# Local Instruction Theory on Decimals: The Case of Indonesian Pre-service Teachers

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# ABSTRACT

This thesis reports on an empirical study in developing an instructional sequence on decimals to promote Indonesian pre-service teachers' content and pedagogical content knowledge on decimals. The study was situated in the context of the current reform effort in adapting Realistic Mathematics Education (RME) theory and looked into the role and issues of incorporating RME tenets into the design of activities for teacher education in Indonesia.

The study was carried out in two cycles of teaching experiments involving 258 pre-service primary and secondary teachers in one particular teacher training institute in Indonesia using Design Research methodology. After the first cycle of 4 lessons, activities and test items were refined for trialling with a new cohort in the following year.

Findings from the two cycles signified the importance of revisiting and improving pre-service teachers' content and pedagogical content knowledge of decimals. This study found that pre-service teachers' knowledge on decimals were characterised as fragmented, with strong reliance on rules without understanding, and strong association with fractions. Pre-service teachers in both cycles made substantial improvement in both content and pedagogical content knowledge and they gained their first experiences of working with physical models and working in groups with class discussion. The nature of pre-service teachers' knowledge of decimals highlighted a challenge in attending to the guided reinvention tenet of RME.

# DECLARATION

This thesis contains no material that has been accepted for any degree in any university. To best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where reference is given in the text. This thesis is less than 100 000 words in length, exclusive of tables, maps, bibliographies, and appendices.

Signature:

Date: 26 June 2008

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# LIST OF PUBLICATIONS

## Publications by candidate on matters relevant to thesis:

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Widjaja, W., Stacey, K., & Steinle, V. (2007). Misconceptions in locating negative decimals on the number line In J. Watson & K. Beswick (Eds.), *Mathematics: Essential Research, Essential Practice* (Vol. 2, pp. 923). Hobart: MERGA.

Widjaja, W., & Stacey, K. (2006). Promoting pre-service teachers' understanding of decimal notation and its teaching. In J. Novotna & M. Kratka (Eds.), *The Proceedings of 30th Annual Conference of Psychology of Mathematics Education* (pp. 385-392). Prague: PME.

Widjaja, W. (2005). Didactical analysis of learning activities on decimals for Indonesian pre-service teachers. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Annual Conference of Psychology of Mathematics Education* (pp. 332). Melbourne: PME.

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# CHAPTER 1 INTRODUCTION

## 1.1. Introduction to the study

Some people might perceive decimals as a simple school mathematics topic. This misleading perception can be attributed to how decimals are approached in textbooks as a simple extension of whole numbers. In many countries, the textbook approach of introducing decimals tends to focus on symbolic manipulation of decimals and less attention is paid to building a sound understanding of decimals based on place value notions. One of the common approaches of teaching decimal operations is by extending syntactic whole number rules to decimals. The extensive practice of applying whole number rules might lead students to think that decimals are just a pair of whole numbers separated by a decimal point (Brekke, 1996). In a similar vein, Brousseau (1997) commented on the inadequacy of the teaching of decimals in France around 1960 which attended more to the discrete nature of decimals by employing smaller units and using measurement contexts as expressed in the following quotation:

> But in fact, these school decimal numbers are really just whole natural numbers. In every measure there exists an indivisible sub-multiple, an atom, below which no further distinctions are made. Even if the definition claims that all units of size can be divided by ten, these divisions are never- in elementary teaching- pursued with impunity beyond what is useful or reasonable, even through the convenient fiction of the calculation of a division. (p. 125)

Contrary to the assumption of the simplicity of decimals, extensive studies from around the world on decimals have documented students' difficulties and weak conceptual understanding of decimals from primary to college levels (e.g., Glasgow, Ragan, Fields, Reys, & Wasman, 2000; Irwin, 1995; Padberg, 2002; Steinle & Stacey, 1998b, 2001, 2002). A weak understanding of place value, coupled by weak notions of the magnitude of decimals and poor performance on estimation tasks, are amongst the indicators of problems in decimals (Sowder, 1997; Steinle, 2004; Steinle & Stacey, 2002, 2003b; Swan, 2001).

Studies investigating pre-service teachers' understanding of decimals have revealed that misconceptions also persisted in this group (Putt, 1995; Stacey, Helme, Steinle et al., 2001; Thipkong & Davis, 1991). The fact that some mathematics teachers themselves have limited understanding of decimals might explain the practice of relying on the memorized procedures. This underscores the need to prepare mathematics teachers with conceptual understanding of decimals in order to enable them to uncover and resolve students' misconceptions. Unfortunately, research involving overcoming pre-service teachers' difficulties in decimals seems to be more limited. The fact that pre-service teachers in their future employment may share their misconceptions with children underscores the need to improve pre-service teachers' understanding of decimals. This provides an impetus for research into designing instruction to improve pre-service teachers' content and pedagogical content knowledge of decimals.

Studies in the design of instruction in decimals (Brousseau, 1997; Hiebert, 1992; Hunter & Anthony, 2003; Irwin, 2001; Lachance & Confrey, 2002; Sowder, 1997; Stacey, Helme, Archer, & Condon, 2001) articulated the importance of building meaningful interpretation of decimals. Hiebert (1992) contends that "A greater investment of time would be required to develop meaning for the symbols at the outset and less emphasis would be placed on immediate computational proficiency" (p. 318). Furthermore he argued that having meaningful interpretation of decimals would enhance performance in computation skills.

Despite extensive studies of decimals in other countries (e.g., Irwin, 1995; Owens & Super, 1993; Padberg, 2002; Peled & Shabari, 2003; Steinle & Stacey, 1998b), a comprehensive study of teaching and learning decimals in the Indonesian context has not been carried out. An analysis of some Indonesian commercial textbooks (e.g., Listyastuti & Aji, 2002a, 2002b) indicates strong reliance on syntactic rules based on whole numbers to teach decimals. The approach to teaching and learning decimals is very symbolic and no attention is given to creating meaningful referents such as concrete models to help students make sense of the place value structure of decimals. The models for learning decimals presented in the textbooks are the more symbolic models such as number lines, emphasising positions of points rather than lengths of lines. Considering that these approaches used in Indonesia are similar to those that have been found inadequate in international research studies, it is posited that Indonesian pre-service teachers' knowledge in decimals will be limited and not well-connected. Hence, this study intends to develop a set of appropriate activities on decimals to promote a conceptual understanding of the topic for pre-service teachers in Indonesia and to strengthen their ideas about how to teach the topic. This study is expected to contribute to the international discussion regarding the development of adequate mathematical knowledge in pre-service teacher education.

Current thinking in Indonesia, influenced by the Freudenthal realistic mathematics education (RME) theory, which will be reviewed in Chapter 2, accepts that improvement in mathematics education will come by increasing emphasis on developing meaning and moving away from teaching based only on rules, and through adopting new teaching methodologies, such as group work, which encourage students to construct mathematical ideas together. The project to adapt RME to the Indonesian context called "*PMRI*" (Pendidikan Matematika Realistik Indonesia) began in 2000. To date 11 teacher education institutions involving 30 Indonesian primary schools have been participating in this reform effort (Sembiring, 2007; Zulkardi & Ilma, 2007). Sanata Dharma University, where the main data collection for the current study took place, is one of the four teacher education institutions that have been involved since the start of the project. The research PMRI is still in infancy but early studies (Fauzan, Slettenhaar, & Plomp, 2002; Hadi, Plomp, & Suryanto, 2002) adapting this approach indicate promising results. The promising impact of PMRI is particularly evident in creating an active and engaging learning atmosphere in mathematics classrooms. However, Armanto (2002) found that teachers' limited knowledge of RME and resistance to the new teaching ideas often resulted in conducting RME lessons with little variation from the conventional Indonesian approach. This underscores the need to introduce and incorporate RME tenets in activities in teacher training institutes in order to prepare and familiarize the pre-service teachers with this new approach. This concern has been voiced by Hadi (2002) as one of the recommendations from his study. Designing activities to be used in teacher education programs would have different requirements from the ones for schools since the pre-service teachers have acquired some knowledge of decimals. Moreover, the activities will need to address not only an understanding of decimal concepts but also prepare pre-service teachers with ideas for teaching decimals in more meaningful ways. Chapter 2 will incorporate further discussion on findings from early studies on the implementation of the RME approach in an Indonesian context.

## 1.2. Definitions and Scope of Key Constructs

### 1.2.1. Local Instruction Theory (LIT)

A local instruction theory (LIT), as used in this study, is defined in accordance with Gravemeijer's (2004) definition. It is "the description of, and the rationale for, the envisioned learning route as it relates to a set of instructional activities for a specific topic" (p. 107). The concept of the LIT defined by Gravemeijer performed a similar function to the Hypothetical Learning Trajectory (HLT) defined by Simon (1995). However, in Gravemeijer's LIT, the rationale for the envisioned learning route should explain "how the instructional activities comply with the intention to give the students the opportunity to reinvent mathematics" (Gravemeijer, 1998, p. 280). This is because Gravemeijer is a leading proponent of RME and guided reinvention is one of the RME principles. An LIT is developed through the iterative process of conducting a thought experiment, implementing experiments in the classroom and reflecting upon the considerations, deliberations, and experiences. The term 'local' is used here to describe "how that specific topic should be taught to fit the basic principles" (p. 280). It is 'local' in the sense of applying to one topic, not to all mathematics teaching. The starting point of developing a LIT is the existing knowledge of students and by "imagining students elaborating, refining, and adjusting their current ways of knowing" (Gravemeijer, 2004, p. 106).

### 1.2.2. Indonesian Pre-service Teachers

The Indonesian Pre-service teachers in this study are pre-service teachers enrolled in the Primary School Teacher Education (Two-Year Diploma Program) and in the Mathematics Education Study Program (Bachelor of Education Program) for secondary teaching at Sanata Dharma University in Yogyakarta. Therefore findings related to the performance of pre-service teachers reported in this thesis will be confined to this scope. It is anticipated that the implementation of the result to other teacher education institutions might require some adjustments. However, the researcher does not know of any features that make Sanata Dharma University pre-service teachers different from other pre-service teachers.

### 1.2.3. Decimals

The word 'decimal' is used to refer to a base ten number that is written with a decimal point. In this thesis, decimal refers to the notation of the numbers not the value. Note that in an Indonesian context, a decimal comma is used instead of a decimal point, as will be observed in samples of work presented along the thesis. Examples of decimals are 1.8, 0.5, and –0.5 but not 1/2. Although nearly all decimals discussed are finite, we also use the term decimals for decimals with infinite lengths.

### 1.2.4. Content Knowledge of Decimals

Content knowledge refers to a subset of knowledge about decimals. It includes understanding of decimal place value, understanding of the multiplication and division by 10, special properties of decimals such as density, and the relationships among decimals, fractions and whole numbers. However, knowledge that underpins the execution of algorithms involving decimals, such as lining up the decimal point is not the focal interest of this study. The exact subset of content knowledge areas in the activities in this study will be discussed in detail in Chapter 2, and Chapter 3.

### 1.2.5. Pedagogical Content Knowledge

Pedagogical Content Knowledge (PCK) refers to one category of Shulman's (1987) PCK definition, which focussed on knowledge of representations, i.e. "knowledge that goes beyond knowledge of subject matter per se to the dimension of knowledge about ways of representing and formulating the subject matter that make it comprehensible to others". Other categories of Shulman's PCK, i.e. "understanding of what makes the learning of a specific topic easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning" (p. 9), were not the focal point of this study. In this study, the activities will focus on the issue of finding ways to represent concepts and properties in decimals that are meaningful and comprehensible for students. This notion will be discussed and defined further in Chapter 2 and in Chapter 4.

### 1.3. Aims of the study in relation to Research Questions

This study aims to develop a *Local Instruction Theory* (LIT) on decimal notation for Indonesian pre-service teachers. Primarily, developing LIT shall be based on empirical studies of elaborating, deliberating and reflecting on a set of activities that are designed to promote and improve Indonesian pre-service teachers' content knowledge (CK) and pedagogical content knowledge (PCK) on decimals.

The first research question is concerned with content knowledge. The learning activities will focus on the key notions and properties that are often not addressed properly in Indonesian education. These have been identified by an analysis of primary school textbooks (e.g., Listyastuti & Aji, 2002a, 2002b). The most important are:

- decimal place value system and interpretation of decimals
- additive and multiplicative structures in decimals including unitising and re-unitising of decimals
- density of decimals
- connections and links between decimals, positive and negative whole numbers and fractions

Further explanations about the above concepts and properties will be presented in Section 2.2. The learning activities will incorporate principles of RME in order to fit with new Indonesian initiatives.

The success of the learning activities to improve pre-service teachers' content knowledge (CK) on decimals is investigated in research question 1 as follows:

Research question 1: To what extent do the activities improve pre-service teachers' content knowledge (CK) on decimals?

In answering this question, the following sub-questions are to be investigated:

a) What is the current state of Indonesian pre-service teachers' CK of decimals?
b) What is the interplay between pre-service teachers' participation in the set of activities on decimals and their CK of decimals?

As is appropropriate in a teacher education course, the activities will also address pedagogical issues and it is to be expected that pre-service teachers' knowledge of how to teach decimals will be improved by their participation. The study will therefore also examine pre-service teachers' knowledge of decimals in the context of teaching it to their future students. This aspect relates to one of Shulman's (1986) categories of PCK, i.e. the knowledge of how to help students to learn concepts or properties in decimal numeration. Shulman's other aspect of PCK, i.e. "understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them" will not be examined in this study. The impact of learning activities on pre-service teachers' pedagogical content knowledge (PCK) will be investigated in research question 2 as follows:

Research question 2: To what extent do the activities improve pre-service teachers' pedagogical content knowledge (PCK) on decimals?

In answering this question, the following two sub-questions are to be investigated:

a) What is the current state of Indonesian pre-service teachers' PCK of teaching decimals?
b) What is the interplay between pre-service teachers' participation in the set of activities on decimals and their PCK of decimals?

Lying behind these two main questions is the author's concern to prepare future Indonesian teachers who are well trained to implement principles of RME. It is the researcher's intention that participation in these learning activities will give pre-service teachers first-hand experience of how the principles of RME can be implemented into their own mathematics teaching. However, there is to date little Indonesian experience of using RME principles in teacher education. This concern is reflected in the following question:

Research question 3: How can teacher education assist Indonesian schools to adapt RME principles?

Research question 3 will be answered by the researchers' reflections on empirical work undertaken in answering Research Question 1 and Research Question 2. Responses to question 3 are taken up in the final chapter.

## 1.4. Relevance and Significance of the study

This thesis will develop a framework of teaching and learning decimal notation for Indonesian university lecturers to use with pre-service teachers. It is expected that this framework could be adapted for use with in-service teachers in Indonesia, thereby contributing to the improvement of both pre-service and in-service teacher education in Indonesia. Future students of these pre-service teachers should benefit from this improved conceptual understanding of decimals and teaching ideas. Moreover, this study will also contribute to the international discussion regarding the issue of teaching and learning decimals in teacher education. LIT will also give insights to the development and dissemination of PMRI in teacher education in Indonesia, and of the application of RME to teacher education.

## 1.5. Outline of thesis

Chapter 2 will review literature on prior research in learning and teaching of decimals pertinent to this study. The discussion of basic tenets of RME and RME instructional principles will also be addressed in this chapter. Chapter 3 contains discussions about the Design Research methodology which is employed in this study. Analysis of findings from different phases in cycle 1 and cycle 2 of the Design Research cycle will be presented in Chapter 4 and Chapter 5 consecutively. In Chapter 6, a review of how the activities evolve in the two cycles, common findings and differences between the two cycles of teaching experiment will be explicated. Chapter 7 concludes with presenting the activities depicting the LIT on decimals. Moreover, recommendations for the role of teacher education institutions in assisting Indonesian schools to adapt the RME principles will be articulated in this chapter.

# CHAPTER 2 LITERATURE REVIEW

## 2.1. Introduction

This chapter starts with a general review of the main areas of conceptions in decimals that need to be addressed in teaching decimals, as documented in past and recent studies. A brief overview of diagnostic items used to detect misconceptions of decimals is included. The main focus in reviewing the literature on difficulties will be placed on identifying common difficulties related to conceptual understanding of decimals rather than computational operations. Discussion on common difficulties about decimals will be taken up at the end of Section 2.2. Section 2.3 will elaborate various teaching ideas to improve understanding of decimals. In Section 2.4, the Realistic Mathematics Education (RME) basic tenets, and teaching ideas about decimals based on RME theory, will be discussed. Designing instruction based on RME principles is of interest in this study because it is consistent with the current reform initiative in Indonesian mathematics education. Finally, Section 2.5 ends with a concluding remark about the main areas on teaching and learning about decimals that will be addressed in this study with respect to the Indonesian context and consistent with the current reform effort to adapt the RME theory.

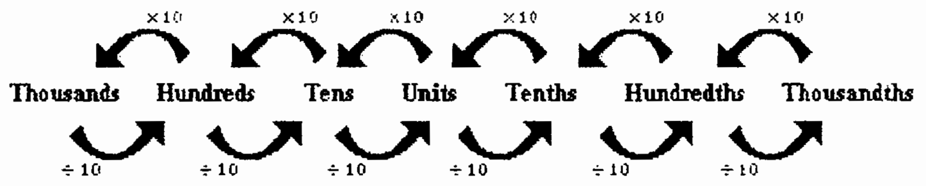
## 2.2. General overview of content areas and difficulties with decimals

This section will start with a review of key notions in content areas for learning decimals. Following that, literature on studies of misconceptions and problems in the content areas of decimals at various levels of education will be discussed.

### 2.2.1 Overview of the key notions in learning decimals

Various studies have examined basic notions in learning and teaching decimals and the inter-relations among those notions. Understanding place value has been widely cited as one of the basic notions in decimal numeration. The term "endless base ten chain" is used by Steinle, Stacey, and Chambers (2002) to refer to the chain of relationships between the values of place value columns. As depicted in Figure 2.1, the value of each place value column is ten times the value of the column to the right (including across the decimal point) and that the value of each column is one tenth of the value of the column to the left. Similar relationships imply that the value of each place value column is one hundred times the value of the column two places to the right. Conversely, the value of each column is one hundredth of the value of the column located two places to the left, and so on. These structural relationships among different place value columns are known as the multiplicative structure of decimals.

Figure 2.1: Endless base ten chain (from Steinle, Stacey, & Chambers, 2006)




Baturo (2000) proposed a model representing three levels of knowledge involved in processing decimal numbers. The first level consists of knowledge of position (involving place value names), the base ten system, and order of the places (hundredths are larger than tenths, etc). This knowledge of 'position' seems to refer to the "place" of place value, rather than the position of decimal numbers on a number line. The second level of knowledge encompasses knowledge of unitising and equivalence of decimals (e.g., 1 ten = 10 ones). Unitising is defined by Baas "identifying singleton of units, e.g., 10 of hundredths = 1 tenth". In the third level of her model are multiplicative structures along with additive structures (e.g., 0.25 = 2 tenths + 5 hundredths) and reunitising. These are considered as the highest level of knowledge because they integrate other knowledge with place value knowledge. Reunitising is defined as "the ability to change one's perception of the unit (i.e., to also see that one whole partitioned into 10 equal parts as five lots of 2 parts and two lots of 5 parts)" (Baturo, 2004, p.96). Baturo (2000) contends that reunitising consists of three processes: partitioning (e.g., 6 tenths = 60 hundredths), grouping (e.g., 60 hundredths = 6 tenths), and re-grouping (e.g., 0.6 = 5 tenths + 10 hundredths) and hence it involves a more complex processes than unitising. This is in line with Behr, Khoury, Harel, Post, and Lesh (1997)'s comment about reunitising. They contend that reunitising "requires flexibility of thinking" and involves more complex process than unitising (p.96). It is established that different level of complexities are involved in unitising and reunitising. this thesis will not attempt to look in detail into various levels of cognitions involved in unitising and reunitising. In this thesis, the two notions are viewed more generally as part of the multiplicative structure of decimals in Figure 2.2. Unitising is defined as grouping of smaller units into a larger unit of decimal fractions. For example, 5 tenths + 10 hundredths = 6 tenths. Reunitising is defined as the process involved in structuring a unit from smaller unit parts of decimal fractions.

Figure 2.2: Key notions for learning decimals covered in activities

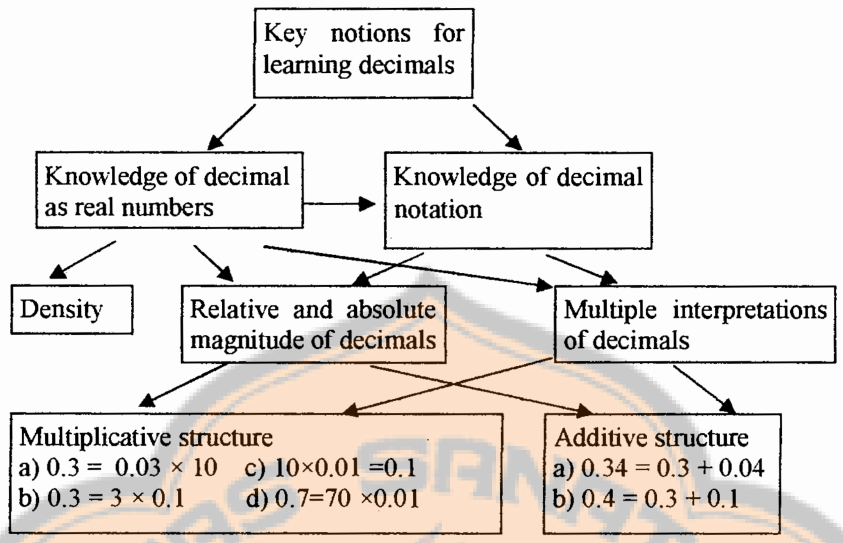



Figure 2.2 above summarises the key notions of decimals covered in the learning activities. Various researchers (see e.g., McIntosh, Reys, & Reys, 1992; Sowder, 1997; Steinle et al., 2006) contend that deep knowledge of place value plays a crucial role in developing meaningful and flexible interpretation of decimals, e.g., 0.651 = 6 tenths + 5 hundredths + 1 thousandth = 65 hundredths + 1 thousandth = 6 tenths + 51 thousandths = 601 thousandths + 5 hundredths. Ability to interpret a decimal into different place value related terms is an important aspect of decimal numeration that relies on place value and structural understanding of decimals (see e.g., McIntosh et al., 1992; Stacey, 2005). Moreover, this knowledge plays a crucial role in determining the relative magnitude of decimals as displayed in tasks such as comparing and ordering decimals.

As will be discussed later in Section 2.2.3, researchers have documented the fact that many students have trouble in judging the relative magnitude of decimals.

Another key notion in learning decimal is knowledge of decimal as real numbers. These features would still hold if decimal numbers were represented not in a base ten place value system but, for example, as continued fractions or in a base 2 system without place value. For the purpose of this thesis, these features encompasses knowledge about density of decimals (see e.g., Reys et al., 1999; Steinle et al., 2002; Swan, 1990) and knowledge about relative and absolute magnitude of decimals. The density property signifies that in between any two decimals, there are infinitely many more decimals. This feature of decimals is independent of the notation that is used to represent them as it is a property of real numbers. Figure 2.2 illustrates various key notions for learning decimals derived from prior studies. It should be noted that Figure 2.2 does not contain exhaustive list of key notions for learning decimals. It mainly serves as a framework to guide the goals of activities in this study.

### 2.2.2 Overview of decimal misconceptions identified using decimal comparison tasks

Misconceptions of decimals and the underlying thinking behind these misconceptions have been well documented in many studies. One of the tools that has been widely used by researchers to reveal understanding and to detect misconceptions of decimals is the task of comparing decimals (see e.g., Nesher & Peled, 1986; Sackur-Grisvard & Leonard, 1985; Steinle & Stacey, 1998b, 2001, 2002). In general, decimal comparison tasks ask students to determine the larger decimal among two or more decimals, or to order two or more decimals from the smallest to the largest. This section will present a brief overview of decimal misconceptions detected through the use of these comparisons and ordering tasks.

There was a long history of use of comparison of decimal tasks, which resulted in various classifications of decimal misconceptions. Early studies to diagnose decimal misconceptions by Sackur-Grisvard and Leonard (1985) and Resnick, Nesher, Leonard, Magone, Omanson, and Peled (1989) discriminated three incorrect rules which students appeared to use based on association with common fractions or whole numbers. The three incorrect rules are referred to as the Whole Number Rule, Fraction Rule, and Zero Rule. Each of these rules arises from a misconception of decimals. In fact, later work has shown a cluster of misconceptions behind each rule. In the Whole Number Rule, decimal digits are interpreted according to overgeneralisation of whole number knowledge, which implied that decimals with longer decimal digits are perceived as larger numbers. In contrast, knowledge of common fractions and place value are incorrectly extended to interpret decimal digits using knowledge of common fractions in the Fraction Rule. Following this rule, decimals with shorter decimal digits are perceived as the larger number. Meanwhile, the Zero Rule is based on awareness of the role of zero as place holder but this knowledge is not well connected with the decimal place value structure. Those who follow the Zero Rule perceive decimals with an immediate zero after the decimal point such as 3.06 as smaller decimals compared to 3.7, but otherwise follow the whole number rule. Resnick et al. (1989) noted that students who applied an Expert Rule might do this with real understanding or they may be applying a correct rule without understanding.

Later studies confirmed the existence of students following those four incorrect rules identified in the earlier studies (see e.g., Brekke, 1996; Fuglestad, 1996; Markovitz & Sowder, 1994; Moloney & Stacey, 1997; Rouche & Clarke, 2004). Yet, careful analysis of large data over several years (Stacey & Steinle, 1998, 1999; Steinle & Stacey, 1998a, 2002, 2003b) reveals that there are more variations of students' erroneous responses and underlying thinking, which can again be revealed using a decimal comparison test, with refined items.

A systematic way of classifying incorrect responses and a refined classification of ways of thinking was then offered. In Steinle's (2004) classification, the term *behaviours* were used to refer to patterns of incorrect responses on DCT on different types of items. Four groups of behaviours and twelve ways of thinking were established which corresponded to a two stage classification process. The terms 'ways of thinking' was used by Steinle to distinguish behaviour from underlying (conceptual) misconception, because earlier studies used misconception in both ways. First, behaviours are classified into one of the four coarse-grained behaviours as follows:

- *Longer-is-Larger* (L behaviour), choosing the decimal with the *most* digits after the decimal point as the largest
- *Shorter-is-Larger* (S behaviour), choosing the decimal with the *fewest* digits after the decimal point as the largest
- *Apparent Expert* (A behaviour), comparing "straightforward" pairs of decimals correctly with or without full understanding
- *Unclassified* (U behaviour), indicating behaviours that does not fit in either *L*, *S*, or *A* behaviour

Second, based on more detailed analysis of patterns of responses, twelve categories as presented in Table 2.1 can be identified. This refined analysis of ways of thinking is presented here because it will be employed as an initial framework for analysing data gathered in this study. The decimal comparison test called DCT3a (pre-test) and DCT3b (post-test) includes items problematic for adults in order to better capture their thinking, as well as items representative of those used in earlier studies. These tests have very closely matched but different items (see Appendix B1 and B2). Its usage has been reported in some studies involving pre-service teachers and nursing students to diagnose their conceptual understanding on decimals (see e.g., Steinle & Pierce, 2006; Widjaja & Stacey, 2006). The DCT3a and DCT3b will be used in this study to diagnose and follow pre-service teachers' misconceptions about decimals.

DCT has been shown as an insightful and reliable diagnostic tool in revealing the nature of some incorrect thinking. However, one limitation lies in the fact that DCT is unable to discriminate whether students have a meaningful understanding of decimals or are just following expert rules (annexing zeros and then comparing as whole numbers or comparing decimal digits from left to right). Hence Steinle and Stacey (2002; 2003a) use the label 'task expert', rather than just simply expert. The next section will review literature on the main areas of difficulties with decimals informed by prior research in this area.

Table 2.1: Ways of thinking and corresponding behaviours adopted from Steinle (2004, p.47-59)

| Way of thinking | Brief description | Fine code | Coarse code |
|---|---|---|---|
| Task expert | Indicating either solid understanding of decimals or following of *annexe zero algorithm* or *left-to-right comparison algorithm* | A1 | A |
| Money | Drawing analogy to money (dollar and cents) or equivalent systems for comparing decimals. | A2 | |
| Unclassified A | | A3 | |
| Decimal point ignored | Choosing the longest sequence of digits as the larger number, ignoring the decimal point | L1 | L |
| Numerator focussed | Concentrating on the *number of parts* and disregarding the *size* of the parts, thinking 6.03 and 6.3 are the same because they both contain 6 wholes and 3 more parts. | | |
| String length | Choosing decimals with longer decimal digits as the larger number | | |
| Zero makes small | Thinking that decimals with a zero or zeros in the first column(s) after the decimal point is "small" | L2 | |
| Column overflow | Squashing the place value columns to the left, e.g., noting that 0.70>0.7 because 70 tenths is larger than 7 tenths | | |
| Reverse thinking | Confusing the decimal place value names with whole number place value names, e.g., identifying 0.35 as 3 tens and 5 hundreds instead of 3 tenths. | L3 | |
| Unclassified L | | L4 | |
| Denominator focussed | Considering the size of the parts in isolation and ignoring the number of parts, e.g., 0.6>0.75 because tenths is larger than hundredths | S1 | S |
| Place value number line | Considering place value columns as a pseudo number line | | |
| Reciprocal | Confusing decimal digits as whole numbers and drawing analogy with reciprocals, e.g., 0.3>0.4 like 1/3>1/4 | S3 | |
| Negative | Associating decimals with negative numbers 0>0.6 like 0>-6 | | |
| Unclassified S | | S4 | |
| Unclassified | Not fitting anywhere due to mixed or unknown ideas | U1 | U |
| Misread | Misreading the instruction | U2 | |
| Misrule | Applying opposite thinking after following expert rules such as *annexe zero* or the *left-to-right digit* comparison algorithms | | |

* *whole number thinking = numerator focussed + string length thinking*

### 2.2.3 Other main areas of difficulties about decimals

Various areas of difficulties about decimals have been identified, including weak knowledge of decimal place value, fragmented links between decimals and fractions, lack of knowledge on density of decimals and confusion about relative position of decimals on the number line. The following paragraphs will expand on evidence of difficulties in each of those areas. Moreover, reviews of studies investigating pre-service teachers' knowledge and difficulties on decimal numeration will be discussed. Whereas the misconceptions in Table 2.1 describe the underlying interpretation of decimals, the studies in this section generally show the consequences of these erroneous interpretations.

**Place value**

Weak knowledge of place value interpretation of decimals (A. R. Baturo, 1997; Carpenter, Corbitt, Kepner, Linquist, & Reys, 1981; Kouba et al., 1988; Lampert, 1989; Sowder, 1997) is one of the main conceptual problems in decimal numeration. This is evident in nearly all of the ways of thinking in Table 2.1. Carpenter *et al.* (1981) recorded that only 21% of 9 year olds in a USA national study and 79% of 13 year olds could correctly identify the place value names of a decimal number, 7.94. Their facility with the reverse task, to match up thirty-seven thousandths with its decimal notation was much lower with only 3% of 9 year olds and 54% of 13 year olds providing correct answers.

Both Hiebert (1992) and Kouba *et al.* (1988) linked lack of sound knowledge on place value in decimals to difficulties in understanding and operating with decimals. Similarly, Grossman (1983) observed that performance on calculations involving decimals was higher than the ability to interpret the meaning of decimals. She noted that more than 50% in a USA college could perform decimal computation (addition, subtraction, multiplication) but only less than 30% could select the smallest decimal from five possibilities which indicated scant knowledge of place value.

In the same vein, Tsao (2005) found from her interview data that performance on computation algorithms did not imply good number sense. She suggested that over-reliance on written algorithms might inhibit the development of important aspects of number sense including flexible interpretation and use of numbers.

Baturo (1997) investigated a more specific aspect of place value, focussing on multiplicative structures in place value of decimals involving 175 Grade 6 students in Australia. She found that a majority of students showed lack of knowledge of multiplicativity in decimal place value with only high performance students showing full knowledge of multiplicative structure.

Similarly, Stacey, Helme, & Steinle (2001) reported evidence of place value column name confusions wherein "whole number" part of the place value system (tens, hundreds, etc) placed to the right of "number line" were mirrored to the "decimal" part of the place value system (tenths, hundredths, etc.). Merging this place value confusion with the knowledge of the placement of positive and negative numbers on the number line resulted in students thinking of decimals as negative numbers. The 'negative thinking' of Table 2.1 is an instance of this.

**Density of decimals**

One of the features distinguishing decimals from whole numbers is the density of decimals. Hiebert *et al.*, (1991) found improving the continuity aspects of decimals' density was particularly difficult. Working with problems involving continuous models in the written tests and the interviews, such as marking a representation of a decimal number on a number line, or finding a number in between two given decimals such as 0.3 and 0.4 were found to be more challenging than working with discrete-representation task utilizing MAB models. Analysis for this finding suggested that an extra step in finding the unit of the continuous models explained the lower performance on continuous-representation tasks. In Table 2.1, A2 (money thinking) describes students thinking like this.

Likewise, Merenlouto (2003) found that only a small portion of Finnish students aged 16-17 years old in her study changed their concept of density. She attributed difficulties with grasping density to students' reference to natural numbers and difficulties in extending their frame of reference to rational or real numbers. Some students relied on the possibility to add decimals in their explanations for recognizing density of decimals. Furthermore, she contended that this kind of explanation was based on an abstraction from natural numbers properties rather than a radical conceptual change from natural to real numbers.

Difficulties with density were also evident in studies involving pre-service teachers. Menon (2004) found only 59% of 142 pre-service teachers recognized the density of decimals. A similar trend was noted by Tsao (2005) who found that of 12 pre-service teachers involved in her study, only the six high ability students demonstrated an understanding of density.

The nature of incorrect responses with regard to the density of decimals is reflected in common misconceptions drawing on analogies between decimals and whole numbers. Clearly density will not make sense to students holding misconceptions identified in Table 2.1 such as money thinking, denominator focussed thinking, reciprocal thinking, and place value number line thinking.

In general incorrect answers in recognizing the density of decimals could be classified in two categories. The first category of incorrect answers is identifying no decimal existing in between pairs of decimals. Fuglestad (1996) found that most students in her study of Norwegian students claimed there were no decimals in between two given decimals such as between 3.9 and 4 or between 0.63 and 0.64. Similarly, Bana, Farrell, and McIntosh (1997) reported that the majority of 12 year olds and 14 year olds from Australia, US, Taiwan and Sweden displayed the same problem. Only 62% of 14 year olds from Australia and 78% of 14 year olds from Taiwan showed understanding of decimal density. This evidence reflected incorrect extension of whole number knowledge that there is no whole number in between two consecutive whole numbers such as 63 and 64. Note that students holding money thinking (allocated to A2 code in Table 2.1) also will have difficulty in grasping the density notion of decimals and identify no decimals in between decimals such as 0.63 and 0.64. However, these students might identify 9 decimals in between 0.6 and 0.7 for instance, if they interpret decimals only as a number system for dollar and cents.

The second category of incorrect answer translates knowledge of multiplicative relations between subsequent decimal fractions. For instance, Hart (1981) reported that 22 to 39% students age 12 to 15 year-old thought there were 8, 9, or 10 decimals in between 0.41 and 0.42. Similarly, Tsao (2005) observed the same phenomenon in her study with pre-service teachers. She found that three pre-service teachers from a low ability group believed there were nine decimals in between 1.42 and 1.43 by sequencing only the thousandths: 1.421, 1.422,..., and 1.429. Along with most of the students in L and S groups (see Table 2.1), some students holding A thinking, such as A2 thinking with reference to metric measures (m, cm, mm) might possibly respond in this way.

