST AND POSTTEST

Pre concept map	**Post concept map**
Count = 31	Count = 31
Mean = 44.13	Mean = 82.03
SD = 12.42	SD = 29.69

Paired one tail *t*-test:

DF = 30

Paired *t* value = 7.49

Prob. (1-tail) = **.0001** (significant <0.05)

Twenty two students increased their scores, five students decreased their scores, and four students got the same scores in the multiple choice test. Thirty students increased their scores and one student decreased his score in the concept map test. As expected most students increased their scores in the concept map.

Twenty five students answered incorrectly multiple choice problem #1 and fifteen students answered incorrectly problem #2. Problem #1 was about independent probability. Students were asked to decide which probability was higher: (a) a person smokes and gets cancer or (b) a person gets cancer. Many students answered (a). Problem #2 was about conditional probability. Which of the following events has more chance of happening: (a) a daughter has blue eyes if her mother has blue eyes or (b) a mother has blue eyes if her daughter has blue eyes? It appears that students still had difficulties with those problems.

Computer time on task vs. students' scores

How much time the students spent on the computer was noted in this study. The time spent was compared with students' scores in the posttest and with students' score difference (posttest score - pretest score). Using simple regression it can be determined if the time spent in computers influenced students' scores. In the time range between 15 to 45 minutes per lab hour, the result was that there was no significant relationship between time on task and students' score ($p > .05$) (see Appendix F).

Summary

The statistical analyses of pretest and posttest from both multiple choice and concept map scores showed that the differences between students' posttest and pretest were statistically significant ($p < .05$). This indicates that students improved their knowledge of probability and randomness. In addition, the amount of time using the computer did not appear to influence students' scores.

WHAT CONCEPTS WERE CHANGED

It was hypothesized that students' concepts of probability and randomness would change as a result of the treatment.

- **FROM CONCEPT MAP DATA**

Most students changed their concept maps in three basic ways:

1. The meaning of randomness, probability, and the relation between randomness and probability were altered.
2. The second concept maps were more specific, more exact, and showed mathematical relationships.
3. The second concept maps included new concepts, new propositions, and additional examples.

Twenty five students changed the meaning of randomness in their concept maps. Nine students who did not explain the meaning of randomness in the first concept maps, gave explanations in the second. In their second concept maps students described randomness as "having no precision, not predictable, having no specific pattern, independent, and having no specific method to determine." Nine students developed the meaning of randomness. In the first concept maps they described randomness as not predictable, but in the final concept maps they discussed randomness as "having no pattern, no causal law, no precision, independent, arbitrary, and no control." Six students changed their notion of randomness from "nothing is random" to "several things are random." This was a strong concept change. For example:

<u>Pre concept map:</u>	<u>Post concept map:</u>
- No explanation of randomness	- Randomness is not predicted. - You never know what you will get. - Randomness has no precision. - Randomness has no specific pattern. - Randomness is independent.
- Randomness is unpredictable.	- Randomness has no pattern. - Randomness cannot be controlled. - Randomness is arbitrary. - Randomness describes the result of rolling dice.
- Nothing is random.	- Diffusion is random. - Rolling a die is random. - Fractals are random.

Twenty nine students changed the meaning of probability in their second concept maps. Originally 12 students did not explain what probability was or just gave a short explanation, but they explained more in their second concept maps. They explained that probability determined and predicted something which was random. Seventeen students also described factors that influence probability in the second maps. They wrote that probability was related to the number of events and affirmations. Probability was given as a ratios. For example:

Pre concept map:	Post concept map:
- no explanation. - probable = can happen.	- Probability is to predict something. - Probability is to determine something random. - Probability is never exact. - Probability governs the result of many walkers. - Probability is changed each time you gain more. - Probability is measured in ratios. - Probability is changed with the # of event, affirmation.

Twenty students changed their concepts maps about the relationship between randomness and probability. In their pre concept maps 12 students did not mention the connection between randomness and probability, but mentioned this relation clearly in their second maps. They wrote in their second maps that probability could be used to predict, control, describe, and make patterns of randomness. In the first maps eight students mentioned the relation between probability and randomness unclearly; but in the second maps they described the relation more clearly and more specifically. In the first map those students only wrote that probability was a random function, but in the second map they wrote that probability described, explained, and summarized randomness. For example: (P = probability, R= randomness)

Pre concept map:	Post concept map:
- No relation.	- P predicts R.
	- P controls R.
	- P makes patterns of R.
	- P decides results of R.
	- P describes R.
- P has relation to R.	- P is a way to find out R.
	- P is concept based on R.
- P is a random function	- P is to describe R.
	- P explains /summarizes R.

Seventeen students explained the concept of classical and frequency probability more specifically, more exactly, and more mathematically or statistically. In the first concept maps those students did not mention how to calculate probability; In the second concept maps however, they mentioned how to calculate probability, such as the probability of flipping a coin was 1/2 and the probability of tossing a die was 1/6. For examples:

<u>Pre concepts</u>	<u>Post concepts map</u>
- No calculation	- P of flipping coin = 1/2.
	- P of toss a die = 1/6.
	- P of getting #>1 on die is 5/6.
	- P of picking a red marble of a jar (6 reds and 14 whites) is 3/7.
	- combination and permutation.

Because most students had new concepts, links, and examples in their second concept maps their concept maps were more developed and complete. Some concepts such as fractals, coin flipping, dimension, combination-permutation, random walkers, coastline, diffusion, chaos, were included more in students' second concept maps than in the first concept maps. Most of the new concepts were studied in the computer programs. Some links such as "predicted, controlled, can be calculated, influenced, figured out" were relations that students learned during the activities. Most students in the first maps used "group, sex, and tossing die or coin" as examples of randomness and probability, but in the final maps they used more "lottery, fractals, diffusion, random walker, many walkers, coin, die, and chaos," concepts connected to the simulations, as examples.

A comparison of one student's pre concept map (Figure 3) and post concept map (Figure 4) illustrates typical changes. This student added new concepts such as permutations and combinations, fractal dimensions, fractals, diffusion, outcomes, random walkers, and bell curves. All the new concepts came from the computer programs and students' activities. There were many new linkages such as "is controlled by, are used to analyze, determined by, has an outcome controlled by, predict random sets of, can demonstrate behavior of." Some old concepts were erased such as mathematics, our group, and predictable. The second concept map was more complicated and had greater contents.

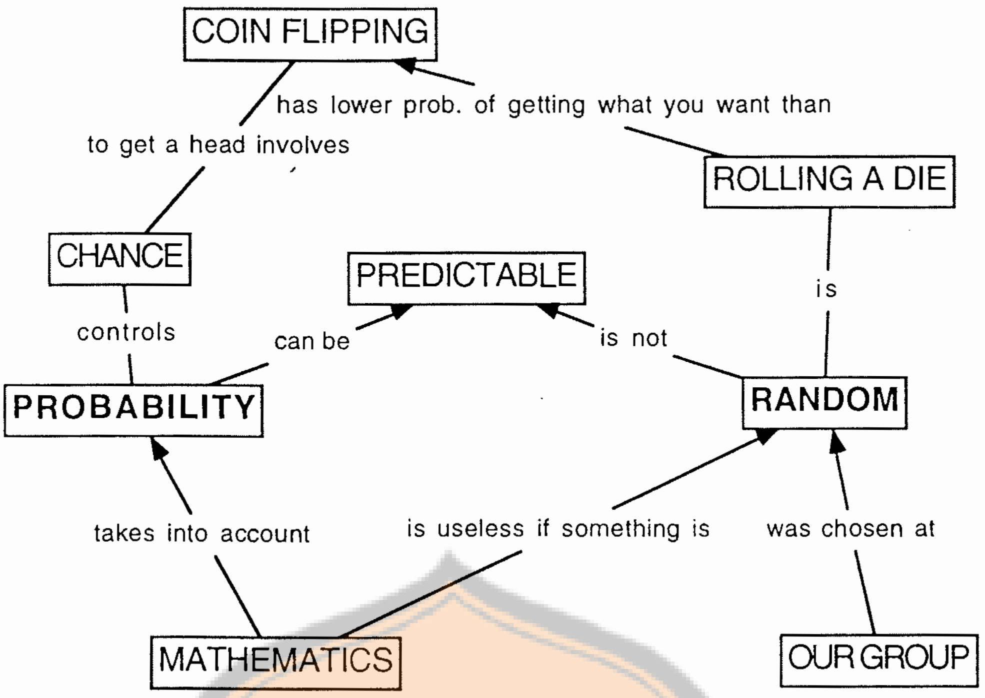

Figure 3. A's pre concept map of probability and randomness.