**Relative and absolute magnitude of decimals**

Various studies have underlined the importance of having a sense of relative and absolute magnitude of decimals, including as shown on the number line (see e.g., McIntosh et al., 1992; Thompson & Walker, 1996; Watson, Collis, & Campbell, 1995). The importance of this knowledge is articulated by Thompson & Walker (1996) as follows:

> For students to understand decimals thoroughly, they need to have an understanding of the relative magnitudes of these numbers. That is, they should have a good idea of where decimal values lies on a number line in relation to other decimals. Furthermore, they need to know how the values of decimals compare to common fractions and whole numbers. They need to know for example, that 0.48 is about ½ and is about halfway between 0 and 1. (p.501)

However, a number of studies (Bana et al., 1997; Kloosterman et al., 2004; Michaelidou, Gagatsis, & Pitta-Pantazi, 2004; Rittle-Johnson, Siegler, & Alibali, 2001; Thipkong & Davis, 1991) have documented persistent problems in locating decimal number on a number line. Kloosterman *et al.* (2004) reported only 39% of fourth graders correctly placed a decimal number on the number line (with divisions not 0.1 apart) in consecutive NAEP studies in 1992 and 1996. The result was statistically improved in year 2000, yet it still indicated weak knowledge as reflected in 48% facility of correct answers.

Likewise, Bana, Farrell, and McIntosh (1997) found that the task of identifying decimals on the number line was particularly challenging for 10 year olds. All performance of students in this age group from Australia, Sweden, and the USA were below 50%, with the USA students performing particularly poorly at only 20% facility.

In the study by Michaelidou, Gagatsis, & Pitta-Pantazi (2004) involving 120 12 year olds students' understanding of the concept of decimals in Cyprus, many students attended to the discrete instead of the continuous aspect of number line as they worked with decimals on the number line. It was found that "they treated number lines not as a continuous model…, but as line segments from which they had to select a part" (p.310). The problem reported in Michaelidou, Gagatsis, & Pitta-Pantazi's (2004) study confirmed Hiebert *et al.*'s (1991) finding that working with a continuous representation model for decimals such as the number line was more difficult, as discussed earlier. Evidence from these studies has underscored the complexity of decimals in understanding relative and absolute magnitude of decimals experienced by both children and adults.

**Studies of decimals understanding in teacher education**

Fewer studies have been carried out to investigate pre-service teachers' understanding of decimals compared to similar studies involving children. As pointed out before, studies have revealed that some of misconceptions observed in children were also apparent in adults, including pre-service teachers (Menon, 2004; Putt, 1995; Stacey, Helme, Steinle et al., 2001; Thipkong & Davis, 1991; Tsao, 2005). However, Steinle's longitudinal study (2004) showed a different prevalence of misconceptions in children of different ages and hence it is likely that there will be a different prevalence for secondary school students and pre-service teachers. This finding implies that the teaching of decimals in teacher education needs to take into account the different nature of pre-service teachers' knowledge and misconceptions on decimals. The misleading common perception of topics in elementary schools including decimals as "simple" mathematics and the constraints on formal teachers' education, such as limited time, were noted by as Ball (1990) as factors contributing to a decision not to revisit this topic in many of the U.S. teacher education institutions. It is the author's view that this practice is also shared in many teacher education programs in Indonesia and is possibly relevant in some other countries as well.

Investigating pre-service teachers' understanding and misconception of decimals entails assessment of an aspect of knowledge, called "pedagogical content knowledge" or PCK (Shulman, 1986). In Shulman's (1986) seminal work, PCK is defined as composing two aspects, the knowledge of students' knowledge and understanding, and knowledge of representations, which are useful for teaching. Knowledge of students' knowledge, including common difficulties and misconceptions in the topic, allows teachers to explain the confusions that occur and to assist students to overcome these problems (Graeber, 1999; Leinhardt, Putnam, Stein, & Baxter, 1991). For instance, in a decimal topic, Ball and Bass (2000) noticed that students often get confused about the '*oneths*' place. In this case, teachers need to be able to explain that this *'expectation'* comes from applying the '*symmetry*' principle around the decimal point, which was also noted by Stacey, Helme, and Steinle (2001). They contend that "helping a fifth grader to understand the "*missing oneths*" requires an intertwining of content and pedagogy, or pedagogical content knowledge" (p.87). Knowledge of students' prior conceptions and misconceptions assist teachers in tailoring their lessons so that the lessons will be more meaningful for students.

Having a wide repertoire of representations enables teachers to flexibly expand on mathematical notions and properties in order to make them more comprehensible for students. Ball and Bass (2000; 2003) relate this knowledge to the importance of the teacher's role in "unpacking" mathematical content knowledge to fit a learner's perspective and in identifying central ideas in teaching mathematics.

Studies by Thipkong and Davis (1991) and Putt (1995) confirmed the trend predicted by Steinle and Stacey's (1998a) study of the changes in prevalence of misconceptions from years 3 to 10; that *shorter-is-larger* (S) behaviours would be likely to continue throughout adulthood. Steinle (2004) found the increasing trends with grade of students who made very few errors in DCT such as in the A categories of Table 2.1.

Stacey *et al.*'s (2001) observation of 553 pre-service primary teachers' content and pedagogical content knowledge of decimal numeration from Australia and New Zealand revealed a high incidence of difficulties of comparing decimals with zero (13%). Similar to Putt's (1995) finding, this study also found evidence of '*shorter-is-larger*' (S) misconception (about 3%) among pre-service teachers. However, many pre-service teachers had little awareness of this misconception even when they held this misconception. Pre-service teachers were able to identify four surface features that make the comparison of pairs of decimals difficult for students: ragged or unequal lengths (long decimals and decimals with unequal lengths), comparison with zero, presence of zero digits, and similar decimal digits (decimals that differ only in the third or fourth decimal digits). However, despite having good ability of identifying these features, pre-service teachers' explanations for the thinking underlying these difficulties were less satisfactory. An understanding of why these errors occur is an important part of PCK.

Pre-service teachers' fragmented knowledge of decimals and fractions was apparent in Tsao's (2005) study, when "students considered fractions and decimals as different entities and they did not necessarily make any connections between them" (p.661). Yet many of the S thinkers believe that decimals and fractions are similar as evident in overgeneralising of fractions knowledge to decimals. Resnick *et al.* (1989) found that different curriculum sequences in different countries generated different pattern of misconceptions. In countries where the teaching of fractions precedes the teaching of decimals such as the U.S.A and Israel, misconceptions resulting from overgeneralising fraction knowledge to decimals (S thinkers) were dominant. However, Steinle and Stacey's Australia data shows that the proportion of S thinkers increased with age. It will be discussed later whether this study indicate the same trend as in Indonesia curriculum, where decimals are taught after fractions. This will be discussed in Chapter 4 and Chapter 5.

This signifies the importance of attending to and resolving the main problematic areas both in content and pedagogical content knowledge in decimal numeration as recommended by prior studies (see e.g., Putt, 1995; Stacey, Helme, Steinle et al., 2001). This study will capitalize on findings on key conceptions in learning decimal notation and likely areas of weaknesses in decimal numeration informed by prior studies in designing the activities and written test items. It will also use items derived from these studies to monitor change. The next section will review different teaching ideas proposed to improve understanding of decimals.

## 2.3 Teaching Ideas about Decimals

A number of teaching ideas have been proposed and tested to improve conceptual understanding of decimal numeration (see e.g., Boufi & Skaftourou, 2002; Brousseau, 1997; Hiebert, 1992; Hunter & Anthony, 2003; K. C. Irwin, 2001; Lachance & Confrey, 2002; Sowder, 1997; Wearne & Hiebert, 1988a, 1988b). Hiebert (1992) expressed the importance of attending to conceptual understanding by noting "A greater investment of time would be required to develop meaning for the symbols at the outset and less emphasis would be placed on immediate computational proficiency" (p. 318). In making this recommendation, he argued that meaningful interpretation of decimal notation would enhance performance in computation skills.

Whilst researchers agree on the importance of designing instruction to instil a deeper meaningful understanding of decimals, there have been a variety of approaches in teaching ideas on decimals in a meaningful way. These studies vary with respect to prior teaching exposure to decimals of the participants, the place of decimals in the curriculum sequence, and scope and length of the studies. This review, however, does not attempt to compare the various teaching ideas about decimals on the basis of the effectiveness or success rate. Considering the differences in students' background knowledge, different lengths and nature of the teaching interventions, it is not sensible to compare various teaching interventions' success.

In this discussion, teaching ideas on decimals will be classified into two categories based on their main emphasis as indicated in their choice of the initial activities. The first category places heavy emphasis on integrating decimals with other related concepts such as ratio or rational numbers or percentage (K. C. Irwin, 2001; Lachance & Confrey, 2002; Moss, 2005; Moss & Case, 1999). The second category focuses on building meaningful understanding of decimal numeration based on place value (Bell, Swan, & Taylor, 1981; Helme & Stacey, 2000; Hiebert et al., 1991; Lampert, 1989). It should be noted that the two camps are not viewed as opposing camps. Both camps underscore the importance of constructing a meaningful link between related ideas such as fractions, and decimals, for instance. The following paragraphs will expand on these ideas and associated findings.

The first camp is characterized by its integration of teaching ideas of decimals with other related concepts such as ratio and proportion or percent and linear measurement contexts or to present it in contextual problems. Examples are Hunter and Anthony (2003), Irwin (2001), Lachance and Confrey (2002), Moss (2005) and Moss and Case (1999).

The basic argument of this position is grounded on the idea that conceptual understanding of decimals presupposes strong connections among related constructs. Furthermore, proponents of this approach observe and criticize the fact that different models (called referents) are used in teaching decimals, fractions, and percentages without explicit links among them. Lachance & Confrey (2002) articulated this concern as follows:

> The "one referent per mathematical construct" set of activities may be too narrow to allow for "real" conceptual knowledge to emerge. Hence, unless work with concrete referents allows students to develop an understanding of the meaning of mathematical symbols and at the same time, explore the connections between various types of mathematical symbols, such experiences may only lead to rather weak, superficial, and narrow understanding of mathematical constructs. (p.508)

However, within the same camp, there is no unique path in sequencing different constructs to arrive at conceptual understanding. Moss and Case (1999) devised a curriculum to introduce rational numbers comprising percentages, fractions, and decimals to a group of 29 fourth graders (aged 10 to 11) in a Canadian school in 20-25 lessons over 5 months. This program was based on intuitive estimation of ratios and representation of length (number-ribbon diagrams) to introduce percentages in linear measurement contexts. One reason for starting the sequence with percentage is to postpone problems of comparing and manipulating fractions with different denominators and complex conversion among percents, fractions, and decimals. The successive halving strategy (used in the context of estimating the fullness of water in glass beakers) was emphasized to link computations involving percents, fractions and decimals. The link to decimals was developed by contextual problems of calculating tax or tips where that could be solved by calculating with multiple of 10% relying on the knowledge of money (dollar and cents).

In this program, two-place decimals were introduced as notation to represent the distance between two adjacent whole numbers as percentage, e.g., 5.25 represented a distance that is 25% of the distance between 5 and 6. The transitional notation called "double decimal notation" such as 5.25.25 was employed intuitively by children to represent a number that is located 25% of the distance in between 5.25 and 5.26. Students were grouped into the experimental curriculum based on intuitive notion of ratio (16 students) and the control curriculum (based on widely-used Canadian textbooks) (13 students). In the control curriculum, the sequence started with fractions followed by one-place decimals and two-place decimals using models such as pie graphs, number line, and place value charts. The experimental group outperformed the control group overall and in most areas including decimal notation, density of decimals, comparing and ordering decimals as well as decimal operations (including conversion among fractions, decimals and percentages). However, no detail was given on comparison of performance from both groups on decimal components. Furthermore, no explanation was given on how this teaching approach addressed other properties of decimals such as density in relation to the transitional notation or "double decimal notation" for instance. The fact that this program avoids complex fractions and decimals raises a concern about how to build students' understanding of more complex decimals.

Hunter and Anthony (2003) adopted Moss and Case's teaching ideas in New Zealand over a six-month period and evaluated it using interviews (4 case studies), and lesson observations with qualitative analysis of the classroom episodes. In line with Moss and Case's (1999) study, the initial activities were designed to capitalize on students' informal understanding of ratio based on a numerical halving strategy. Decimal notation was established as a way to record the whole number of metric units such as metre with part of the unit as percentage (e.g., 3 metres and 47% of a metre was linked with 3.47).

Misconception in decimals was observed when a student recorded 761 metres and 4 cm as 761.4 instead of 761.04 and noting the extra 4 cm as 40%. In addressing this problem, the teacher used percentage as a scaffold by comparing 40% and 4% in relation to 100%. This "dropping back" (p.457) action of going back to the initial context allowed students to correctly link 0.40 with 40%, and 0.04 with 4%. Yet, in my opinion, it is not clear whether this approach indeed resolves students' misconception of place value in decimals and the interpretation of its magnitude in metric contexts because the explanations seem to rely on comparison of the corresponding percentages.

Another study within the same camp by Lachance & Confrey (2002) involving a 6-week instruction sequence on decimals was built around three open-ended, contextual problems grounded on the multiplicative notion called "splitting". This splitting notion, evident in actions such as sharing, folding, and magnifying, was introduced by Confrey in 1994. The first contextual problem explored different notational systems (including base 2, 4, and 6) before investigating base 10. The second contextual problem, called the '*Decimal Olympics*', involved computations (addition and subtraction) using decimal notation in measurement context (metric rulers) and included discussion on ways of noting quantities in metric system. The last problem, called the '*Domino Problem*', explored issues in computing with decimals using rate and ratio reasoning. In this study, 20 Grade 5 students (working in 5 small groups during instruction) were assessed on the following areas: (1) meaning of decimal notation, (2) the ordering of decimals, (3) the conversion from fraction to decimal and vice versa, and (4) computations with decimals. Interestingly, despite significant improvement on the written tests (overall facility increased from 35.6% to 80.3%), a majority of students struggled in working with contextual problems, particularly in making sense of decimal notation such as 0.7625 in the context of the problem. In my opinion, while contextual problems allowed rich exploration of decimal notation and computation, the complexity of the contexts seemed to inhibit the development of meaningful understanding of decimal notation.

Irwin (1996; 2001) designed an intervention study on decimals for 16 students (ages 11 and 12) working in pairs on problems in everyday contexts such as decimals in metric measurement systems and money contexts. Another group of students worked on similar decimal-fraction problems without context. In each contextual problem, two hypothetical answers were presented, one answer derived from a misconception and the other, the correct answer. Concrete models, as illustrated in the contextual problems, and a calculator were provided during this intervention study. Both groups of students participated in pre- and post-testing, involving non-contextual problems so as not to favour students in the intervention group. The main findings of this study suggested that students who worked on the contextualized problems improved on their knowledge of decimal fractions more than those on non-contextualized problems. Note that in this case, decimal fractions are limited to rational decimal numbers with finite lengths. Furthermore, Irwin justified her choice of using everyday settings as follows:

> Problems presented in everyday settings provided the context that students needed for reflection on the scientific concept of decimal fractions… Such problems may have provided the reflection required for expanding their knowledge of decimal fractions. (p. 416)

However, limitations and problematic aspects of using everyday contexts such as money in interpreting decimals notation were observed and reported. This problem emerged in conversation between pairs of students on how to solve a conversion rate problem between New Zealand and Australian dollars (1 NZ$ = 0.9309 AU$). At the end of the discussion, both students were uncertain whether 0.9309 represented 93.09 cents or 93.09 dollars. The fact that the explanation drew on analogy of another conversion rate (New Zealand dollar to Indonesian rupiah) instead of on the multiplicative relations on decimals highlighted the problematic aspect of working with the money context where the understanding of parts of a unit was not necessarily due to the availability of smaller units.

Whilst other researchers (Thompson & Walker, 1996) take a similar position as Irwin and Lampert with regard to the relevance of everyday contexts, many researchers (Brekke, 1996; Brousseau, 1997) have pointed out the problematic aspects of metric measures and money in teaching decimals. They claimed that the use of metric measurement or money context in teaching decimals could lead to interpretation of decimals as pairs of whole numbers as expressed in Brekke's comment:

> When children first experience decimal numbers, usually in connection with money and measurement, they may lead to believe that the decimal point is introduced to separate two units of measurement. From the teaching of fractions they know that the fraction bar is used to split for example a part from a whole. They are also told that there is a relationship between fractions and decimal numbers. It is therefore not a great step further for them to conceive that the decimal point as a separator also. (p.142)

Common practice, such as introducing decimal notation as composed of whole number parts and decimal parts, might also lead students to think of the decimal point as a separator of two different units: for example, separating the dollars from the cents or the metres from the centimetres. Another concerning note from Brekke (1996) is his observation that this kind of teaching approach has a lasting impact on children and impedes their proper understanding of decimal place value. He commented, "It seems that the teaching of decimals as one number which can contain tenths, hundredths, thousandths, etc of a unit, does not replace this first decimal experience with money and measurement" (p.2-138). Therefore it would be better to build understanding of decimal notation based on place value understanding rather than rely on these metric contexts.

Similarly, Stacey & Steinle (1998) found evidence that students who showed over-reliance on the money context had difficulties in comparing pairs of decimals such as 4.4502 and 4.45:

> When the numbers are the same in the same spot I get very confused ... Does the number get bigger or smaller with more numbers on the end?...When the number after the decimal point is different the question is easier but when they are the same, I don't know what rule to apply. (p.60)

Reliance on the money context indeed could provide a hindrance to the development of a robust decimal concept as evident in a case above, because money is a discrete system, not a continuous one. Students can deduce 4.4502 = 4.45 because they are the same amount of money.

In the same vein, Basso, Bonotto, and Sorzio (1998) reported a problem of utilizing the metric system in teaching decimals. Their study showed that in comparing 8.1 and 8.15, one child responded that 8.1 was larger because it was composed of 8 dm and 1 cm and cm was a larger unit compared to mm. Apparently, the student interpreted a decimal number as composed of pairs of whole metric units and associated metric units based on the length of decimal digits, 1-place decimal with cm and 2-place decimals with mm. Focussing on the size of the parts, in this case comparing cm and mm in general, led students to conclude that 8.1 was larger than 8.15. This is like the denominator focussed thinking (S1) of Table 2.1.

Moreover, Brousseau (1997) pointed out that the use of decimals to express measures of cardinality of finite sets such as population or in the metric measures where all units of size can be divided by ten, emphasized the discrete nature and hence might mask the continuous nature of decimals including density.

> Under these conditions, decimal numbers retain a discrete order, that of the natural numbers; many students using this definition will have difficulty in imagining a number between 10.849 and 10.850. (p.125)

In summary, teaching ideas which integrate decimals with other constructs often face problems originating from either the complexity of the contexts or weak knowledge of other constructs which interfere with the effort to build understanding of decimals through these constructs. These studies also highlight the limitations and problematic issues of using contexts such as money and metric systems in teaching decimals. Therefore taking into account the problematic sides of such contexts informed by prior studies, my study opted for contexts which explored basic notions of decimals without links to metric measures. These links must be made at some point, but not necessarily in these lessons.

Teaching ideas from the other camp (Hiebert, 1992; Hiebert et al., 1991; Stacey, Helme, Archer et al., 2001; Steinle et al., 2006; Wearne & Hiebert, 1988a) focused on developing meaningful understanding of decimal notation. These teaching ideas shared the same stand in creating meaningful interpretation of decimal notation as the initial step in activities in which models played a crucial role as referents. This was based on the fact that many misconceptions and difficulties reflected scant meaningful understanding of decimal notation.

The emphasis in creating meaningful understanding of decimal notation is reflected in Wearne and Hiebert (1988a)'s four stages of developing understanding of decimal notation, particularly in the first two stages. The last two stages link the first knowledge of the meaning of decimals with the procedures and rules for operations with decimals:

> (1) creating meaning for notation by connecting them to familiar or meaningful referents;
> (2) developing symbol manipulation procedures in which "procedures are developed as actions on referents are extended and reflected unto the symbols";
> (3) elaborating (means extending syntactic procedures to other appropriate contexts) and routinizing procedures (memorizing and practicing rules until they become automatic and can be executed with little cognitive conflict;
> (4) using symbols and rules (in a familiar system) as referents (in constructing more abstract system). (p. 372-3)

Wearne and Hiebert (1988a) argued that building on the first two stages, called "semantic processes", will lead to a better transfer and understanding of links between fractions and decimals, and decimal magnitude including ordering decimals. In the first stage of creating the meaning for decimal notation, Dienes base 10 blocks (referred to as MAB later in this thesis) were employed as a referent (see e.g., Hiebert et al., 1991; Wearne & Hiebert, 1988a). However, Hiebert et al. (1991) were aware of the fact that MAB models attended more to the discrete nature of decimals. Hence, a number line was utilized to address the continuous nature of decimals such as density.

The MAB model has been widely used for teaching whole number place value and operations as well as decimals. English and Halford (1995) pointed out that prior use of MAB for teaching whole numbers added an extra cognitive load when MAB model was utilized for teaching decimals. The Stacey, Helme, Archer, and Condon (2001) study confirmed this prediction and an alternative model called Linear Arithmetic Blocks (LAB) was proposed for teaching decimals.

In contrast to MAB, which is a volume-based model, LAB represents decimals by the quantity of length (not metric length such as metres and centimetres). It consists of long pipes that represent a unit and shorter pieces that represent tenths, hundredths, and thousandths in proportion. Pieces can be placed together to create a length modelling a decimal number and can be grouped or decomposed (for example to show 0.36 as 3 tenths + 6 hundredths or as 36 hundredths).

In line with Hiebert et al.'s (1991) and Wearne and Hiebert's (1988b) stance on the importance of building a meaningful interpretation of decimal notation, some educators proposed teaching ideas which started with explorations of base ten structures of decimals using the LAB model (see e.g., Helme & Stacey, 2000; Steinle et al., 2006). Similar to Hiebert et al. (1991), they also recommended a number line for teaching decimals, particularly in incorporating discussion of various sets of numbers such as whole numbers, common fractions, negative numbers and decimals. The role of a game such as the 'Number Between' game in addressing density notion of decimals has been pointed out by Tromp (1999). The LAB model leads easily to the number line.

Stacey, Helme, Archer, and Condon's (2001) study demonstrated that LAB was a more effective model than MAB for teaching decimals because the model relied on length, a cognitively simpler quantity than volume. It also highlighted the superiority of LAB to model density of decimals and to demonstrate the principle of rounding (due to its linear nature). Neither LAB nor MAB model positional place value due to the fixed value of the pieces. However, MAB demonstrates multiplication by a power of ten better than LAB. In this study, students found the LAB model more acceptable and accessible for three reasons:

(1) the newness of LAB in contrast to the prior use of MAB for whole numbers;
(2) the simplicity of LAB as a length-based model in contrast to MAB as a volume-based model
(3) the complexity caused from switching various MAB components such as mini, long, flat, block/cube with their associated apparent dimensions of 0, 1, 2, and 3 respectively.

Based on these observations, Helme and Stacey (2000) and Steinle (2004b) recommended LAB as a starting model in teaching decimals instead of MAB.

Review of the literature in Section 2.2 has established the need to address decimal misconceptions in teacher education. Yet most of the proposed teaching ideas were targeted at the primary school level. Studies on teaching ideas to improve pre-service teachers' knowledge and teaching ideas on decimals are needed particularly concerning pedagogical knowledge pertinent to pre-service teachers. Moreover, the fact that pre-service teachers already have knowledge on decimals, which might be fragmented or incomplete in their training requires a different approach than teaching decimals to primary students who are learning decimals in school for the first time.

This study will try to fill in this gap by capitalizing on ideas from different sources to fit with the target audience in teacher education. This is in line with the process of devising local instruction theory using design research methodology (Gravemeijer, 1998, 2004), which will be discussed in Chapter 3. This study will follow this line of thought by employing LAB in the initial activities. The linear nature of LAB fits with the didactical analysis of the context problems presented in the starting activity which will be explicated in Chapter 3. More symbolic models such as number line and number expander (a symbolic model which show various expansion of a decimal number in different place value terms) will also be employed.

## 2.4 RME basic tenets and teaching ideas on decimals

Considering the current reform effort to improve mathematics education in Indonesia by adapting Realistic Mathematics Education (RME) theory, this section will discuss in more detail the RME basic tenets and teaching ideas grounded on RME theory.

The basic tenets of RME refer to guided reinvention, didactic phenomenology, and mediating model principles. All these tenets are inspired by Freudenthal's (1973; 1983; 1991) foundational principle of 'mathematics as a human activity'. This notion places a heavy emphasis on students constructing their own knowledge with the guidance of teachers in the learning experience of mathematics.

In addition to the three basic tenets of RME, five basic principles of instructional design (Treffers, 1987) will be reviewed briefly. Treffers (1987) articulated these five basic principles which guide both how learning is constructed and principles for teaching that support learning process in RME lessons.

### 2.4.1 The basic tenets of RME

Each of the RME basic tenets will be reviewed and teaching ideas on decimals consonant with the RME basic tenets will be discussed.

**Guided reinvention**

The guided reinvention principle is advocated by Freudenthal in response to teaching '*mathematics as a ready-made system*', where the end results of the work of mathematicians are taken as the starting points of mathematics education (Gravemeijer & Doorman, 1999). Freudenthal (1973; 1991) contends that mathematics should be undertaken as an activity for students to experience mathematics as a meaningful subject and to better understand mathematics. In his opinion, mathematics should not be presented as ready made.

The guided reinvention principle places importance on mathematics as a process in which students experience learning mathematics in activities guided by the teachers or their peers. The idea is "to allow the learner to come to regard the knowledge they acquire as their own private knowledge, knowledge for which they themselves are responsible" (Gravemeijer & Doorman, 1999, p. 116). The inspiration for the reinvention route could be grounded on the history of mathematics or students' spontaneous solution strategies. For instance, Freudenthal (1983) observed that in the approach to decimal notation by Simon Stevin (1585), decimals was closely connected to a decimal system of measure. This may provide a starting point for teaching about decimals. However, as Gravemeijer (1998) pointed out it might not be sensible to expect the reinvention of decimal notation since this would involve a lengthy and complex process.

In teaching ideas on decimals developed based on RME tenets (see e.g., Gravemeijer, 1998; Keijzer, van Galen, & Oosterwall, 2004), the guided reinvention tenet was translated into reinventing the salient base-ten positional system in decimal notation. Both Gravemeijer (1998) and Keijzer et. al (2004) followed this idea by choosing the context of refining measurement units to measure quantities more precisely as the initial point of departure of reinvention route. For instance, Keijzer *et al.* (2004) employed a problem of measuring with a rope of a certain unit where repeated partitioning was explored in the context of refining the measurement.

However, it was found that repeated *halving* (repeated partitioning into 2 equal parts) was more natural to students than repeated refinement into ten equal parts (called '*decimating*').

The present author has observed the same phenomenon with teacher education students and trainee nurses, which will be discussed in Chapter 4 and 5. The novel issue of how guided reinvention tenet could be interpreted in teacher education level will be explored in Chapter 6 and 7.

**Didactical phenomenology**

The didactical phenomenology principle concerns finding problem situations that allows generalizations and provides a basis for linking solutions to concepts or properties in mathematics. The choice for the initial contexts is not restricted to real-world situations; even the world of mathematics and fairy tales, can serve as a 'real context' when these are understandable by students (see e.g.,Treffers, 1987; van den Heuvel-Panhuizen, 2001). Gravemeijer (1994a) expounded on this tenet as follows:

> Didactical phenomenology points to applications as a possible source. Following on the idea that mathematics developed as increasing mathematization of what were originally solutions to practical problems, it may be concluded that the starting points for the re-invention process can be found in current applications. The developer should therefore analyse application situations with an eye to their didactic use in the reinvention process. (p. 179)

Freudenthal (1983) observed that a possible source of applications for common and decimal fractions involves problems of measuring an object where there exists a remainder but a greater precision and a more systematic procedure is required. Based on this analysis he contends that "length is one of the concepts by which common and decimal fractions can operationally be introduced" (p.26). This idea is followed by both Treffers (1987) and Streefland (1991) who suggested measuring with remainders, which elicit refinement of a given measuring unit as one way of introducing decimals in relation to common fractions. In similar vein, Streefland (1991) and Gravemeijer (1998) proposed the use of metric measurement as potential contexts of the repeated decimating strategy. Moreover, Gravemeijer (1998) included money as another potential context for decimals although you do not want to get smaller and smaller amounts.

Teaching decimals from the money context will not find relevant applications in the Indonesian context because the currency system in Indonesia reflects only the whole number system and does not recognize units smaller than one, such as cents which represent one hundredth of a dollar. As discussed in Section 2.2, prior studies (Brekke, 1996; Brousseau, 1997) have voiced concerns about the use of money for teaching decimals. Brekke (1996) found that use of money context in teaching decimals, particularly in primary school, might inhibit students from developing a deep understanding of the decimal concept:

> Teachers regularly claim that their pupils manage to solve arithmetic problems involving decimals correctly if money is introduced as a context to such problems. Thus they fail to see that the children do not understand decimal number in such cases, but rather that such understanding is not needed; it is possible to continue to work as if the numbers are whole ... It is doubtful whether a continued reference to money will be helpful, when it comes to developing understanding of decimal numbers; on the contrary, this can be a hindrance to the development of a robust decimal concept. (p. 138)

Hence, the idea of repeated refinement into 10 smaller units might be best pursued outside linear metric measure contexts. The didactical phenomenology is of great significance in generating problem structures for instruction, but evaluation remains essential at this stage. This will be discussed further in the design of activities in Section 3.3.

**Mediating model**

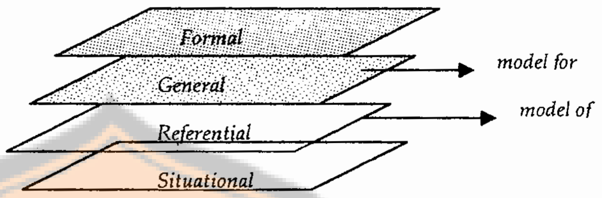
According to RME theory, models are not limited to concrete models but also include situational models (such as fairy tales stories that serve the purpose of illustrating mathematical principles) or mathematical relations (see e.g., Gravemeijer, 1994a, 1998; van den Heuvel-Panhuizen, 2001; 2003). Models are first linked with the contextual problems and then, by gradually by solving similar problems, students will be led to more formal mathematics. Ideally, models in RME emerge from students' own activities and then gradually serve as a catalyst for a growth process to more formal knowledge (Gravemeijer, 1998). Gravemeijer noted that it is not always possible to have students re-invent models on their own. Sometimes, models are given to students but in that case, these models should support the transition of students' thinking of more formal mathematics.

Gravemeijer (1997; 1998) linked the transitions of models with the four levels of how reinvention process was structured:

i. a situation level, where domain-specific, situational knowledge and strategies are used within the context of situation;
ii. a referential level, where models and strategies refer to the situation described in the problem;
iii. a general level, where a mathematical focus on strategies dominates over the reference to the context;
iv. a formal mathematics level, where one works with conventional procedures and notations. (p. 286-287)

The following figure depicts how the concept of models relates to different levels of activities in RME:

Figure 2.3: Levels of models in RME (derived from Gravemeijer, 1998, p. 286)




In RME approach, the starting point should be connected to the knowledge of students, through what is called the situational activity. As discussed above, the context of measurement, which relates to how decimals are used in daily life, served as a situation level in RME teaching ideas.

Gravemeijer (1998) points out that the number line is an important model in RME for teaching decimals because it allows the shifts from informal to more formal mathematical activity. At the situational level, he argued that the use of a 'simple ruler' (without refined units of tenths markings) offered alternatives of refinement such as halving or refinement by ten. A more precise ruler with fixed refined units was used on the next level.

This study will try to accommodate the basic tenets of RME in the design of the activities, as will be discussed further in Section 3.3. The following few paragraphs will review some teaching ideas on rational numbers which included discussion of teaching ideas on decimals. The purpose of this review is to gain more ideas about the interpretation of teaching ideas consonant with RME, which will be helpful in design and refection of activities in this study.

Boufi and Skaftourou (2002) adopted teaching ideas from a unit "Measure for Measure" of the Mathematics in Context (Encyclopaedia Britannica Educational Corp., 1998) for teaching decimals to a fifth-grade classroom in Athens. Decimals in this study were introduced in the context of measuring the turn of the wheel. Interestingly it was reported that repeated halving was easily invented by students but not repeated division by ten. This trend was also similar to findings reported in other studies discussed above (see e.g., Keijzer et al., 2004). The teacher in this study announced the wheel was repeatedly divided by ten and students accepted this as an efficient way to measure more accurately the result of the measurement. The exploration of decimal relations in the problem of measuring the turn of the wheel was extended to a problem of exploring subunits' relationships on the metre stick.

Following Gravemeijer (1998), this study capitalized on the use of the double number line, integrating two units of metric measures such as metre and centimetre on the number line in the next sequence. In this study, Boufi and Skaftourou observed a phenomena indicating reliance on numerical patterns instead of reasoning with repeated decimating on the number line. They pointed out the need for children "to reflect on their decimating activity in connection with decimals numbers" to advance their understanding of place value, otherwise students' understandings of place value "remain instrumental" (p.157).

Hadi (2007) devised learning materials focussing on fractions which integrated lessons on decimals and percentage, grounded on RME theory for Indonesian primary school children. In these learning materials, a fair sharing context of dividing cakes serves as an initial activity to develop a sense of magnitude of fractions. The context problem of measuring length was utilized to explore repeated halving and then was followed by exploring repeated division into 10 in the context of measuring a turn of a wheel. One-place decimals was introduced in relation to the corresponding decimal fraction, such as $\frac{1}{10} = 0.1$, $\frac{2}{10} = 0.2$, etc and two-place decimals were linked to percentage. This study reported positive reactions to the lessons in the two trials of the activities as documented on students' questionnaires. The use of real contexts in a fair-sharing problem, models and interactivity depicted some characteristics of RME lessons.

However, no explicit report was given on the impact of the activities on students' knowledge on the constructs addressed in the activities including decimals.

The review of teaching ideas on decimals consonant with RME tenets signified the importance of attending to repeated base ten partitioning in decimal notation. This idea will be adapted in the design of the starting contextual problem in this study. In addition to attending to basic tenets of RME discussed in Section 2.4.1, implementing activities in the classroom context is guided by RME learning and teaching principles which will be addressed in the following section.

### 2.4.2 RME Learning and Teaching principles

In addition to the three basic tenets of RME, Treffers (1987) articulated five principles for teaching and learning as follows:

1. Phenomenological exploration in which the concrete context and real phenomena are explored as starting points and applications.
2. Bridging by vertical instruments using models or concrete situations to bridge the gap between the informal and formal levels
3. Student contribution, emphasizing active role of students in constructing their own knowledge;
4. Interactivity, placing importance on explicit negotiation, intervention, discussion, cooperation, and evaluation as means of progressing from informal to more formal knowledge
5. Intertwining, incorporating applications and dealing with learning strands in a problem solving.

The first principle relates to the Didactical Phenomenology tenet and the Guided Reinvention tenet, whereas the second and the fifth principles relate to the Mediating Model tenet. The third and the fourth principles reflect the pedagogy characteristics of RME lessons.

Treffers (1987) particularly emphasizes the importance of the third principle, i.e., the pupil's own contribution to the learning process with the interactive instruction principle serves as a catalyst for the learning process to occur in the classroom situations. He noted that "in realistic programs and textbooks, instruction is didactically organised in such a way that interaction and cooperation between the pupils and with the teacher are coexistent with individual work" (p.261). The two pedagogical principles (principle 3 and 4) are in line with the widely growing vision of classroom in reform movements around the world. The vision underlying many of the reform movements in Western countries view learning mathematics as a process of constructing and negotiating meaning where students actively engage in the learning process as a community (see e.g., Kilpatrick, Swafford, & Findell, 2001; Victorian Department of Education and Early Childhood Development, 2007).

In line with the current reform effort in mathematics education in Indonesia, this study tries to establish this classroom culture by encouraging students to engage and contribute in discussion and negotiation throughout the activities.

## 2.5 Concluding Remark

It is well established, based on the reviews of past and current studies, that teaching and learning decimals is a complex topic for both children and adults. Improving pre-service teachers' content and pedagogical content knowledge on decimal numeration in their training is expected to assist in breaking the cycle of misconceptions on decimals. The main areas of concern as reviewed in Section 2.2 need to be addressed.

However, to date, insufficient studies have attended to design instruction to improve understanding of decimals and PCK for teaching decimals in teacher education context. Hence there is a gap in studies that are aimed to improve decimal understanding in primary or secondary schools and in teacher education. This study will try to fill in the gap, as will be explicated in Chapter 3, 4, and 5.

Consistent with the current reform in Indonesian to adapt RME principles in mathematics pedagogy, teaching ideas consistent with RME tenets are of particular interest in this study. The challenges that arise in designing curriculum and teaching materials that are consistent with RME tenets and that fits Indonesian contexts is an issue that needs work (Hadi, 2007; Zulkardi & Ilma, 2007; Zulkardi, Nieveen, van den Akker, & de Lange, 2002).

Moreover, the key role of teacher education in dissemination of PMRI (see Section 1.1) in Indonesia has been articulated (Hadi, 2002; Sembiring, 2007; Zulkardi & Ilma, 2007). This again underscores the importance of preparing the set of activities for pre-service teachers to revisit and to improve their content and pedagogical ideas on decimals. The elaboration of this idea will be discussed in Chapter 3.

# CHAPTER 3 METHODOLOGY

## 3.1 Introduction

This chapter contains a description of the methods to address the research questions and provides details of the procedures for analysing and interpreting data in this study. Section 3.2 contains a general overview of design research methodology and its iterative cyclic phases of design, teaching experiment and retrospective analysis. Section 3.3 presents a summary of main activities involved in cycles and phases of the design research in this study. In Section 3.4, the instruments utilized in this study including the framework for analysing the data obtained from the instruments will be explicated. Section 3.5 will discuss the methodological concern about the validity and reliability in design research. Finally Section 3.6 concludes with an overview of data sources and their relations to the research questions and the goals of this study.

## 3.2 Overview of Design Research

Design research (Cobb, Stephan, McClain, & Gravemeijer, 2001; Edelson, 2002; Gravemeijer, 2004; Kelly, 2003; Research Advisory Committee, 1996) is considered as an emerging paradigm which aims to develop sequences of activities and to grasp an empirically grounded understanding of how learning works. Other terms such as "design experiments" (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Wood & Berry, 2003) or "developmental research" (Gravemeijer, 1994b, 1998; van den Akker, 1999) have been used to refer to design research methodology. Despite different terms and interpretations in describing design research, the iterative cyclic character of design research and its role in developing domain specific theories are the shared key characteristics of the design research.

In design research, the process starts with an anticipatory thought experiment by the researcher to formulate a hypothesis about how activities will be employed to promote the identified goals. In constructing a provisional design, the researcher adopts ideas from different sources available from previous studies, in this case in teaching and learning decimals, and makes them fit the purpose of the study. Identifying the central idea of the study in design research is accomplished by drawing on and synthesizing prior literature (Cobb et al., 2003; Gravemeijer, 1998). Gravemeijer (1994b; 1998; 2004) referred to this approach as 'theory-guided bricolage'.

Moreover, Edelson (2002) points out that the process of adopting ideas is informed by knowledge of gaps in current understanding of the area. The hypothesis is then tested, elaborated and refined during and after a series of deliberations on, and trials of, the activities in the classrooms. The knowledge formed is then used to construct a recommended sequence of activities. Edelson (2002) articulated the cyclic processes involved in design research as follows:

> In this theory development approach, the design researchers begin with a set of hypotheses and principles that they use to guide the design process.... design researchers proceed through iterative cycles of design and implementation, using each implementation as an opportunity to collect data to inform subsequent design. Through a parallel and retrospective process of reflection upon the design and its outcomes, the design researchers elaborate upon their initial hypotheses and principles, refining, adding, and discarding-gradually knitting together a coherent theory that reflects their understanding of the design experience. (p.106)

Gravemeijer (1994b; 2004) contends that the process of cyclic alternation between thought and practical experiments can be considered as the concrete sediments of a local instruction theory. Cyclic alternation between thought experiment and teaching experiment in design research contribute to the strength of design research, i.e. "the explanatory power" and "their grounding in specific experiences" (Edelson, 2002, p. 118).

The next section will expand on design research in this study by articulating the process involved in various cycles and phases of the design research in this study.

## 3.3 Design Research in this study

The design research in this study adheres to Gravemeijer's account (2004) whereby a set of instructional activities for decimals is devised through a cycle of design, teaching experiment, and retrospective analyses. The starting point for devising the instructional activities is taken from the likely existing knowledge of the students and by hypothesizing their learning trajectories at the design phase. At this phase, the learning goals are determined and the set of planned activities are devised to achieve the learning goals. Following that, a conjectured learning path of how students' understanding will evolve from their prior knowledge in working with the activities is developed. This process constitutes a conjectured Local Instruction Theory or LIT (Gravemeijer, 1998, 2004).

There are two main reasons for choosing design research methodology in this study. Firstly, there is, as yet, no existing elaborated theory of teaching and learning decimals available for pre-service teachers in the Indonesian context. Hence, design research suits the purpose of developing a prototype for local instruction theory on decimal topic in teacher education level grounded in Indonesian context. Secondly, design research enables the researcher to study the learning process of participants and to find the extent to which activities impact on pre-service teachers' understanding of decimal notation, which was consistent with the aim of this study.

The justification of design research involves not only in selecting methods of data collection but also in the structure of reporting the findings from both cycles as will be explicated in Chapter 4 and Chapter 5. The rationale for determining goals of the activities, choosing the test items and selecting parts of empirical data to focus on the analysis will be made available along the way.

### 3.3.1 Cycles of design research

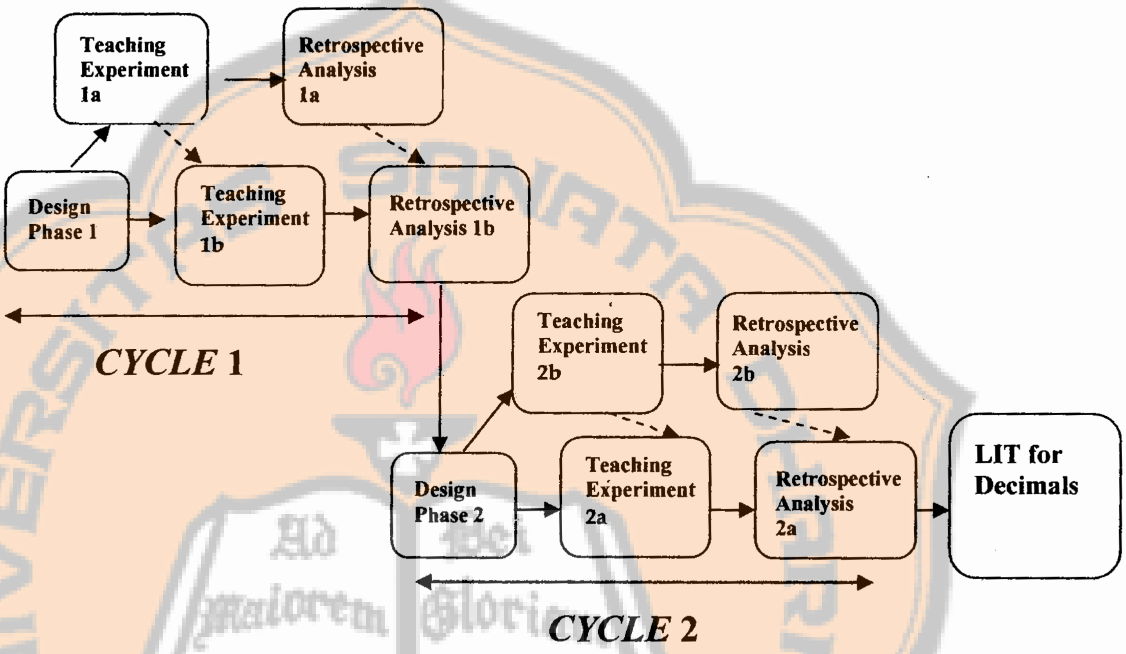
The main study was carried out in two cycles with each cycle consisting of three phases. One cycle here refers to a complete process of the design, implementation of the design in the teaching experiment, and reflection of the design and its implementation in the retrospective analysis phase as illustrated in Figure 3.1.

There are two reasons for having only two cycles in this study. Firstly, the researcher's restricted time (i.e., approximately three years) and resources limited the number of cycles. Secondly, the courses that embedded activities of decimal topic were only offered once a year. Whilst more cycles may be desirable, testing and refining the activities and its sequence with pre-service teachers from two different programs across two cycles with multiple classes provided a considerable insight into the learning of decimals. The adequacy of two cycles will be assessed in Chapter 6.

The first cycle was carried out in 2005 involving three participating classes (*N*=118), with the second cycle in 2006 involving four classes (*N*=140) of the next cohorts of Sanata Dharma University pre-service teachers. The first cycle implemented activities and subsequent retrospective analysis provided a recommendation for the improvement of the activities in the second cycle. The second cycle implemented and tested the refined activities developed in the first cycle. Findings from the analysis of data from the second cycle provided the final recommendation of this study.

An overview of the phases involved in the design research of this study is presented in Figure 3.1. Note that the 1 and 2 referred to the cycle. The letters a and b refer to the two separate cohorts of pre-service teachers involved in each cycle: a for the primary cohort and b for the secondary cohort.

Figure 3.1: Cycles and Phases of Design Research




In cycle 1, teaching experiment with the pre-service primary was carried out earlier than the secondary cohort due to the time arrangement with volunteer lecturers. The dotted arrow from Teaching Experiment 1a to Teaching Experiment 1b signified feedback from Teaching Experiment 1a that was accommodated in Teaching

Experiment 1b. Similarly, data from retrospective analysis 1a pertinent to the pre-service secondary cohort were fed into retrospective analysis 1b.

In contrast, time arrangement with the lecturer in charge of the three primary cohort classes in cycle 2 did not allow for the same arrangement to take place. Teaching experiments in the secondary cohort started first and consequently adjustments coming from teaching experiments from the secondary cohort were fed immediately to the primary cohort. This will be discussed further in Chapter 5.

### 3.3.2 Phases of design research

This section presents summary of key things involved in each phase of design research in this study.

***Design phase:***

The main purposes of design phase are to develop a sequence of activities on decimals and to design tools for evaluating the learning processes of pre-service teachers. The first stage in devising the activities comprised determining the goals of the activities, selecting the activities and conjecturing pre-service teachers' learning pathways in achieving those goals.

The goals for the activities, as mentioned in Chapter 1, were determined based on analysis of the teaching approach on decimals addressed in commercial Indonesian primary mathematics textbooks. Details of the analysis carried out in cycle 1 will be presented in Section 4.2. Key areas on teaching and learning of decimals which have been identified in prior studies (see Chapter 2) served to identify the likely gaps in knowledge that need to be addressed in the activities. The activities in this study also aimed to improve pre-service teachers' pedagogical ideas on decimals. Accommodating the basic tenets of RME in the design of the activities was another important feature of this phase and was principally carried out by review of literature and published RME materials. The details of the process involved in the design phase in each cycle will be elaborated later in Section 4.2 and Section 5.2.

Table 3.1 presents the links among general learning goals, ways of evaluating the achievement of these goals and corresponding items in the written tests, which will be discussed in Section 3.4.

Table 3.1: Links among areas in learning goals, measure of achievement and corresponding items

| **Areas in learning goals** | **Measure** | **Part of written test** | **Item no or group of items*** |
|---|---|---|---|
| Meaningful interpretation of decimals | Comparing pairs of decimals | A | DCT3a |
| | Decomposing and unitising decimals into place value terms | B | 2a,b,c,d, 3a, b |
| Explicit knowledge of place value | Identifying place value names of a decimal digit | B | 1a,b,c,d |
| Knowledge on density of decimals | Identifying number of decimals in between two given decimals | B | 5, 6 |
| Knowledge on relative magnitude of decimals and links among decimals, common fractions, and whole numbers | Ordering and sequencing decimals | B | 3, 4 |
| | Placing decimals on the number line | B | 7, 8, 9 |
| | Finding closest decimals to a given decimal number | B | 10, 11 |
| Interpretation of decimals and operation involving decimals in various contexts | Solving word problems involving decimals | B | 12, 13, 14, 15, 16 |
| Knowledge of teaching ideas and representations for teaching decimals | Exploring teaching ideas to compare decimals, to diagnose decimal misconceptions, to divide decimals by 100, and to link common fractions and decimals | C | 17, 18, 19, 20 |

*based on cycle 1 list of written test items

Teaching ideas from various studies (see e.g., Condon & Archer, 1999; Gravemeijer, 1998; Steinle et al., 2002) were adapted to fit the Indonesian context and the basic tenets of RME comprising guided reinvention, didactical phenomenology, and mediating model tenets. Connections to Indonesian contexts and incorporation of concrete models in learning decimal notation were evaluated in relation to their potential to assist pre-service teachers in revisiting and reconstructing both their understanding in various content areas of decimals as well as their pedagogical ideas.

The design phase is also concerned with the choice or development of instruments such as written tests and interview items to measure the impact of the activities. These instruments were designed to enable the identification of changes related to the goals of the program. Details on adoption or development of these instruments will be addressed in Section 3.4.

As part of the design phase, prior to each cycle in the main study, volunteer pre-service teachers at the University of Melbourne and Sanata Dharma University trialled the written tests and the activities. The volunteer pre-service teachers were approached by their lecturers. Group discussions during the trial of the activities were audio-recorded or video-recorded and the completed learning worksheets were collected. Outcomes from the trial of activities were used to refine the activities prior to their use in the main classrooms where the data collection took place.

***Teaching experiment phase:***

The term ‘teaching experiment’ does not refer to a formal experiment and control group design. In this study, the teaching experiment phase encompasses all activities involving direct interactions with the participants as can be observed in Figure 3.1. In this phase, the designed activities were implemented and the extent to which these activities improve pre-service teachers’ content and pedagogical content knowledge was observed and reported. Moreover, factors contributing to the success or lack of success of the activities were investigated. Details of the activities devised in each cycle and report of the enactment, analysis, and interpretation of the outcomes will be elaborated in Section 4.5 and 5.3.

The teaching experiment phase in both cycles involved two cohorts of pre-service teachers, i.e., the primary and the secondary cohort. Different traits of these two groups were seen to provide opportunities for the researcher to test and see how the activities work with different target audiences. Furthermore, this added to the depths of the data in the study and reduced the effect of limited sampling as will be discussed further in Section 3.3.

Observations prior to the enactment of the activities were conducted in 2 to 3 meetings in every participating class to allow pre-service teachers to get familiar with modes of learning and with the observation protocols (video-recording of group works). These observations focused on the interactions and the exchange of ideas during group works and served the purpose of selecting video-recorded groups (1 group in each class). The criteria in selecting video-recorded groups was based on practical considerations such as consent to be video-recorded, adequate communication skills of group members, and high engagement level in group activities and discussion.

Group work was chosen as the dominant mode of delivering the activities during the teaching experiment phase. Pre-service teachers worked together in groups of 4 to 6 of their own choice in discussing and solving problems presented in the activities. The choice of group work was consistent with RME instructional principle of students' contribution and interactivity discussed in Section 2.4.2. This mode of learning was a new experience for many of Indonesian pre-service teachers. Lecturers assigned for the course carried out the activities during their regular classes. The lecturers' role in this study is more as a facilitator for delivering activities and leading group presentations and discussions. This mode of delivering activities is consistent with Freudenthal's notion of guided reinvention, which encourages active participation of pre-service teachers in constructing meanings for themselves. This way, pre-service teachers were expected to explore more ideas and get firsthand experience of new methodologies for their future career. The role of lecturer and researcher will be explained in Section 3.3.2.

Information about pre-service teachers' current state content knowledge (CK) and pedagogical representation knowledge (PCK) was obtained by administering one-hour pre-test (see Appendix B1 for cycle 1 or Appendix B5 for cycle 2) and by conducting interviews (see Appendix B3 for cycle 1 or Appendix B7 for cycle 2) with approximately 5 pre-service teachers from each class and Post-tests (see Appendix B2 for cycle 1 or Appendix B4 for cycle 2) intended to be carefully matched but non-identical items to pre-test were administered following the completion of the activities to observe the impact of designed activities on pre-service teachers' CK and PCK on decimals. A post-course interview (see Appendix B4 for cycle 1 or Appendix B8 for cycle 2) was conducted with after the post-test to gain in depth insights about the impact of activities towards their CK and PCK and to elicit information about the thinking involved in incorrect responses in the written test responses. In this respect, the interview items were guided mainly by explanations for certain pattern of incorrect responses. All interviews were audio-recorded or video-recorded and were transcribed. Selected parts of the interviews were analysed based on their relevance in answering the research questions, and in improving subsequent activities. Discussions about the components in the written tests and interview items will be addressed in Section 3.4.

***Retrospective analysis phase***

The retrospective analysis phase was carried out once the teaching experiment was completed. The retrospective analysis phase (see Figure 3.1) comprised data analysis, reflections, interpretation of findings, and formulation of recommendations for the next cycle as illustrated in Figure 3.1. The aims of the retrospective analysis phase are to evaluate the success of the enacted activities, to observe the learning progress of pre-service teachers and to inform the improvement of the activities for the next cycle.

The retrospective analysis phase involved elaborating data from various sources and looking for trends from various data. Observation of video-recorded group and whole class discussions, which provide insights into learning process in working with the activities, complemented the analysis of group worksheets. These observations and group worksheets were analysed against the initial goals and expectations set in the design phase. Success and lack of success in the enactment of the activities were reported. Factors contributing to success or lack of success were gathered by analysing the discourse during group work, or derived from researcher's observation notes in observing discourse among various groups. Analysis of activities also aimed at identifying problems and gaps found in the design of activities and its enactment during the teaching experiment and accounting for these problems and gaps.

Likewise, responses to the written tests guide the interview questions as interviews also aim to gain in depth insights on pre-service teachers' thinking in solving the written tests. Comparing results of pre- and post-tests as well as pre- and post- interviews (when applicable) provided information about evolutions of pre-service teachers' content and pedagogical representation knowledge and aspects of the learning that were successful or less successful. Selected segments from audio/video recording of observations and interviews were transcribed and analysed based on their relevance to the research questions and contributions to the improvement of activities and written tests or interview items. Space and time limitations preclude the presentation of all this data in this thesis: some representative and some particularly insightful comments are given throughout.

Findings pertinent to research questions were triangulated using the whole data set to check for any inconsistency or deviant cases using constant comparative method (Strauss, 1987; Strauss & Corbin, 1990). When differences existed between expectations and findings, then explanations or interpretations of the causes were sought. The findings were then summarized and illustrated by prototypical examples in Section 4.5 and Section 5.3. Based on analysis and reflections on the findings, ideas to improve activities and instruments used in the next design phase were devised. In this study, the improvement of content knowledge and pedagogical content knowledge was the criterion for optimization from retrospective analysis of cycle 1 to design phase of cycle 2.

Refinement on research instruments were carried out based on analysis of the findings in cycle 1. As this study involved only two cycles, the result of the retrospective analysis of the second cycle served to give a final recommendation from this study. A detailed explanation of the analysis and discussions of findings from each cycle will be addressed in Chapter 4 and 5. An overview of findings from the two cycles will be addressed in Chapter 6.

### 3.3.3 Conduct of the teaching experiments

The main data collections for this study were carried out in Sanata Dharma University, Yogyakarta, Indonesia, the normal employment place of the researcher. This convenient purposive sampling was chosen based on two considerations. First, Sanata Dharma University is one of the higher education institutions that have been involved in developing PMRI in Indonesia since the early stage. The lecturers volunteered in this study were affiliated to PMRI development team which implied they had certain degree of familiarity with the basic tenets of RME. This was an important aspect as the activities designed in this study tried to accommodate the RME basic tenets. Second, the lecturers in this institution were willing to fit the activities designed by the researcher into their lessons with their pre-service teachers. The willingness of lecturers to carry out these activities was a critical factor in determining the selection of the institution. Ethics approval was obtained from the institution and consent from both pre-service teachers and lecturers were sought prior to the teaching experiment.

The pre-service teachers participated in this study were the ones attending 'Teaching and Learning Mathematics for Primary School' from two different programs, the Primary School Teacher Education Program (PSTEP) and the Secondary Mathematics Education Program (SMEP) study program in semester 1, year 2005 and semester 1, year 2006. The pre-service primary teachers were undertaking a two-year diploma program run by elementary teacher training department, whereas the pre-service secondary teachers were enrolled in a 4-year Bachelor of Education program run by the mathematics and science education department. For pre-service primary teachers, 'Teaching and Learning Mathematics for Primary School' was a required course as part of their training. Meanwhile, 'Teaching Mathematics in Primary School' course was an elective course for pre-service secondary teachers. The decision to choose this course to embed the instruction was based on the relevance of decimal topics in this course. It should be noted that the nature of their participation and the teaching intervention for both the primary and secondary were the same. However, in general the mathematical ability of pre-service teachers from PSTEP was lower than that of the pre-service teachers from SMEP, as expected from the lower mathematical entrance score to the programs. In analysing the data, the researcher took into account the different levels of knowledge of the subjects when evaluating the impact of the activities designed in this study. How these different traits relate to the outcomes of the activities in this study will be illuminated in Chapter 4, 5, and 6.

This study operated within the time constraints of PSTEP and SMEP. The lecturers permitted only 4 classroom meetings for the complete program. Discussions on the arrangement and delivery of the activities were held between lecturers and the researcher prior to the enactment of the activities. However, it should be noted that the lecturers were fully responsible of classroom management and all decisions during the classroom activities including presentations Feedback about the activities from the lecturers was obtained through informal interviews prior to and after the enactment of activities. Any interesting phenomena observed during the enactment of activities and group or whole class discussions were also discussed with the lecturers to accommodate them during classroom discussion when applicable. Having the lecturers conduct the activities provides a realistic test of their effectiveness in practice. However, as will be described in Chapter 4 and 5, on a few occasions the intention of the activities was not fully appreciated.

The researcher was present in the classroom as an observer and directed the video recordings of group and whole-class discussions accompanied by two technical assistants. Two video-cameras were used during the observations, one was directed to a group (in each class) that was followed during the whole set of activities. The other video camera moved around the class to capture the variety of responses around the class and to get a general impression of how the activities worked in the classroom situations. During the whole-class discussions led by the lecturers, both cameras were used to capture the public classroom discussions and presentations of ideas by the lecturer and pre-service teachers. The group worksheets accompanying the activities were collected after each meeting for subsequent analysis. Researchers' notes during the observation supplemented the group and whole class discussion for the purpose of ongoing analysis. These data allowed the researcher to note phenomena and trends of difficulties which guided the retrospective analysis phase.

Consent to participate in this study was sought from pre-service teachers, with options to contribute to the data by taking parts in written tests, interviews, and video-recorded activities. The participants were made aware at the beginning that their participation in this study would not affect their grade in the subject they were taken. Moreover, they were allowed to withdraw their participation at any stage of data the research.

## 3.4 Instruments

This section will explicate the design of the written tests and interview items utilized in this study and ways of analysing the findings in relation to the research questions and to the improvement of the activities.

### 3.4.1 Written tests

The first part of the written test (labelled as Part A) comprised the 30 item decimal comparison tests, DCT3a and DCT3b (see Appendix B1 for detail of DCT3a and Appendix B2 for detail of DCT3b). This test was adopted to identify ways of thinking in interpreting decimal notation. Research has shown that DCT is an insightful instrument to diagnose misconceptions on decimals reliably. Similar to the earlier versions, DCT3a and DCT3b classify ways of thinking based on the performance on different item types and not based on the total score.

The items in DCT3a and DCT3b belong to ten different item types, listed in Table 3.2. Responses to item type 1 and type 2 in DCT3a and DCT3b served as a core criteria in identifying coarse codes (L, S, A, or U) and pattern of responses to item types 1, 2, 3, 4, 4R, and 5 determined the fine code (see Table 3.2 for item types). The remaining 9 items of DCT3a and DCT3b (from type 8, 9, 10, and 11) provided information on pre-service teachers' knowledge on the role of zero in decimal numbers and relations between zero and decimals. Change in ways of thinking was observed by comparing the thinking classification from the pre-test to the post-test. The DCT3a and DCT3b were analysed following the classification criteria spelled out by Steinle and Stacey (2004b) as illustrated in Table 3.3. Explanations about various ways of thinking adopted from Steinle (2004) have been given in Section 2.2.

Table 3.2: Types of decimal comparison items and number of items in DCT3a

| Type | Number of items | Example | Brief description of item type |
|---|---|---|---|
| 1 | 6 | 3.92/3.4813 | Unequal length. The larger decimal is the shorter |
| 2 | 6 | 0.6/0.73 | Unequal length. The larger decimal is the longer |
| 3 | 2 | 4.08/4.7 | A zero in the tenths column of one number, which would otherwise be the larger |
| 4 | 2 | 4.4502/4.45 | One decimal is a truncation of the other |
| 4R | 2 | 3.7/3.77777 | One decimal is a truncation of the other |
| 5 | 3 | 0.3/0.4 | Equal length decimals |
| 8 | 2 | 0/0.6 | A comparison of a positive decimals with zero |
| 9 | 2 | 0.0004/0.4 | Unequal length decimals. |
| 10 | 3 | 0.8/0.80000 | Decimals with the same value but different lengths |
| 11 | 2 | 3.72/3.07 | Equal length decimals with a zero in the tenth column of one number |

Table 3.3: Item types and classification of ways of thinking (fine codes) in DCT3a and DCT3b

| | Item type (number) | | Examples | Fine Codes | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | L1 | L2 | L3 | L4 | S1 | S3 | S4 | A1 | A2 | A3 | U1 | U2 |
| Core | 1 | (6) | 0.41/0.362 | Lo | Lo | Lo | Lo | Hi | Hi | Hi | Hi | Hi | Hi | | |
| Core | 2 | (6) | 5.73/5.847 | Hi | Hi | Hi | Hi | Lo | Lo | Lo | Hi | Hi | Hi | | |
| Non-Core | 3 | (2) | 3.72/3.073 | Lo | Hi | Lo | | Hi | Hi | | Hi | Hi | | | |
| Non-Core | 4 | (2) | 1.1503/1.15 | Hi | Hi | Hi | | Lo | Lo | | Hi | Lo | | Else | Else, few correct* |
| Non-Core | 4R | (2) | 3.7/3.77777 | Hi | Hi | Hi | Else | Lo | Lo | Else | Hi | Lo | | | |
| Non-Core | 5 | (3) | 0.7/0.6 | Hi | Hi | Hi | | Hi | Lo | | Hi | Hi | Else | | |

*Hi–High (at most one error in the set of items for that type)*
*Lo–Low (at most one item correct in the set)*
* *up to 6 correct answers*

The researcher constructed Part B and Part C of the written tests (see e.g., Appendix B1 and Appendix B2 for the complete test items in cycle 1). The construction of items in Part B was guided by main difficulties in content areas of decimals identified in prior studies as discussed in Section 2.2. The items were commonly used in studies such as NAEP studies to examine knowledge in content areas of decimals. Part B was analysed according to areas of content knowledge. Similar weights were assigned to different areas of content knowledge to reflect the importance of these content areas of decimals (see Table 3.4). For each item, the incorrect responses were marked as 0 and the correct responses were marked as 1. Paired t-tests were carried out to observe any changes and to identify areas of improvement. Note that problems involving decimals in word problems context were excluded from the analysis of this part due to the confounding factors such as knowledge of the contexts. Responses on these word problems revealed that pre-service teachers made a number of errors and alternative interpretations that were related to the contexts instead of their understanding of decimals. Therefore these items were not useful in tracking pre-service teachers' understanding that was pertinent to this study.

Patterns and common trends of difficulties and misconceptions in various content areas were surveyed and reported. Variations in the number of items involved in the two cycles reflected the refinement of the instruments after retrospective analysis in cycle 1. This information was then utilized in refining the activities as well as the written test items. Note that the different number of items reflected the refinement between cycles.

Table 3.4: Distribution of items in various areas of content knowledge assessed in Part B

| Areas of content knowledge | Item number on the written test Part B (total score) | | | |
|---|---|---|---|---|
| | Cycle 1 Appendix B1 | Total marks | Cycle 2 Appendix B2 | Total marks |
| Identifying place value names | 1a, b, c | (3) | 1a, b, c, d | (4) |
| Decomposing of decimals | 2a, b (8 alternatives) | (4) | 2 (4 alternatives) | (4) |
| Unitising decimals | n/a | n/a | 3a, b | (4) |
| Ordering and sequencing decimals | 3a, b, 4a, b | (4) | 4a, b | (4) |
| Density of decimals | 5, 6 | (4) | 5, 6 | (4) |
| Relative magnitude of decimals on a number line | 7a, b, 8a, b, 9a, b, c, d | (4) | 7a, b, 8a, b, 9a, b, c, d | (4) |
| Closeness of decimals to a decimal | 10, 11 | (4) | 10, 11 | (4) |

n/a: not applicable (not included in the written tests of cycle 1)

Part C of the written tests was designed to assess pedagogical ideas on decimals (see Appendix B1, B2, B5, and B6 for the copy of the written tests). Scoring rubrics were devised to classify and to quantify responses in Part C as presented in Table 3.5. A second researcher who was not involved in data collection of this study repeated the scoring for some of data from Part C in order to test the consistency of the scoring criteria and to establish the reliability of this scoring. After quantifying the data, a paired 2-tailed t-test was carried out to identify areas of improvement in teaching ideas. Various representations posed in teaching ideas were classified and shifts in trends of dominant representations utilized in teaching ideas were surveyed and reported.

Table 3.5: Scoring criteria for various areas of pedagogical ideas assessed in Part C

| Low (0 out of 2 or 1 out of 3) | Medium (1 out of 2 or 2 out of 3) | High (2 out of 2 or 3 out of 3) |
|---|---|---|
| ▪ Blank or | ▪ Give correct answer based on reliance on rules without meaningful explanations or | ▪ Indicate understanding of basic notion in decimals such as place value and incorporate proper model in teaching ideas or |
| ▪ Indicate misconception on CK or | ▪ Make links to appropriate concept but give no proper models or | ▪ Justify teaching approach that go beyond reliance on "expert rules" and incorporate proper model in teaching ideas |
| ▪ General teaching ideas, e.g., teaching it slowly, repeat explanations, etc | ▪ Include teaching ideas with models but not much explanation | |

To illustrate the scoring criteria, an example of various responses to teaching ideas to Item 15 in the pre-test of cycle 2, i.e., "Explain your ideas for teaching primary school students to find the larger number between 0.8 and 0.8888. Include any models that you can think of in your teaching ideas!" is given in Table 3.6:

Table 3.6: Illustrative samples of scoring criteria in Part C

| Response | Score | Rationale |
|---|---|---|
| Teaching 8/10 > 8888/10000 without explanation | 0 | Indicate a misconception, i.e., overgeneralising that tenths are larger than ten thousandths. |
| Annexing zeros to 0.8, and telling students that 0.8888 > 0.8000. No model is proposed | 1 | Give correct answer based on reliance on rules without meaningful explanations. |
| Suggest both 0.8 and 0.8888 by 100 to conclude that 88.88 > 80. Suggest abacus as a model | 1 | Make links to appropriate concept but suggests no proper model. |
| Decomposing 0.8 = 0 ones + 8 tenths, 0.8888 = 0 ones + 8 tenths + 8 hundredths + 8 thousandths. Suggest ruler as a model | 2 | Indicate understanding of basic notion in decimals such as place value and incorporate proper model for teaching. |

Finally, out of the total score of 9 in Part C, the score are classified into low (score 0 to 3), medium (score 4 to 6), and high (score 7 to 9).

### 3.4.2 Interview items

The interview was designed to examine the current state and the progress of pre-service teachers' content and pedagogical content knowledge on decimals and to elicit pre-service teachers' thinking behind some of the incorrect or unexplained answers on the written tests. List of interview questions along with the rationale for each question for pre- and post-course interviews for cycle 1 and cycle 2 could be found in Appendix B3, B4, B7, and B8. A "Think aloud" procedure (audio- or video-recorded) was employed during the interviews as students worked through the problems in the presence of the researcher, who observed and asked further probing questions. Pre-service teachers were asked to write some part of their explanations, which were kept by the researcher. In cycle 1, investigations of prior schooling experiences were included as part of the interviews prior to the enactment of the activities, but this was excluded in cycle 2.

The interview responses were grouped based on the main content areas or pedagogical ideas and their relevance to the research questions. Segments of interviews that provided clear insights into pre-service teachers' thinking processes or indicated evolutions of their understanding would be reported. Common trends of difficulties or misconceptions observed during the interviews in two cycles were reported in Chapter 4 and 5. Detail discussion on findings from the interviews from samples of pre-service teachers will also be reported in Chapter 6.

## 3.5 Discussion of the methodology

In this section, discussions about methodological and practical issues concerning the research design will be addressed. Moreover, some constraints beyond the control of the researcher that may affect the result of this study will be highlighted.

### 3.5.1 Methodological Issues

The large data corpus gathered from this study consisting video-recording of group discussions, worksheets of group activities, written tests and interviews imposed a challenge to integrate the data across the two cycles. This raises the fundamental issues in design research concerning data reduction and analysis.

Collins, Joseph, & Bielaczys (2004) pointed out that one of the challenges in design research was large amounts of data collected over the number of cycles that require a lot of resources to analysed. In addressing this challenge, the analysis was conducted only on data that would directly contribute to answering the research questions and that provided ideas for the improvement of the activities. For instance, a decision was made not to analyse in detail all data gathered during the teaching experiment. The dimensionality of the qualitative data in this study such as the interview data and video observation data were reduced via exploratory thematic analysis whereas descriptive statistics were employed to reduce the dimensionality of the quantitative data such as Part B of the written tests.

The situated aspect of implementing design research in "real" classrooms created another issue of uncontrolled variables that affect the success or failure of the design. The nature of this study depended largely on voluntary participation of lecturers and pre-service teachers. Thus, establishing and maintaining a respectful and collaborative partnership with lecturers in the research process played a critical role in the success of carrying out the design.

Another characteristic related to the nature of the teacher education program was the limited number of meetings that could be devoted to this study, i.e., 4 to 5 meetings in each class. There are many promising situations which pre-service teachers could explore to deepen knowledge of decimals. However, implementing the design in the real context of teacher education with its limited time allocation forced the researcher to focus the lessons on the most important issues to be addressed. In this sense, the limited time could be considered as strength of the program because there was little benefit in developing a set of activities that could not be implemented because they required too much time particularly in the teacher training context.

### 3.5.2 Justification and Trustworthiness

Following the proponents of design research (Cobb et al., 2003; Cobb et al., 2001; Edelson, 2002; Gravemeijer, 1994b; Research Advisory Committee, 1996; The Design-Based Research Collective, 2003), justification in this study relied on an argumentative character of the thoughts in designing the provisional activities in addition to the interpretation of the empirical findings. Unlike the experimental research, justification in design research methodology is not merely confined to empirical testing but also is in thought experiments (Gravemeijer, 1994b). Thus, documentation of the enactment of the activities provided critical evidence to establish warrants for claims about why outcomes occurred which related to the nature of justification in design research.

The justification in the design research comprised of "an analysis on the area of subject matter, an intrinsically substantiated characterization of the structure and content of the course, paradigmatic examples (of student works and interaction) and a reflection on the realistic calibre of the whole" (Gravemeijer, 1994, p.291). Freudenthal (1991) commented on the importance of reflection and reporting the researcher's thoughts and experience as way of justifying design research or what he referred to as developmental research as follows:

> Developmental research means: 'experiencing the cyclic process of development and research so consciously, and reporting it so candidly that it justifies itself, and that this experience can be transmitted to others to become like their own experience.
> (Freudenthal, 1991, p. 161)

Graveimeijer contends that justification in design research relates to the learning process of the research team in a notion called '*trackability*' by reporting both failures and successes, on the procedures followed, on the conceptual framework, and on the reasons for the choices made. Rationale for choices and interpretation of the empirical data forms part of justification in design research. By so doing, other researchers can retrace the learning process of the researcher and enter into a discussion. This thesis followed this line of justification in reporting the findings and interpretation of the findings in this study as will be discussed in Chapter 4 and Chapter 5.