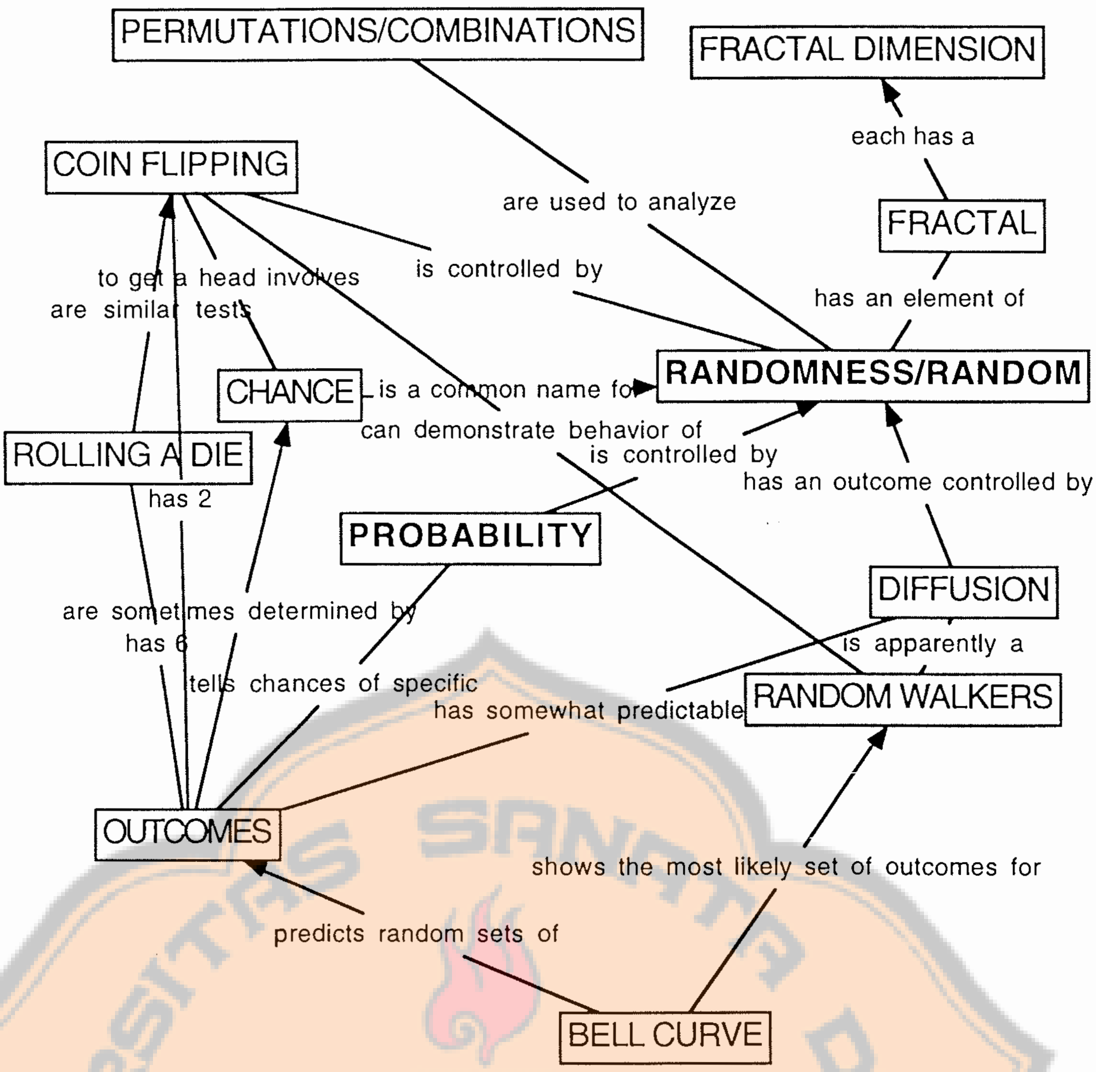

Figure 4. A's post concept map of probability and randomness.

From a composite analysis of all students' concept maps we can conclude that several general concepts of randomness and probability were changed. Those concepts were:

1. the existence of random events. Some students changed their ideas of the existence of random events from "there is no random" became "there are random events."

2. the meaning of random. Many students developed a better understanding of the meaning of random. Students not only explained "random" as unpredictable, but also that random events had no precision, no pattern, no causal law, independent, had the same chance of occurring, and were arbitrary.

3. classical and frequency probability. Most students gave calculations of classical and frequency probability that were more exact, mathematical, and specific. Students displayed understanding of several characteristics of probability more such as: probability is given in ratios, probability is changed with the number of events.

4. relation between probability and randomness. Many students changed their concepts about the relation between probability and randomness. They said that probability predicted, controlled, made patterns, described randomness in their last concept maps. In their first concept maps they did not understand these relationships.

5. expansion of the concept of probability and randomness. Many students expanded their ideas of probability and randomness by connecting those ideas to new concepts such as: fractals, diffusion, molecules, random walkers, and chaos.

- **FROM WHY-QUESTION DATA**

The data on reasoning in the multiple choice items both on the pretest and posttest were compared. Six codes were used in the classification:

N+ : Number of students who did not change the reason and answered correctly.

N- : Number of students who did not change the reason and answered incorrectly.

C+ : Number of students who changed their reasoning and changed their answers to the correct answers.

C- : Number of students who changed their reasoning and changed their answers to the incorrect answers.

CR+ : Number of students who changed their reasoning even though they still answered correctly

CR- : Number of students who changed their reasoning even though they still answered incorrectly.

If the sum of N+ and N- was smaller than the sum of (C+, C-, CR+, CR-), the researcher classified this event as a conceptual change. If C+ or CR+ individually was greater or equal to 9, it was also classified as a conceptual change. Based on those criteria the result of why-questions were able to be classified (see Table 6).

Table 6

STUDENTS' CONCEPTUAL CHANGE IN REASONING

PROBLEM NUMBER	N+	N -	C+	C -	CR+	CR-	PATTERN
# 1	3	20	2	3	1	2	no change
# 2	8	10	2	3	6	2	no change
# 3	20		2	1	7		no change
# 4	14	4	4		7		no change
# 5	10	2	3		12	2	change
# 6	9	2	9	4	1	3	change
# 7	16		8		7		no change
# 8	17		7		7		no change
# 9	5	2	11	5	6	1	change
# 10	16		7		5	1	no change
# 11	13	1	12	1	2		change
# 12	22		1		7		no change
# 13	13	1	1		15		change
# 14	5	2	8	4	9	2	change
# 15	20				11		change *
# 16	11	4	2	5	7		no change
# 17	20		2		9		change *
# 18	18		4		8		no change
# 19	10		11		9	1	change
# 20	18			3	9		change *

Many students changed their reasoning in the multiple choice why-questions. Many of them changed the reasoning even though they had answered correctly in both cases. There were about ten problems in which the greatest change of reasoning was detected (problems #'s 5, 6, 9, 11, 13, 14, 15, 17,19, and 20) (see Table 6). From all the data above, the researcher was able to identify what kind of concepts of randomness and probability were changed.

There were six concepts of randomness and probability that many students changed:

1. concept of not very random
2. unpredictable randomness
3. independent random event
4. degree of probability
5. classical probability
6. frequency probability.

In several cases problems were answered correctly in both pretest and posttests (problems #'s 3, 7, 8, 10, 12, 15, 17, 18). In problems #1, most students did not change their reason and they still gave wrong answers (see Table 6). From the data, the researcher investigated what kind of concepts of randomness and probability were not changed.

Nine students changed their ideas about a third category: not very random in problem #6. Students were asked whether they agreed about a third category (not very random) between random and not random. In the pretest students agreed with the third category, but in the posttest they did not agree. They changed their idea from "there is a third category" to "there is no such third category". They said that something could only be random or not random. This was a strong conceptual change. They wrote in their tests:

In pretest: "An event may appear random but also quite possible it is not as far as others."

In posttest: "Things are either random or not random."

In pretest: "Yes, something can appear random for a long time which really are not and should go into this category."

In posttest: "Something is either predictable or not. Either it is random or not random."

Twelve students changed their reasoning in problem #5. It was a problem about an unpredictable random event. The question was:

A random event is uncertain and unpredictable. That is why we cannot use random events to predict some physical phenomena.

a. True. *b. False.*

Students changed their reasoning even though they had the correct answers. Their reasoning was more specific and clear. They explained clearly that even though random events were unpredictable, they could be used to predict some physical phenomena. They said:

In pretest:	"They are predictable about a certain event."
In posttest:	"It is uncertain but predictable to a certain degree <u>i.e. flipping coin will produce heads around 50% of the time</u>."

In pretest:	"When repeating the same event you may understand a situation."
In posttest:	"As seen in the fractal coastline and other phenomena, <u>random events are very useful in determining certain things</u>."

Eleven students changed their reasoning on problem #15 about the independence of random event. They said that random was independent and that the previous flips had no effect on future flips. They reasoned more explicitly about the meaning of randomness. The problem was:

You toss one coin 4 times and get HHHH. If you toss again, what will it be?

a. H b. T c. cannot tell.

They chose (c) with reasoning:

In pretest: "It is random."

In posttest: "Previous flips have no effect on future flips. The events are independent from each others."

Many students changed their answers and their reasoning in some concepts about the degree of probability (problem #'s 7, 9, 11) such as: impossible, unexpected, possible, likely, and certain. Their reasoning was more exact and their explanations used statistical data in the posttest. They wrote more accurately about what was happening in the world, so they could calculate the degree of probability of an event better. For example, in problem #9:

There will be no war in the world during the next two years.

a. It is definite.

b. It is impossible.

c. It is likely.

d. It is possible.

e. It is unexpected.

Eleven students changed their reasoning and changed their answers. They changed their answer from "possible" to "impossible" because they understood that right now there is a war. They knew more realistically what was happening in the world right now. They said:

In pretest:	"We cannot predict the mental moods of the men in power."
In posttest:	"War is a part of human culture. We have a war going on right now."

In pretest:	"We don't think it is true but there is no way of knowing."
In posttest:	"As I am writing the question, one is definitely occurring."

In the problem about probability of "two students in their school have the same birthday", twelve students changed their reasoning and changed their answers. Most of them gave more exact data about the number of kids in their school and compared this to the number of days in a year. They answered more definitely. For example:

In pretest:	"There is a high probability with the number of kids."
In posttest:	"Since there are only 365 days and there are 400 students, some birthdays must double."

Some students changed their concepts of classical probability (problem #'s 13, 14) about the probability of rolling a die and probability of tossing coins. In the posttest, they explained their reasoning using statistical terms. Their reasoning showed their understanding of probability. For example, in problem #13:

What are the odds of getting number 3, when you roll a die? Students answered 1/6 with a reason:

In pretest: "6 possibilities."
In posttest: "All 6 outcomes are equally likely therefore 3 being one of the 6 outcomes has chance of 1/6."

In problem #14: "*You toss 3 coins at once. What is the chance that you will get 2 heads?"* nine students changed their reasoning even though they had answered correctly. Eight students changed their reasoning and changed their answers. They changed their reason using statistical evidence. They answered 3/8 with reason:

In pretest: "You held it or not. 1/2."
In posttest: "Possible outcomes 8 (HHH, HTH, THH, HHT,TTT, THT, TTH, HTT), three of them have 2 heads."

Some students changed their reasoning in problem #19 dealing with the concept of frequency. They answered more explicitly and mathematically in their posttest. The question was:

You flip 6 coins at once. You do it 1000 times. What number of heads has the most chance?

a. one head.

b. three heads.

c. five heads.

d. six heads.