Following Drijvers (2004), this study perceived internal validity of the data as the measure of "quality of data collection and the soundness of the reasoning that lead to the conclusions" (p. 23). Internal validity of the instruments and content validity of instruments was obtained by reviewing the instruments used in this study by experts in teaching and learning decimals and by having the instrument trialled prior to the implementation stage. The internal validity of the data was also ensured by sharing crucial findings with colleagues for peer examination in departmental seminars to gain different perspectives.

External validity concerns "the bearing of the results on other situations", which is led by a question "how certain elements of the results will apply to other situations" (Gravemeijer, 1994, p.455). In this study, reporting and sharing the findings of the study in publications and conference contributions to gain feedback about the quality of the reasoning promoted external validity (Widjaja, 2005; Widjaja & Stacey, 2006). Moreover, complementary triangulation of data and repeated analysis across the two cycles account for reliability of the findings (The Design-Based Research Collective, 2003).

Trustworthiness in design research is concerned with the reasonableness and justifiability of the inferences and assertions. To ensure this, the analysis of the data generated during the teaching experiment was carried out in systematic and thorough ways. Following Cobb et al (2001), inferences in this study were treated as provisional conjectures that were continually open to refutations. Analysis was documented and reported in such a way that is open to criticism from other researchers to ensure trustworthiness in design research.

## 3.6 Overview of data sources

This section outlines the data sources involved in different phases of the design research cycle and their relevance to the research questions and goals of this study as presented in Table 3.7. Timeline of the research in cycle 1 and cycle 2 are presented in Figure 3.2. Links between various data sources and the research questions are given in Table 3.8.

Figure 3.2: Timeline of the research in cycle 1 and cycle 2

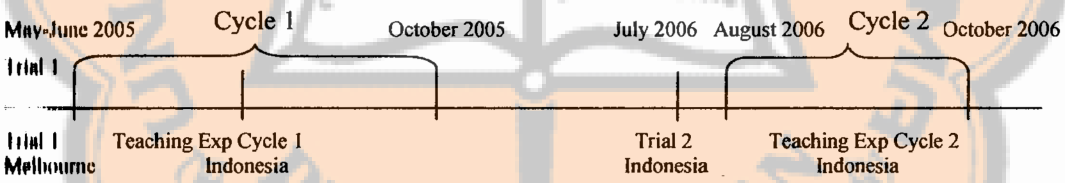


Table 3.7: Outline of data in relation to the research questions and goals of the study for both cycles

| Phase | Data collection technique | Goals or Relevance to Research Questions |
|---|---|---|
| 1 | | ***Design Phase*** |
| | Design instruments and activities | To develop and adapt activities to promote knowledge (CK and PCK) of PSTs* on decimals.<br>To develop instruments that can provide indications of any progress in understanding. |
| | Trial activities | To observe how the activities work and to identify any problem in carrying out the activities. |
| | Trial of instruments with volunteer PSTs | To gain feedback to improve the instruments |
| | Informal Classroom Observation | To familiarise researcher to PSTs involved in the study |
| 2 | | ***Teaching Experiment Phase*** |
| | ▪ Pre-tests | To gain understanding of existing knowledge (CK and PCK) of PSTs (RQ 1a & 2a) |
| | ▪ Pre-course interview (audio/video-taped and transcribed) | To gain knowledge of the PSTs previous learning experiences on decimals (RQ1a, cycle 1 only)<br>To gain knowledge on PSTs' ideas in teaching decimals (PCK) (RQ2a) |
| | ▪ Classroom observation (Video-recording groups discussion and transcribed) | To observe the conduct of the activities, e.g., whether they have been carried out as intended<br>To observe PSTs' reaction to the activities<br>To capture any impact of the activities to PSTs' CK and PCK (RQ 1b & 2b)<br>To identify PSTs' difficulties on content areas and pedagogical ideas (RQ 1b & 2b) |
| | ▪ Post-course Interviews (audio/video-taped and transcribed) | To learn about the impact of activities for PSTs' CK and PCK on decimals (RQ 1b & 2b). |
| | ▪ Group worksheets of activities | To find any indication of progress of PSTs' CK and PCK and their changes (RQ1b & 2b) |
| | ▪ Post-tests | To assess the impact of the activities on PSTs' CK and PCK (RQ1b &2b)<br>To identify areas of improvement in CK and PCK (RQ1b & 2b) |
| | | ***Retrospective Analysis Phase*** |
| 3 | Triangulate the data gathered from teaching experiment phase | To elaborate on the data gathered from teaching experiment phase to find a general pattern in various data.<br>To gain insights on how to refine and reorganize the activities for the next cycle. New ideas from previous activities will also be put forward into the design phase of the next cycle. |

*PSTs: pre-service teachers

Table 3.8: Links between research questions and various data sources

| ***Research Question*** | ***Data Sources*** | | | | | |
|---|---|---|---|---|---|---|
| | Pre-test | Pre-course interview | Observation | Group worksheets | Post-test | Post-course interview |
| 1a. What is the current state of Indonesian pre-service teachers' CK of decimals? | ✓ Part A, B | ✓ | - | - | - | - |
| 1b. What is the interplay between pre-service teachers' participation in the activities and their CK of decimals? | - | - | ✓ | ✓ | ✓ Part A, B | ✓ |
| 2a. What is the current state of Indonesian pre-service teachers' PCK of decimals? | ✓ Part C | ✓ | ✓ | - | - | - |
| 2b. What is the interplay between pre-service teachers' participation in the set of activities and their PCK of decimals? | - | - | ✓ | ✓ | ✓ Part C | ✓ |

# CHAPTER 4 GOING THROUGH PHASES IN CYCLE 1

## 4.1 Introduction

This chapter contains a description of different phases leading up to and involved in cycle 1 and findings gathered in each phase. The description will explicate the research instruments comprising written tests, interview questions and activities involved in different phases. Section 4.2 addresses the design phase focusing on the initial development of the Local Instruction Theory (LIT) and findings from the trial phase of the instruments used prior to cycle 1 along with lessons learned from the trial phase. Refinements and adaptations made as a result of the trial phase will be explicated in Section 4.2. Section 4.3 presents the research instruments comprising written tests and interview items. In section 4.4, the design and the enactment of the activities and the findings during the teaching experiment phase are discussed. Section 4.5 discusses the findings from pre-test and post-test as well as from pre-course interviews and post-course interviews that indicate any evolution of pre-service teachers' content knowledge and pedagogical content knowledge. Finally, this chapter ends with retrospective analysis phase of cycle 1 and feed-forward recommendations for the second cycle in Section 4.6.

Before discussing the first design phase in Section 4.2, I will present the details of the data collection methods in relation to the research questions and goals, which are summarised in Table 4.1. A total of 31 pre-service teachers were involved in the trial phase. A total of 136 pre-service teachers sat for the pre-test. After the teaching intervention, a total of 129 pre-service teachers sat for the post-test. From these two tests, longitudinal data on 118 pre-service teachers, 67 from the primary cohort and 51 from the secondary cohort were obtained. There were 16 pre-service teachers who participated in the pre-course interviews; but 2 of them gave very little information and refused to be audio-recorded, even though they had noted otherwise on the consent form so 14 useful interviews were obtained. Selection of the interviewees in the first cycle was based only on the responses on the pre-test and on consent to be interviewed. The same pre-service teachers who participated in the pre-course interviews were invited to participate in the post-course interviews. However, 5 pre-service primary teachers and 1 pre-service secondary teacher did not attend the post-course interviews so that there were 10 pre-service teachers properly interviewed before and after the enactment of the activities. During the enactment of the activities, the pre-service teachers worked in small groups of 4-6 people. The total number of groups varied from meeting to meeting, as some pre-service teachers were occasionally absent. All activities were carried out in the span of 4-5 meetings lasting approximately 2 hours each during the teaching experiment phase.

Table 4.1: Overview of data sources in relation to research questions and goals in cycle 1

| Methods | Cohort | Number of participants | Research questions * | Goals |
|---|---|---|---|---|
| Trial phase | Volunteers | 5 (Melb)<br>9 (Indo) | - | Trial test items and identify problems on test items |
| Observation of trial of activities (Appendix A1, A2) | Volunteers | 2 groups (Melb-9)<br>2 groups (Indo-8) | - | Test and identify problems in the activities |
| Pre-test (Appendix B1) | Primary<br>Secondary | 72<br>64 | 1a<br>2a | Identify current state of CK<br>Identify current state of PCK |
| Post-test (Appendix B2) | Primary<br>Secondary | 73<br>56 | 1b<br>2b | Identify the evolved CK<br>Identify the evolved PCK |
| Pre-course interviews (Appendix B3) | Primary<br>Secondary | 11<br>5 | 1a<br>2a | Clarification of pre-test responses<br>Identify prior learning experience<br>Identify initial ideas for future teaching |
| Post-course interview (Appendix B4) | Primary<br>Secondary | 6<br>4 | 1b<br>2b | Clarification of post-test responses<br>Gain feedback on activities<br>Identify evolved ideas for future teaching |
| Observation of activities (video and audio-recordings) | Primary<br>Secondary | 3 groups<br>2 groups | 1b<br>2b | Identify aspects in the activities that contribute to improvement of CK and PCK |

* see Section 1.4

## 4.2 Design phase 1

In this section, I first explain about the initial development of the LIT, which is based on the analysis of decimal notation, and analysis of the potential use of the models. The decision to incorporate models is based on the aim to introduce pre-service teachers to a less symbolic teaching and learning approach on decimals. The starting point for devising the instructional activities is taken from the likely existing knowledge of pre-service teachers and by hypothesizing their learning trajectories. The existing knowledge of pre-service teachers was posited through analysis of how the decimal topics were commonly approached in primary mathematics textbooks, which will be described in the following section.

### 4.2.1 Textbook Approach on Decimal Notation

Analysis of some Indonesian commercial school textbooks (e.g., Khafid & Suyati, 2004a, 2004b; Listyastuti & Aji, 2002a, 2002b) indicates a very symbolic approach in teaching decimals. Common fractions starting with one tenth are utilized to introduce decimal notation ($\frac{1}{10} = 0.1$) followed by exercises to convert fractions such as $\frac{1}{2}, \frac{3}{5}$ into decimal notation. No attention is given to creating meaningful referents such as concrete models to help students make sense of the place value in decimal notation. Moreover, strong reliance on syntactic rules based on whole numbers dominates the approach in comparing two decimals and in carrying out operations with whole number algorithms. Two and three digit decimals are introduced through finding conversion of fractions with denominator 4 and 8 to denominators 100 or 1000. This approach clearly overemphasizes operations of fractions as the basis and overlooks place value understanding in building understanding of decimal notation. Rules such as moving a decimal comma when dividing or multiplying by 10 are stated as shortcuts without any justification or illustration.

Moreover, decimals with repeating digits are "given" without explicating the division process, for instance $\frac{2}{3}$=0.6666...; $\frac{2}{6}$=0.3333... ; $\frac{5}{6}$=0.8333... ; $\frac{7}{9}$=0.777.... Textbooks also emphasise rounding to two digit decimal numbers, e.g., $\frac{2}{3}$= 0.67 without providing much justification for the use of rounding. Steinle (2004) found that this teaching approach contributes to lack of understanding of decimals with longer digits and infinite repeated digits. Moreover, this practice in learning decimals does not develop meaningful understanding of decimal notation based on important ideas such as place value.

Therefore, utilizing this analysis and insights from prior research in teaching and learning decimals (Hiebert, 1992; Hiebert & Wearne, 1986, 1987; Stacey, Helme, & Steinle, 2001; Steinle & Stacey, 2001, 2002), the focus of the activities in this study was determined. Prior studies in teaching and learning of decimals suggested the importance of building understanding of decimal notation based on place value understanding and a focus on structural characteristics of decimal system. Hence this study focuses on building understanding of decimals and basic notions in decimals such as an understanding of density of decimals, and structural relations including additive and multiplicative structures which are crucial in building meaningful interpretation of decimals.

### 4.2.2 Determining Goals for the Activities

Activities were selected with the purpose of improving understanding of the key notions of decimals that were not appropriately addressed in the Indonesian primary school mathematics textbooks. Below are the lists of goals that guide the development of activities in the trial phase of cycle 1:

- Develop an understanding of decimals based on place value concepts; that is to recognise decimal digits in terms of place value.
- Develop an understanding of additive and multiplicative structures of decimals; that is to recognize that decimals can be represented as a linear combination of powers of 10 and to recognize the base ten multiplicative structures of decimals.
- Develop an understanding of equivalent decimals and multiple ways of interpreting decimals, e.g., 2.35 as composed of 2 ones, 3 tenths, and 5 hundredths but also 23 tenths and 5 hundredths, and 235 hundredths.
- Develop an understanding of density of decimals; that is to recognize that there are infinitely many decimals in between any two decimals.
- Building links among decimals, fractions and whole numbers and a sense of relative magnitude of those numbers, including knowing relative magnitude of decimals on the number line.

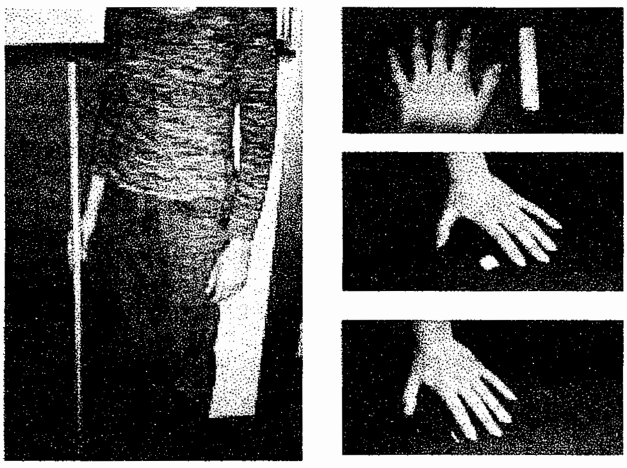
### 4.2.3 Designing activities and conjecturing learning paths

Based on the above goals, sets of activities were devised and conjectures about pre-service teachers' learning paths in working with these activities were developed. Guided by the didactical phenomenology tenet of RME, the context of measuring length was chosen in order to explore the basic notion of repeated refinement into ten. This idea was inspired by activities on decimals consonant with RME basic tenets devised earlier (see e.g., Gravemeijer, 1998; Keijzer et al., 2004). Gravemeijer (1998) proposed the use of ruler with metric measures (such as m and cm) and number line as mediating models for teaching decimals. In contrast, Keijzer et al.(2004) utilised a less standard measurement tool such as a rope and small strips of paper in their initial teaching activities. As discussed in Chapter 2, reference to metric measures in teaching decimals allowed a decimal number to be interpreted using two separate units. Hence, an understanding of decimals as part of a whole is missing.

In this study, a linear concrete model based on length, called Linear Arithmetic Blocks (LAB) (see Figure 4.1) was employed as a learning tool for decimals. The linear nature of LAB fits with the chosen context of measurement in this study. LAB consists of long pipes that represent a unit and shorter pieces that represent tenths, hundredths, and thousandths in proportion. Pieces can be placed together to create a length modelling a decimal number and can be grouped or decomposed (for example to show 0.23 as 2 tenths + 3 hundredths or as 23 hundredths). The LAB model has been explored in prior studies on teaching and learning decimals (Stacey, Helme, Archer et al., 2001; Steinle et al., 2006) and suggested as a powerful model in learning decimals. LAB represents decimal numbers by the quantity of length (not measured length such as metres and centimetres) and not volume such as Multi Arithmetic Blocks (MAB). Moreover, the simplicity of LAB and its linear nature allowed the extension to a more abstract model of the number line.

The researcher hypothesized that exploring the relationships between different pieces of LAB in the context of measuring a length of a table and different ways of naming the pieces would be a useful initial task to create a meaningful interpretation of base ten relations. The longest piece is called "*one rod*" and by observing the relationships of the shorter pieces to the longest one, it is expected that pre-service teachers establish the name that reflect the relationships such as "one tenth of a rod", "one hundredth of a rod", also "one tenth of one tenth of a rod", etc (see Appendix A1 for the complete set of trial version activities).

Figure 4.1: LAB pieces*



**Trial Stage of Activities**

Results from the trial of Activity 1 in Set 1 showed that establishing the names for each of the pieces involved a more complicated process unforeseen by the researcher. Establishing the names for shorter LAB pieces involved a long discussion and yet the chosen names did not necessarily express the relationships to one rod as the longest piece. One of the trial groups established the names based on the physical appearance of the models to objects familiar with them like "*a stick*" for a tenth of a rod, "*a cheezle*" for a hundredth of a rod, and "*a splinter*" for a thousandth of a rod. This finding suggested that different interpretations of the activity to label the pieces shifted the focus on this activity away from the more important exploration of the relations between the pieces of LAB. Thus, the researcher determined to introduce the longest piece as "one" and ask pre-service teachers to explore and find the relationship between shorter pieces to "one" for the initial task in the main study in cycle 1. This more straightforward approach was expected to set the focus on the relations between the pieces, more than asking for names of the pieces.

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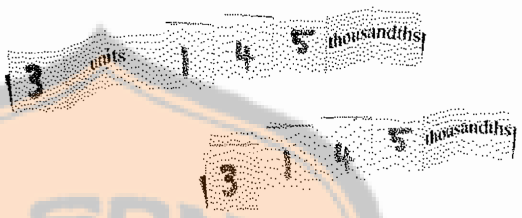
* LAB pictures are taken from Steinle, Stacey & Chambers (2006) Teaching and Learning Decimals CD version 3.1. Melbourne: The University of Melbourne

In the trial phase, promoting the understanding of additive structures of decimals were addressed in Activity 1, 2, and 3 in Set 2 (see Appendix A1). Pre-service teachers were asked to find various ways of constructing a decimal using LAB model, e.g., making 0.213 by 2 tenths and 13 thousandths or 21 hundredths and 3 thousandths. Following this activity, a number expander model was introduced to observe various ways of expanding decimals. A number expander works on the symbolic representation of decimals. It displays the extended notation of a number in different ways as can be seen in Figure 4.2 below. The outcome of the trial indicated the majority of volunteer pre-service teachers, except one pre-service teacher, were able to give more than one of way constructing a decimal 0.213 after the intervention. The number expander was perceived by the researcher as a model to help them in seeing that the same decimal number can be expressed in different ways.

Figure 4.2: Various expansions of 3.145 using the number expander model (from Steinle et al., 2006)




The multiplicative structure of decimals was explored in Set 3 activities of exploring the endless base ten chain pattern (see Section 2.2) by looking at different conversions from ones to tenths, tenths to hundredths, etc. and vice versa using the number expander model. The iterative process of finding the base-ten multiplicative structure pattern was expected to afford pre-service teachers to arrive at more abstract understanding of the endless base ten chains of decimals. Results indicated that these tasks did not assist pre-service teachers to observe the multiplicative structures in a more meaningful way. This was evident as one pre-service teacher applied a memorized strategy of moving a decimal point in expressing the relations among 3.07 in tenths, hundredths and thousandths. Consequently, these tasks were omitted in cycle 1.

Set 4 in this trial stage (see Appendix A1) aimed to explore density of decimals (by finding decimals in between pair of decimals) and links among decimals, fractions and whole numbers using a number line model. The task asked pre-service teachers to locate different numbers on the same number line, e.g., locating –2.65, ½. 0.6, –0.9999, 0.9999, and 0.501. It was expected by discussing and working together in locating those numbers on the number line, they learn from each other about the relative magnitude and positions of those numbers on the number line.

An important amendment made after the trial was the more explicit inclusion of pedagogical aspect of teaching decimals as a focus of the study. Whilst working on problems with models during the trial of the activities, one pre-service teacher pointed out the importance of finding ways to help her students to solve the problems. Attending to the need of finding ways to make a topic more comprehensible for students relates to one aspect of pedagogical content knowledge defined by Shulman (1987), i.e. knowledge of "ways of representing and formulating the subject matter that make it comprehensible to others" (p. 9). This particular knowledge is referred to as Pedagogical Content Knowledge (PCK) in this thesis. Section 1.2.5 lists various other components of Shulman's PCK, but only one aspect is being considered here. The development of PCK during the teaching experiment will be tracked.

Exposing pre-service teachers to new concrete models in this study is expected to encourage them to revisit their knowledge about decimals and to enhance their knowledge about different ways of teaching decimals more meaningfully. By investigating and exploring the principle of partitioning into ten smaller units in establishing the names of the models, pre-service teachers are expected to revisit and reinvent their understanding of decimals and to promote meaningful teaching ideas. However, the fact that pre-service teachers bring with them prior strong syntactic knowledge of decimal notation might challenge their openness to developing new knowledge.

In this trial stage, the researcher observed that pre-service teachers with a strong reliance on syntactic knowledge showed higher level of resistance in working with new concrete models to revisit their understanding of decimals than others who have less reliance on syntactic knowledge. This was in line with Wearne & Hiebert's (1988a) prediction of this tendency in the following comment:

> Theoretically, students who have already routinized syntactic rules without establishing connections between symbols and referents will be less likely to engage in the semantic process than students who are encountering decimal symbols for the first time" (p. 374)

It is significant that none of the subjects in this thesis fall into the class of encountering decimals symbols for the first time.

Moreover, Hiebert, Morris, Berk, and Jansen (2007) contend that prior learning experiences heavily influence pre-service teachers' perceptions and interpretations of what they learn at the teacher education level. As the prior teaching experience of Indonesian pre-service teachers is predominated by syntactic rules, a challenge in revisiting the decimal notation and its basic properties is recognized. This understanding of the possible impact of prior learning experience is accommodated in the design of the activities and instruments.

## 4.3 Research instruments

The section will report on findings from the trial phase of research instruments both written tests and interviews prior to the main study in cycle 1. The description below includes the rationale for the selection of the instruments and surveys the concepts evaluated by the instruments.

**Instruments**

1. **Written tests**

The written tests (see Appendix B1 and Appendix B2) were trialled with 5 pre-service teachers in Melbourne and 9 pre-service teachers in Indonesia. The trial of written test items with pre-service teachers in Melbourne was conducted in face to face basis and pre-service teachers were asked to think aloud when solving the questions. The main aims of trialling the research instruments were to gain feedback on the test and to observe any ambiguity in the written test questions. Seven out of the nine pre-service teachers involved in the trial of instruments in Indonesia pointed out confusions in understanding item 2 in Part B of the pre-test. Hence, adjustments were made to improve the instruction on Item 2 in Part B in the Indonesian translation for in both the pre-test and the post-test items.

2. **Interviews**

The pre-course interview in cycle 1 (Appendix B3) examined the current state of pre-service teachers' content and pedagogical content knowledge of decimals as well as their prior schooling experiences. Moreover, the pre-course interview aimed to elicit pre-service teachers thinking behind some of the incorrect or unexplained answers on the pre-test. The pre-course interviews were carried out after the pre-test. "Think aloud" procedure (audio-recorded) was employed during the interviews and pre-service teachers were asked to write some of their explanations, which were kept by the researcher.

Post-course interviews in cycle 1 (see Appendix B4) were carried out after the post-test and focussed mainly to gain feedback for improving the activities. Moreover, the post-course interviews also aimed at eliciting pre-service teachers' underlying thinking behind some incorrect answers on the post-test that were indicative of certain misconceptions. During these post-course interviews, pre-service teachers were asked to identify three different models for teaching decimals and to rank the models according to their levels of accessibility.

## 4.4 Reorganising activities after the trial phase

In contrast to the common approach of teaching and learning decimals in Indonesia which puts a heavy emphasis on symbolic manipulation, activities in this study were designed to utilize concrete models in assisting pre-service teachers to revisit their understanding of decimal notation. The enactment of activities during the teaching experiment involved a limited amount of lecturing, which is in line with a goal to develop meaningful understanding and interpretation of decimals. The main role of the lecturer in this study is to facilitate the discussions in small groups and in the whole class by emphasising the main points of the activities. This approach emphasizes active engagement in group work, which is consistent with the RME instructional approach (see discussion in Section 2.4). It is expected that this approach will encourage more engagement in exploration of ideas, and pre-service teachers will get firsthand experience of new methodologies for their future teaching.

**Set 1**

Utilizing the didactical phenomenology tenet of RME, the measurement context was chosen to explore base ten relationships, i.e., that parts of units hold a specified size relationship to the unit: tenth of a unit, hundredth of a unit, as a basis for understanding decimals. The choice of the measurement context was in line with Freudenthal's (1983) position who commented on the context for teaching decimals and common fractions. He pointed out that "length is one of the concepts by which common and decimal fractions can be operationally introduced" (p. 26).

In an effort to reflect the guided reinvention tenet, Set 1- Activity 1 introduced the longest piece of LAB as one and asked pre-service teachers to explore the relationships of shorter pieces to the one piece and establish the verbal names for each piece (see Figure 4.3). It was posited that pre-service teachers will capitalize on the "divide by ten" relationship found between one and a tenth, a tenth and a hundredth, and a hundredth and a thousandth to establish the names for all the shorter pieces. The direct focus on the relationships between pieces was in response to the finding in the trial. The more open activity of 'naming pieces' led to imaginative but not mathematical discussion.

Figure 4.3: Set 1- Activity 1 worksheet of cycle 1

| **Pieces** | **Name** |
|---|---|
| | One |
| | ..... |
| | ..... |
| | ..... |

The aim of Set 1- Activity 2 was to link the pieces of LAB with verbal names, decimal notation and fraction notation by matching up the pieces and the associated symbolic representations (see Appendix A2). It was expected that exploring the relationships among different LAB pieces, discussion about the link between decimal and fractions would help pre-service teachers to create a meaningful link between decimals and fractions.

Figure 4.4: Set 1- Activity 3: Measuring length and width of a table using LAB pieces

In Set 1- Activity 3 (see Appendix A2), ideas to measure the length and width of a table using various LAB pieces to get an accurate result were explored. Furthermore, in Set 1- Activity 4, explanations about the reasoning and justification as well as the result of the measurement were called for. Note that for this activity, some steel pipes were available to conveniently join different LAB pieces together (see Figure 4.4). In set 1- Activity 5 (see Figure 4.5), various ways of sketching out representations for three decimals to emphasize that the value of a decimal digit depends on its place. The notion of rounding in decimal notation was addressed in Set 1, activity 6 (see Figure 4.6), in which pre-service teachers were asked to find the number of hundredth pieces closest to the length that represent 0.666 and 1.55569 by using only hundredth pieces.

Set 1- Activity 7 (see Figure 4.6 and Appendix A2 for more detail) was designed to expand the use of LAB as a thinking tool to compare two decimals based on length. It was expected that erroneous thinking such as 0>0.6 and 1.666 = 1.66, which might have been uncovered by DCT would be challenged and resolved.

Figure 4.5: Set 1- Activity 5, 6: Sketching out representations of 2.06, 0.26, 0.206

5. *Sketch the construction of LAB representations of the following decimals.*

| **Numbers** | **Sketch** |
|---|---|
| 2.06 | |
| 0.26 | |
| 0.206 | |

- What could you conclude from the construction process above?
- What is the value of 6 in 0.26 ? ………………………………………………………………

Is it the same as the value of 6 in 2.06? Yes/No. Why?

Is it the same as the value of 6 in 0.206? Yes/No. Why?

6. If you measure a length of something using only hundredth pieces of LAB, answer the following questions:
- How many of hundredth pieces of LAB are needed to represent a length closest to 0.666?
- How many of hundredth pieces of LAB are needed to represent a length closest to 1.55569?

Observing the reflections of pre-service teachers of new learning experiences was expected to bring out insights about the new learning ideas of decimals that the pre-service teachers perceived as meaningful. Set 1- Activity 8 (see Appendix A2) asked pre-service teachers to articulate their new learning experiences about decimals. Moreover, insights about prior or initial knowledge of decimals might be gathered in these reflections. Meanwhile, Set 1- Activity 9 was expected to disclose information about translation of ideas gathered from pre-service teachers' own learning experience to ideas for future teaching of decimals. This activity is an example of activities to elicit PCK of pre-service teachers.

Figure 4.6: Set 1- Activity 7, 8, and 9

7. Explain your idea on how to decide the larger decimals between the given pairs of decimals below using the LAB model.

| | |
|---|---|
| 0.9 | 0.90 |
| 0 | 0.6 |
| 1.666 | 1.66 |
| 1.9912999 | 1.9912 |

8. Based on your learning experience last week, explain your new learning experience of decimals.

9. Explain your ideas to teach decimals in primary school based on your learning experience.

**Set 2**

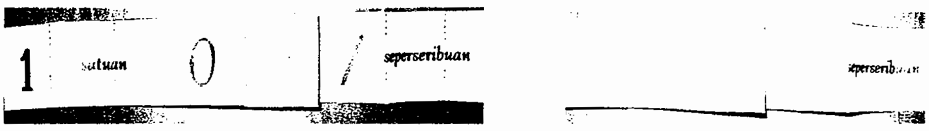
Set 2 activities (see Appendix A2) started with Activity 10 to explore different ways of decomposing two decimals 0.123 and 1.230. For each decimal number, pre-service teachers were asked to draw sketches of how to decompose the decimal numbers using various pieces of LAB models. Columns to decompose the number in up to 8 ways into ones, tenths, hundredths, and thousandths were provided for each decimal (see Appendix A2). It was expected that this activity would encourage pre-service teachers to explore various ways of interpreting decimals using LAB as a thinking tool. In the process of finding different ways of decomposing decimals, the researcher posited that pre-service teachers would "re-invent" structural relations (additive and multiplicative) amongst ones, tenths, hundredths and thousandths in the process of finding different alternatives. It was also expected that different ways of decomposing 0.123 would be capitalized in finding multiple ways of decomposing 1.230.

Having pre-service teachers' reflecting on Set 2- Activity 10, Set 2- Activity 11 explored various ways of decomposing a decimal number in symbolic way as shown in Figure 4.7. Following this activity, each group was given a number expander to work with and pictures of how the number expander displayed the extended notation of a decimal 1.027.

Figure 4.7: Set 2-Activity 11: Decomposing decimals into various extended notations

0.213 = .... ones + **2** tenths + .... hundredths +........thousandths
0.213 = .... ones + **2** tenths + **0** hundredths +........thousandths
0.213 = .... ones + **0** tenths + ..... hundredths +........thousandths
0.213 = .... ones + **1** tenths + ..... hundredths +........thousandths
0.213 = ...... tenths + ...... thousandths
0.213 = ....... hundredths + ...... thousandths
0.213 = ........ hundredths
0.213 = ......... thousandths

Figure 4.8: Various expansions of 1.027 using the number expander




A number expander was first introduced to explore various ways of decompose decimals in Set 2 - Activity 12 and to check the answers for solving problem in Figure 4.8 above. Moreover, discussion about similarities and differences between a number expander and LAB was sought in Set 2- Activity 13. In Set 2- Activity 14 written reflections of the learning experiences on activities in Set 2 were called for. Finally in Set 2- Activity 15, teaching ideas about decimals in primary school based on pre-service teachers' own learning experiences was probed (see Appendix A2). It was expected that in the reflections of the pre-service teachers' own learning experience, remarks indicating the evolved CK or PCK could be observed.

**Set 3**

Set 3 activities were designed to address density of decimals and links between decimals including negative decimals, whole numbers, and fractions on the number line as shown in Figure 4.9. This set started with the use of concrete model LAB which then followed by the use of number line as a more symbolic model in locating decimals and addressing density of decimals. Note that LAB represents decimals by length, whereas number line represents decimals by length and also position. These activities were expected to elicit knowledge and misconceptions on the links among decimals, whole numbers and fractions, such as knowledge of equivalence relation between 2¼ and 2.25, and that 0.6 was not equal to $\frac{1}{6}$, which might be taught by some pre-service teachers identified by DCT3a and DCT3b as having S3 (reciprocal) thinking (see Table 2.1).

Unfortunately, the significant amount of time devoted to group presentations of their responses to activities and sharing of ideas following group discussion elongated the enactment of activities in cycle 1. Hence Set 3 activities were not carried out in the first cycle. However, performance on these areas was observed in both pre-test and post-test so information about pre-service teachers' understanding on this aspect could still be inspected. It is at least possible that an improved understanding of an advanced topic such as density may occur as result of attention to "the basics" knowledge of decimals.

Figure 4.9: Set 3 activities: Density of decimal numbers

Set 3

16. Use the LAB to construct pairs of the given decimals in Set A (see below). For each pair, please check whether you could find decimals in between the pair of numbers. If yes, please name the number, and explain how you find the number of decimals.

| | |
|---|---|
| 0.9 | 1 |
| 0.66 | 0.666 |
| 1.21 | 1.23 |
| 1.5 | 1.51 |
| 1 | 1.001 |

Set A

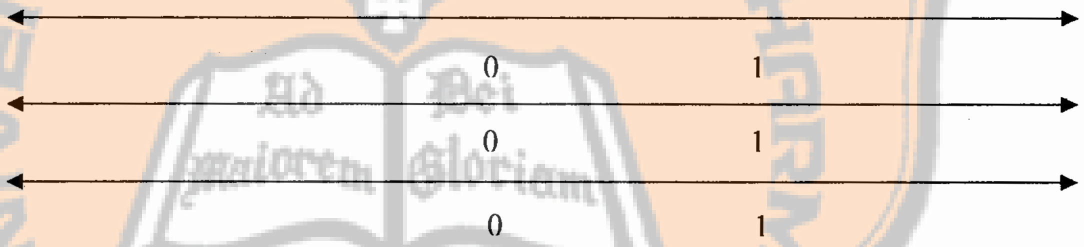
17. Use the number line to locate the pair of decimals given in the Set B, and discuss whether it is possible to find any number in between a given pair.

| | |
|---|---|
| 0.1 | 0.11 |
| 0.7501 | 0.7501 |
| 0.600 | 0.60001 |
| 2.2452 | 2.245201 |
| 0.366666 | 0.3666601 |

Set B

18. What can you conclude from working with the problems above?
19. Can you find any decimal that is bigger than 0.36666601?
20. Can you find any decimal that is bigger than 99.999999?

21. Locate the following numbers in the number line below:
    a) 2, 2 ¼, -1, $\frac{1}{3}$, 0.3333333, 0.3334, 2.25
    b) 1.5, $\frac{1}{5}$, 0.21, 1/10, 0.1, 0.010, 0.100
    c) –1.5 $\frac{1}{6}$, 0.6, –0.9999, 0.9999, 0.501



## 4.5 Teaching experiment phase

The aim of the first teaching experiment is to investigate how the LIT plays out in the participating classes and whether the implementation of activities improve pre-service teachers' CK and PCK of decimals. Section 4.5.1 will discuss findings from classroom observations and responses in worksheets of the activities. Note that not all the activities designed to be implemented in cycle 1 were carried out. Activities in Set 3 which were designed to address density of decimals and relative magnitude of decimals had to be left out due to time constraints. Section 4.5.2 will present findings from pre/post-written tests along with insights gathered from pre/post course interviews that deepen our understandings about the results on the content knowledge. In Section 4.5.3, findings related to PCK from both written tests and interviews will be discussed.

### 4.5.1 During the teaching experiment

This section describes the findings during the teaching experiment gathered mainly from observation of the groups during the whole class discussions and repeated viewing of the videotapes of 5 groups and written responses to the activities. Most of pre-service secondary groups consisted of 5-6 people due to the large size of the class (64 people at the start of teaching experiment). In contrast, pre-service primary teachers work in groups of 3-5 people as the size their classes allowed for smaller groupings (range from 36-38 per class). In line with the Guided reinvention tenet, the activities in cycle 1 were carried out with limited formal teaching. Pre-service teachers were expected to construct new knowledge through their engagement in carrying out and discussing various activities. Opportunities to address and resolve misconceptions were taken up during the whole class discussions led by the lecturers.

#### 4.5.1.1 Outcomes from Set 1

As predicted in the LIT, in Set 1- Activity 1 and 2 explored relationships between different pieces of LAB and connections of LAB pieces with decimal and fractional notation, could be completed without much difficulty. Note that unlike in the trial phase when the longest piece was called "one rod", in the main study of cycle 1, the longest piece was just called "one". This is to avoid the complicated problem of naming the pieces based on their physical appearance as found in the trial stage. Interestingly, most groups utilized one tenth relations between LAB pieces of subsequent length to establish the names for the other pieces. For example, the observation of group discussions revealed that most groups started with finding the relationship between the longest piece (one) and the one-tenth pieces. By noting that ten of the one tenth pieces made one, most groups were able to establish the name one tenth. Successively, because ten of the shorter pieces made one-tenth pieces and utilizing previous relation that ten of one-tenth pieces made one, the name for that piece as a hundredth was established. During their group discussion of this approach, a possibility of children associating one-tenth with different pieces was raised by one member of the video-recorded group. This remark emphasized the importance of a referent unit (one) in partitioning and establishing the names of the other pieces.