They answered (b) with the reason:

In pretest: "It is quite probable that you will get 1 head out of 6 pennies. The probability decreases as the number of heads increases."

In posttest: "With 1/2 probability and 6 coins, it will most likely average out 3 heads and 3 tails per toss."

Concepts that were unchanged

There was no evidence that students changed their answers and reasoning to problem #'s 3, 7, 8, 10, 12, 15, 18 because they answered correctly in both the pretest and posttest. The concepts were <u>independence of random event</u> (#'s 3, 15, 18), <u>classical probability</u> (#12), and <u>the degrees of probability</u> (#'s 7, 8, 10). It seemed that students understood the concept and the meaning of randomness and some probability related terms such as: impossible, probable, possible, uncertain, and unexpected. Twenty students answered problem #1 incorrectly and did not change their answers in their posttest. The problem was about independent probability. It seemed the students did not know that their ideas about the independent probability were wrong. Expose to the simulations did not alter these concepts.

- **MISCONCEPTIONS OF RANDOMNESS AND PROBABILITY**

Several misconceptions about randomness and probability were found in students' concept maps and pretests. Seventeen students had misconceptions about a third category of randomness (problem #6). They thought that there was a third category of randomness, i.e. not very random. They believed that there were three categories in randomness: random events, not random events, and not very random events. Their reasons were:

"Some events depend on certain things but still also have a random factor."

"There are always gray areas in everything, something may be independently random but depend on other things."

Six students believed that we could not use random events to predict some physical phenomena because random events were uncertain and unpredictable (problem #5). They said: "We cannot use something that is unpredictable to predict something." Several students believed that there was nothing that was really random. For them, randomness did not exist.

Six students had misconceptions about random sequences of having five children (problem #16). They were asked to choose which sequence is more likely to occur for having 5 children, if the chance of having a boy baby is 1/2. The sequences are: (a) BGBGB, (b) BBBGB, (c) GGBGG, and (d) equal. They chose (a) with reasons: (B = boy, G = girl).

> "Outcomes tend to be closest to probable chance, 2/5 is close to 1/2."
>
> "Out of two babies are boys, so alternating boys and girls is very likely."

They thought that the random sequence should occur alternatively between boys and girls. They did not think that the former events do not influence future events in random cases.

Twenty four students had misconceptions about independent probability (problem #1). Students thought that the probability of people smoking and getting cancer is higher than the probability of people getting cancer. Students thought causality in this problem. They said that smoking causes cancer. They did not understand that the probability of people smoking and getting cancer is lower than the probability of people getting cancer only.

Twelve students had difficulties in finding the probability of getting 2 heads in flipping 3 coins at once (problem #14). The students answered 1/2. Their reasons are:

"You either get 2 heads or 2 tails."

"You held it or not."

Students solved the problem by thinking the problem as a problem of flipping only one coin at once.

Several students had misconceptions about frequency probability (problem #19). Students were asked to find out what number of heads has the most chance, when they flipped 6 coins at once and they did it 1000 times. Those students answered that one head was the most probable, even though the correct answer was three heads. They gave reasons such:

"Getting one is easier than getting more."

"It is quite probable that you will get one head out of six pennies. The probability decreases as the number of heads increases."

Twelve students were confused about the meaning of "probable" and "definite" (problem #11). They gave a correct reason in the problem about the probability of two students having the same birthday in their school, but they chose an incorrect term for that reason. They explained:

"There are only 365 days in a year and there are 400 students in our school."

Those twelve students described two students having the same birthday as a probable event, not as a definite event. The correct term for those events is "definite". It seemed that students did not understand the term "probable" and "definite" clearly.

The study indicates that computer simulations overcame some misconceptions such as the third category of randomness, the existence of randomness, unpredictable randomness, random sequences, frequency probability, and some definitions of probable. But the computer simulations seemed not to overcome misconceptions about independent probability in problem #1.

Summary

From the data found in concept maps and multiple choice tests, it was evident that students showed changed concepts related to randomness and probability. Some randomness concepts such as: the third category (not very random), unpredictable randomness, and independent randomness were changed. Students changed their ideas of the existence of randomness from "nothing is random" to "some things are random." Some concept changes were strong and some were weak.

Several concepts of probability such as: classical probability, frequency probability, and the degree of probability were changed. Students explained their concepts of probability more mathematically, statistically, completely, and clearly. Some students explained the characteristics and the meaning of probability more in the posttest than in the pretest.

The concept of the relationship between probability and randomness was changed too. In the pretest some students did not mention the relationship, but in their posttest, they explained some of the relationship. Students changed their ideas through new concepts, examples, and links. Some new concepts such as: many walkers, fractals, random walkers, diffusion, and chaos were used more in students' final concept maps. In addition, several other misconceptions about randomness and probability were examined.

WHY DID STUDENTS CHANGE THEIR CONCEPTS?

It was hypothesized that students changed their concepts of probability and randomness because by using simulations they could learn more concepts, links, and examples that related to the concepts of probability and randomness. Students changed their ideas about probability and randomness because they thought that their old concepts and reasoning were not appropriate to their new experiences or situations.

Twenty six students gave reasons why they changed their concept maps. Seventeen students said that they understood probability and randomness better by the time they completed their second maps. They had a better grasp of what probability and randomness were. They said:

> . . . the second is more detailed. It contains permutation and combination which I did not know about when we stated the unit. I believe it was a <u>better grasp</u> of what randomness and probability are.

> . . . my second concept maps showed a greater understanding and a number of specific events. . . I was able to create a more detailed concept maps because I had developed a <u>greater understanding</u> of randomness and probability through the unit.

Seven students used the term "have learned more" beside "better understanding" to explain why they changed their concept maps.

> . . . I had more freedom over the last one, . . . , of course I had <u>learned more</u> about probability.

. . . I added a lot more concepts and connections, these were added because from the time we made the first map to the time we made the last, we learned and explored a lot more new ideas.

Some students expressed better their understanding of probability and randomness in four classifications: (1) They understood more details; (2) they knew more examples; (3) they understood more ideas, concepts, and relations; and (4) they understood the error concept.

. . . my second concept map is much more detailed than my first. I have revised some previous notion which have been shown to be incorrect after our rigorous study of probability and randomness. . . I have also added many new concepts and terms to my second concept map which I have picked up and learned about since my first map. . .

. . . I changed my concept map because my first one didn't have enough examples and also wasn't very clear.

. . . on my first map the only relationship involving randomness stated that nothing is random. During the unit, we learned that even coin flipping and dice-rolling are random as to how their outcome is. On my new map even probability is connected to random, along with things like fractals, coin-flipping, and ice rolling.

From fieldnotes and students' learning logs, it was discovered that some students changed their old ideas because they were inconsistent with their former ideas. Their old ideas were wrong or their old ideas were not supported by their experiments or their computer simulation results. For example, before the experiments most students said that the probability of getting two heads and of getting three heads in flipping 10 coins are the same. Some students said that the probability of getting two heads are bigger than of getting three heads. After experiments and preparing a histogram, students understood that the probability of getting three heads are greater than the probability of getting two heads.

Before doing electrodeposition, the teacher asked students to predict what kind of shape will occur in the experiment. Most of students said: "The shape will be circle around the wire, a pattern of atomic charge, an indefinite pattern, a more random shape." In their observation, students saw that the shape was fractal. They measured its dimension and found that the dimension was not an integral number. They became aware that their old ideas conflicted with what they saw in their experiment.

In the diffusion experiment, students were asked whether molecules move randomly or ballistically? If they move ballistically, then the distance and time when the reaction happens will be linear. If they are random, their time and distance are not linear. The problem was:

> Suppose we were using the program to model the gas experiment you did recently. By having one side take steps of size 2 and the other take steps of size 1, you can have one set of molecules move twice as fast as the other. Describe below how to predict both WHERE the chemical reaction will occur, and WHEN. Then test your predictions using the model.

In their learning logs several students wrote:

> If the motion were ballistic, the reaction would take place 1/3 of the way from the side with the particles moving one step at a time, and 2/3 of the way from the other side. Since both sets of molecules move randomly, the random motion is negligible and the same result as above occurs. We still cannot figure out how to predict the time.... In the computer simulation, we found that the reactions occurs approximately .9 away, on a scale from 0 to 1, from the starting point of the gas moving twice as fast.

Some students wrote:

> Our guess of when the reaction begins, was really off. It only took 35 steps for the first collision to take place, as opposed to our guess of 4096 steps for the average molecule to collide.

It is clear that some students experienced that their predictions or their old ideas were different than their experiments. These differences encouraged students to reform their old concepts.

Summary:

Improved understanding was the principal reason that students changed their ideas about probability and randomness. Because they had learned more, they understood better. Better understanding was showed in their detailed and clear concepts, their awareness of error, and their new content knowledge. Students changed their old ideas because they experienced that their old ideas were wrong and not appropriated with their experiments or computer simulation results.

HOW DID STUDENTS CHANGE THEIR CONCEPTS?

By examining the changed concepts, it was evident that there were three methods by which students changed their understanding of probability and randomness: (1) by adding new concepts, new propositions, and new examples; (2) by eliminating old concepts, old links, and old examples; (3) by changing their reason in the multiple choice questions. For example, in comparing the pre concept map (Figure 5) and post concept map (Figure 6) it could be noted that:

1. New concepts were added in the second map such as "fractals", "nature coastline", "knowledge", "financial decisions", "death", "coin flipping." New links were added in the second map such as "is controlled by", "is affected by", "is changed each time you gain more", "described things which are", "are born out of."

2. Old concepts were erased from the first map such as "nothing", "heredity", "temperature of the school", and "X is arriving on time to math."

Some students changed their ideas or at least became more confident in their ideas after doing experiments with computer simulations or hands-on activities. Students predicted what would happen with their experiments. Some students became more certain when their experiments were the same as their predictions. Some students changed their predictions when their experiments differed from their predictions. For example, students changed their predictions about a thumbtack's probability and became sure about their predictions.