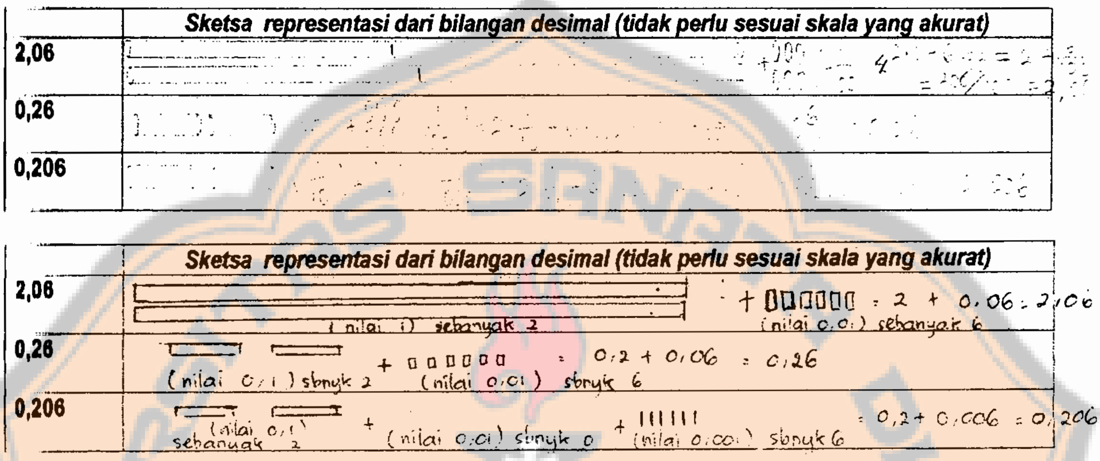
Despite familiarity with symbolic notation of decimals and fractions, the activity of establishing the names assisted pre-service teachers in understanding the meaning of "one tenth", "one hundredth", and "one thousandth". Written test and interview data in Section 4.5.2 documented evidence on this phenomenon. In this respect, this activity enabled pre-service teachers to revisit their understanding on the meaning of notation 0.1, 0.01, and 0.001.

The context of measuring a table or a chair (Set 1, Activity 3 and 4 in Appendix A2) capitalized on the linear nature of LAB. Adding and subtracting strategies to get the total length were observed. Interestingly, the majority of groups employed written fraction operations to find the total length and converted the answers into decimals at the end, e.g., $\frac{6}{10}+\frac{3}{100}+\frac{5}{1000}=\frac{600}{1000}+\frac{30}{1000}+\frac{5}{1000}=\frac{635}{1000}=0.635$. It was posited that strong focus on computational fluency and lack of emphasis in understanding of decimal notation system in their prior schooling led to this preference to fraction operations. Three groups (2 primary and 1 secondary) linked LAB with a ruler, a reference to the metric system, to find the total length in cm and mm after measuring the length of a tenth piece LAB in mm (approximately 1.1 mm). Apparently, a ruler is a standard tool in solving measurement problems and these groups focus on finding the length in metric measures. Whilst the use of ruler in the measurement context seemed natural, the design of LAB was not intended to directly link to the metric measures. The ruler has multiple units such as cm and mm, whereas LAB has only one reference unit.

Consequently all measurement result should be expressed in relation to the one. However, both LAB and a ruler share a similarity in representing base ten relations.

In Set 1- Activity 5 (see Figure 4.10, Appendix A2) of constructing three decimals that have same digits in different places using LAB, most groups note different value of decimal digits in different places by observing different length of LAB model. Some groups linked their sketches with the corresponding "expanded notation" as shown in Figure 4.10. The first sketch illustrates a preference of fraction notation and operations whereas the second sketch illustrates the link between pieces of LAB model with the decimal place value system and its notation. Interestingly, more groups employed fraction notation and operations (8 groups) than decimal notation (5 groups). The rest of the groups only provided sketches of representations. This finding depicted a strong association of decimal notation with fraction notation and operations as commonly reflected in many Indonesian primary school textbooks.

Figure 4.10: Sketches of representations for 2.06, 0.26, and 0.206 in worksheet of Set 1- Activity 5




The following excerpt of discussion from one video-recorded primary group in working with Set 1- Activity 5 showed evidence of pre-service teachers' evolving understanding of decimal notation and place value. An understanding of the additive structure of decimals and the realization that place value for decimals could go beyond ones and thousandths were particularly evident. Stacey, Helme, Archer et al. (2001) articulated this as one of the strengths of LAB for teaching decimals. Moreover, the notion of place value was articulated as the determining factor distinguishing three decimals given in this task as documented in group discussion excerpts of one of the video-recorded groups:

| | |
|---|---|
| Susilo: | A decimal number is a number that is composed by addition of ones, tenths, and hundredths. |
| Maya: | But if you said that a decimal number is a number composed of ones, tenths and hundredths, how about this example (i.e., 0.206), this decimal number is also composed of thousandths? |
| Susilo | Yes, it can be composed by thousandths, ten thousandths, and so on. |
| Nia: | That's right, because if we have twenty comma something, that is also decimals, right? I think we can say that most primary school children might think that the value of 2, 0, and 6 are the same in these numbers, but in fact their values are not the same because their place value are different. |

Responses to a task of rounding to hundredths in the context of length in Set 1 - Activity 6 (see Figure 4.5) indicated some confusion in the interpretation of the problem. Two groups interpreted this question as finding how many hundredths are in the numbers and use rounding in their answers so finding 7 hundredths in 0.666 and 6 hundredths in 1.55569 instead. One group answered with division by 100 instead and found 66.6 hundredths and 155.569 hundredths. Another group revealed a misconception, by noting that using only hundredths, the closest length to 1.55569 is $\frac{155}{100}+\frac{569}{1000}$. This indicated that this group thought of the number 1.55569 could be obtained by adding or joining two decimals 1.55 and 0.569, ignoring the place value concept and associating decimals with fractions based on the length of decimal digits. Despite the fact that this activity was able to uncover a decimal misconception, misinterpretations to this activity suggested that this activity was ambiguous. Hence Set 1- Activity 6 was omitted in cycle 2.

Responses to Set 1- Activity 7 (see Appendix A2) about ideas to use LAB for comparing two decimals revealed that most groups capitalized on decomposing decimals in place value related terms of the decimal pairs before comparing the length of LAB pieces needed to make the two decimals. For instance, noting that the value of each digit in 0.9 and 0.90 in order to notice that both numbers have 0 ones, 9 tenths. Hence both numbers could be represented with 9 tenths pieces of LAB. Eight groups compared the length of 9 tenth LAB pieces and 90 hundredths LAB pieces and capitalized on the relationship that 1 tenth LAB = 10 hundredth LAB pieces to concluded that 0.9 = 0.90. Meanwhile seven groups utilized the multiplicative relations between tenths and hundredths, i.e., 9 tenths = 90 hundredths in comparing decimals. Three other groups showed reliance on syntactic procedures such as cancelling out zero in 0.90 knowing that $\frac{90}{100} = \frac{9}{10}$ so 0.90 = 0.9 or by writing zero at on the right end of 0.9, which resulted in both decimals have the same number of decimal digits in their solutions. This procedure is often referred to as the annexe zero algorithm or simply annexing zeros. One group revealed a misconception of thinking 0.90 as 90 tenths. This misconception has been labelled as 'column overflow' because of an analogy of overflow to the left columns in whole numbers to decimals (see e.g., Stacey, Helme, & Steinle, 2001; Steinle & Stacey, 1998a) and is one of the ways of thinking associated with code L in DCT3a and DCT3b (see Table 2.1).

Similarly, most groups chose to decompose decimals in expanded notation using LAB for comparing decimals with repeating digits such as 1.666 and 1.66. For instance, explaining that 1.666 = $\frac{1666}{1000}$ is smaller than 1.66 = $\frac{166}{100}$ because $\frac{1666}{1000} = 1 + \frac{6}{10} + \frac{6}{100} + \frac{6}{1000}$ could be represented by one piece, 6 of tenth pieces, 6 of hundredth pieces and 6 of thousandth pieces of LAB, whereas $\frac{166}{100} = 1 + \frac{6}{10} + \frac{6}{100}$ was represented by one piece, 6 of tenth pieces, and 6 of hundredth pieces. Hence 1.66 is smaller than 1.666 because the LAB representation of the number is shorter. Only three groups indicated reliance on the 'whole number strategy' of multiplying both 1.666 and 1.66 with 1000 and comparing 1666 and 1660 to determine the larger decimals, i.e., 1.666. These explanations indicated positive impacts of the activities since most answers in the pre-test in comparing decimals relied on either the annexe zeros algorithm or by comparing the equivalent common fractions.

Another common strategy in determining the larger of pairs of decimals with common initial decimal digits and decimals with repeating digits such as in comparing 1.1503 with 1.15 or in comparing 1.777 with 1.77, were the use of rounding or truncating rule. Findings from interview and written tests, which will be discussed in Section 4.5.2, confirm these strategies.

The outcomes of Set 1- Activity 8 on reflections of learning experiences in Set 1 and Set 1- Activity 9 on the articulation of future teaching ideas will be discussed together with reflections and teaching ideas from Set 2 activities in Section 4.5.1.3.

#### 4.5.1.2 Outcomes from Set 2

Set 2- Activity 10 (see Figure 4.11, 4.12, and 4.13 for illustrative responses) requires pre-service teachers to explore various ways of interpreting a decimal number by sketching out representations of the given decimal. The sketches and ways of decomposing a decimal number reflect whether base ten structures are observed in different representations for the same number. Worksheets of Set 2- Activity 10 documented that most groups could find 5 or more ways to express 1.230 or 0.123. However, their sketches depicted different mathematical understandings, which can be categorized as showing 10-grouping, 5-grouping and no-grouping (see Figure 4.11, 4.12, and 4.13). Note that in Indonesia, a decimal comma is used instead of a decimal point to mark the ones column.

Of 29 groups that handed in their written work, only 6 groups reflected the 10-grouping in their sketches. Four groups showed a combination of 5- and 10-grouping in their sketches with dominant 5-grouping, and 19 groups showed no particular grouping. This suggested that even though most groups could complete many possible alternatives for decomposing decimals, they did not emphasize base ten structures in their solutions, which was very important for teaching. The researcher also observed that most groups did not work with the LAB model when sketching decimal representations. Instead, they found solutions arithmetically by using addition, subtraction, multiplication, and division. Prior learning experiences in decimals with heavy emphasis on symbolic manipulations might cause them to be more comfortable working on the problems arithmetically.

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Figure 4.11: Sketches showing no particular grouping

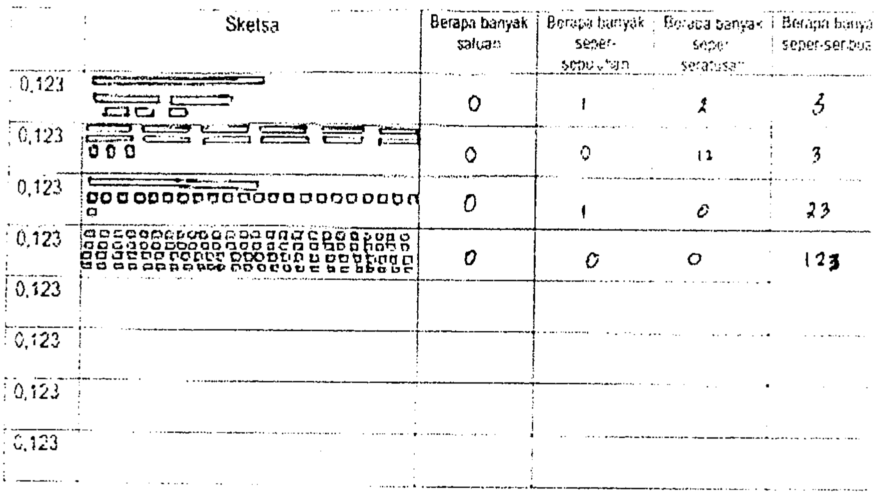



Figure 4.12: Sketches showing five-grouping

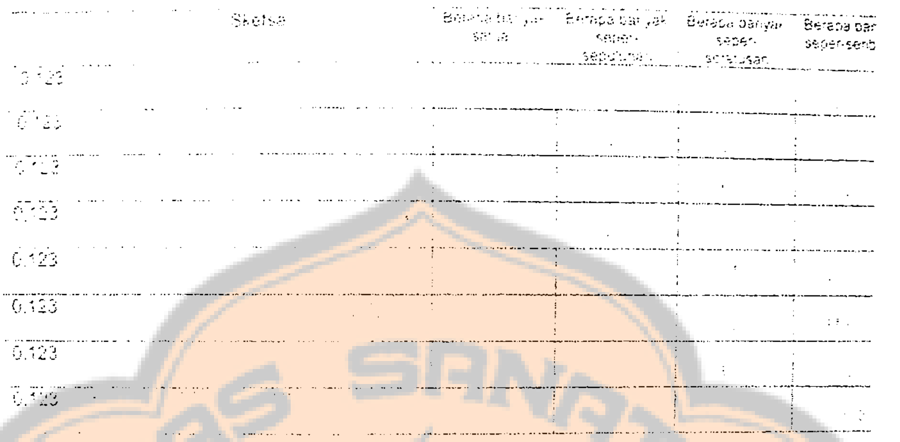
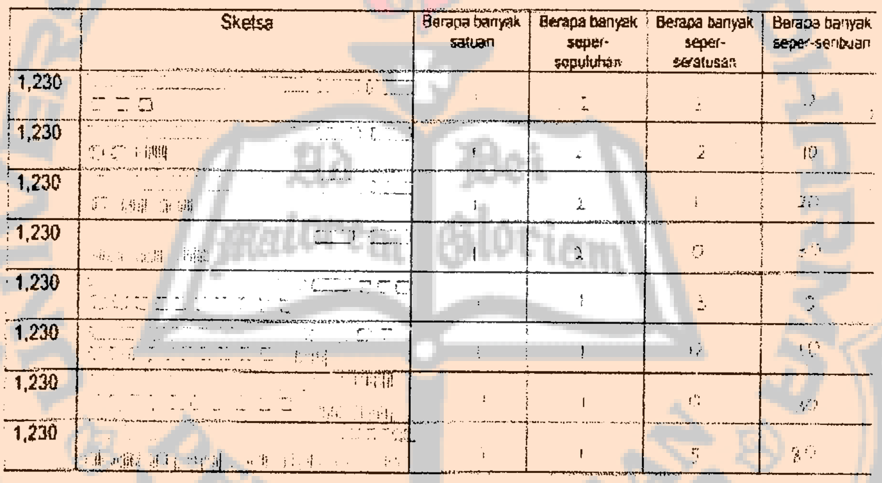


Figure 4.13: Sketch showing ten-grouping



Capitalizing on experience in working with Set 2- Activity 10, pre-service teachers were asked to decompose a decimal number into various extended notations symbolically in Set 2- Activity 11 (see Figure 4.7). One common difficulty recorded in responses of 11 groups (out of 29 groups), noting that 0.213 = 21 hundredths instead of 21.3 hundredths. As revealed in observation during group discussions, these groups thought the answers should be in whole numbers which suggested that this activity was ambiguous.

Findings in Set 2- Activity 12 with number expander to observe various ways of expanding decimals were not satisfactory. The majority of groups (12 out of the 29 groups) attended to the technical aspects of working with the number expander instead of focussing on the mathematical ideas. For instance, one group explained that with the number expander finding the value of ones can be done by unfolding the ones, finding the value of tenths by unfolding the tenth column etc. Seven groups linked the number expander with multiple ways of decomposing decimals in related place value terms. Refining this activity so that pre-service teachers could focus more on the mathematical ideas in using the number expander and its connection with LAB was one of recommendations for cycle 2.

#### 4.5.1.3 Reflections on new learning experiences and ideas for future teaching

The novelty of using concrete models such as LAB and number expander for learning decimals and their role in creating more active and engaging learning process were two most common features noted as new learning points both in Set 1- Activity 8 and Set 2 -Activity 14. For instance, one of the primary cohort groups commented that experience with the concrete materials helped them to move away from reliance on rounding rule in comparing pairs of decimals as recorded in the following reflection note in Set 2- Activity 14.

> We learnt that decimal numbers which are used to be taught only using numbers can be represented using concrete materials so that students can actively engage in the learning process. In comparing decimals such as 0.123 and 0.1231, I used to think that 0.123=0.1231 using the rounding rule but after the learning experience, I know that 0.123<0.1231 because if I use LAB then 0.1231 is longer than 0.123.
>
> (Hery's group- Primary cohort)

Similarly, the following comments from the reflection notes of Set 2- Activity 14 worksheets illustrated different aspects of the activities that were perceived as most valuable by pre-service teachers. The first and the third quotes confirmed that the concrete models such as LAB and number expander were perceived as valuable learning tools. It was clearly expressed in the first comment that experience with concrete models and modes of learning during the teaching experiments had expanded pre-service teachers' ideas about new ways of teaching decimals. The second and third comments below showed that various ways of decomposing decimals were new to them. Moreover, this learning experience led to knowledge of different ways of interpreting decimals.

> This is the first time for us to use concrete models in learning decimals. This helped us to become more creative in finding other models to learn decimals like a piece of paper, plasticine, which will be a concrete way to learn place value in decimals. We also experienced a new approach in learning decimals, namely by finding it for ourselves, sharing amongst groups and gaining feedback from the lecturer.
>
> [Veni's group- Secondary cohort]

> We learnt how to differentiate different place value, ones, tenths, hundredths, thousandths, etc. Also we learnt finding different ways of decomposing the same decimals, for instance: 1.025 = 1 one, 0 tenth, 2 hundredths, and 5 thousandths but it also can be composed of 0 ones, 10 tenths, 0 hundredths, and 25 thousandths.
>
> [Anik's group- Secondary cohort]

> We were pleased to learn new experience that we could not imagine before that there are different ways of presenting a decimal number… Learning decimal numbers was much easier when we use media or concrete models. We will use concrete models in teaching decimals in the primary school.
>
> [Diana's group – Primary cohort]

Some groups perceived the mode of learning, which encouraged active engagement with the content, (e.g., Hery's group and Veni's group) as an insightful approach. They also commented that this approach implied the shift of role for teachers to act more as a facilitator in the learning process, which was in line with the intention of the study on the method of delivering the activities.

In teaching ideas articulated in Set 1- Activity 9 and Set 2- Activity 15, incorporating the use of concrete models to help students in creating meaningful understanding of decimals was dominant. However, many groups voiced concerns about the affordability of providing LAB in teaching decimals on the reflections in Set 1-Activity 7. They suggested the use of models similar to LAB but made from materials such as straws, bamboo sticks, or wood. For the teaching experiment in this study, LAB was made from aluminium steel. Concern about logistics in arranging pieces of LAB with primary school children in the classroom was another point highlighted by pre-service teachers. Whilst these issues on practical and technical aspects of the use of models in classroom situations were important, reflections on the mathematical principles observed in the models are preferable but fewer were found in teaching ideas of cycle 1.

Interestingly one group from the primary cohort proposed the use of money (ten thousands rupiahs, one thousand rupiahs, and one hundred rupiahs) to show a tenth, a hundredth and a thousandth relations as teaching ideas in Set 1- Activity 9 (see Appendix A2). Despite the fact that Indonesian money system works on the basis of whole numbers, this idea indicated an understanding of the act of combining and partitioning into ten as the basis of decimal system. It should be noted that this response was an exception as this group was able to link the decimal relations in whole numbers and not only in decimal numbers.

Note that there Set 3 was not carried out in cycle 1 due to the time constraints. Hence Set 3 will be trialled for the first time in cycle 2.

### 4.5.2 Findings from tests and interviews on Content Knowledge

As noted earlier, both tests and interviews were administered on two occasions during the teaching experiment phase, prior to and after participation in the set of activities. Discussion of findings from pre and post-tests as well as pre and post-interviews will focus on data gathered from 118 pre-service teachers who sat both tests in order to gain a better indication of the impact of the activities (see Section 4.1). Pre-service teachers' content knowledge on various areas of decimals was evaluated by examining performance in DCT3a and DCT3b and Part B of the tests as well as responses in the pre- and post course interviews. Analysis of responses to DCT3a and DCT3b identified the most problematic item types for pre-service teachers and predicted pre-service teachers' underlying thinking behind the observed patterns of errors on various item types.

#### 4.5.2.1 Decimal Comparison Test

Both cohorts showed improvement in their performance of DCT3a as shown by the increased percentage of pre-service teachers who made no errors in DCT3a from 45.7% to 61% in DCT3b. Pre-service teachers from both the primary and the secondary cohort recorded improvement with the secondary cohort outperformed the primary cohort as shown in Table 4.2.

Table 4.2: Number and percentage of pre-service teachers with no errors in two cohorts

| Cohorts | Total number of PSTs | Pre-test | | Post-test | |
|---|---|---|---|---|---|
| | | Number of PSTs with no errors | % no errors | Number of PSTs with no errors | % of no error |
| Primary | 67 | 15 | 22.4% | 27 | 40.3% |
| Secondary | 51 | 41 | 80.4% | 45 | 88.2% |
| TOTAL | 118 | 56 | 45.7% | 72 | 61.0% |

Table 4.3 presents the distribution of pre-service teachers' ways of thinking diagnosed according to Steinle & Stacey (2004a) and given in this thesis in Figure 4.3. The proportion of A1 pre-service teachers ('Apparent experts') increased to approximately three quarters of the combined cohorts. There was only one pre-service primary teacher who was identified as holding any form of 'Longer-is-Larger' thinking (L1, L2, L3, L4). He moved to unclassified category (U1) in the post-test.

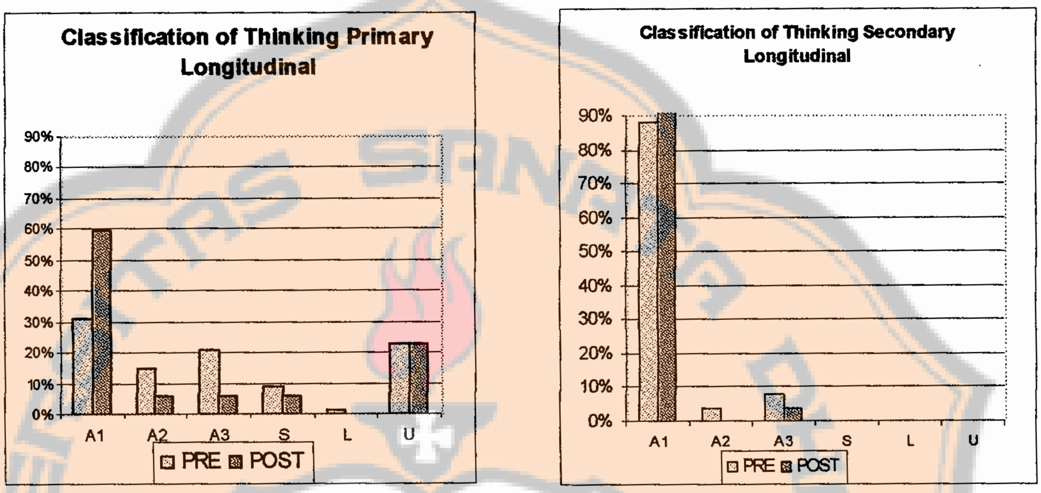
Table 4.3: Number and percentage of pre-service teachers in various thinking classification

| Thinking Classification | Pre-test | | Post-test | |
|---|---|---|---|---|
| | Number of PSTs | Percentage of PSTs | Number of PSTs | Percentage of PSTs |
| A1 | 66 | 55.9% | 89 | 75.4% |
| A2 | 12 | 10.2% | 4 | 3.4% |
| A3 | 18 | 15.3% | 6 | 5.1% |
| L3 | 1 | 0.8% | 0 | 0.0% |
| S1 | 4 | 3.4% | 2 | 1.7% |
| S3 | 1 | 0.8% | 2 | 1.7% |
| S4 | 1 | 0.8% | 0 | 0.0% |
| U1 | 14 | 11.9% | 15 | 12.7% |
| U2 | 1 | 0.8% | 0 | 0.0% |
| Total | 118 | 100% | 118 | 100% |

There were a total of 6 pre-service teachers from the primary cohort identified as holding various ways of 'Shorter-is-Larger' thinking (S1, S3, S4). Of those 6 pre-service teachers holding 'Shorter-is-Larger' thinking, one third of them persisted as holding 'Shorter-is-Larger' thinking, one third moved to A1 thinking and one third moved to Unclassified (U1) category. This trend is in line with Steinle's (2004) prediction that 'Shorter-is-Larger' thinking is more prevalent in post-school students than in 'Longer-is-Larger' thinking and also Steinle & Pierce's study (2006) in involving student nurses.

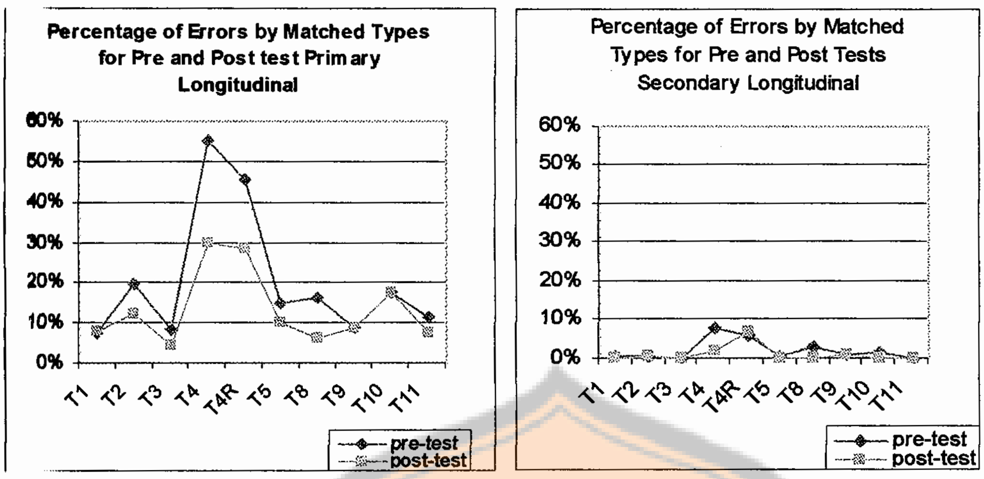
Figure 4.14 shows that both cohorts improved as the percentage of pre-service teachers holding A1 thinking rose and the percentage of pre-service teachers holding one of the error patterns of thinking decreased except for U categories in the primary cohort which remained the same. The primary cohort showed a wider range of variability in difficulties in interpreting decimal notation.

Figure 4.14: Classifications of responses to DCT3a and DCT3b from two cohorts in cycle 1




Both cohorts demonstrated misconceptions associated with inappropriate use of rounding/truncating rules, apparent in type 4 and 4R items (see Figure 4.15) which consists of decimals with the same digits in the first two decimal digits or finite repeated digits such as comparing 4.45 with 4.4502 or 3.7 with 3.77777. Those who are not holding L or S thinking but showing consistent error patterns in these items were identified as holding A2 thinking (see Table 2.1). As noted in the previous section, lack of exposure to decimals with more than three decimal digits coupled with strong reliance on rounding and truncating in schools contributed to the dominance of rounding/truncating strategy. These misconceptions relate to pre-service teachers' understanding of only the first few decimal places. Figure 4.15 presents the percentage of pre-service teachers' errors in matched item types of DCT3a and DCT3b from both cohorts. Note that T1 refers to type 1 items, T2 refers to type 2 items of DCT3a and DCT3b (see Table 3.2 for detail of item types).

Figure 4.15: Percentage of pre-service teachers' error on DCT3a and DCT3b by matched item types




Findings from the pre-course interviews also confirmed reliance on rounding and truncating strategy in solving type 4 and type 4R items as articulated by five of the fourteen interviewees. The following interview transcripts showed this tendency of one primary pre-service teacher, Hery, who utilized rounding and noted that 4.4502 and 4.45 were equal. However, he also showed a misconception associated with thinking of decimal digits as reciprocals. This suggested that Hery applied a mixed of strategies related to A2 and S3 thinking which explained his being diagnosed in Unclassified category (U1) in the pre-test:

Researcher: So could you explain your thinking in solving this problem? [Referring to a problem to choose the larger decimal between 4.4502 and 4.45]
Hery: Well, I round the numbers to two decimal places 4.45 so they are the same.
Researcher: But if you don't round the numbers, are they the same?
Hery: Different, of course 4.45 is larger
Researcher: Why is that?
Hery: Because 4.4502 is 4 and $\frac{1}{4502}$, whereas 4.45 is 4 and $\frac{1}{45}$ so when we divide, this one (4.4502) is smaller.

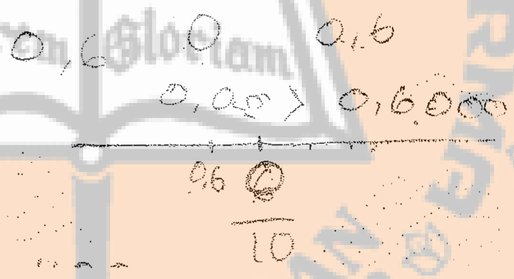
Hery was classified as holding A3 thinking in the post-test and correctly answered 28.454 was larger than 28.45 (type 4 item) showing he had overcome his previous misconception revealed by type 4 items. Furthermore, he made reference to the use of LAB model for comparing decimals during the post-course interview. However, his reliance on the rounding rule was still evident in comparing decimals with repeating digits by answering 1.777 was equal to 1.77 (type 4R) in the post-test. This indicated that his knowledge was isolated and fragmented as various strategies were applied to solve different types of items.

Besides reliance on rounding rules, a misconception of thinking 0 as larger than decimals with 0 in the ones column was recorded in both pre-test as error rates in items type 8, as shown in Figure 4.16. Two pre-service primary teachers provided explanations for thinking that 0>0.6 as expressed in the following quotes:

> Yulius : I think that it is zero point something that it has another digit behind a comma whereas 0 is a whole number, so I think 0 is bigger because 0.6 has one decimal digit. The larger number will be the number without a comma.
>
> Ismi : The way I solve this is by using a number line, 0 is located here and 0.6 is around here. Hence comparing 0.6 and 0, I decide 0 is larger because it is more to the right side... Or similarly I know that 0 is the same as 0.00 and 0.6 is the same as 0.6000, this way is the same, here the 0 is closer to the right.

The first quote from Yulius indicated an overgeneralisation that decimals (and fractions) are smaller than whole numbers whereas Ismi's confusion seemed to stem from mixed of confusions about decimals were placed on the number line (see Ismi's interview note in Figure 4.16).

Figure 4.16: Ismi's misconception about decimals



It was interesting the way she noted the relationships between 0 and 0.00 and between 0.6 and 0.6000. Her conclusion of 0.00 > 0.6000 indicated denominator focussed thinking (S1), based on overgeneralisation that any ten thousandths is smaller than any hundredths. Note that pre-service teachers who made mistakes in type 8 only showed a serious problem in their understanding of decimals. However, they might be still classified as A1, on their responses to basic items.

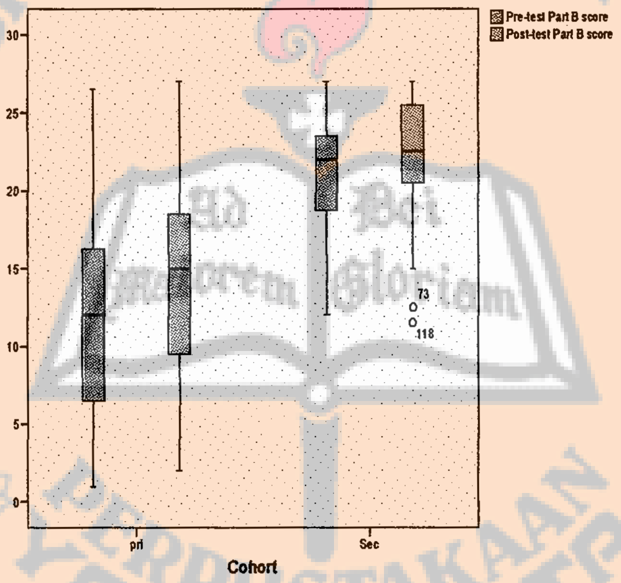
#### 4.5.2.2 Results of Part B of the written tests

Overall, pre-service teachers from both cohorts showed improvement in their content knowledge of decimals in cycle 1 as shown in Table 4.4, as measured by Part B of the written test (see Appendix B1 and Appendix B2). These significant improvements suggested positive impact of the activities. However, the mean of total scores in Part B, particularly of the primary cohort signified inadequate knowledge of content areas of decimals assessed in this study. Even at the post-test, the mean score for the primary cohort was only around 50% (14.34 out of 27).

Table 4.4: Paired t-test on the total score of Part B (out of the total score of 27)

| Cohort | N | Df | Pre-test | | Pos-test | | t value | p value |
|---|---|---|---|---|---|---|---|---|
| | | | Mean | Standard dev (SD) | Mean | Standard dev (SD) | | |
| Primary | 67 | 66 | 11.98 | 6.5 | 14.34 | 6.39 | 8.844 | $p$=0.000 |
| Secondary | 51 | 50 | 21.03 | 3.53 | 22.25 | 3.94 | 8.313 | $p$=0.010 |

Figure 4.17: Box plot representing Part B total scores in pre-post of two cohorts



Examining performance in various content areas, the results of paired t-tests in Table 4.5 and Table 4.6 showed similarities and differences in trends of improvements in content areas. The secondary cohort consistently outperformed the primary counterparts in all content areas. This superior performance of the secondary cohort was not surprising as in general they had higher initial entrance score of mathematics. The high initial mean score of the secondary cohort in some content areas also implied there were not much room for improvement to be observed. However, as discussed later, the high mean scores could also be due to the lack of sensitivity of the written tests in picking up the incorrect thinking. This will be expanded in the discussion of findings in each of the content area and changed for cycle 2.

Both cohorts recorded significant improvement on decomposition of decimals and showed significant decline on sequencing of decimals. Moreover, both cohorts showed no significant improvement on relative magnitude of decimals, measured in content areas of placing decimals on the number line and finding the closest decimal to a given decimal. The difference between the two cohorts was recorded on identifying place value names of a decimal digit, where the primary cohort showed significant improvement whilst the secondary cohort had little room for improvement.