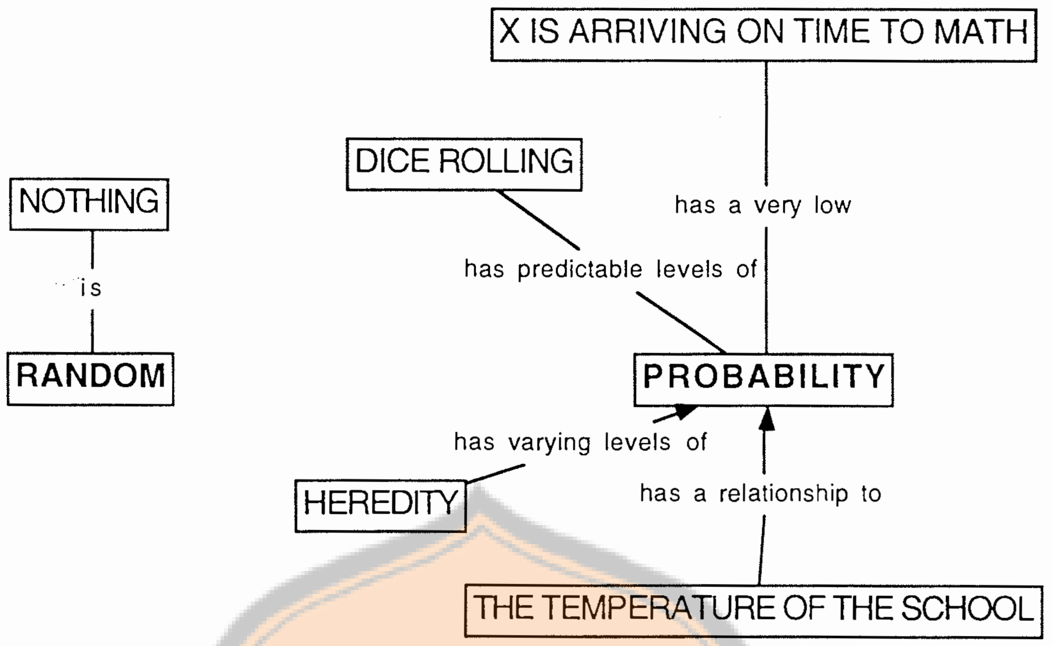

Figure 5. B's pre concept map of probability and randomness.

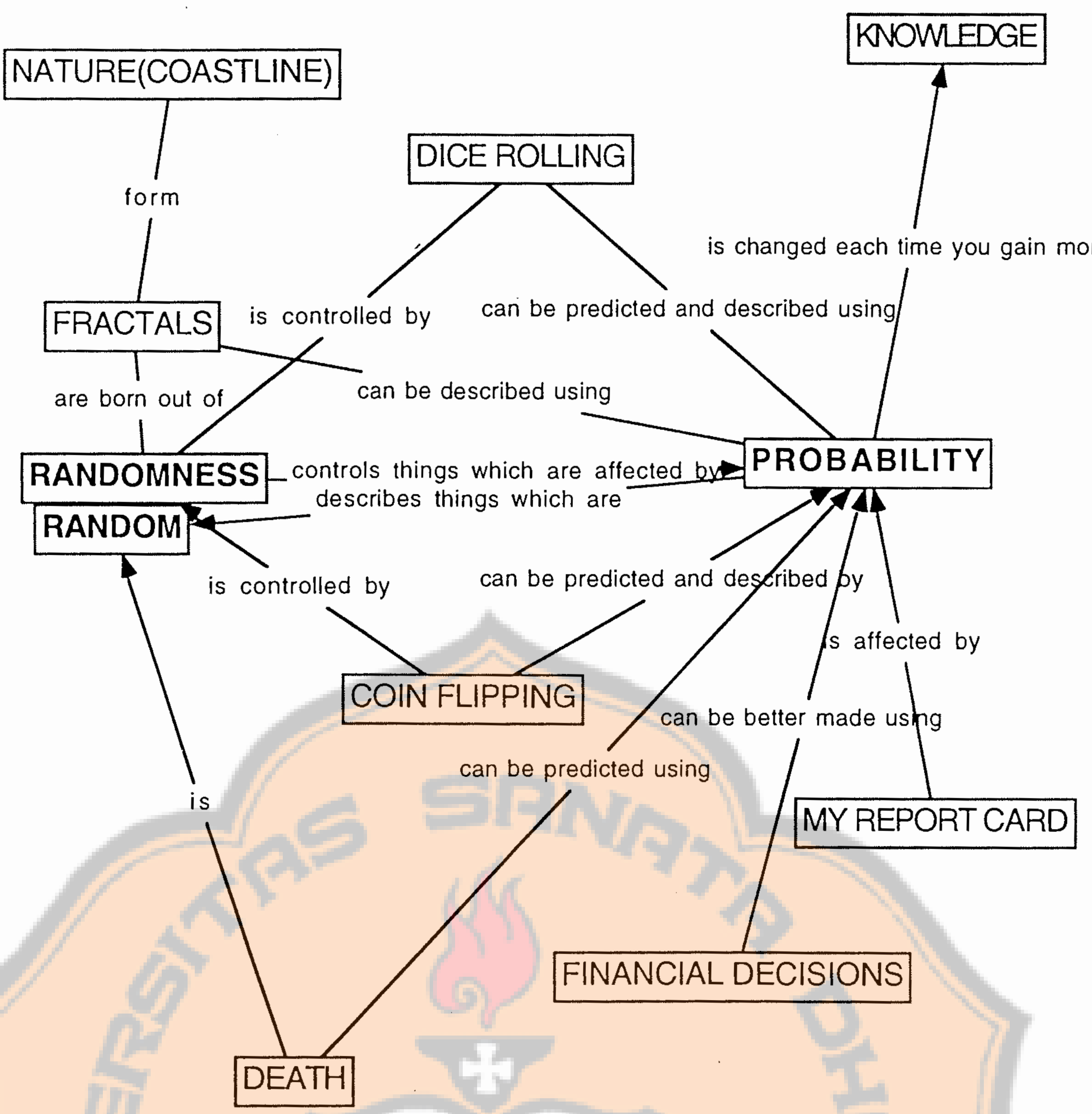

Figure 6. B's post concept map of probability and randomness.

Students were asked to predict the probability of flipping a thumbtack and to check their predictions with experiments. In their learning-logs some students wrote:

> The probability is relatively equal because a thumbtack is apparently stable in either position, face down and face up. Therefore it is equally likely that it will face up as it will land face down. Probability is then 1/1. We <u>have revised our probability</u> based on further knowledge to 7/10. Meaning that 7 times out of 10 the tack will land heads up. We decided on this probability because the tack is not like the coin in that both sides are identical, therefore the tack is more arbitrary. <u>After testing the tack many times we have concluded that 7/10 is a more reasonable probability</u>.

Completions the simulations made some students more certain about probability levels. They predicted that the probability of flipping a thumbtack was 0.5 and found in their experiments that the probability was 0.5. They wrote:

> We feel that the probability is around 50%. We flipped a tack <u>many times</u>, and <u>we found that it landed pin up about half the time and point down half the time</u>.

We believe that this event is random in the sense that the outcome has many factors and is unpredictable. We believe that whether the tack lands up or down depends upon the angle at which it hits the table, the number of bounces it takes, and the surface of the table. If we had to assign a probability to it, we would make it 1/2 for both up and down. We <u>came to the conclusion because we did a trial and found the outcomes to be approximately</u>.

There is 50/50 chance of getting either outcome. We tried the experiment several times with 10 tacks each time. Usually we got the above stated probability. <u>After doing some more, we found that almost always</u> if the pin was dropped pin down it landed pin up while if it was dropped pin up if landed pin down. So the randomness in the problem could be said to lie in how you drop the pin just like you drop a coin.

After using the Many Walker program and discussing results with their teacher in learning logs some students experienced conceptual changes. Students were asked to predict the values of the average x, average |x|, and average squared x after W walkers take N steps. Most students predicted the average x zero and they experienced in the computer programs the same. Some students changed their predictions on the values of average absolute x and average of squared x. For example, Students: The value of average x will remain around 0. The values of average absolute x and averaged squared x steadily increase with the number of steps but are bounded by the number of steps.

Teacher: You stated that the average x remains around zero which seems sensible. You also claimed both the other measures are bounded by the number of steps taken. I can agree for the |x| but why for the x^2? It seems to me that squaring 7 steps can give 49! Why do you feel it won't exceed 7?

Students: We were incorrect. The average x^2 is bounded by the number of steps.

Another example,

Students: As N increases, average x stays near and around 0; average |x| increases slowly, and average squared x increases faster than absolute x. As W increases, there is no substantial effect on the average values.

Teachers: . . . You have noted that average x stays close to 0. Can you predict values for the others (say, after 250 walkers take 25 steps). Can you think of others ways of measuring dispersion?

Students: The maximum will be 25 for |x|.

Teachers: You predict average x = 0. Average x^2 = ??? Average |x| = ?? (One of these two is not that hard to predict. Study the many Walkers program a bit more!)

Students: The average |x| is hard to predict, it seems to increase steadily, about .1 every step, after 25 steps it is around 4 and 5. The <u>average squared x after 250 walkers taking 25 steps is around 25</u>.

Summary

Most students changed their concepts by adding new concepts, relations, and examples in their second concept maps, eliminating old concepts, relations, and examples from their first concept maps, and changing the reasoning in the why-questions more exactly, clearly, and completely.

Students changed their concepts after doing computer simulations several times. They understood better the patterns which appeared in the computer simulations. In addition, students changed their ideas after discussion of their predictions with their teachers in their learning-logs.

WHAT DID STUDENTS THINK AND EXPERIENCE ABOUT THE COMPUTER SIMULATIONS

It was hypothesized that the computer programs were helpful and useful for students to understand better probability and randomness. Students seemed to like the programs and changed their ideas about the use of computers in learning science.

• QUESTIONNAIRE DATA

From the questionnaires about the computer programs, all students (31) wrote that most of the programs were useful for them to learn probability and randomness. All the programs were interconnected. Each program covered different aspects of probability and randomness. Each program taught different things, and helped them to understand the concepts of probability and randomness more fully. Programs like fractal coastline and diffusion linked the course to the real world. A student explained how he was helped by the visual aspect of the computer programs:

> . . . They were useful in learning about randomness and probability because they gave me a visual aid to support my understanding. I am a visual learner, so they were of great use to me. The different movements and results of the programs were easy to understand and very explanatory.