Table 4.5: Mean pre- and post-test Part B scores of the primary cohort (N=67, paired t-tests)

| **Content Areas** | ***df*** | **Pre-test** | | **Pos-test** | | | |
|---|---|---|---|---|---|---|---|
| | | **Mean** | **SD** | **Mean** | **SD** | **t value** | **p value** |
| Identifying place value names | 66 | 1.60 | 1.060 | 2.43 | 0.908 | 6.022 | 0.000 |
| Decomposition of decimals | 66 | 0.73 | 0.931 | 2.85 | 1.340 | 13.027 | 0.000 |
| Density of decimals | 66 | 1.28 | 1.799 | 1.52 | 1.778 | 1.158 | 0.251 |
| Sequencing of decimals | 66 | 2.06 | 1.516 | 1.13 | 1.445 | 1.952 | 0.000 |
| Ordering of decimals | 66 | 2.24 | 1.859 | 2.72 | 1.695 | 4.504 | 0.055 |
| Decimals on the number line | 66 | 2.47 | 1.269 | 2.51 | 1.138 | 1.280 | 0.205 |
| Closeness to a decimal | 66 | 1.81 | 1.203 | 1.97 | 1.058 | 2.157 | 0.035 |

Table 4.6: Mean pre- and post-test Part B scores of the secondary cohort (N=51, paired t-tests)

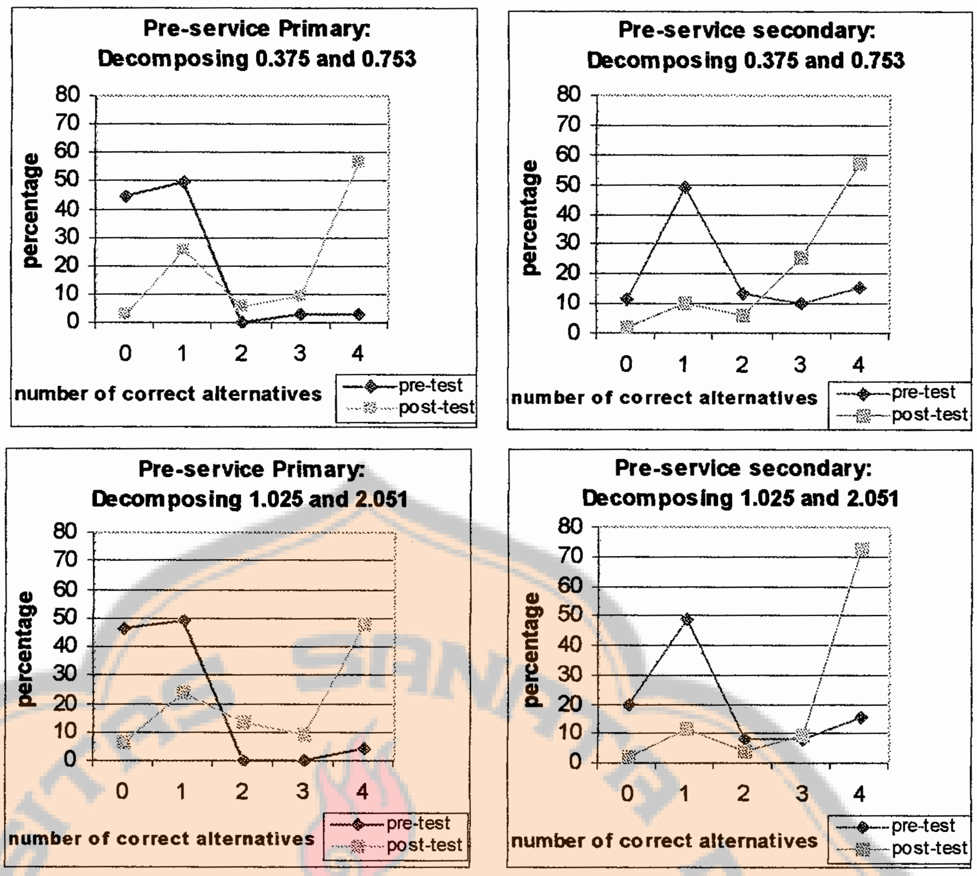
| **Area** | ***df*** | **Pre-test** | | **Pos-test** | | | |
|---|---|---|---|---|---|---|---|
| | | **Mean** | **SD** | **Mean** | **SD** | **t value** | **p value** |
| Identifying place value names | 50 | 2.61 | 0.493 | 2.71 | 0.701 | 0.927 | 0.358 |
| Decomposition of decimals | 50 | 1.69 | 1.334 | 3.45 | 1.083 | 9.040 | 0.000 |
| Density of decimals | 50 | 3.14 | 1.456 | 3.25 | 1.495 | 0.622 | 0.537 |
| Sequencing of decimals | 50 | 3.69 | 0.735 | 2.90 | 1.404 | 0.704 | 0.001 |
| Ordering of decimals | 50 | 3.84 | 0.543 | 3.76 | 0.651 | 3.606 | 0.485 |
| Decimals on the number line | 50 | 3.33 | 0.712 | 3.22 | 0.808 | 0.979 | 0.332 |
| Closeness to a decimal | 50 | 2.82 | 1.396 | 3.10 | 1.285 | 1.188 | 0.240 |

Pre-service teachers from the primary cohort improved significantly in explicit names of place value (see row 1 of Table 4.5). The initial mean score of this cohort (1.60 out of 3) indicated pre-service primary teachers' weak knowledge of place value. Moreover, a common error pattern to identify the place value of a decimal digit based only on the length of decimal digits signified lack of place value understanding in the pre-test (Item 1a, b, c in Part B, see Appendix B1). The value of digit 1 in 9.31 was identified as a hundredth (apparently correctly), and the value of digit 1 in 5.1064 was identified as a ten thousandth. There were 45 pre-service teachers showed this error pattern in the pre-test of cycle 1 and a majority of them were from the primary cohort. Fortunately this error pattern was not persistent, with only 8 pre-service teachers indicated this problem in the post-test. Observation of an error pattern in identifying place value names also suggested lack of sensitivity of the written test items to pick up this error pattern in cycle 1. The high initial mean score of 2.61 out of 3 of the secondary cohort which gave little room for improvement might be due to this lack of sensitive items. Hence, confirmation about this error pattern will be tested with an additional test item in cycle 2.

Both cohorts recorded the weakest performance in Item 2a and 2b (see Appendix B1) on decomposing decimals (see row 2 of Table 4.5 and 4.6) in the pre-test but made the most significant improvements on this area in the post-test. Lack of place value understanding, higher proportion of blank answers, and unfamiliarity with various ways of interpreting decimals were amongst the factors contributing to this weak performance in this area. Scant knowledge of place value was indicated by re-ordering decimal digits, e.g., decomposing 0.375 = 5 one + 7 tenths + 3 hundredths + 0 thousandths, or 0.375 = 0 one + 5 tenths + 7 hundredths + 3 thousandths, or 0.375 = 0 one + 7 tenths + 3 hundredths + 5 thousandths. This is evidence of a classic misconception that is referred to as 'reverse thinking' by Stacey and Steinle (1998). However, the DCT is known not to identify this misconception very well.

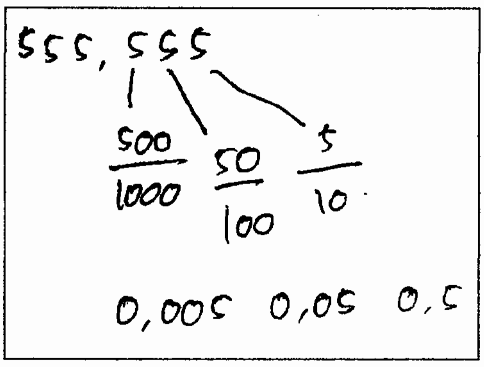
Furthermore, responses in the pre-test showed that most pre-service teachers from both cohorts were not aware of multiple ways of decomposing a decimal number. Pre-service teachers' unfamiliarity of different ways of decomposing decimals was confirmed in the interview transcripts. Some interviewees noted that they were only familiar with one standard way of interpreting a decimal number in their schooling experience. Consequently, it was not surprising that both cohorts started with half of the cohorts giving only one correct alternative, i.e., 0.375 = 0 one + 3 tenths + 7 hundredths + 5 thousandths. Both cohorts gained advantage from the teaching as majority of pre-service teachers could provide more than 4 correct alternatives for each decomposition item as can be seen in Figure 4.18.

Figure 4.18: Performance in decomposing decimals (Item 2a,b in Part B) from both cohorts




Similarly, the pre-course interview data with Andin, a pre-service primary teacher, provided a confirmation of her identifying the right most digit as the tenth (reverse thinking) (see Figure 4.19 below). This thinking was evident in her decomposing decimals in Part B item 2a and b. An improvement was observed in the post-test as she could give one correct alternative in decomposing decimals (Item 2a and 2b, Appendix B2).

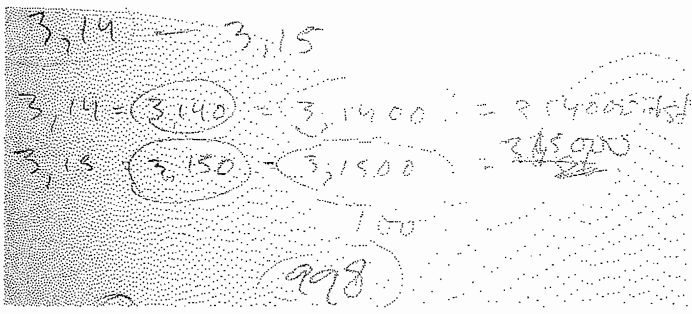
Figure 4.19: Andin's explanation for her erroneous interpretation of 555.555




Density of decimals was another content area that both cohorts showed no significant improvement (see row 3 of Table 4.5 and Table 4.6). However the mean scores in both tests for the two cohorts showed wide gap between their performances. The performance of the primary cohort in both tests was particularly alarming on this area. In contrast, the mean score of the secondary cohort of 3.14 (out of 4) indicated that pre-service secondary teachers might have a good knowledge about density of decimals. However, explanations on the written tests and interview data, items to assess density of decimals in cycle 1 (item 5 and 6 of Part B) revealed the limitation of the items to discriminate clearly those who thought there were finitely or infinitely many decimals in between the pair of given decimals.

A majority of incorrect answers from the primary cohort to density items reflected incorrect association of decimal digits with whole numbers. Interestingly, some pre-service teachers employed subtraction to find the number of decimals in between two decimals in the pre-test of Part B Item 5 and 6 (see Appendix B1), particularly in the primary cohort responses. This response might be triggered by the question format of asking how many numbers in between two decimals. Some pre-service teachers suggested no decimal number in between 0.899 and 0.90 and noted that 0.899 was the same as 0.90 suggesting inappropriate use of rounding. The textbook approach, which overemphasized rounding to two or three decimal digits, seemed to delimit pre-service teachers' understanding of density of decimals as well as giving the impression that digits beyond the second or the third have no meaning. This lies behind A2 thinking (see Table 2.2). Another incorrect answer to Item 5 and 6 was related to a common practice of working with decimals numbers of the same lengths as showed by Novo's written explanations (see Figure 4.19) recorded during the pre-course interview excerpt below:

Figure 4.20: Novo's explanation in respond to pre-test Item 5 in Part B about density of decimals




Novo: First I thought that since 3.14 equals to 3.140 equals to 3.140000 and so on. Similarly 3.15 is the same as 3.150 and 3.15000 and so on. Then the interval between them can be 10 and between this one (3.1400) and this one (3.1500), the interval is in thousandths (sync) so there should be more than 200 because thousandths implies that there will be more than 200. I personally think that there are 998 but because the option is only more than 200 then I choose the one with more than 200 as an option.

Apparently Novo's thinking showed reliance on strategy of working only with decimals with the same length, which consequently led to the answer there were finitely many decimals in between two decimals. A similar way of thinking was also evident in Maya's and Nara's pre-course interview responses below:

Maya: I think that between 3.14, and 3.15 there is a difference like between 2 and 3 so I think starting from 3.14 there must be 3.141, 3.142, 3.143, and so on until 3.15. How many are there? Well, last time I counted … perhaps there were about nine?

Nara: 3.142, 3.143, 3.144 and so on until it get close to 3.15. I think there are finite numbers in between because 3.15 is a fixed number.

Pre-service teachers who answered there were infinitely many decimals in between two decimals also failed to provide satisfactory justifications both in the pre-test responses to Part B item 5 and 6 and also in the pre-course interviews. The language employed such as "adding more digits" or "infinite zeros behind the last decimal digit" in the following pre-course interview excerpts which reflected reliance on a procedural approach.

Ayi: I remember that behind 3.14 there are infinitely many zeros, it can go very long so I imagine that from 0 we can make infinite digits. It can start from unit to tenths so clearly there are infinitely numbers in between.

Tinton : Because the number of digits behind the decimal comma are infinite.

Sasti: Because in between 3.14 and 3.15 there are tenths, hundredths, and thousandths and the decimal digit that is larger than 5 will be rounded up.

Only a small number of pre-service teachers provided sound explanations about the density of decimals as illustrated below. Apparently some explanations concerning the density property marked different traits of the two cohorts in terms of mathematics subjects they took up during their training. The fact that the pre-service secondary cohort had more exposure to properties of different number systems including the real numbers allowed them to provide explanations using knowledge obtained from other courses.

Susilo: Because after 3.14, there are 3.1401, 3.1402, 3.140021 and if we continue up to 3.15, there will be so many of them. It is infinite because this 3.14 is in hundredths, if we add one more digit; it will be thousandths, 3.141 and if we add more digits until perhaps hundred thousandths or millions then there will be so many. There will be infinitely many of them.

Retno: In between 3.14 and 3.15 there are more than 5 numbers, one of them is 3.142. In between 3.14 and 3.142 there are also more than five decimal numbers so following this thinking there are infinitely many numbers in between 3.14 and 3.15.

Ayi: Because decimals are a subset of real numbers so they satisfy the dense property of real numbers.

Besides positive improvement observed, both cohorts recorded decline on sequencing of decimals (row 4 of Table 4.5 and Table 4.6). High cognitive load of problems of finding the sequence of the next three decimals (Item 4a, 4b of Part B) and lack of comparability between pre-test and post-test items were the identified factors to explain the decline on this content area items assessing knowledge in sequencing decimals. Moreover, the common incorrect answers in cycle 1, such as sequencing decimals 0, 0.05, 0.10 after 1.5, 1, 0.5, indicated weak knowledge of place value and negative decimals. The common incorrect answer in the post-test item of finding the next three decimals in the sequence 1.4, 1, and 0.6, i.e., 0.2, 0.08 and 0.04 confirmed pre-service teachers' difficulties in working with negative decimals. Hence, a decision to refine the written test items in cycle 2 on sequencing of decimals was made.

Similarly both cohorts showed no significant improvement in ordering decimals (row 5 of Table 4.5 and Table 4.6). The majority of pre-service teachers showed no difficulty with ordering five decimals in Item 3a and 3b of Part B (see Appendix B2). In contrast, the primary cohort showed a variety of incorrect responses as also observed in DCT responses. Based on this, it was decided to omit ordering of decimals from written test of cycle 2.

Weak knowledge on relative magnitude of decimals was evident particularly in the primary cohort as reflected in low initial mean scores in both identifying or locating decimals on the number line (row 6 of Table 4.5 and Table 4.6, item 7, 8, 9 from Part B Appendix B2) and finding the closest decimal to a given decimal (row 7 of Table 4.5 and Table 4.6, item 10, 11 from Part B Appendix B2). Moreover, difficulties in locating negative decimals on the number line were found in the primary cohort and were reflected in the low mean scores on this content area in both pre- and post-tests.

Note that word problems (item 12, 13, 14, 15, 16 from Part B Appendix B1 and Appendix B2) were excluded from the paired t-test analysis of cycle 1. The decision was made based on the fact that most responses focused on finding the exact answers and ignored the context set in these problems. For instance, when asked how many package of 100 grams need to be bought for 1.25 kg of flour, many answered 12.5 instead of 13. Prior learning experiences which lack exposure to contextual items and have strong emphasis on computational skills played a role in low rate of success with contextual items as suggested in the following interview excerpt:

> Susilo: In the past, we were directly taught to find the decimal 0.6. My teacher did not focus on the understanding of decimal notation. We were given a lot of exercises to do that but no real life examples given, just sample problems and explanations how to solve the problems.

In summary, the activities in cycle 1 improved pre-service teachers' knowledge of place value and various ways of interpreting decimal numbers. These areas were the main focus of the activities in this cycle. Content areas that indicated no improvement such as density, relative magnitude of decimals, ordering of decimals were not covered in the activities in this cycle due to lack of time. Hence lack of improvement in these areas did not reflect the effectiveness of activities. Meanwhile the decline in sequencing decimals could be explained by higher level of difficulties in the post-test items. Similarly, difficulties with word problems suggested a tendency to focus on computational fluency and ignoring the context of the problem. These findings will be taken into account in the design of the activities in the second cycle. Section 4.6 will address these issues.

### 4.5.3 Findings from tests and interviews on Pedagogical Content Knowledge

In examining the pedagogical ideas on the teaching and learning decimals, the responses to Part C of the written tests (see Appendix B1 and Appendix B2) were scored and classified into low, medium and high based on the scoring rubric (see Table 3.5). Common themes recorded in teaching ideas will be reported and illustrative examples will be presented to get a complete picture of pre-service teachers' evolutions in teaching ideas for decimals.

Improvement in pedagogical ideas was indicated by the progress of the 118 pre-service teachers in various classifications based on their scores in Part C of the written tests. The total score of Part C was classified into low, medium, and high based on the following criteria. Total score 0 to 3 out of 9 are classified as low, total score of 4 to 6 out of 9 are classified as medium, and total score 7 to 9 out of 9 are classified as high PCK. The proportion of pre-service teachers who were identified as having low PCK dropped from 55.9% in the pre-test to 32.2% in the post-test, and the percentages of those identified as having high PCK increased from 2.5% to 13.6% in the post-test (see Table 4.7).

Table 4.7: Distributions of pre-service teachers in various categories in Part C of cycle 1 (N=118)

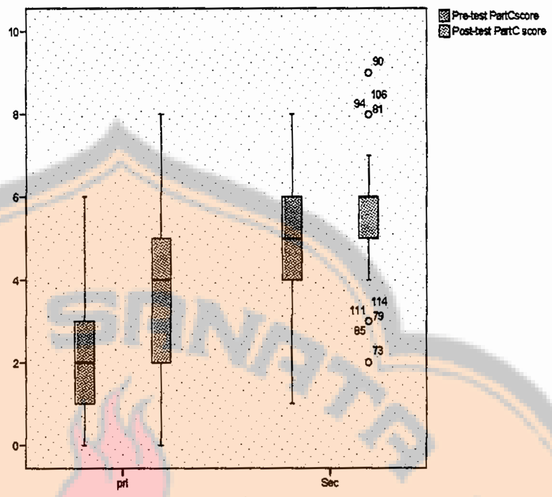
| Pre-test (%) | Post-test (%) | | | Total |
|---|---|---|---|---|
| | Low | Medium | High | |
| Low | 27.1 | 25.4 | 3.4 | 55.9 |
| Medium | 5.1 | 27.1 | 9.3 | 41.5 |
| High | 0 | 1.7 | 0.8 | 2.5 |
| Total | 32.2 | 54.2 | 13.6 | 100 |

Both cohorts improved significantly from the pre-test to the post-test. Significant improvement of pre-service teachers' pedagogical ideas was confirmed by the result of paired t-tests on the overall score of Part C as presented in Table 4.8. However, despite significant improvement in the post-test, both cohorts still documented a lack of satisfactory explanations in teaching ideas as reflected in the low mean scores of both cohorts.

Table 4.8: Mean pre- and post-test Part C score (total score out of 9)

| Cohort | N | df | Pre-test Mean | Pre-test Standard dev (SD) | Pos-test Mean | Pos-test Standard dev (SD) | *t*-test | *p* value |
|---|---|---|---|---|---|---|---|---|
| Primary | 67 | 66 | 2.00 | 1.303 | 3.61 | 2.181 | 7.228 | 0.000 |
| Secondary | 51 | 50 | 4.69 | 1.378 | 5.39 | 1.484 | 2.944 | 0.005 |

Figure 4.21: Box plot representing PCK scores in pre-post of two cohorts




One of the common problems in both cohorts was lack of attention to teaching ideas. The performance of the primary cohort was particularly alarming. A high proportion of blank answers as well as fragmented content knowledge contributed to this low performance in this area. For instance, 21 pre-service primary teachers did not provide any teaching ideas in the pre-test of Part C Item 19 (see Appendix B1) to diagnose and to resolve students' misconception in ordering 0.3<0.34<0.33333. The fact that no blank responses were recorded in the secondary cohort ruled out the possibility of lack of time in completing the test to explain this trend in the primary cohort.

Table 4.9: Paired t-test results of various items in Part C items of cycle 1

| Cohort | No | Area | *df* | Pre-test[a] | | Pos-test[b] | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Mean | SD | Mean | SD | t value | p value |
| Primary | 17 | Teaching ideas for comparing pairs of decimals | 66 | 0.76 | 0.676 | 1.18 | 0.716 | 4.378 | 0.000 |
| Primary | 18 | Teaching ideas on division of decimals by 100 | 66 | 0.39 | 0.491 | 0.52 | 0.587 | 1.584 | 0.118 |
| Primary | 19 | Diagnosis of students' error in ordering decimals and teaching ideas to resolve it. | 66 | 0.40 | 0.698 | 1.25 | 0.990 | 6.305 | 0.000 |
| Primary | 20 | Teaching ideas on the links between fractions and decimals | 66 | 0.45 | 0.634 | 0.66 | 0.617 | 2.117 | 0.038 |
| Secondary | 17 | Teaching ideas for comparing pairs of decimals | 50 | 1.31 | 0.583 | 1.51 | 0.505 | 1.697 | 0.096 |
| Secondary | 18 | Teaching ideas on division of decimals by 100 | 50 | 0.86 | 0.401 | 0.96 | 0.631 | 0.868 | 0.389 |
| Secondary | 19 | Diagnosis of students' error in ordering decimals and teaching ideas to resolve it. | 50 | 1.24 | 0.551 | 1.55 | 0.856 | 2.177 | 0.034 |
| Secondary | 20 | Teaching ideas on the links between fractions and decimals | 50 | 1.27 | 0.777 | 1.41 | 0.669 | 1.155 | 0.254 |

a: Pre-test items are given in Appendix B1
b: Post-test items are given in Appendix B2

As revealed in Table 4.9, of four PCK items covered in Part C items 17, 18, 19, and 20 (see Appendix B1 for details of the items), the highest improvements were observed in teaching ideas to compare decimals and to diagnose students' errors in ordering decimals (examined in item 17 and 19). Rounding to two or three decimals digits was the most common strategy shared by both cohorts in teaching ideas to rectify errors in ordering decimals in the pre-test item 19. The striking difference in the post-test was reference to LAB model and extended notation of decimals for comparing and ordering decimals.

Clearly both cohorts gained advantage from the models introduced during the teaching experiment as indicated by the fact that many of them could provide three different models for teaching decimals. Interestingly, the majority of pre-service teachers proposed the use of linear models similar in principle to LAB such as bamboo sticks, straws along with other models such as paper strips, plastic rope, and number line. All pre-service teachers who participated in the post-course interviewees listed LAB as one of the models for their future teaching ideas in decimals.

Despite significant improvement observed in the post-test, extending the link between pieces of LAB model with finding a decimal notation of a given fraction was particularly challenging for a majority of pre-service teachers. Some pre-service teachers made a remark that decimal notation for a fraction $\frac{1}{3}$ should be limited to only two or three decimal places to avoid confusion and complexity for primary school students in the pre-test. This comment reflected the common practice in the primary school textbooks, where discussions of decimals were often limited to thousandths. In response to Part C post-test item 20, only two pre-service teachers could extend their learning experience with models and incorporate it in their ideas for future teaching such as explaining with LAB the link between the division algorithm and extended notation for $\frac{1}{6}$. Nonetheless these findings indicated a promising impact of activities in developing ideas for teaching decimals.

Teaching ideas in Part C Item 18 about explaining how to divide a decimal number by a power of 10 (e.g., 100) indicated no significant improvement. Fragmented memorized knowledge of the standard algorithm was evident in errors involving invert and multiply algorithm to solve division of 0.5 by 100. Incorrect answers such as; $0.5 \div 100 = \frac{5}{10} \div \frac{100}{1} = \frac{5}{10} \times 100 = \frac{10}{5} \times 100 = 50$ or $0.5 \div 100 = \frac{5}{10} \div 100 = \frac{10}{5} \times 100 = 200$ documented in the pre-tests showed evidence of the reliance on algorithms without referencing to underlying meaning.

Only a few pre-service teachers were able to translate their extended notation of decimals for ideas to teach division of decimals by 100 in the post-test. These pre-service teachers expanded 0.3 as 0 + 3 tenths = 0 ones + 0 tenth + 30 hundredths = 0 ones + 0 tenths + 0 hundredths + 300 thousandths and then divided 300 thousandths by 100 to obtain 3 thousandths or 0.003 in the post-test. Obviously this explanation signified an understanding of place value including additive and multiplicative structures of decimals. Two pre-service teachers proposed similar teaching ideas by utilizing LAB model and straws to represent 0.3 and then divided it into 100 smaller pieces to get to 3 thousandths or 0.003. One pre-service teacher opted to explain this to children by representing 0.3 not using tenths pieces but thousandths pieces, which depicted her understanding of multiplicative structure between tenths and thousandths, i.e., there are 100 thousandths in 1 tenth.

In response to cycle 1 post-course interview question on value judgement of the models for teaching decimals (see Appendix B4), a majority of pre-service teachers ranked LAB as the most accessible model for students as it reflected base ten relationships among ones, tenths, hundredths and thousandths clearly and commented on simplicity of the model due to its linear nature as expressed in the following representative comments. Note that Ayi's comment reflected the influence of an additional task given by the lecturer in the secondary cohort to think about other models for teaching decimals. This lecturer presented pre-service secondary teachers with a task to compare LAB model with an area model or a volume model. Note however, that the other lecturer did not present the task with pre-service teachers from the primary cohort.

> Hery: LAB is the easiest one because the relationships such as tenths, hundredths, etc are reflected in the models. Meanwhile with a concrete model (from daily life) such as rock melon you need to (*imagine*) cut it into pieces and with number expander students need to fill in the numbers first. (Primary cohort)

> Ayi: It is easier to use LAB because even though it looks like a three-dimensional model, the prominent feature is length whereas with cube we have to consider the volume. With a paper [area based model] there is a problem of consistency in dividing into shorter/smaller pieces. Dealing with the spatial model is even more complex. (Secondary cohort)

> Ismi: LAB is easier…. with LAB, I came to understand why we call these pieces tenths, because there are ten tenths in a one. Similarly a hundredth consists of ten tenths, and a thousandth consists of ten hundredths. (Primary cohort)

Most of the models proposed in the post-course interviews were models introduced in the activities. Whilst some comments tended to focus on the logistic and physical characteristics of concrete models instead of the mathematical principles, it was encouraging that many responses noted that the linearity of LAB as helpful in noticing the decimal relations.

Examinations of common teaching ideas for decimals highlighted some interesting similarities and differences between the two cohorts. The difference was that the primary pre-service teachers incorporated more use of concrete models in their teaching ideas than the secondary pre-service teachers in both pre- and post-tests. However, pre-service teachers' own confusions were depicted in some of these teaching ideas using concrete or pictorial models. The use of models often signalled weak knowledge of links between fractions and decimals as captured in the following pre-course interview excerpts. This explained the lower scores of the primary cohort despite the fact that they showed more awareness of teaching pedagogy as they took into account the developmental stages of children's thinking in their teaching ideas. The following excerpts illustrated incomplete knowledge of two pre-service primary teachers observed during their individual pre-course interviews.

> Dian: Using concrete materials, for instance using a cake as an example... Ehm, but a cake is more like to help children learn fractions and not decimals. For decimals, we can multiply the numbers by tenths. (With the cake) well, 0.6 means that we have to find,... like if we need to find percents... how much is 0.6 of 1 so we divide the cake into equal pieces from one. Multiply it by six tenths of it, so ... [we need to find] how many parts, right?... Probably six parts... I don't know... I am weak in mathematics.

> Ismi: Perhaps we can use daily experience, for primary school children, using word problem such as sharing a cake. One cake is divided for several students; for instance there is one cake and there are ten students then we just need to divide one cake into 10 parts. [Interviewer: How will you represent 0.6 then?] One part is a tenth so if we are asked... if each child gets a tenth of one, then how many are ... for instance there are four children. Four children means, ehm... it means that there are ten cakes, then if each child gets 2 cakes, then the total is 8 and we have 2 left over. Then we divide 2 into 4 children, so each child gets two and a half.

Dian, a primary pre-service teacher, apparently had incomplete ideas about the relation between fractions and decimals. Her fragmented knowledge was indicated in her first comment to suggest cake as a model for teaching fractions but not for teaching decimals. Similarly, Ismi, a fellow pre-service primary teacher, also proposed sharing a cake as a context to represent decimals. She was able to explain a model for one tenth correctly but she showed difficulties in utilizing the same context to represent 0.6. Even when she proceeded with an attempt to explain the model for ¼, she was unable to link this with the corresponding decimal notation. She explained (see excerpt above) that a quarter of 10 objects in 2 ½ objects but did not link to the question about decimals.

In fact, both cohorts showed lack of awareness about the use of models in teaching and learning decimals could be attributed to limited exposure to models in learning decimals as captured and expressed below by Ayi, a secondary pre-service teacher:

Ayi: From our learning experiences in primary school, we never learnt decimals using concrete models. Teacher used to explain in the black board without manipulative that could help us to understand decimal concept. We were not encouraged to participate actively in the learning process and to express our thinking.

Teaching ideas from the secondary cohort were dominated by the computational approach and the proposed models were more abstract such as the number line. The fact that the secondary cohort possessed better skills with some standard algorithms such as the division algorithm might explain their stronger reliance on syntactic procedures in their teaching ideas. However, interview transcripts revealed cases of pre-service teachers from both cohorts who memorized the relations $\frac{1}{3} = 0.333\ldots$ and could not explain this equivalence relation.

In summary, despite some improvements observed in teaching ideas for decimals such as reference to basic notion of place value and structural properties of decimals, the pedagogical content knowledge of most pre-service teachers was far from satisfactory.

## 4.6 Retrospective analysis phase

This section starts with an overview of findings from various data sources in answering the research questions. Reflections on the achievement of goals for the activities and the contributing factors that afford or inhibit the attainment of those goals will also be discussed in Section 4.6.1. Feed-forward recommendations for the next research cycle will be articulated in Section 4.6.2.

### 4.6.1 Overview of Research Findings

There was an overall trend of improvement in both CK and PCK from both cohorts after the teaching experiment. Performance on content areas of decimals covered in the activities such as decomposition of decimals in expanded notation, and ideas for comparing decimal numbers indicated clear improvements. However, findings from pre-test and pre-course interviews identified some major difficulties, i.e., overgeneralization of rounding, and evidence of 'Shorter-is-Larger' misconceptions in the primary cohort. Performance in some areas of decimal numeration that were not addressed in the teaching such as sequencing of decimals declined in the post-test. Besides lack of teaching, lack of comparability between pre- and post-test items might explain the decline. A better match between pre- and post-test items as well as attention to these areas of decimals is expected in the design of the next cycle. Meanwhile low scores and lack of significant improvement in content areas such as density and knowledge of relative magnitude of decimals on the number line signified the importance of attending to these areas in the activities for the next cycle.

The initial state of pre-service teachers' PCK for both cohorts was characterized by the strong reliance of syntactic computational procedures and lack of ideas for incorporating models in teaching decimals. Pre-service teachers' knowledge of decimal representation was limited to only the standard form of representation. This characteristic of pre-service teachers' PCK reflected influence of common teaching approach in Indonesian classroom practice. Areas identified in the goals of the activities such as decimal place value, interpretation of decimals, and continuous nature of decimals were found to be problematic for most pre-service teachers. Hence the importance of addressing those topics in the activities was confirmed.

Post-course interview data also revealed clear improvements in the ideas for the use of models for teaching decimals. The use of concrete models in particular LAB was observed by the researcher to mediate between the abstract notion of decimal notation and its interpretations through hands on explorations and group discussions. However, we found that the use of models might have not been well integrated in the activities as many pre-service teachers still relied on syntactic rules in solving the problems. Moreover, extending and linking the models with the standard algorithm proved to be challenging for most pre-service teachers. This suggested that linking concrete models with the standard algorithm was a mathematically challenging task. The fact that no activity in cycle 1 was devoted in particular to assist pre-service teachers in linking their experience with models and the standard algorithm explained pre-service teachers' difficulty. In contrast, activities providing a clear direction in integrating models with a mathematical task for comparing decimals (see Set 1 Activity 7) resulted in better teaching ideas in corresponding topic. Hence more careful planned activities with a clear direction to explore concrete models in making sense of standard algorithms followed by classroom discussion might improve learning on this aspect.

Interactive modes of learning that focused on active participation in finding own solutions and group discussions, and the use of concrete models, were noted as contributing factors to the improvement. These factors were articulated by pre-service teachers as documented in the reflections comments in Set 1 Activity 8 and Set 2 Activity 15 were consistent with some of the RME instructional principles (Treffers, 1987). However, interpretations of basic tenets of RME such as guided reinvention, mediating model and didactical phenomenology were not yet clearly observed in this cycle. This raised a concern about the integration of RME tenets in the design of the activities in the first cycle. Hence integrating RME basic tenets in the design of the activities needs to be improved in the next design. These aspects will be addressed in more detail in Chapter 5.

Concerning the interpretation of didactical phenomenology, I hypothesised that the measurement context used in Set 1 Activity 1 was a good context for introducing decimals. However, building the connections between the context situation and the formal decimal notation required some improvement. The preference of pre-service teachers to rely on a computational approach suggested that the strong interference of pre-service teachers' existing knowledge of decimals. The mediating models tenet was not well reflected in the design of the first cycle. For instance the link between the LAB model and with the more abstract models such as number expanders and the number line was not addressed well in cycle 1.

### 4.6.2 Feed forward for cycle 2

Findings from the first cycle signified the need to adjust the sequence of the activities. In the introductory activities to explore LAB and establish the names for each piece, focus should be placed on linking the materials with the symbolic representation of decimals. Observation of group discussions revealed that some groups utilized a standard measurement tool such as a ruler in solving the problem. Hence discussion about the idea of partitioning into ten smaller parts in both models (LAB and rulers) might be helpful in understanding the idea of decimals. An issue regarding the integration of models in the activities was one of the concerns in the design of activities for cycle 1. This challenge was in part caused by the fact that pre-service teachers had already acquired symbolic knowledge. Encouraging pre-service teachers to use models as a thinking tool in solving the problems will be made clearer in cycle 2. As different models are involved, attention to the links among models such as LAB, number line, and number expander become crucial. A challenge to refine activities so that pre-service teachers can focus more on the mathematical ideas in using the models is one of the feed-forward ideas for cycle 2.

Many pre-service teachers also showed difficulties in identifying decimal numbers on the number line and have little idea about density of decimals. Hence extending the LAB model and linking it with the number line model in the early stage of teaching experiment will be a feature of cycle 2. Moreover, an activity to find a decimal number in between two given decimals will challenge some misconceptions with decimals including rounding/truncating thinking and the associated lack of meaning of the decimal places beyond the first two or third. Attending to pre-service teachers' misconceptions and problems was one of the aspects that should be improved in cycle 2.

The fact that some pre-service teachers left answers for PCK items blank in pre-test and post-test indicates difficulties in finding ideas for teaching decimals. Incorporating diagnostic questions based on hypothetical situations as part of the teaching activities might encourage deeper engagement with the issues. Posing diagnostic questions in the interview questions is another idea to gain more insights into the thinking and misconceptions held by pre-service teachers. The post-course interview in cycle 1 also missed out on addressing pre-service teachers' self-evaluation about their own evolution in understanding of decimals.

The importance of guiding group and classroom discussions to focus on key concepts addressed in each goal was particularly important. Findings from cycle 1 indicate the activities made a positive impact on both cohorts despite some different traits of the primary and secondary cohort. Further investigation in cycle 2 will either confirm or not the different traits of the two cohorts and the impact of the activities on these cohorts. Note that even though data collection will take place in the same university and involve both primary and secondary pre-service teachers, the individual pre-service teachers participating in the next cycle will be different. Where activities remain unchanged, their effects will be trialled again with a new set of pre-service teachers. Conjectures made based on the findings from cycle 1 regarding the nature of the activities and the cohorts involved in the study will be tested and refined in cycle 2.

# CHAPTER 5 GOING THROUGH PHASES IN CYCLE 2

## 5.1 Introduction

This chapter describes the phases leading up to and involved in cycle 2 and findings gathered in each phase. The description will follow similar structure in reporting phases and findings of cycle 1 in Chapter 4. First, Section 5.2 gives a summary of the refined research instruments comprising written tests, interview questions, activities and reasons leading to the changes. Moreover, I will explicate findings from the trial phase of the refined instruments prior to cycle 2 along with lessons learned from the trial in this cycle. Section 5.3 discusses the enactment of the activities and describes the findings during the teaching experiment phase. In section 5.4, results of the pre-test and the post-test, pre-course interviews and post-course interviews will be discussed, with an indication of any evolution of pre-service teachers' content and pedagogical content knowledge. Finally, Section 5.5 concludes with the retrospective analysis of cycle 2 and presents reflections on the overall findings gathered in cycle 2.