Fourteen students said that "Many Walker" was the most useful, followed by "Coin flipping" (9), "Fractal Coastline" (9), "Diffusion" (9), "Fractal Dimension" (8), "1-D Walker" (7), and "Pascal Triangle" (2). Only two students said that "Pascal Triangle" was useful.

According to some students, Many Walkers was quite useful because it used many numbers of random walkers, gave them a realistic picture of randomness. It was good for visualizing and interpreting general patterns of randomness. It illustrated the concept of randomness in a simplistic way. The outcomes determined by randomness were clear, so students could observe general trends of the random walkers. A student wrote:

> Many Walkers is very useful because with a display of such a high number of walkers on screen at once, you could really see how the individual movement was random, as well as see how the average of everyone could be predicted.

Some students said that coin flipping provided a good starting point for learning probability and randomness because of its simplicity and concreteness. Coin flipping, Fractal coastline, and Diffusion place the concepts of randomness and probability into real world situation. Fractal Coastline showed the randomness of natural occurrences and helped students to form an image of fractals and to manipulate and derive data from them. The Diffusion program showed how random events actually can be predicted like the movement of gases. It showed some concepts of randomness and how events actually random. "Molecules actually do move randomly and I was able to understand this," said a student. Fractal Dimension was useful because it showed the randomness of natural occurrences and how to calculate fractal dimension. Most programs illustrated simple concepts in a very interactive way and with a nice visual interface.

From questions C.4: "Suppose that you are asked to teach and explain probability and randomness to your friends in an other school, which programs would you want to use," seventeen students said that they wanted to use Many Walker, followed by Fractal Coastline (10), Coin flipping (9), Fractal dimension (9), Diffusion (7), all programs (6). Only 2 students would use Pascal Triangle.

Some students chose all the programs because they all gave different views of probability and randomness. They illustrated a different aspect of probability and randomness. Seventeen students chose Many Walker because it best illustrated randomness, displayed basic probability, and was simple. It was interesting and easy to understand. It showed general trends in repeated events. It demonstrated random walk, and faster than the other. It taught students a lot about probability and randomness in a clear way.

Fractal Coastline, Fractal Dimension, and Diffusion were realistic, entertaining, interesting, clear, and easy to use and understand. They were fun and illustrated how randomness worked. People would enjoy using the program in order to learn. They allowed involvement by the students. Coin flipping was basic for all the other programs. It displayed the basis of probability and randomness.

- **FROM LEARNING-LOG AND EVALUATION**

In most learning-logs students wrote that the teaching-learning using computer programs was enjoyable, fun, and a nice variation from the normal classroom routine. The computer programs presented a visual model to follow, were interesting, and became an effective way to teach probability and randomness. The computer programs helped students to understand concepts of randomness and probability. In addition, the programs allowed students to explore. Students wrote:

> I have enjoyed this unit a lot. I find it a nice variety to be able to look forward to more than just the classroom. It is nice to look forward to finding an interesting question or problem in the log to do. . . I would like to continue doing some more stuff

like this as the year winds down because the variety is a nice change.

I found this work a whole lot more interesting. I enjoy working in groups and using the computer. They are both helping me understand and want to learn more. With visual aid and a group that has really helped me to understand, I feel like a much more competent math student.

This was a very effective way to learn how to explore different ways of finding solutions and to help us understand math from a different perspective. . . this was a nice compliment to our classroom activities, and we all enjoyed it.

In the last evaluation about teaching-learning using computer programs, students explained their experiences with the programs. Twenty students said that they had positive experiences. They liked the teaching-learning method because it was fun, refreshing, and exciting. Students experienced that computer programs were helpful to understand the concepts. Some students liked the programs because of their visual representations of events. It was much better than only listening to a teacher. Students discovered themselves. Some students said:

I really liked this method. It was a refreshing change from coming to a classroom and listening to a teacher for the whole period. This way we got to explore things ourselves and with others.

Tell me, I forget. Show me, I remember. Involve me, I understand. I think this definitely helped me solidify some core knowledge on probability.

I liked using the computers often because they enabled us to discover for ourselves what would have been taught to us otherwise.

I really enjoyed this unit. I loved working with my peers to know how to communicate with me and explain to me in a way that I would understand. I liked working at the computers. They made learning a lot more exciting. I understood this unit better than I have often understood many other things.

Summary:

Most computer programs were useful and helped students to learn probability and randomness. Many Walkers best illustrated the concept of randomness. Coin Flipping was the most simple and the basis of all other programs. Fractal Coastline and Diffusion illustrated randomness and probability in the real world.

The computer programs were exciting, fun, enjoyable, and refreshing. They helped students to understand better. The programs involved students to learn actively and discover for themselves, instead of only listening to the teacher. Most students liked the programs and the teaching-learning using computers.

SUMMARY

This chapter presented the results of conceptual change studies on probability and randomness. Students improved their knowledge of probability and randomness. Students also changed their ideas of probability and randomness in their concept maps and also in their posttest multiple choice responses. They changed their understanding of probability and randomness, explained the concepts more exactly, explicitly, and clearly through statistical terms. They changed their ideas by adding new concepts, examples, and propositions, and by eliminating the old concepts, links, and examples.

Most students changed their ideas because they understood the concepts better. This happened because they had learned more. They knew more concepts, some details, and saw some of their errors.

Students changed their old ideas because these ideas were not supported by new experiences.

Most students said that the computer programs helped them to understand probability and randomness. They liked the teaching-learning method using computers. The method was more fun, exciting, and helped them to discover things for themselves.

CHAPTER 5
DISCUSSION OF THE RESULTS

This final chapter reviews the findings of the study. The results of the study are summarized and some comments are presented. The implication of the results in theory, research, and practice of science and mathematics education, especially in teaching-learning probability and randomness, is explained. Some recommendations for future study are proposed.

OVERVIEW OF THE RESULTS AND COMMENTS

This study investigated high school students' conceptual change on probability and randomness after the students ran computer simulations. More specifically the research examined: (1) whether students' understanding of probability and randomness improved, (2) what students' concepts were changed; (3) why students changed their concepts; (4) how students changed their concepts; and (5) what students thought and experienced about computer simulations. Based on the results in chapter 4, the researcher summarizes the findings of the study in a schema as in Table 7. The findings of each question are discussed one by one.

Table 7. RESEARCH FINDING SCHEMA

QUESTIONS	RESULTS	COMMENTS
1. Did students' understanding improve?	- significantly difference p = .0002 for multiple choice p = .0001 for concept maps (significance level = .05) - time vs. scores is not significant	- Students' understanding improved. - equal time on task
2. What concepts were changed?	Several concepts were changed: - existence of randomness - third category of randomness - unpredictable randomness - independent randomness - classical and frequency probability - degree of probability (terms) - relationship between prob. and random - expansion of concepts	- Computers improved changes. - strong and weak changes - Some did not change because they are correct. - Some did not change because they are difficult.
3. Why did students change their concepts?	- better understanding - learned more details, examples, concepts, relations, errors. - old ideas are wrong. - dissatisfactions/anomaly.	- improved contents - dissatisfaction with new experiments and situations
4. How did students change their ideas?	- adding, eliminating, changing concepts, links, and examples. - changing "why" reasons. - experienced anomaly: in experiments, discussing with teacher and friends.	- assimilation - accommodation
5. What did students think and experience computer simulations?	- useful for study and improved their knowledge. Many Walkers is the best for learning randomness and probability. - Students enjoyed, were interesting, discovered for themselves.	- Computers can be used for learning random and prob.

• **STUDENTS' KNOWLEDGE IMPROVEMENT**

The difference between students' posttest and pretest knowledge was statistically significant. It indicated that students improved their understanding of probability and randomness.

The result indicated that the computer simulation programs were able to help students improve their understanding of probability and randomness. The result was similar to Sterling and Gray's (1991) study about the effect of computers on understanding introductory of statistics. Sterling and Gray showed that computer simulations improved college students' knowledge of introductory statistics. In this study, computer simulations affected students' understanding of probability and randomness in the high school level.

Based on these results, the computer simulation programs could be used as a means of learning-teaching probability and randomness on the high school level. The result fulfilled the suggestion from Garfield and Ahlgren (1988) and Shaughnessy (1983) to use activities that allowed students to explore data. The computer simulations allowed students to explore, collect data, analyze data, and to make conclusions.

The computer simulation programs emphasized the concept of probability and randomness. They did not stress mathematics' calculations or formulas. So the programs could be used for non-mathematics majors as well as for mathematics majors. Science major students could also use the programs.

The result showed that probability and randomness could be learned in an interdisciplinary course. Usually, probability is studied in mathematics courses as a part of mathematics. In this program, probability and randomness were studied through physics, chemistry, biology, and mathematics as well. In addition, the probability and randomness were studied as a part of fractals in nature, a new topic that is relevant to many disciplines.

Twenty-two students increased their scores, five students decreased their scores, and four students had the same scores in the multiple choice test. In the concept map test, thirty students increased their scores and only one did not. It seemed that more students increased their scores in the concept map test than in the multiple choice test. The reasons could be: (1) students had more free time to express their ideas in the concept maps than in the multiple choice test; (2) some questions in the multiple choice test still were difficult for students.

Why were the computer time on task and students' scores not significant? That the length of time using a computer did not clearly have an effect on the students' scores could be analyzed below.

1. Students run computer simulation programs more or less in the same time length, in the range of 15-45 minutes per lab. So the length for each student was about the same. That is apparently why computer time did not have an effect.

2. The intelligence and ability of the students was diverse. Some students needed little time with the computer, while other students needed a longer time.

3. Students have different learning styles in working and studying. There is no single most-effective method in teaching-learning (Claxton, in Osborne, 1993). Students who did not like the programs might learn better in different ways and methods. For them, time of use had no effect.

- **WHAT CONCEPTS WERE CHANGED**

Concepts of randomness and probability were changed. Some students changed radically, some not at all. Some students did not change, because their old ideas were right or because they did not know the right one.