Details of the data collection methods in relation to the research questions and goals are summarised in Table 5.1. A total of 172 pre-service teachers sat for the pre-test. After participating in the sets of activities, a total of 155 pre-service teachers sat for the post-test which then formed a total of 140 longitudinal cases, 94 from primary cohort and 46 from secondary cohort. Of 20 pre-service teachers who participated in the pre-course interviews, 3 did not attend the post-course interview. One pre-service teacher, who did not participate in the pre-course interview, demonstrated an interesting way of thinking during group discussions and expressed her interest to participate in an interview so she was involved in post-course interview. Her thinking and strategies in the pre-test and changes she experienced after the teaching experiments were extracted during the post-course interview. Following the same design structure as in cycle 1, pre-service teachers worked in small groups of 4-6 people during the enactment of activities. All the activities were carried out in the span of 4-5 meetings each lasting approximately 2 hours. Similar to the conduct of teaching experiment in cycle 1, the main strategies in delivering activities rely on pre-service teachers' engagement in activities on group works. There were 3 classes of the primary pre-service teachers and one class of the secondary pre-service teachers.

Table 5.1: Overview of data sources in relation to research questions and goals

| Methods | Cohort | Number of participants | Research questions * | Goals |
|---|---|---|---|---|
| Trial phase Pre-test | Volunteers | 2 (Indo) | - | Trial test items and identify problems on test items |
| Observation of trial of activities | Volunteers | 1 group (Indo) | - | Test and identify problems in the activities |
| Pre-test (Appendix B5) | Primary<br>Secondary | 119<br>53 | 1a<br>2a | Identify current state of CK<br>Identify current state of PCK |
| Post-test (Appendix B6) | Primary<br>Secondary | 108<br>47 | 1b<br>2b | Identify the evolved CK<br>Identify the evolved PCK |
| Pre-course interviews (Appendix B7) | Primary<br>Secondary | 14<br>6 | 1a<br>2a | Confirmation of the pre-test<br>Identify initial ideas for future teaching |
| Post-course interview (Appendix B8) | Primary<br>Secondary | 13<br>5 | 1b<br>2b | Confirmation of the post-test<br>Feedback for activities<br>Identify evolved ideas for future teaching |
| Observation of activities (video and audio-recordings) | Primary<br>Secondary | 5 groups<br>1 group | 1b<br>2b | Identify aspects in the activities that contribute to improvement of CK and PCK |

* see Section 1.4

## 5.2 Design Phase 2

In this section, I will explicate the fine-tuning of instruments based on analysis of findings and recommendations from cycle 1. The main refinements of different research instruments will be explicated starting with refinements of activities, followed by refinements of some of the written tests and interview questions. See Appendix A4 for complete record of changes and refinement of LIT and Appendix B9 for record of changes in written test items.

### 5.2.1 Refinement of the activities

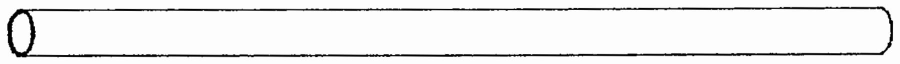
A complete set of activities in cycle 2 can be found in Appendix A5 (Indonesian version). As pointed out in the retrospective analysis of cycle 1, the activities in cycle 2 aims to better reflect the basic tenets of RME. In Set 1 Activity 1 of cycle 1, the process of partitioning the longest LAB pipe into ten, hundred, and thousand shorter pieces of equal length was given. Based on exploration of the relationships among various pieces, pre-service teachers were expected to establish verbal names, fraction and decimal notation of those pieces. Yet in the light of RME guided reinvention tenet, this activity did not allow much opportunity for pre-service teachers to reinvent the decimal structure by themselves and to connect this decimal structure with the formal decimal notation. Experiencing the process of partitioning one successively by ten was expected to guide pre-service teachers to understand the connection between this act of partitioning and the decimal notation. Therefore, Activity 1 in cycle 2 started with finding ways of partitioning the longest LAB piece into equal parts and asking for justification of choosing certain ways of partitioning. Moreover, ideas about further refinement into shorter pieces were probed in the context of making the measurement more accurate (see Figure 5.1).

Following the above activities, Set 1- Activity 2 to link LAB pieces, verbal names, decimal notation, and fraction notation and to record the result of measuring a table was explored as in the first cycle. Articulation of new ideas for future teaching based on reflection of their engagement in the activities was more emphasised in cycle 2. This is in line with the aim of the study to engage pre-service teachers in constructing meaningful understanding of decimal notation and pedagogical ideas of various representations in this area. Moreover, the sequence of the activities was modified to ensure that density of decimals was attended in this cycle, after being omitted from cycle 1 because of time constraints. Hence in cycle 2, this activity was trialled for the first time.

Figure 5.1: Comparison of Activity 1 in cycle 1 and in cycle 2 (from Appendix A4)

**Set 1 Activity 1 and Activity 2 in Cycle 1**

1. We assign the longest LAB piece as a one and agree to name the length of the longest piece as one. Explore the relationships between different pieces of LAB and establish the names of the pieces (verbally).

| Pieces | Verbal name |
|---|---|
|  | |
|  | |
|  | |
|  | |

2. Match up the following cards with the LAB pieces and fill out the following table with corresponding verbal names, fraction and decimal notation. From now on we will focus on the use of the decimal notation.

a tenth | $\frac{1}{10}$ | 0.1 | $\frac{1}{100}$ | a hundredth | 0.01 | 1 | a thousandth | $\frac{1}{1000}$ | 0.001 | one

**Set 1 Activity 1, 2, and 3 in Cycle 2**

1. You are given this piece and we agree to call it one. If you were to measure the length and width of your table with the piece, how many parts would you divide the piece into? Explain why do you choose this? What is the advantage and limitation of your choice?
2. If you want to get a more accurate result compared to the one before, what do you need to do?
3. Now if you need to measure the length of an eraser, how would you divide the piece?

### 5.2.2 Refinement of the test and the interview items

Basically refinement of the test and interview items aimed at getting better data. For instance, analysis of responses to Part B Item 1 in cycle 1 (see Figure 5.2) highlighted one common error pattern, i.e., the fact that place value of a decimal digit was determined based on the number of decimal digits. An additional item was included in cycle 2 as shown in Figure 5.2 to test the conjecture of the error pattern observed in cycle 1. As will be discussed in section 5.4, these new items worked well in revealing and confirming our conjectured pattern of thinking.

Figure 5.2: Refinement of Item 1 Part B from cycle 1 to cycle 2

| Pre-test B1cycle1 | Post-test B1cycle1 |
|---|---|
| 1. Tick the boxes that indicate **the value of digit 1** in the following decimal numbers: | 1. Tick the boxes that indicate **the value of digit 3** in the following decimal numbers: |
| a) 9.31 | a) 9.31 |
| ☐ a tenth ☐ a hundredth ☐ a thousandth ☐a ten thousandth | ☐a tenth ☐ a hundredth ☐a thousandth ☐ a ten thousandth |
| b) 23.001 | b) 23.001 |
| ☐ a tenth ☐a hundredth ☐a thousandth ☐a ten thousandth | ☐ a tenth ☐ a hundredth ☐ a thousandth ☐ a ten thousandth |
| c) 5.1064 | c) 5.1064 |
| ☐ a tenth ☐a hundredth ☐a thousandth ☐a ten thousandth | ☐ a tenth ☐a hundredth ☐a thousandth ☐ a ten thousandth |

| Pre-test B1cycle2 | Post-test B1cycle2 |
|---|---|
| 1. Tick the boxes that indicate **the value of digit 1** in the following decimal numbers: | 1. Tick the boxes that indicate **the value of digit 3** in the following decimal numbers: |
| a) 9.31 | a) 9.31 |
| ☐ a tenth ☐ a hundredth ☐ a thousandth ☐ a ten thousandth | ☐ a tenth ☐ a hundredth ☐ a thousandth ☐ a ten thousandth |
| b) 23.001 | b) 23.001 |
| ☐ a tenth ☐a hundredth ☐a thousandth ☐ a ten thousandth | ☐ a tenth ☐a hundredth ☐a thousandth ☐ a ten thousandth |
| c) 5.1064 | c) 5.1063 |
| ☐ a tenth ☐ a hundredth ☐ a thousandth ☐ a ten thousandth | ☐ a tenth ☐ a hundredth ☐ a thousandth ☐ a ten thousandth |
| d) 2.318 | d) 2.183 |
| ☐ a tenth ☐ a hundredth ☐a thousandth ☐ a ten thousandth | ☐ a tenth ☐ a hundredth ☐a thousandth ☐ a ten thousandth |

Similarly, analysis of cycle 1 Part C data (pre-service teachers' PCK) revealed that pre-service teachers tended to focus on explaining what they do rather than the reasons behind it (e.g. find the larger number by comparing digit by digit) rather than on the teaching ideas. Hence in cycle 2, explicit teaching ideas were probed to overcome those problems, as illustrated by two examples in Figure 5.3.

Figure 5.3: Refinement of Pre-test from Part C, Item 17 & 20 of cycle 1

| Pre-test Item C17 cycle1 | Pre-test Item C15cycle2 |
|---|---|
| What is your idea of teaching your students to find the larger number between 0 and 0.6. | Explain your ideas for teaching primary school students to find the larger number of 0.8 and 0.8888. Include any models that you can think of in your teaching ideas. |

| Pre-test Item C20cycle1 | Pre-test item C18cycle2 |
|---|---|
| How will you assist your students in converting 1/3 into a decimal number? | When asked to solve 1/3 x 100 000, a student answered 1/3 x 100 000 = 33 000. Do you agree with this student? How would you, as a teacher, help this student? |

Concerning the interview questions, questions that failed to reveal much about conceptual understanding were omitted from cycle 2. On the other hand, items that revealed a progress of understanding or challenged certain misconceptions were retained. Some changes in interview items are illustrated in Figure 5.4 below but a complete list of changes can be found in Appendix B9.

Figure 5.4: Changes in interview items and their justification

| Pre-course Interview | Action | Justification for changes |
|---|---|---|
| How was your experience in learning addition of two decimals? For example how did your teacher teach you to add 1.8 and 1.31?<br>Was the reason for lining up the decimal comma explained to you?<br>What are your ideas for teaching addition of decimals in the future? (Pre-course interview) | Omit questions | Findings confirm the prediction about the reliance of rules of lining up the decimal point. Don't reveal much about the conceptual understanding of decimal notation. Not required in cycle 2 to establish memories of primary school learning. |
| What is your idea to help children find the decimal expansion of 1/6? | Adjust question | Reveal ability to translate and link understanding to algorithm. |
| In answering a question to compare two decimals 1.66666 and 1.66, a student said that 1.66666 = 1.66. Do you agree with this answer? Why do you think she answer that way? What is your idea to resolve this problem? Can you think of any model that will be helpful in addressing/resolving this problem? | New Question | Reveal and challenge rounding/ truncating thinking (money thinking according to Steinle and Stacey, 1998). Also address the pedagogical aspect of resolving misconceptions involving decimals with repeated digits |

Incorporating all changes above, instruments and the initial activity were trialled out with 2 volunteer pre-service teachers in Sanata Dharma University, Yogyakarta. It should be noted that these two volunteers were not involved in the teaching experiment of cycle 1. The fact that the previous cohort year participated in cycle 1 and most of the fourth year pre-service teachers were undertaken teaching practice in schools as well as semester break (mid August) contributed to low participation of the trial in cycle 2. It was realized that this trial would not be representative of the research participants. However, responses and feedback from this trial were valuable particularly in ensuring that the problems were comprehensible and time provided for the test was enough. Minor refinements in the Indonesian translations were made in the wording of Part C Item 15 (see Appendix B5, B6), as volunteers suggested a possible misinterpretation of the problem in the Indonesian version.

Trial of the interview items revealed that the new items were easily understood so no further change was made. However, trial of the activities could only cover the first few activities due to limited time that could be afforded by volunteers. The outcome of the trial of Set 1 Activity 1 confirmed a conjecture about preference of halving over decimating in partitioning the longest LAB piece in the initial refined activity (see Section 4.2.3). However, this preference over intuitive halving strategy was not perceived as a hindrance to the exploration of decimating.

## 5.3 Teaching Experiment 2

This section will explicate findings during the cycle 2 teaching experiment from observation of group discussions, written responses to the activities, and the whole class discussions. Section 5.3.1 starts with the administration of the teaching experiments and discusses some changes that occurred in the teaching experiment of cycle 2. Section 5.3.2 presents findings from the classroom observations and the worksheets of the activities during the teaching experiment. Meanwhile Section 5.3.3 discusses the findings from the written tests along and the interviews on various areas of content knowledge. In Section 5.3.4, findings from the written tests and interviews on pedagogical content knowledge will be elaborated.

### 5.3.1 Administration of Teaching Experiment 2

Two lecturers were initially involved in the teaching experiment of cycle 2. It should be noted that there was a change of lecturers from cycle 1 to cycle 2. This was not an ideal situation but it was beyond the researcher's control to decide. One lecturer, who participated in cycle 1, was again in charge of the class of the secondary cohort participated in this project. Yet, due to his busy schedule, he was unable to allocate extra time to prepare and discuss the teaching experiment. Consequently, the researcher took over his class for 5 meetings to conduct the activities. The former lecturer provided his insights before the teaching experiment in this cycle started based on cycle 1 experience. The inability of the normal lecturer to take the secondary class presented the researcher with a dilemma. The researcher appreciated that her role would shift from mainly lesson observation to include ensuring pre-service teachers' engagement in activities, and leading class discussion. Consequently, the researcher's role in observing group discussions during was limited by this shift of role. On the other hand, the researcher was able to ensure that the activities and discussion had the intended focus. Despite the fact that the class had been quite supportive during the research process on a voluntary basis, the researcher's lack of authority over the class impacted on the level of commitment level of students to this project.

In the primary cohort, one lecturer, who was not involved last year, was in charge of all three parallel classes this year and wished them all to be involved in the project. Similar to other lecturers who participated in cycle 1, this lecturer has been involved in the PMRI project. In all three classes, the researcher maintained the observer role and directed the video-recording during the whole teaching experiment.

### 5.3.2 During the teaching experiment

This section reports on findings from the trialled of cycle 2 teaching experiments. As discussed in Section 5.2, a change was made in the initial activity of cycle 2.

#### 5.3.2.1 Outcomes from Set 1

Following the introduction of the longest piece of LAB as one in Set 1 – Activity 1 (see Appendix A4), pre-service teachers were asked to explore ways of partitioning the one piece in the context of measuring dimension of a table so that it allowed them to compare and record the measurement result accurately. It was expected that this exploration would guide pre-service teachers to observe that partitioning into ten subsequently relates to decimal notation and reasons for favouring this approach.

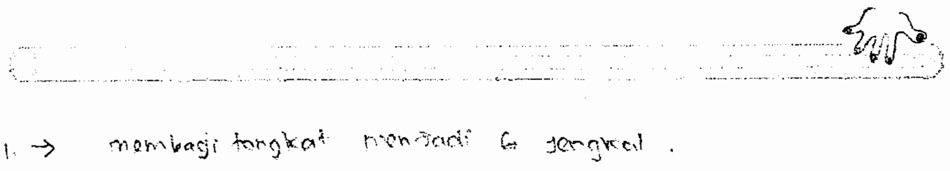
Preference for halving was predicted as this approach offered a more intuitive and practical approach. However, as pre-service teachers held prior knowledge of decimals, an idea about partitioning into ten parts was expected to emerge during the discussion besides the other approaches.

Confirming the prediction in the conjectured LIT, worksheets documenting response to Set 1 Activity 1 (see Appendix C4) recorded that most primary cohort groups (11 out of 23 groups) opted for some sort of halving strategies, varying from partitioning the one into 2 equal parts, 4 parts, 8 parts, 16 parts, or 32 parts by successive halving. Most groups articulated the practical ease of finding a half of the length as the strength of this approach, for instance, by utilizing a pivot point as illustrated in Figure 5.5. Meanwhile lack of accuracy or higher error rate, and having to divide many times in halving were noted as weaknesses of this approach. These in fact the disadvantages of halving compared to decimating strategies.

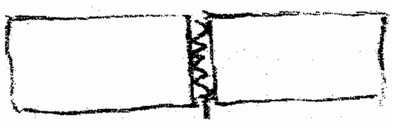
**Figure 5.5: Pivoting the LAB pieces to divide into 2 halves**

In exploring ways of partitioning the one, a majority of groups in the secondary cohort tended to engage more on 'how to' partition using non-standardized measurement tools such as chalks, hand span, and length of a pen as illustrated in Figure 5.6. Only 2 out of the 10 secondary groups opted for the halving strategy in partitioning the longest LAB piece or the 'one', half of the groups in the secondary groups partitioned one using hand span (see Appendix C4 for distribution of ways of partitioning one in Set 1- Activity 1 in cycle 2). Meanwhile 70% of the secondary pre-service teachers groups employed a non-standard shorter measurement tools such as hand span, chalk, tiles, or pen's length as illustrated in Figure 5.6.

Figure 5.6: Illustrations of intuitive approaches in partitioning one




*Using hand span in partitioning one into 6 parts*



*Using floor tiles in partitioning one*

Utilizing hand span was not only popular in the secondary cohort but also in the primary cohort as 3 secondary cohort groups and 4 primary cohort groups employed this strategy in dividing the one into 6 parts. This strategy emerged because the longest LAB piece was approximately the length of 6 hand spans. The nature and length of the LAB employed in this study, i.e., made of aluminium steel pipe 110 cm long, might promote this strategy. Practicality, time-efficiency, and the fact that hand span was readily available were noted as the strengths of this approach. On the other hand, inaccuracy and difficulty in doing further refinements were perceived as the limitation of this approach.

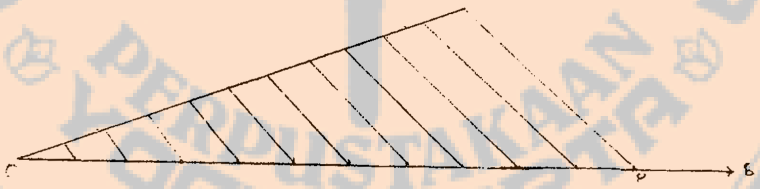
Meanwhile 7 groups utilized partitioning into ten ('*decimating*') comprising 5 groups from the primary cohort groups and 2 groups from the secondary cohort. The reasons for choosing decimating were based on familiarity with base ten structures observed in whole numbers and relation to decimal notation. Justification for 'decimating' approach relied on the ease of calculations and the connections to decimal numbers. In contrast, technical difficulty in division process was seen as a weakness of this approach.

Responses to Set 1 Activity 2 about further refinement of the longest LAB piece in the context of measuring a dimension of shorter length such as measuring an eraser showed that 6 of the 7 groups that opted for decimating in the first refinement consistently applied decimating in their next refinement. The remaining one group suggested the use of metric ruler for the second refinement which indicated an association of metric measures as a standard measurement tool for length. This showed that pre-service teachers utilized their prior knowledge of metric measures in responding to the activity. However, as the length of the longest LAB piece is deliberately not 1 metre, a further refinement using a metric ruler did not fit with the original length of the one piece. In contrast only 3 groups out of the 12 groups that applied halving utilized the same strategy in the next refinement in Set 1 Activity 2. Moreover, one group solely employed partitioning of one into 4 parts and consistently partitioned into 4 in the second refinement. The inconsistency in the next refinement was evident in all groups who employed an intuitive approach in partitioning.

Explanations of ways to divide one into equal parts reflected different traits of the primary and the secondary cohorts in terms of their mathematical content knowledge. Two groups from the secondary cohort that opted for decimating approach referred to geometry property of parallel lines to partition one as illustrated in Figure 5.7. This reflected a higher level of formal mathematical knowledge of the secondary cohort groups. Meanwhile one of the primary groups who opted for decimating used halving by dividing each half into 5 parts using trial and error (by folding a scotch tape with same length as the LAB pipe one). Justifications for 'decimating' from the primary cohort groups showed links to prior knowledge such as extending the power of ten patterns in whole number to decimals, link to decimal notation and ease of calculations.

Overall responses in Set 1- Activity 1, 2 worksheets suggested that activity of partitioning one and refining ways of partitioning with the purpose of recording measurement in higher accuracy were not strongly associated with decimal notation. The fact that exploring ways of partitioning was not clearly linked with the ease in calculating and recording the result of measurement might explain the lack of association to decimals.

**Figure 5.7: Illustration of partitioning one into 10 parts from Secondary cohort responses**



Responses to Set 1- Activity 3 about the relationships among various LAB pieces and the verbal names (one, tenths, hundredths, and thousandths) and the corresponding notation suggested that the majority of groups observed the base ten relations (one tenth, one hundredth, etc) among various pieces as predicted. Interestingly in Set 1- Activity 4 to record the measurement of a table (length and width) using various LAB pieces, 23 out of the 34 groups capitalised on fraction notation and operations and made the link to decimal notation. The trend of preference to the computational approach with fractions before decimals was also observed in cycle 1. Only 5 groups capitalised on decimal notation. This trend reflected the common approach to decimals in the Indonesian curriculum which was strongly linked to fractions operations.

In Set 1- Activity 5 (see Figure 5.8), place value understanding of decimals and additive and multiplicative structures of decimals were explored through sketching out representations of three decimals. Surveying the worksheets responses to this activity suggested that sketching out representations of decimals was quite simple but articulating the pattern from this activity in Set 1- Activity 7 was not straightforward (see Figure 5.9 for samples of responses to Activity 6). Similar to findings in cycle 1, some groups depicted association of decimal notation with fraction notation and operations in their sketches of decimal representations.

Figure 5.8: Set 2, Activity 5-7 Cycle 2 investigating place value of decimals

5. Sketch the construction of LAB representations of the given decimal numbers.

| Numbers | Sketch |
|---|---|
| 2.06 | |
| 0.26 | |
| 0.206 | |

6. Can you observe an interesting pattern from sketching the three decimals?
7. What can you conclude from sketching out those numbers?

Overall, the goal of attending to place value of decimals was achieved, however, fewer groups observed the structural relationships between 2.06 and 0.206 as compared to superficial observations as documented in Table 5.2. As can be seen, the comments varied from technical, superficial observations such as those related to sketching to structural relations of decimal place value. The fact that some groups attended to mathematical relations in various decimal place value was quite encouraging. Similarly, responses in worksheet of Set 1- Activity 7 also showed that the majority of groups attended to place value notions by concluding that representations of those decimals reflected the different values of decimal digits.

Figure 5.9: Sample of worksheet response to Set 1 - Activity 5 in cycle 2

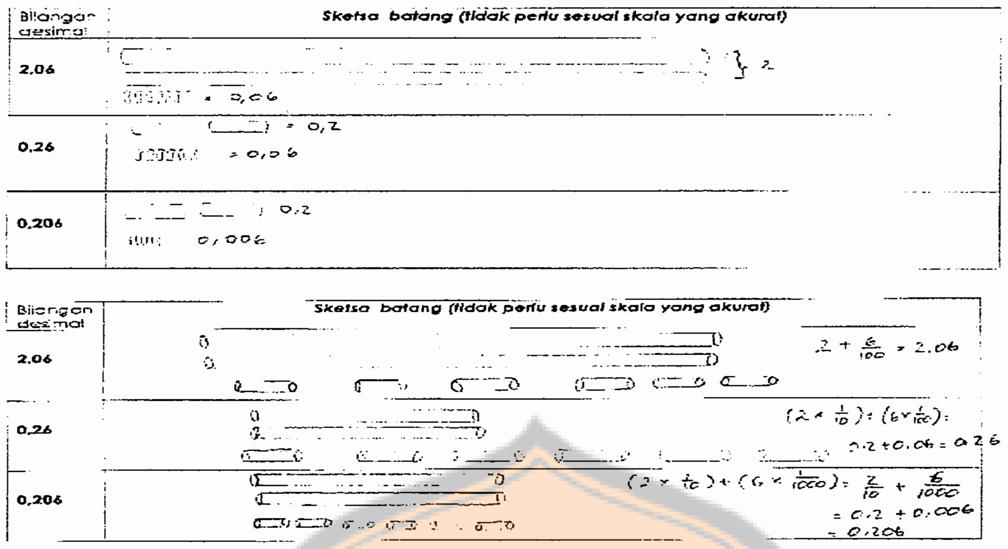



Table 5.2: Representative comments on patterns observed in Set 1- Activity 7

| | Patterns observed | Cohort |
|---|---|---|
| Superficial | It is easier to explain sketch of 2.06 because it is simpler | Primary B |
| | The longer the decimal digits, the more complicated the construction process | Primary B |
| | In constructions, they all use 2 different pipes, there are 2 longer pipes and 6 shorter pipes | Primary C |
| Structural relations | All decimals contain digits 2, and 6 but their places and values are different | Primary A |
| | Same digit represents different place value reflected in length of pipes representing digits | Primary C |
| | All three numbers have same digits 2 & 6 but different decomposition. 2.06 = 2 ones and 6 hundredths, 0.26 has 2 tenths and 6 hundredths, whereas 0.206 has 2 tenths and 6 thousandths | Secondary |
| | Different position of decimal digits implies different values and 2.06 = $10 \times 0.206$ or $0.206 = \frac{1}{10} \times 2.06$. | Secondary |

In Set 1- Activity 8 (see Appendix A4) further reflections of the meaning of decimals such as 1.23456 was probed by linking it with the activity of constructing the number using LAB pieces. Worksheet of this activity showed that the majority of groups attended to place value either by decomposing decimals into various place value terms or by identifying the value of each decimal digit. Future teaching ideas probed in Set 1- Activity 9 will be discussed later, together with teaching ideas from Set 3 – Activity 21 in Section 5.3.2.4. The decimal number 1.23456 cannot be physically represented by LAB model, which represent nothing smaller than a thousandth. Stacey, Helme, Archer et al (2001) noted that it is relatively easy for students to extend the physical model to a mental model.

#### 5.3.2.2 Outcomes from Set 2

Set 2 activities in cycle 2 (see Appendix A4), started with playing the 'Number Between' game with the whole class wherein pre-service teachers were asked to find a number in between pair of numbers. Note that this activity was not carried out in the first cycle due to the time constraint. Given a pair of whole numbers at the start, a number in between the pair was sought consecutively until the class perceived the density of decimals. Observations of whole class discussions during the 'Number Between' game showed that pre-service teachers engaged in the activity and noticed the density property of decimals through this game. Worksheet responses to Set 2 Activity 10 (see Appendix A4) recorded that 25 out of the 34 groups articulated density of decimals, (i.e., that in between two decimals there are infinitely many decimals as their learning point. Moreover, 2 groups among these 25 groups contrasted density of decimals with whole numbers, by noting that there are no whole numbers in between two consecutive whole numbers. However, worksheet of this activity also recorded two groups which indicated S thinking. This was evident as they noted "the longer the number of digits behind the comma, the value of the number is getting smaller" as the property of decimals instead. Nonetheless the fact that a significant proportion of groups (74%) indicated a positive impact of addressing density of decimals in the teaching experiment.

Teaching ideas proposed in Set 2- Activity 11 was dominated by the inclusion of number line and ruler to find a decimal number in between two decimals. Three main strategies were observed in worksheet responses to Activity 12 to find and locate decimals in between given pair of decimals (see Figure 5.10).

Figure 5.10: Set 2 - Activity 12 of Cycle 2

12. For each given pair of decimals in Table A, find decimals in between those pairs if they exist. Explain how do you find the answer and locate your answer on the number line.

| | |
|---|---|
| 1.5 | 1.51 |
| 0.99 | 0.999 |
| 1.7501 | 1.75011 |

Table A

Is there any model or game that you can use to assist children to solve this problem?

13. What can you conclude from Activity 12 above?
14. Can you find a decimal number larger than 0.36666001? If yes, how many decimals can you find? Give some examples!

The most popular strategy (recorded by 17 groups) was to segment the interval between pair of decimals into 10. The second common approach (recorded by 6 groups) was to find a mid-point of the pair of decimals. Interestingly, 2 primary cohort groups converted decimals into fractions with common denominators to determine decimal fractions within this range. However, by counting on the thousandths only in between $\frac{990}{1000}$ and $\frac{999}{1000}$, these two groups noted there were 8 or 9 decimals in between 0.99 and 0.999. Yet, at the same time by counting on ten thousandths in between $\frac{9900}{10000}$ and $\frac{9990}{10000}$, they noted there were 89 decimals in between the same pair of decimals (see Figure 5.11). This approach worked well in finding decimals in between pair of decimals, however, it led to an idea that there are only finitely many decimals in between two given decimals which was in conflict with developing understanding of density of decimals. Number line, ruler and LAB were the three main models proposed to assist children in finding decimals in between in Set 1- Activity 12. The majority of groups observed the density of decimals and noted that there were infinitely many decimals larger than 0.36666001 recorded in the worksheet of Set 2- Activity 13 and Activity 14. However, there are also infinitely many natural numbers larger than 0.36666001, so this particular activity did not get at density of decimals.

Figure 5.11: Responses to Set 2 Activity 12 of Cycle 2 from one primary cohort group

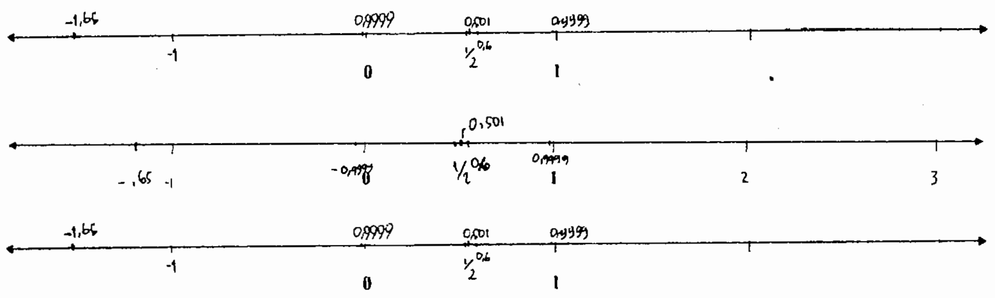
Responses to Set 2 – Activity 15 to explore the relative magnitude of decimals, including negative decimals, common fractions, and whole numbers and their positions on a number line showed that many groups had difficulties with negative decimals and finite decimals with long repeating digits such as 0.3333333333. Nearly 52% of 33 groups from both cohorts marked $\frac{1}{3}$ and 0.3333333333 at the same location on the number line and 2 groups in the primary cohorts noted that $\frac{1}{3}$ = 0.3333333333. Whilst marking $\frac{1}{3}$ and 0.3333333333 was understandable considering the scale of the numbers, the fact that two groups noted that $\frac{1}{3}$ = 0.3333333333 indicated they used rounding or truncating strategies. This strategy was confirmed during the whole class discussion in one of the primary cohort class as recorded in the following excerpt:

| | | |
|---|---|---|
| Lecturer | : | A few groups answered that 1/3 is the same as 0.3333333333 last time. Do you think that they are the same? |
| Rita | : | They are the same |
| Aris | : | Not the same, they are close, 1/3 is the same as 0.3333 but with 3 repeating forever, whereas 0.3333333333 the digit stop. |
| Lecturer | : | How about the rest of the class? Do you think Aris is right? According to a few other groups in the worksheet, 1/3 is the same as 0.3333333333 but from this discussion they are not the same. How about the group who says they are the same, could you explain how do you arrive to this conclusion? |
| Rita | : | We are thinking in simple way, if 1/3 is converted to decimals then it is the same as 0.333 so we thought they are the same because of rounding. |
| Lecturer | : | Do you still think that they are the same now? |
| Rita | : | Not really. |

Meanwhile, nearly one third of the total cohort (10 out of 33 groups completing worksheet in Set 2 -Activity 15) placed negative decimals incorrectly on the number line as illustrated in Figure 5.12. This problem with negative decimals was particularly prominent in the primary cohort as 9 of the 10 groups were from the primary cohort.

Analysis of written test responses which will be discussed in Section 5.3.2.6 confirmed difficulties with negative decimals. The fact that the design of activities in cycle 2 did not address negative decimals in particular was an important omission. This finding highlighted the importance of attending to negative decimals in the future teaching. A paper further analysing pre-service teachers' behaviours in placing negative decimals is in preparation (Widjaja, Stacey, & Steinle, in preparation).

Figure 5.12: Evidence of difficulties in locating negative decimals on the number line



#### 5.3.2.3 Outcomes from Set 3

Set 3 activities were trialled in cycle 1 (see Set 2 activities in Section 4.5). These activities were refined and trialled again with different set of pre-service teachers in cycle 2 (see Appendix A4 for the refined LIT from cycle 1 to cycle 2). Set 3 started with Activity 16 to sketch out various decompositions of decimal representations using LAB pieces. More than 56% of 32 groups sketched out LAB representations without grouping of the pieces in the sketch, only 3 groups employed grouping by 10 and 4 groups utilized grouping by 5. There were 5 groups that showed a combination of both grouping by 5 and 10 in their sketches. This finding was similar to the finding in cycle 1 (see Section 4.3), which also documented lack of attention in grouping by 10.

Despite lack of awareness of the decimal structure in grouping representations of decompositions, it was satisfying that the average facility of decomposing decimals from the worksheet was quite high, i.e., 94.5%. Decomposing decimals smaller than 1, in this case 0.123 was found to be slightly more difficult, as shown by the lower facility, i.e., 91% compared to 98% for decomposing 1.230. Steinle and Stacey (2003a) also noted students have more difficulty with decimals between 0 and 1 than others. The differences might be due to the fact that thousandths could be more difficult than hundredths. Moreover, 2 groups provided 10 ways of decompositions and 1 group gave 9 ways of decompositions suggesting some understanding or application of 10-structure.

In Set 3- Activity 17, various ways of expanding a decimal number 0.213 revealed one problematic item to express the decimal number 0.213 in hundredths. There were 3 groups left this item blank in the worksheets and 1 group answered this as 21 hundredths. Similar to finding in cycle 1, those groups who had difficulties in this item perceived that the answers should always be in whole numbers. In Set 3- Activity 18, the majority of groups pointed out that the number expander could not be used in checking one of a non-canonical expansion of decimals (i.e., 0.213 = … ones + 1 tenth + … hundredths + … thousandths).

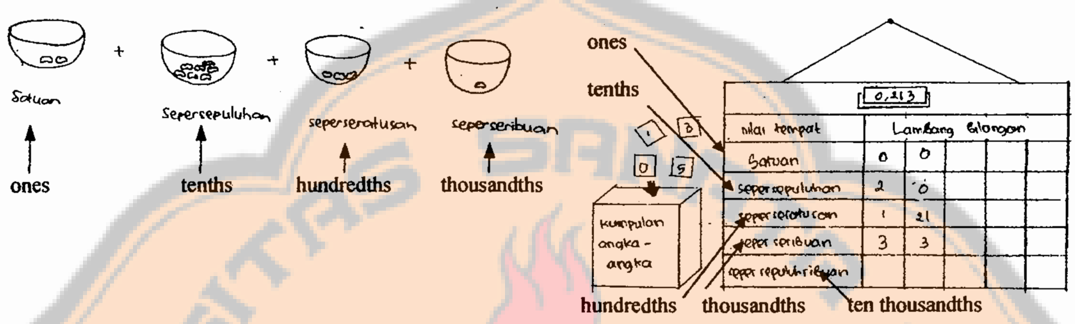
In Set 3- Activity 19, pre-service teachers were asked to observe any patterns in expanding the same decimal number (see Appendix A4). Worksheets documented that 11 groups mentioned various ways of expanding the same decimals into related place value without changing the value of the decimals, e.g., 0.213 = 21 hundredths + 3 thousandths and 0.213 = 2 tenths + 13 thousandths, as their observed pattern in expanding the same decimals. Set 3- Activity 20, which aimed at linking the LAB and various expansions of decimals using the number expander was quite straightforward. All groups noted they could use LAB to illustrate expansions of decimals as found using the number expander. The more interesting finding was documented in respond to Set 3- Activity 20 on future teaching ideas after Set 3 activities, which will be discussed in the following section.