Some students substantially changed their ideas on randomness, probability, and relation between randomness and probability. They restructured their ideas and did reconceptualization (Dykstra et al., 1992). They accommodated their old ideas, not just assimilated with the new situations (Posner et al., 1982). In some ways, the radical restructuring was consistent with Kuhn's paradigm theory in which students had to find a new paradigm because the old paradigm did not work. Some concepts of randomness and probability which were significantly changed were: "randomness is unpredictable" changed to "predictable", "nothing is random" became "their is something random", "there is a third category of not very random" became "there is no such category", "there is no relation between randomness and probability" became "randomness describes and predicts probability." Most students also changed their probability calculation in problems #'s 9, 11, and 19. These problems were about probability of a war, the same birthday in school, and flipping six coins. They changed their answers because they were familiar with data that were needed to solve those problems.

Some students only changed their ideas weakly. They made a differentiation or class extension of the old ideas (Dykstra et al., 1992). According to Posner et al. (1982), they assimilated their old ideas but did not accommodate them. They expanded their ideas because they had more information to be included in their old ideas. The expansion of ideas of probability could be classified in some categories:

1. Their new ideas became more exact.
2. Their new ideas became more mathematical/statistical.
3. Their new ideas became more specific and clear.
4. Their new ideas had new concepts and relations, so they were more complex.

Some students did not change their old ideas of randomness or probability. They said the same thing in their reasoning or they chose the same number in the multiple choice questions. There were two phenomena in this unchanged situations:

1. Students did not change their idea because it was right and they did not need a new complement for the idea.

2. Students did not change their idea because they did not know that their old idea was wrong and they did not know the right one.

Most students did not change their concepts on problems #'s 3, 12, 15, 17, because they answered correctly on the pretest. The problems were about the meaning of randomness (#3), probability of flipping a coin (#12, #15), and probability of marbles (#17). Problem #3 asked students to decide whether flipping a coin is random because it has only two possible outcomes. Problem #12 asked students to calculate the probability of flipping a coin and in problem #15, students used randomness for guessing future events in a coin flipping. In problem #17, students calculated the probability of drawing a yellow marble from a container with three red, three yellow, three white, and three blue marbles. It seemed that most students had known the concept or the meaning of randomness and probability of flipping coins before the programs. They understood how to calculate the probability of a coin flipping and a marble drawing. It could be they had instruction on the topic in elementary school or junior high school. It could be they knew from their everyday life experiences. If so, it was consistent with Konold's finding that students had preconceptions on probability and randomness.

Most students did not change their answers in problem #1, even though they answered wrong. The problem was about independent probability. Most students said that the probability of persons smoking and getting cancer were higher than persons getting cancer. Most of them gave the same reason: smoking causes cancer. They did not understand that the problem was one of independent probability. According to the independent probability, the probability of persons smoking and getting cancer is lower than the probability persons getting cancer. Kahneman and Tversky explained that students used available heuristic thinking in the problem. Students made decisions on probability based on the available data. In this case, students recalled that some people who got cancer were smokers. Students also thought of the causality that smoking causes cancer. According to Tversky and Kahneman and Shaughnessy, it takes time to change students' misconceptions. Beliefs and concepts are slow to change (Shaughnessy, 1977). Sometimes, the misconceptions were unchangeable for them. In addition, in the computer simulation programs, the specific misconceptions were not directly addressed, so students might not make association between the misconceptions with what they were doing in the computers.

- **WHY STUDENTS CHANGED THEIR CONCEPTS**

Students changed their concepts because they understood better and had learned more about randomness and probability. Their better understanding was showed in their detailed concepts, clear concepts, new concepts, and awareness of their errors. They changed because they experienced dissatisfaction or an anomalous situation. with their new experiments.

Students changed their ideas about randomness and probability because they understood better. They understood more concepts, relations, and examples, so they could build different concept maps or they answered differently in the posttest. They understood the ideas of probability and randomness more clearly, in more detail, and more exactly, so that they could explain the ideas more clearly, exactly, and with greater complexity. They understood that some concepts were wrong, so they changed or revised the wrong concepts. For example, they understood that "the third category of randomness" was wrong, so they changed in their final test by saying that "there is no such a third category."

Students experienced dissatisfaction and anomalous situations (Posner ét al., 1982), or cognitive dissonance (Novak, 1977) or contradiction experiences (Hirsch & O'Donnell, 1993), so that they were encouraged to change their ideas. The dissatisfaction and anomalous situations were found in teacher's questions, data from computer simulations that were different from students' understanding or predicting. The anomalous situation encouraged students to change their ideas, so that the new concepts were appropriate to the new situation. The result was similar to the Posner and Chinn theory about strong conceptual changes.

The dissatisfaction was also discovered in discussion with other students, either in group or in classroom. In discussion, students made a comparison between their ideas and someone else's ideas. They changed their ideas, when they found that the majority of students had different opinions. The results indicated that groups where students were learning were very important. Like Cobb (1994), the results indicated that knowledge was constructed individually and socially. The interaction in the classroom had an effect in the construction of students' knowledge.

Why did some students change their ideas and some students not change their ideas? According to Chinn (1993), people were free to face the anomalous situation. They can accept the new data and change their old ideas, or they can reject the new data and not change the old ideas. Some students did not change their error ideas because they did not know that their ideas were wrong. They still thought that their ideas were correct. In this situation, students did not experience dissatisfaction or cognitive dissonance.

- **HOW STUDENTS CHANGED THEIR CONCEPTS**

Students changed their ideas by adding concepts, links, and examples; changing concepts, links, and examples, and erasing old concepts, links, and examples. They changed their reasoning in open questions. They changed through assimilation and accommodation.

The result showed that some students changed their ideas by changing concepts, links, and examples, and erasing old concepts, links, and examples. Some of the changes were strong conceptual changes. It was an accommodation, where students felt that their old ideas in their concept maps had to be changed (Posner et al., 1982).

Students changed their ideas by adding new concepts, links, and examples. Some of the changes were assimilation. They expanded the meanings, fulfilled the ideas more exactly, more specifically, mathematically, and statistically. This changing process was consistent with schema theory because students restructured their knowledge by adding schema or developing new conceptualization from the old schema. Some students changed their reasoning in the posttest more exactly, clearly, and completely.

Students changed their ideas of probability and randomness when they felt dissatisfaction with what they previously thought. The dissatisfaction happened when they discussed their ideas with their teacher. The questions from the teacher increased doubts in their mind and made them change their old ideas. The discussion with others sometimes made them uncomfortable with their old ideas. The situations provided conditions for changing.

- **WHAT STUDENTS THOUGHT ABOUT COMPUTER PROGRAMS**

Students thought and experienced that computer simulation programs were useful and helpful in studying probability and randomness. They liked the programs because they made class more fun, more enjoyable, and more exciting.

The study had different results from what Baturo (1993) found in his Australian study. Baturo found that students did not have a purpose for studying probability and did not enjoy probability lessons by using hands on activities. On the contrary, this study found that most students liked the computer programs and learning probability and randomness through the computer simulations. Students stated that the programs helped them to understand better probability and randomness. Students felt that the classes were exciting, fun, enjoyable, and refreshing. Most students said that they discovered themselves engaged in their own learning, instead of listening to their teacher.

Schroeder (1983) studied how computer games could be useful helping elementary school students learn probability. In this study, computer simulations, which had game aspects, helped high school students learn probability and randomness.

Some programs were more useful and interesting for students like "Many Walkers", "Coastline Dimension", and "Coin Flipping." One program - "Pascal's Triangle" - was not fun and not useful according to those students. According to students, Pascal Triangle was boring, not concrete compared to the other programs, and it just reiterated what many walkers explored. The diffusion program was difficult for some students and was a little confusing because it was not very well explained. Some students who did not take chemistry course said that diffusion was not useful for them, but some students who took chemistry course said the contrary. In this case, it seemed that students who took only mathematics thought exclusively that they should only learn mathematics.

Six students preferred the lecture method, twelve students preferred computer methods, and eleven students liked to use both lecture and computers. Two students who preferred lecture said that they liked to study alone. They said that computer simulations spent too much time and they could not grasp as many matters in the course as in a lecture. They said that in a lecture things are to the point and you can ask questions when you do not understand. It seemed that the students considered learning as collecting and recalling data which teachers gave. This type of learning has a different basis from the computer simulation programs: i.e. learning is active and students construct their own knowledge.

IMPLICATION OF THE RESULTS IN THEORY, RESEARCH, AND PRACTICE

- **THEORY**

The results of the study support Ausubel's theory of learning. According to Ausubel, there are two types of learning: meaningful learning and rote learning. Meaningful learning is a process in which new information is related to an existing relevant aspect of an individual's knowledge structure. During meaningful learning, new information is assimilated into an existing relevant knowledge structure in the individual cognitive structure. New meaningful learning results in further growth and modification of an existing knowledge structure. If the relevant concepts do not exist in the cognitive structure of an individual, new information must be learned by rote. In rote learning, new information is not associated with existing concepts in cognitive structure. Rote leaning is necessary when an individual acquires new information in knowledge areas completely unrelated to what he already knows (Novak, 1977; Ausubel et al., 1968). The results of the study indicate that students changed their existing concepts when they gained new information. The changing of the concepts indicated the assimilation process in meaningful learning.

The results of the study support the schema theory of learning. According to the theory, knowledge is stored in a schema that comprise student mental constructs of ideas. In learning process, students restructure their knowledge by adding schema or developing new conceptualization from the old schema (Jonassen et al., 1993). This study showed that students developed their schema about probability and randomness either by radically changing their old schema or by adding new concepts to their old schema.

The results of this study are consistent with the conceptual change theory. According to the theory, conceptual change would be encouraged by anomalous situations (Posner et al., 1982) or cognitive dissonance (Novak, 1977). The computer simulations in the study provided some anomalous situations that promoted conceptual change. The study also showed that there was strong conceptual change and weak conceptual change. There was accommodation process and assimilation process in changing. Some students changed radically in some concepts, but some students just developed their concepts, but not radically.

The study maintained that a constructivist theory of learning was in operation. According to the constructivist theory, learning is an active process where students construct their own knowledge. Computer simulation programs allowed students to be actively involved in their own learning. Students made predictions, collected data, analyzed data, and made conclusions. Students got new knowledge in the analysis and conclusions. They felt that they found their own knowledge, not just listened to their teacher. The study also indicated that constructivist theory should be individual and social. The programs were run in groups of three. In groups they discussed and compared what they found. It seemed there were effects of the groups in the construction of individual knowledge.

The study also added a question about the different style in students' learning. Even though only a few students did not like the methods, there were some. This difference is consistent with the belief that there is not only one learning style. That's why even though many students were helped by the computer simulation, there were two who were not helped. They needed a different approach.

Bruner (1977) explained the importance of intuitive thinking in learning science and mathematics. Intuitive thinking is characterized by not advancing in careful and well defined steps. It involves an implicit perception of the total problem. Intuition is immediate apprehension or cognition. It implies the act of grasping the meaning and it looks the global. Guessing may develop intuitive thinking. Thus, he suggested that guessing is given place in the learning process. The result of the study supports the idea of Bruner in the sense that students always have an opportunity to guess or predict what will happen with their experiments and computer simulation problems. In the beginning of experiment students were asked to predict what will happen.

Inhelder explained why the teaching of probabilistic reasoning is hardly developed in our educational system before college. It is because most school syllabi followed scientific progress with deduction and analytic. It also depends on the belief that the understanding of randomness phenomena depended on the learner's grasp of the meaning of the rarity or commonness of events. Her research indicated that the random phenomena requires the use of certain concrete logical operations within the grasp of the young, free from mathematical expression. Games involving a guessing distribution of outcomes are all ideal for giving the child a basis to grasp the logical operations needed for thinking about probability. In such games, children will discover an entirely qualitative notion of chance (Bruner, 1977). The computer simulation programs which allow students to guess and stress on the concepts than mathematical expression are appropriate to Inhelder's suggestion for study probability and randomness.

- **RESEARCH**

The study fulfilled Shaughnessy's suggestion in his review of research on stochastics. This results add a study on probability and randomness on the high school level in the United States. It also added research on the effect of computer simulations on probability teaching on the level of high school. Computer simulation programs improved students' understanding of probability and randomness.

To study conceptual change, researchers should evaluate several steps of a student's concepts, not only the beginning and the end of product. Researchers should study the process of development. As Vygotsky and Sakharov suggested: "research should study the development of concepts and not finished products" (van Der Veer, p.259). To accomplish this ideal, further research should evaluate more than two concept maps of each student.

In this study, students learned probability and randomness through computer simulation in groups of three. Some students experienced group study as helpful. They learned from friends and they were encouraged by friends. Others were not helped by working in group. Further research on the effect of group study, especially with computer simulation programs on the conceptual change of students, should be done. Is group study better than individual study in the computer simulation programs? Comparison of the individual versus group study is needed to decide which one is more helpful in practice.

- **PRACTICE**

The study showed that the computer simulation programs could be used as a means to teach and learn probability and randomness. Beside the programs improving students' knowledge in probability and randomness, the computer programs provided learning situations which are more fun and interesting. The programs could be used for non mathematics majors as well as mathematics majors because they emphasize the concepts of probability and randomness, not the mathematical formulas. The students would like the programs and would be interested in the probability course.

The programs could be used in an interdisciplinary course. Probability was not only studied in mathematics courses, but in sciences such as biology, physics, and chemistry. They could be studied in social sciences courses. The interdisciplinary approach is suggested by the Association for the Advancement of Science in the "Project 2061: Science for all Americans."

There are many styles of learning. Some students like to learn through computers, some students learn better through lectures, others through reading books, etc.. There is no one style of learning that can satisfy all students. So, to improve the individual's need, in practice, the computer simulation programs should combine with another approach such as lecture, homework, reading assignments, and so forth.

According to students, learning logs were very helpful in the study. Students felt that they learned probability and randomness from questions, directions, and comments from their teacher in the learning logs. For giving the result more individually, students should make their learning logs individually. Even though they work together, they should all write their learning logs individually. For giving maximum encouragement, the questions, directions, and tasks in the learning logs need to be well prepared.

The computer simulation programs need teachers' creativity. Teachers should understand how to run the programs. They should understand how to help their students in running computer programs, especially if the computer crashes. The teachers should construct questions, directions, and comments on the students' learning logs. Teachers should spend time reading and commenting on students' learning logs. A workshop for teachers who want to use the computer simulation programs in their classroom should be provided.

To expand the use of the computer simulation programs more widely, the programs should not be limited to the Macintosh computers. Many schools outside the United States especially in South-East Asia, use PC as well. So the programs should be written in other computer languages if we want to help and influence many high school students around the world.

RECOMMENDATIONS FOR FUTURE STUDY

The findings of this study provide a basis for identify additional research questions. These are itemized in the following paragraphs.

1. A comparison study between the effect of hands-on activities and hands-on activities with computer simulations to students' knowledge improvement should be done. Are the hands-on activities with computer simulations better than the activities only for learning probability and randomness? One group of students using hands-on activities and an other group using hands-on activities with computer simulation programs can be used as samples. The study could identify some aspects of the effectiveness of computer simulations.

2. The next study could be a comparison between group and individual work with the computer programs. Is there any different between using the computer simulations individually and in a group? Some students like to study alone, some like to study in a group. The study could help to decide whether the computer programs are better to be used individually or in groups for learning probability and randomness. The study can be combined with a study about the effect of students' learning style on using computer simulation programs. Students have different learning styles. Does the different style influence the computer use? This study will help educators to choose whether some students should learn using computer programs or different methods.

3. A study to determine the relationship between each computer programs should be done. The effects of each program to the concept improvement can be studied. This study could help teachers and educators to choose several programs which are more useful and have better effect in improving students concepts.

4. Another study should include interviews of students to investigate the reasons why each student changed their concepts. By asking the students more freely, we can find more data about the process and the reason of their conceptual changes. We can trace students' reason in each concept change.

5. Are the computer simulation programs helpful for middle school students in studying probability and randomness? How can the programs be adapted to the level of middle schools?

6. A study among teachers would be interesting. Do the computer programs improve teachers' conceptual changes of randomness and probability? How can the computer programs be effectively used by both teachers and students? What kind of interrelations among teaches and students will have the best effect in learning process using computer simulations?

7. The computer programs are built in the western culture and thinking. Would the programs have the same effect for students from

different cultures. The comparison among them will help in generalizing the impact of the computer programs for learning probability and randomness.

SUMMARY

The study developed step by step. The researcher's curiosity attending a fractals in nature seminar at Boston University one and half years ago raised some questions about the effects of computer simulations programs on students' understanding probability and randomness. Examination literature identified specific questions for this study. The main question was to find whether students experienced conceptual change on probability and randomness after they used computer simulation programs built by Boston University Center for Polymer Study. The result of the study indicated that students experienced conceptual changes. Specifically, students improved their knowledge on probability and randomness. They changed their concepts of probability and randomness. They changed their ideas because they better understood and they experienced cognitive dissonance. They changed their ideas by adding concepts, links, examples or replacing the old concepts, links, and examples from their concept maps. They changed their ideas by changing their reasoning in the posttest. They liked the computer programs and they enjoyed the activities.

This chapter has described the results of the study and some analysis of the results. The results indicated some theories about conceptual changes: why students changed, how they changed, and the kind of changes; such were emphasized in the study. The results showed that constructivist theory and schema theory provided good basis for active learning and conceptual change. The results also strongly indicated that computer simulation programs which use scientific methods could provide students' changes. Some implications of the study on theory, research, and practices in science education have been discussed. In addition, several recommendations for the future research are listed.

APPENDICES

INSTITUTIONAL APPROVAL FORM APPENDIX A

XXX HIGH SCHOOL

January 11, 1995

For two years the Mathematics Department of XXX High School has used curriculum materials in classroom. The curriculum materials consist of computer simulation programs created by the Center for Polymer Studies, Boston University. How effective and useful the computer programs for students to understand the concepts of random events, probability, and fractals need to be evaluated. A systematic study on those materials is necessary.

I welcome the opportunities of the study to be conducted in the spring of 1995. The Mathematics Department of XXX High School will arrange for this survey in classroom instruction situation. I give an approval to PAULUS SUPARNO from Boston University, Science Education, who wants to investigate students' conceptual change on probability and randomness after the students use the computer simulation programs. I hope that he will find something useful in this study and a benefit for the development of the curriculum materials for students. I agree to allow my students participate in this study.

MR. YYYZZZ
XXX High School's Principal

INFORMED CONSENT **APPENDIX B**

PARENTAL CONSENT FORM

Dear Parent,

For the past four years the Center for Polymer Studies and the Science and Mathematics Education Center at Boston University have been developing educational materials for high school students to learn random events and fractals as patterns in nature. The Mathematics and Science Honors Program at XXX High School has used these materials consisting of computer simulation programs. In the Spring of 1995 your son/daughter will use these programs to study probability, randomness, and fractals.

In conjunction with the faculties at XXX High School the Science Education Program at Boston University's School of Education is conducting a study which examines the conceptual change on probability and randomness among high school students. We are particularly interested in how students' understanding of probability and randomness can improve and how they might change their concepts on randomness and probability after they use computer simulation programs in a classroom.

While your son/daughter are using those programs, they will be asked to complete questionnaires, surveys, learning logs, and some assignments that will help us to gain a better understanding of what your son/daughter have experienced and learned. When the results of this study are published any descriptions of you son or daughter's identity will remain anonymous.

Participation in this study is voluntary and your son/daughter may decide at any point to withdraw from the study. I hope that you will agree to allow your son/daughter to participate in helping the Honor Program at XXX High School and the Science Education Program at Boston University gather this necessary and valuable information. If you have any questions regarding this study, please contact me at 617-552-8217.

Thank you for your cooperation and help,

Paulus Suparno
Research Assistant
Science Education

My son/daughter .... has my permission to participate in the study of conceptual change on randomness and probability at XXX High School.

Date: ....
Signature

PRETEST **APPENDIX C**

STUDENT SURVEY I

This survey consists of 20 multiple choice questions and one concept map question.

A. MULTIPLE CHOICE QUESTIONS

For each number you should choose one of the answers that is most close to your understanding (circle the number of the answer) and write your reason why you choose the answer.

1. Which of the following statements has more chance of happening?
   a. A person smokes and gets cancer.
   b. A person gets cancer.
   Why? ....
   ....

2. Which of the following events has more chance of happening?
   a. A daughter has blue eyes if her mother has blue eyes.
   b. A mother has blue eyes if her daughter has blue eyes.
   c. Equal
   Why? ....
   ....

3. Flipping a coin is not random, because it has only two possible outcomes.
   a. True
   b. False
   Why? ....
   ....

4. Rolling a 6-sided die is random, because it has more than two outcomes.
   a. True
   b. False
   Why? ....
   ....

5. A random event is uncertain and unpredictable. That is why we cannot use random events to predict some physical phenomena.
   a. True
   b. False
   Why? ....
   ....

6. Two friends agree that they can identify random and not random events. A third friend says that there should be a third category: not very random. Do you agree with the third friend?
   a. Agree
   b. Disagree
   Why? ....
   ....

7. I will live for 90 years.
   a. It is impossible.
   b. It is possible.
   c. It is unexpected.
   d. It is likely.
   e. It is certain.
   Why?..
   ....

8. If it is cloudy, it will rain. The statement is
   a. undoubted
   b. impossible
   c. probable
   d. possible
   e. unexpected
   Why? ....
   ....

9. There will be no war in the world during the next two years.
   a. It is definite.
   b. It is impossible.
   c. It is likely.
   d. It is possible.
   e. It is unexpected.
   Why? ....
   ....

10. If the outdoor temperature is 100 ^{o}C, it will snow. The statement is
   a. certain
   b. possible
   c. likely
   d. unexpected
   e. impossible
   Why? ....
   ....

11. There are two students in your school who have the same birthday.
    a. It is uncertain.
    b. It is definite.
    c. It is probable.
    d. It is impossible.
    e. It is unexpected.
    Why? ....
    ....

12. You flip a coin once. What is the chance of getting a head?
    a. 0
    b. 1 / 2
    c. 1
    d. 2 / 1
    Why? ....
    ....

13. You roll a die. What is the odds of getting number 3 ?
    a. 0
    b. 1 / 6
    c. 1 / 2
    d. 1
    Why? ....
    ....

14. You toss 3 coins at once. What is the chance you get 2 heads?
    a. 3 / 8
    b. 1 / 2
    c. 2 / 3
    d. 1
    Why? ....
    ....

15. You toss one coin 4 times and get HHHH. If you toss again, what will it be? (H=head, T=tail)
    a. H
    b. T
    c. cannot tell
    Why? ....
    ....

16. If the chance of having a boy baby is 1/2, then which of these following sequence is more likely to occur for having 5 children? (B=boy, G=girl)
    a. BGBGB
    b. BBBGB
    c. GGBGG
    d. equal
    Why? ....
    ....

17. You have a container with 12 marbles inside: 3 red marbles (R), 3 yellow marbles (Y), 3 white marbles(W), and 3 blue marbles(B). You mix them up. What is the chance that you draw a yellow marble, without seeing it?
    a. 1 / 6
    b. 1 / 4
    c. 1 / 3
    d. 1 / 2
    Why? ....
    ....

18. After you draw the yellow marble, you return it back to the container and you mix them up again. What is the chance that you draw the yellow marble in the second time?
    a. 1 / 2
    b. 1 / 3
    c. 1 / 4
    d. 1 / 6
    Why? ....
    ....

19. You flip 6 coins at once. You do it 1000 times. What number of heads has the most chance?
    a. one heads
    b. three heads
    c. five heads
    d. six heads
    Why? ....
    ....

20. You flip one coin 6 times. What number of heads will you get?
    a. one head
    b. three heads
    c. five heads
    d. six heads
    e. cannot tell
    Why?..
    ....

B. CONCEPT MAPS

Construct a concept map based on the concepts: probability and randomness. Add as needed other concepts, links between concepts, and examples which you have understood.

POSTTEST **APPENDIX D**

STUDENT SURVEY II

This survey consists of 20 multiple choice questions, two concept map questions, and some computer simulation program questions.

A. Multiple choice questions

For each number you should choose one of the answers that is most close to your understanding (circle the number of the answer) and write your reason why you choose the answer.

1. Which of the following statements has more chance of happening?

   a. A person smokes and gets cancer.

   b. A person gets cancer.

   Why? ....

   ....

2. Which of the following events has more chance of happening?

   a. A daughter has blue eyes if her mother has blue eyes.

   b. A mother has blue eyes if her daughter has blue eyes.

   c. Equal

   Why? ....

   ....

3. Flipping a coin is not random, because it has only two possible outcomes.

   a. True

   b. False

   Why? ....

   ....

4. Rolling a 6-sided die is random, because it has more than two outcomes.
   a. True
   b. False
   Why? ....
   ....

5. A random event is uncertain and unpredictable. That is why we cannot use random events to predict some physical phenomena.
   a. True
   b. False
   Why? ....
   ....

6. Two friends agree that they can identify random and not random events. A third friend says that there should be a third category: not very random. Do you agree with the third friend?
   a. Agree
   b. Disagree
   Why? ....
   ....

7. I will live for 90 years.
   a. It is impossible.
   b. It is possible.
   c. It is unexpected.
   d. It is likely.
   e. It is certain.
   Why?..
   ....

8. If it is cloudy, it will rain. The statement is
   a. undoubted
   b. impossible
   c. probable
   d. possible
   e. unexpected
   Why? ....
   ....

9. There will be no war in the world during the next two years.
   a. It is definite.
   b. It is impossible.
   c. It is likely.
   d. It is possible.
   e. It is unexpected.
   Why? ....
   ....

10. If the outdoor temperature is 100 degrees, it will snow. The statement is
   a. certain
   b. possible
   c. likely
   d. unexpected
   e. impossible
   Why? ....
   ....

11. There are two students in your school who have the same birthday.
    a. It is uncertain.
    b. It is definite.
    c. It is probable.
    d. It is impossible.
    e. It is unexpected.
    Why? ....
    ....

12. You flip a coin once. What is the chance of getting a head?
    a. 0
    b. 1 / 2
    c. 1
    d. 2 / 1
    Why? ....
    ....

13. You roll a die. What is the odds of getting number 3 ?
    a. 0
    b. 1 / 6
    c. 1 / 2
    d. 1
    Why? ....
    ....

14. You toss 3 coins at once. What is the chance you get 2 heads?
    a. 3 / 8
    b. 1 / 2
    c. 2 / 3
    d. 1
    Why? ....
    ....

15. You toss one coin 4 times and get HHHH. If you toss again, what will it be? (H=head, T=tail)
    a. H
    b. T
    c. cannot tell
    Why? ....
    ....

16. If the chance of having a boy baby is 1/2, then which of these following sequence is more likely to occur for having 5 children? (B=boy, G=girl)
    a. BGBGB
    b. BBBGB
    c. GGBGG
    d. equal
    Why? ....
    ....

17. You have a container with 12 marbles inside: 3 red marbles (R), 3 yellow marbles (Y), 3 white marbles(W), and 3 blue marbles(B). You mix them up. What is the chance that you draw a yellow marble, without seeing it?
    a. 1 / 6
    b. 1 / 4
    c. 1 / 3
    d. 1 / 2
    Why? ....
    ....

18. After you draw the yellow marble, you return it back to the container and you mix them up again. What is the chance that you draw the yellow marble in the second time?
    a. 1 / 2
    b. 1 / 3
    c. 1 / 4
    d. 1 / 6
    Why? ....
    ....

19. You flip 6 coins at once. You do it 1000 times. What number of heads has the most chance?
    a. one heads
    b. three heads
    c. five heads
    d. six heads
    Why? ....
    ....

20. You flip one coin 6 times. What number of heads will you get?
    a. one head
    b. three heads
    c. five heads
    d. six heads
    e. cannot tell
    Why?..
    ....

B. Concept maps

1. Compare your first concept map of probability and randomness with your last concept map.

2. Is your concept map the same as your first concept map? Why did you change your concept map?

C. Computer simulation programs

1. Which of the following computer programs: Coin Flipping, 1-D Random Walk, Pascal's Triangle, Many Walkers, Diffusion, Fractal Coastline, Fractal Dimension, were very useful in your learning about probability and randomness? Why?

2. Which of the above computer programs were not useful in your study on randomness and probability? Why?

3. What are your suggestions about those programs?

4. Suppose that you will teach and explain probability and randomness to your friends in an other school; which programs do you want to use? Why? Explain.

APPENDIX E

EVALUATION OF TEACHING-LEARNING PROCESS

a. How did you experience and think about the teaching-learning using computer programs? Did the teaching-learning help your study?

b. Do you prefer a lecture in classrooms or a teaching-using computer programs? Why?

c. How do you think about learning logs? Were they helpful for your study? How did they really help you?

d. How did your group work together? Did your group help you in this study?

e. How did you participate in your group? Did you feel satisfied with your group?

f. How many minutes did you usually run or used computer programs in each lab?

I much appreciate your participation in this study. Thanks a lot.

APPENDIX F

STATISTICAL ANALYSIS OF STUDENTS' SCORES VS. COMPUTER TIME USE

SIMPLE REGRESSION

X: TIMÉ Y: SCORE DIFFERENCE

ANALYSIS OF VARIANCE TABLE

Source	DF:	Sum Squares	Mean Square	F-test
REGRESSION	1	162.43	162.43	.201
RESIDUAL	27	21801.02	807.45	p=.657
TOTAL	28	21963.45		

SIMPLE REGRESSION

X: TIME Y: POSTTEST SCORE

ANALYSIS OF VARIANCE TABLE

Source	DF:	Sum Squares	Mean Square	F-test
REGRESSION	1	392.55	392.55	.433
RESIDUAL	27	24464.90	906.11	p=.516
TOTAL	28	24857.45		

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