#### 5.3.2.4 Reflections on new learning experience and ideas for future teaching

Similar to the findings in cycle 1, incorporating concrete models in teaching decimals and emphasising place value of decimals were the main features recorded in ideas for future teaching on worksheet of Set 1- Activity 9 in cycle 2. Reference to LAB or other materials (e.g., ropes, straws, or bamboo sticks) using a similar principle to LAB to represent ones, tenths, hundredths and thousandths in decimal numbers was recorded by 12 groups. Interestingly, 6 groups in cycle 2 proposed the fair sharing as a context to introduce decimals through fractions. In these teaching ideas, sharing a cake among 10 people was chosen to introduce the fraction 1/10 and then linked it with a decimal 0.1 by the division algorithm. This approach of teaching reflected the curriculum sequence in teaching decimals after fractions in Indonesian curriculum.

The worksheet for Set 3- Activity 20 showed that the primary cohort groups were more receptive to the use of concrete models in teaching. For instance, reference to LAB or models using a similar principle to LAB made of different materials such as straws, bamboo sticks, ropes, and number expander were proposed by 12 groups. In contrast, teaching ideas from the secondary cohort encompassed more symbolic models such as number lines, number expander, and place value charts or column as illustrated in Figure 5.13. Interestingly the place value column chart dominated the model proposed for teaching decimals in this cohort with only one group included the use of LAB in the teaching idea. This model was not introduced in the classroom during the teaching experiment. It was proposed by one of the group during whole class discussion. The different nature of the training, wherein pre-service secondary teachers are prepared mostly for teaching secondary school explains this preference.

Figure 5.13: Teaching idea incorporating place value column chart/matrix




## 5.4 Findings from written test and interviews

This section will discuss findings from the written tests and the interviews to observe the impact of the activities on pre-service teachers' knowledge of decimals. The discussion in this section is divided into two sections; Section 5.4.1 addresses findings related to content knowledge whereas Section 5.4.2 attends to pedagogical content knowledge of pre-service teachers.

### 5.4.1 Findings on Content Knowledge

Discussion of findings from pre and post-tests as well as pre and post-interviews will focus on data gathered from 140 pre-service teachers who sat both tests in order to gain a better indication of the impact of the activities. Pre-service teachers' content knowledge on various areas of decimals was evaluated by examining performance in DCT3a and DCT3b and Part B of the tests as well as responses in the pre- and post course interviews.

#### 5.4.1.1 Decimal Comparison Test

Both cohorts showed improvement in their performance on DCT3a as shown by the combined percentage of pre-service teachers who made no errors in DCT3a increasing significantly from 45.0% to 65.7% in DCT3b as shown in Table 5.3. The secondary cohort outperformed the primary cohort in both the pre- and post-tests.

Table 5.3: Number and percentage of pre-service teachers with no errors in two cohorts

| Cohorts | Total number of PSTs | Pre-test | | Post-test | |
|---|---|---|---|---|---|
| | | Number of PSTs with no errors | Percentage of PSTs with no errors /prim | Number of PSTs with no errors | Percentage of PSTs with no errors/sec |
| Primary | 94 | 31 | 33.0% | 49 | 52.1% |
| Secondary | 46 | 32 | 69.6% | 43 | 93.5% |
| TOTAL | 140 | 63 | 45.0% | 92 | 65.7% |

Improvement was also evident in the increase of proportion of pre-service teachers who were identified as Apparent experts (A1) from a total of 55% in the pre-test to almost 79% in the post-test (see Table 5.3). Only 2 primary pre-service teachers were identified as holding any '*Shorter-is-Larger*' thinking, which was associated with denominator focused thinking (S1). One of them retained S1 thinking in the post-test, whereas another pre-service teacher moved to A1 thinking. No pre-service teacher was diagnosed with '*Longer-is-Larger*' thinking in cycle 2.

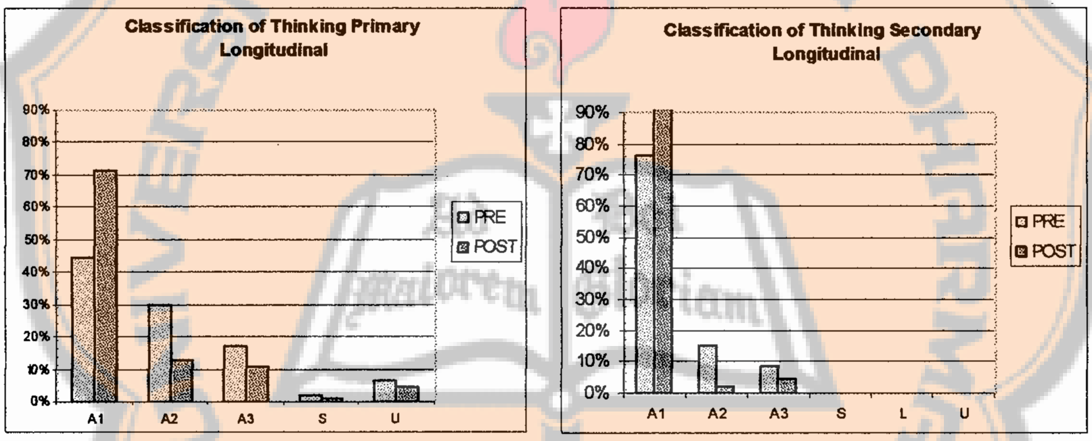
Interestingly all secondary pre-service teachers who participated in both tests were identified holding some sort of A-thinking (A1, A2 or A3 thinking). A large proportion of pre-service teachers (76.1%) held A1 thinking in the pre-test and increased to 93.5% in the post-test. However, as will be discussed later in this section, data from Part B of the written test showed that 32% of the 77 A1 pre-service teachers demonstrated weak knowledge in content areas of decimals in the pre-test. As noted earlier, the classification A indicates expertise only on the comparison task. Only 11% of 110 A1 pre-service teaches showed weak content knowledge in the post-test which indicated a positive outcome of the teaching experiment. The substantial percentage of A1 pre-service teachers showing weak content areas indicated that they might master 'expert' rules for comparing decimals without much understanding.

Table 5.4: Number and percentage of pre-service teachers in various thinking classification

| Thinking Classification | Pre-test | | Post-test | |
|---|---|---|---|---|
| | Number of PSTs | Percentage of PSTs | Number of PSTs | Percentage of PSTs |
| A1 | 77 | 55.0% | 110 | 78.7% |
| A2 | 35 | 25.0% | 13 | 9.3% |
| A3 | 20 | 14.3% | 12 | 8.6% |
| S1 | 2 | 1.4% | 1 | 0.7% |
| U1 | 3 | 2.1% | 4 | 2.9% |
| U2 | 3 | 2.1% | 0 | 0.0% |
| Total | 140 | 100% | 140 | 100% |

Both cohorts demonstrated misconceptions associated with inappropriate use of rounding or truncating rules. These misconceptions appeared in both the primary and the secondary cohorts and were identified as A2 thinking (see Figure 5.14). In contrast, a few pre-service teachers from the primary cohort held S thinking.

Figure 5.14: Classifications of thinking from two cohorts comparing pre- and post-tests



Interestingly, the interview data showed that applying a blend of strategies was a common approach used by many pre-service teachers in comparing decimals on DCT3a. Mixed strategies were common in pre-service teachers' explanations during the interviews. The following interview excerpt with Vita, a pre-service secondary teacher, was an example of this. She was diagnosed as holding A2 thinking in the pre-test. In general, she annexed zeros in dealing with comparisons such as 4.8 with 4.62 which gave a correct answer (4.80 versus 4.62). However, Vita applied a rounding rule for comparing type 4 items (such as in comparing 4.4502 with 4.45 so that 4.4502 became 4.45) and used truncating rule in dealing with type 4R items (such as in comparing 3.77777 with 3.7 so that 3.77777 became 3.7) in the pre-test. In the post-test Vita showed improvement but still indicated reliance on rounding rules in finding the closest decimals on item 10 and 11 of Part B of the written tests. As noted by Steinle and Stacey (2002), the repeating digits are a distinct cause of difficulty.

| | | |
|---|---|---|
| Researcher | : | Can you explain why do you think decimals with same initial digits are the same? |
| Vita | : | I was actually a bit confused last time. Because I thought 4.666 could be rounded to 4.7 so can 4.66 so they can be the same number. Also 3.77777 is a repetition (sic) of 3.7. |
| Researcher | : | But did you think that 4.4502 and 4.45 can be equal? |
| Vita | : | Yes, because according to the rounding rule, 02 can be ignored |
| Researcher | : | How about your strategy in comparing other pairs of decimals? |
| Vita | : | For example in comparing 4.8 and 4.62, by looking at 8 and 6 we know that 4.8 is larger than 4.62. In comparing 0.8 and 0.74, just compare - 0.80 and 0.74 and I know that 80 is larger than 74 so 0.80 is larger than 0.74. |
| Researcher | : | Do you always apply this strategy consistently? |
| Vita | : | No because in comparing 4.66 and 4.666 if we use this strategy, we can conclude that 4.666 is larger but in the case of 4.666 and 4.66, they can be thought as the same number because of the repeating digits. |

Similarly, Yanti, a primary pre-service teacher, applied truncating thinking in solving type 4 and 4R items despite her strong tendency of 'Shorter is larger' thinking otherwise as recorded in the following pre-course interview excerpt:

| | | |
|---|---|---|
| Researcher | : | So what is your strategy in comparing decimals (4.8 versus 4.63)? |
| Yanti | : | Just look at the number behind comma, here 4.8 is the same as 4 and $\frac{8}{10}$ whereas 4.63 is the same as 4 and $\frac{63}{100}$ |
| Researcher | : | How about in comparing 3.7 and 3.77777, do you use the same strategy or a different one? |
| Yanti | : | If there are more than 3 digits behind comma, we just pay attention to 2 digits or less. Since 3.7 has only one digit behind comma, so just compare the first digit of both numbers. |
| Researcher | : | So how about this one, how did you come to conclusion that 7.63 is larger than 7.942? |
| Yanti | : | Because it depends on the comma (meaning decimal digits), this one is $\frac{63}{100}$ and the other one is $\frac{942}{1000}$ so the larger one is hundredths. |
| Researcher | : | So what is your assumption? |
| Yanti | : | Tenths are larger than hundredths and hundredths are larger than thousandths |

This tendency to apply mixed rules in comparing decimals was also observed in cycle 1 data as reported in Section 4.5. The fact that reliance on rules and the tendency to apply mixed rules without understanding was observed in both cycles suggests that these are common features of pre-service teachers' knowledge.

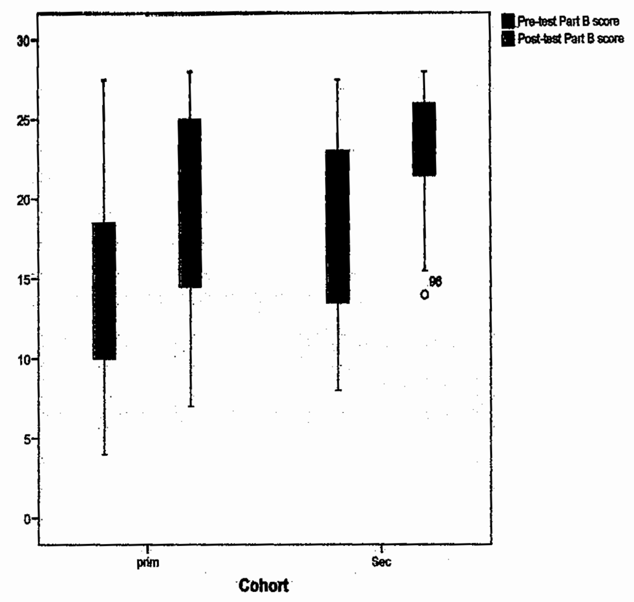
#### 5.4.1.2 Results of Part B of the written tests

Both cohorts showed improvement in their content knowledge as indicated in their improved mean scores shown in Table 5.5. These results suggested that both cohorts gained advantage from the activities. Similar to finding in cycle 1, the secondary cohort showed a stronger performance than the primary counterpart in both pre- and post-tests. Note that the total score of 28 in cycle 2 as opposed to total score of 27 in cycle 1 (see Table 4.4) was due to the additional item in examining place value names of a decimal digit (Item 1d in Part B, see Appendix B5 and B6).

Table 5.5: Mean pre- and post-tests from Part B in cycle 2 (out of the total score of 28)

| Cohort | N | Df | Pre-test | | Pos-test | | t value | p value |
|---|---|---|---|---|---|---|---|---|
| | | | Mean | Standard dev (SD) | Mean | Standard dev (SD) | | |
| Primary | 94 | 93 | 14.43 | 5.38 | 19.45 | 6.18 | 5.02 | 0.000 |
| Secondary | 46 | 45 | 18.60 | 4.99 | 23.47 | 3.73 | 4.87 | 0.000 |

Figure 5.15: Box plots of pre-test and post-test score of Part B




As recorded in Table 5.6 and Table 5.7, pre-service teachers from both cohorts in cycle 2 improved significantly in areas such as explicit identification of place value names (row 1), decomposing decimals (row 2), and density of decimals (row 3). The fact that improvement was observed in density of decimals in cycle 2 showed the positive impact of attending to density of decimals in the activities of cycle 2. As reported in Section 4.5, there was no significant improvement on these areas in cycle 1 when these areas were left out due to lack of time.

There were two content areas i.e., unitising decimals (row 3) and sequencing decimals (row 5) in which both cohorts indicated no significant improvement in cycle 2. The difference between the two cohorts in pre-service teachers' knowledge on relative magnitude of decimals as reflected in locating decimals on the number line and finding the closest decimal to a given decimal. Similar to the finding in cycle 1, the primary cohort showed difficulties with placing negative decimals on the number line and no significant improvement was observed on this area (row 6). In contrast, the secondary cohort with initial score of 3.34 out of 4 indicated significant improvement on this area. Further discussions on the nature of difficulties will be presented in the next two paragraphs.

Table 5.6: Mean pre- and post-test Part B of the primary cohort in cycle 2 (N=94, paired t-tests)

| Area | *df* | Pre-test | | Pos-test | | | |
|---|---|---|---|---|---|---|---|
| | | Mean | SD | Mean | SD | t value | p value |
| Identifying place value names | 94 | 2.70 | 1.115 | 3.54 | 1.114 | 6.239 | 0.000 |
| Decomposition of decimals | 94 | 1.06 | 0.841 | 3.02 | 1.270 | 13.962 | 0.000 |
| Unitising of decimals | 94 | 2.34 | 1.915 | 2.32 | 1.908 | 0.100 | 0.921 |
| Density of decimals | 94 | 1.62 | 1.907 | 2.57 | 1.775 | 4.359 | 0.000 |
| Sequencing of decimals | 94 | 2.53 | 1.284 | 2.55 | 1.449 | 0.127 | 0.899 |
| Decimals on the number line | 94 | 2.83 | 0.922 | 2.95 | 0.922 | 1.284 | 0.202 |
| Closeness to a decimal | 94 | 1.34 | 1.388 | 2.49 | 1.515 | 6.203 | 0.000 |

Table 5.7: Mean pre- and post-test Part B of the secondary cohort in cycle 2 (N=46, paired t-tests)

| Area | *df* | Pre-test | | Pos-test | | | |
|---|---|---|---|---|---|---|---|
| | | Mean | SD | Mean | SD | t value | p value |
| Identifying place value names | 45 | 2.83 | 0.996 | 3.76 | 0.639 | 5.853 | 0.000 |
| Decomposition of decimals | 45 | 1.61 | 1.374 | 3.89 | 0.482 | 10.867 | 0.000 |
| Unitising of decimals | 45 | 2.52 | 1.761 | 3.00 | 1.563 | 1.856 | 0.070 |
| Density of decimals | 45 | 3.17 | 1.495 | 3.65 | 0.971 | 2.119 | 0.040 |
| Sequencing of decimals | 45 | 2.70 | 1.280 | 2.91 | 1.092 | 1.044 | 0.302 |
| Decimals on the number line | 45 | 3.34 | 0.708 | 3.60 | 0.620 | 2.890 | 0.006 |
| Closeness to a decimal | 45 | 2.43 | 1.515 | 2.65 | 1.402 | 0.927 | 0.359 |

The pattern of erroneous responses in identifying place value name of a decimal digit detected in cycle 1 (see discussion in Section 4.5) was confirmed with an additional item, i.e., Item 1d in Part B in cycle 2. The addition of item 1d in Part B of cycle 2 meant that this error pattern was better diagnosed and this added more confidence in the results. In the pre-test of cycle 2, both cohorts recorded the existence of this error pattern, where place value names of a decimal digit were determined by the length of the decimal digits. It was recorded in 72 cases in the pre-test and 9 cases in the post-test of cycle 2. Interview data offered a confirmation on the prediction about associating place value of a decimal digit based on the length of the decimal digits as recorded in the following interview excerpt:

Researcher : Can you explain how did you find the place value of each decimal digit in Item 1?

Vita : I made a mistake when I solved this, I think because 9.31 is the same as 931/100 so I just choose hundredths. When there are three decimal digits, I just choose thousandths etc. I just consider the length of the decimal digits.

Similar to the finding in cycle 1, both cohorts showed the highest improvement on decomposing decimals (row 2 of Table 5.6 and Table 5.7). The pre-test responses on this item showed that a majority of pre-service teachers from both cohorts only offered one way of decomposing decimals. Weak knowledge of place value in decomposing 0.375 as 5 ones + 7 tenths + 3 hundredths + 0 thousandths ("reverse" thinking), found in cycle 1, was again observed in the pre-test of cycle 2. Lack of familiarity about multiple ways of decomposing decimals was one factor that explained the low mean scores in the pre-test of both cohorts.

An additional item (Item 3a, 3b, Appendix B5 and Appendix B6) on unitising decimals was included in the written test of cycle 2 to inspect the relations between knowledge of decomposing decimals and unitising decimals. The results showed that many pre-service teachers were unable to link decomposing and unitising decimals. Data from written tests showed that 15% of pre-service teachers could offer 3-4 ways of decomposing a decimal but were unable to unitise correctly in the post-test of cycle 2. Even though the activity with the number expander model (see Figure 4.7 in Chapter 4) tried to link various ways of decomposing decimals and the standard decimal notation, a larger portion of the activities attended to decomposing decimals. This finding indicated the fragmented knowledge of pre-service teachers where their improved knowledge in decomposing decimals was not linked to knowledge of unitising decimals. This underscored the need to guide pre-service teachers to see the link between the two activities better.

The following interview excerpt with Rian revealed her fragmented knowledge of place value wherein the place value names of a decimal digit was associated with the whole number place value names of the numerator of the corresponding fraction.

| | | |
|---|---|---|
| Rian | : | I'm confused with this problem of finding digits of different place value terms for 0.375. |
| Researcher | : | So could you tell me why in the pre-test here you put 5 first? |
| Rian | : | At that time, it seems to me that sometimes I have a mix of ideas, sometime. I think that the first digit represents ones but occasionally I think that the last digit represents ones. |
| Researcher | : | So reflecting back now, how did you analyse your own thinking? |
| Rian | : | Why did I do it that way? Ehm ...because if I write it as $\frac{375}{1000}$, I think 5 here represents 5 ones, 7 here represents tens and 3 represents hundreds. |

Scant place value knowledge was also reflected in a common error of unitising decimals, e.g., noting that 2 ones + 6 tenths + 15 hundredths + 3 thousandths = 2.6153.

Another evidence of lack of decimal place value was captured when the order of digits were reversed. In this case, 0 ones + 7 tenths + 1 hundredths + 12 thousandths was written as 1.2170. During the pre-course interviews, some pre-service teachers who answered this way explained that they reversed the digits, which signified their lack of understanding of place value.

In contrast to findings in cycle 1, both cohorts recorded significant improvement on density of decimals in cycle 2, which indicated the positive impact of addressing this topic in the activities in this cycle. Note that despite the fact that both cohorts recorded significant improvement on density, the gap between the mean scores of the two cohorts was quite wide (see Table 5.6 and Table 5.7). The primary pre-service teachers recorded a high proportion of blank responses (about 21% of blank responses) and showed difficulties with density of decimals. The incorrect responses on density of decimals could be classified into three different categories. The first category indicated association of decimal digits with whole numbers which resulted in identifying no decimal in between two given decimals. The second category showed knowledge on the link between decimals and fractions but this knowledge was limited by working with equivalent common fractions with the same denominators. The curriculum sequence reflected in the common mathematics primary school textbooks to introduce decimals of the same lengths might explain this approach. The third approach in the third quote showed reliance on rounding rule which was observed in finding the number of decimals in between 0.799 and 0.80. The tendency to rely on rounding rule in this case could be affected by the decimals with repeating decimal digits in 0.799. The following responses were taken from answers to Item 5 and 6 in the pre-test of cycle 2 to illustrate the three categories of incorrect idea on density of decimals:

Riri: There is no decimals in between 3.14, and 3.15 because 3.14 and 3.15 are consecutive numbers.

Agus: There is no decimals in between 3.14, and 3.15 because $3.14 = 3\frac{14}{100}$ and $3.15 = 3\frac{15}{100}$ and there is no number in between $\frac{14}{100}$ and $\frac{15}{100}$.

Igni: There is no decimal in between 0.799 and 0.80 because 0.80 is the result of rounding of 0.799.

Both cohorts in cycle 2 recorded no significant improvement and performed at about the same level on sequencing of decimals (row 5 of Table 5.6 and Table 5.7). However, better items in cycle 2 added more confidence on the results of this cycle.

Difficulties in placing decimals on a number line (particularly negative decimals) were recorded in the worksheets of activities and observed during group and whole class discussions, particularly in the primary cohort (see Figure 5.12 in Section 5.3.2). These difficulties were also documented in responses to written tests and were further confirmed in the pre-course interview data. Table 5.7 presented the distribution of responses from locating negative decimals in the pre- and post-tests from all pre-service teachers in cycle 2. As noted before the difficulties with locating negative decimals on the number line was mainly observed in the primary cohort as reflected in the low pre-test and post-test scores on this area (see Table 5.8). The column "response" indicates the actual positions of the locating mark (interpreted by the researcher).

**Table 5.8: Distribution of responses in locating negative decimals in the pre- and post-test (N=140)**

| Locating –1.2 in pre-test | | Locating –1.3 in post-test | | Locating –0.5 in pre-test | | Locating –0.35 in post-test | |
|---|---|---|---|---|---|---|---|
| Response | % | Response | % | Response | % | Response | % |
| –1.2 | 65.7 | –1.3 | 78.6 | –0.5 | 80.0 | –0.35 | 72.9 |
| –0.8 | 28.6 | –0.7 | 16.4 | –1.5 | 3.6 | 0.35 | 5.0 |
| Blank | 5.0 | –1.7 | 0.7 | 0.5 | 4.3 | –0.65 | 3.6 |
| Other | 0.7 | –0.97 | 0.7 | –0.05 | 2.9 | –0.035 | 2.9 |
| | | Blank | 2.1 | 0.05 | 1.4 | –1.35 | 2.9 |
| | | Other | 1.4 | Blank | 7.1 | –1.65 | 0.7 |
| | | | | Other | 0.7 | Blank | 5.0 |
| | | | | | | Other | 7.1 |
| Total | 100 | Total | 100 | Total | 100 | Total | 100 |

Finding a decimal closest to a given decimal (row 7 of Table 5.6 and Table 5.7) indicated reliance on rounding rules. This reference to rounding rules was recorded in responses of both cohorts. However, performance of the primary cohort was particularly alarming in the pre-test with only 35% facility. However, the primary cohort recorded significant improvement in this area as shown in the increased facility of 62%. In contrast, the secondary cohort showed no significant improvement despite their better performance. The following pre-course interview excerpt with Vita, a pre-service secondary teacher, showed a reliance on rounding and truncating thinking in solving DCT3a and DCT3b items. Strong reliance on rounding rules in the secondary cohort might explain the lack of significant improvement in finding the closest decimal.

Researcher : Can you explain how did you find closest decimal to 8.0791, why did you choose that answer?
Vita : I thought 8.0790001 was also close to 8.0791 but I chose 8.08 because I rounded it up.

In summary, the activities in cycle 2 were successful in improving pre-service teachers' knowledge of place value, decomposition of decimals, and density of decimals. These content areas were addressed in the activities of cycle 2. However, fragmented nature of pre-service teachers' knowledge was observed in their inability to link their knowledge of decomposing and unitising decimals. Both cohorts in cycle 2 recorded no significant improvement on unitising of decimals. Reliance on rules such as rounding rule was a common feature from both cohorts and was observed in pre-service teachers' explanation to find the closest decimal to a given decimal as well as density of decimals.

### 5.4.2 Findings on Pedagogical Content Knowledge

Responses of Part C in cycle 2 showed that both cohorts improved significantly on their teaching ideas in decimals. This improvement was also reflected as the proportion of pre-service teachers who were identified as having low PCK (i.e., scores 0 to 3 in Part C of the written tests) dropped from 32% in the pre-test to 11% in the post-test. Similarly, the proportion of those identified as having high PCK (i.e., scored 7 to 9 in Part C of the written tests) increased from around 14% to almost 38% in the post-test (see Table 5.9).

**Table 5.9: Distributions of pre-service teachers in various categories in Part C of cycle 1 (N=140)**

| Pre-test (%) | Post-test (%) the total cohort | | | Total |
|---|---|---|---|---|
| | Low | Medium | High | |
| Low | 7.9 | 15 | 9.3 | 32.1 |
| Medium | 2.9 | 31.4 | 19.3 | 53.6 |
| High | 0.7 | 4.3 | 9.3 | 14.3 |
| Total | 11.4 | 50.7 | 37.9 | 100 |

Both cohorts improved significantly from the pre-test to the post-test on the overall score of Part C as shown in Table 5.10 (using paired t-tests). Despite the significant improvement, lack of satisfactory explanations in their teaching was observed in both cohorts and this was reflected in the low mean scores. Interviews with 20 pre-service teachers before the start of teaching experiments confirmed their inadequate experience in incorporating models in learning and teaching both in prior schooling and in their current training.

Table 5.10: Mean pre- and post-test of Part C scores (total score ranges from 0 to 9)

| Cohort | N | df | Pre-test | | Pos-test | | *t*-test | *p* value |
|---|---|---|---|---|---|---|---|---|
| | | | Mean | Standard dev | Mean | Standard dev | | |
| Primary | 94 | 93 | 3.72 | 2.076 | 5.50 | 1.955 | 7.339 | 0.000 |
| Secondary | 46 | 45 | 5.48 | 1.560 | 6.41 | 1.784 | 3.441 | 0.001 |

Most of them recalled their learning experience of decimals as being very symbolic, relying on operation to convert from common fractions to decimals and vice versa as expressed by Vita (secondary cohort) and Ana (primary cohort) in the following script:

Vita : In fact at this point, using models in teaching decimals hasn't crossed my mind. During my training, the lecturers don't use much manipulatives or models in our learning. It is more like a general remark or suggestion that in the future we need to use models for our teaching but don't explicate in details how the models should be used. It is limited to observation, but we need also to share our observation and put our own thoughts and be familiar with models. I found it is still lacking.

Ana : The media for learning is inadequate… usually in learning decimals we either start with fractions or use division algorithm to convert between common fractions and decimals, for instance finding that $\frac{1}{2} = 0.5$ using division… That's how my teacher taught me, it was quite confusing but I supposed we get used to it. If I have to teach it to primary school children, I am still confused myself, like this 5 is 5 ones but then how come it can be written as 50?

In cycle 2, improvements were observed in all PCK areas examined in Part C (see Table 5.11). Table 5.11 reports results for items 15 to 18 of Part C (see Appendix B5 and B6) and scored as explained in Section 3.4.

Similar to the finding in cycle 1, written test responses in Part C showed the least improvement in teaching ideas to give reasoning for a commonly practiced procedure such as moving a decimal point in dividing decimals by power of ten (Item 16) and linking decimals and fractions (Item 18) although the improvement is still statistically significant for the primary cohort. In contrast, the impact of activities was better observed in teaching ideas for comparing decimals (Item 15) and diagnosing an error for comparing decimals (Item 17).

Table 5.11: Paired t-test results of various items in Part C items of cycle 2

| Cohort | No | Area | *df* | Pre-test | | Pos-test | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Mean | SD | Mean | SD | t value | p value |
| Primary | 15 | Teaching ideas for comparing pairs of decimals | 93 | 0.80 | 0.784 | 1.50 | 0.652 | 7.012 | 0.000 |
| | 16 | Teaching ideas on division of decimals by 100 | 93 | 0.68 | 0.659 | 0.95 | 0.678 | 2.965 | 0.004 |
| | 17 | Diagnosis of students' error in ordering decimals and teaching ideas to resolve it. | 93 | 1.50 | 0.992 | 2.02 | 0.904 | 3.884 | 0.000 |
| | 18 | Teaching ideas on the links between fractions and decimals | 93 | 0.74 | 0.747 | 1.03 | 0.822 | 3.017 | 0.003 |
| Secondary | 15 | Teaching ideas for comparing pairs of decimals | 45 | 0.76 | 0.673 | 0.96 | 0.868 | 1.386 | 0.173 |
| | 16 | Teaching ideas on division of decimals by 100 | 45 | 1.20 | 0.654 | 1.39 | 0.537 | 1.386 | 0.173 |
| | 17 | Diagnosis of students' error in ordering decimals and teaching ideas to resolve it. | 45 | 2.09 | 0.865 | 2.48 | 0.913 | 2.139 | 0.025 |
| | 18 | Teaching ideas on the links between fractions and decimals | 45 | 1.43 | 0.688 | 1.59 | 0.541 | 1.635 | 0.109 |

Improvement in teaching ideas for comparing decimals in item 15 was indicated by the shift of main teaching strategies proposed in the written tests. It should be noted that the low facility of item 15 in the pre-test was partly caused by 33 blank answers. In the pre-test, common teaching ideas for comparing decimals were dominated by reference to rounding rules and annexing zeros. Limited ideas to incorporate models in teaching ideas for decimals were apparent in all proposed teaching ideas. In the case of comparing decimals, a fair sharing situation of one cake with 8 people was proposed as a model for representing the decimal 0.8, when in fact it represented a fraction 1/8 instead. This inappropriate model situation was documented in 6 pre-service teachers' responses in the pre-test. Ruler and number lines were other models proposed for comparing decimals. In contrast, reference to place value notions by decomposing decimals was central in teaching ideas and this was observed in 42 post-test responses to Item 15. Moreover, the inclusion of models such as LAB, number line, and place value charts signified the positive outcome of the activities on teaching ideas in the post-test. Interview data recorded the progress of pre-service teachers' teaching ideas as illustrated in the following post-course interview responses:

> Sam : In the pre-test I just mentioned the use of number line for teaching ideas but in the post-test I also included LAB. With LAB, using ones, tenths, hundredths, thousandths and ten thousandths, we can show that for 0.7777 we need 0 ones, 7 tenths, 7 hundredths, 7 thousandths, and 7 ten thousandths to show that 0.7777 is longer than 0.770.

The pre-test responses in diagnosing errors to order decimals in Item 17 showed that only 24% pre-service teachers from the whole cohort were able to diagnose correctly and proposed teaching ideas that address basic notions of decimals such as place value. Similar to responses in Item 15, rounding rules and annexing zeros as well as comparing equivalent fractions were common in the pre-test responses to resolve an error in ordering three decimals 0.3<0.34<0.33333. Teaching ideas to decompose decimals in related place value terms and the use of LAB or models similar to LAB (made from different materials such as straws, bamboo sticks, etc) as well as the use of number line marked the improvement in teaching ideas in the post-test. Moreover, comparing equivalent fractions with the same denominators was the second common strategy proposed in teaching ideas in the post-test.

Analysing written responses to Item 16 about the procedure of moving a decimal comma in solving dividing 0.5 by 100, a majority of pre-service teachers from both cohorts agreed with this procedure because it gave correct answers quickly. 'Invert and multiply' algorithm after converting the decimals into its equivalent fraction and division algorithm were two common approaches proposed as alternatives to solve division of decimals in the pre-test. Similar to findings in cycle 1, inadequate mastery of the 'invert and multiply' algorithm was evident in some explanations, such as $0.5 \div 100 = \frac{5}{10} \times \frac{1}{100} = \frac{10}{5} \times \frac{100}{1} = \frac{1000}{5} = 200$ or $0.5 \div 100 = \frac{1}{2} \div \frac{1}{100} = \frac{1}{2} \times 100 = 50$. These incorrect interpretations of invert and multiply algorithm showed not only reliance on the algorithm without understanding but also suggested pre-service teachers' lack of number sense.

The alternative teaching ideas proposed in the post-test also showed reliance on invert and multiply algorithm after conversion to fractions. Similar to the finding in cycle 1, only a small number of (8 out of the 140) pre-service teachers capitalized on place value notion and LAB models in making sense of the division process as illustrated in the following responses in the post-test:

> Ken : We can use a wood cut into 10 parts to represent tenths and then used 3 of tenths wood to represent 0.3. To divide it by 100, we can divide it into 100 parts. From this process, students can understand that the result of 0.3÷100 will be 100 times smaller than 0.3.

> Erni : Explain the meaning of 0.3÷10 first, when 0.3 is divided into 10 parts, each represents 0.03. Then explain when 0.03÷10, there are 10 parts with each represents 0.003 because 0.3÷100 = 0.03÷10 so there are 10 parts, each represents 0.003. So 0.3÷100 = 0.003.

Teaching ideas in the pre-test Item 18 (Appendix B5) showed that the majority of pre-service teachers recommended teaching fraction operation and division algorithm to find the correct answer to the question. However, similar to responses in Item 16, both cohorts showed lack of satisfactory explanations in teaching ideas to help students in realizing the risk of rounding in the problem of 1/3 ×100 000 = 33 000. Fragmented and incomplete knowledge on algorithm involving multiplication of fraction were documented in the post-test responses, such as 1/7 × 100 000 = 7/1 × 100 000 = 70 000. Similarly weak links between fractions and decimals were evident in responses such as 1/7 = 0.7777… (an overgeneralisation from 1/3 = 0.3333…) or 1/7 = 1.7. Improvement was reflected in responses which indicated place value understanding in explaining the process involved in carrying out the division algorithm.

Interestingly, teaching ideas documented in Part C of the written tests also conveyed different traits of the two cohorts. More symbolic teaching approaches such as subtraction, conversion to equivalent common fraction with common denominators, multiplication of decimals by power of ten were recorded in the secondary cohort's teaching ideas in both the pre- and post-tests in cycle 2. Models incorporated in teaching approaches also reflected more symbolic and formal teaching approaches such as the use of the number line with some reference to LAB models. During one of the post-course interviews, one interviewee from the secondary cohort revealed that he didn't bear in mind teaching ideas in the context of primary level; even though the item clearly specified it. Instead, he addressed his explanation to the researcher whom he knew was going to evaluate his responses. The fact that secondary pre-service teachers possessed better skills with standard algorithms such as the division algorithm might explain their stronger reliance on algorithms in teaching ideas. In contrast, the primary cohort suggested concrete models such as LAB or other models such as straws, ribbons, ropes or bamboo sticks in similar principle to LAB, or place value columns made of different materials. Pre-service primary teachers' own difficulty in comprehending abstract and formal mathematical concepts along with their pedagogical awareness to primary school students' psychological development were attributed as factors leading most of the primary cohort to rely on concrete models instead of more abstract models.

Overall, the primary pre-service teachers were more receptive to new teaching ideas compare to the secondary counterparts. The fact that the primary pre-service teachers had more exposure to practical training in schools might explain the higher pedagogical awareness. In contrast, the secondary cohort had done no practical teaching in schools.

## 5.5 Retrospective analysis

Findings from implementing 3 sets of activities with different pre-service teachers and lecturers in cycle 2 revealed an overall trend of improvement in both CK and PCK from both cohorts. Key notions of decimals covered in the activities such as decomposition of decimals in expanded notation, place value, ideas for comparing decimals, and density of decimals indicated a clear improvement. However, performance in areas such as placements of decimals on the number line, sequencing of decimals, and unitising of different place value parts of decimal numbers only showed a slight improvement. Examination of areas of PCK showed significant improvements in all areas. The positive impact of the activities were reflected meaningful explanations which showed linked to basic notions such as place value and reference to the use of concrete models in teaching ideas.

Similar to findings in cycle 1, different traits of the two cohorts were also observed in cycle 2. In general, the secondary cohort outperformed the primary cohort in regard to the content knowledge. However in regard to the pedagogical ideas, the primary cohort turned out to be more receptive to the teaching ideas particularly in incorporating the use of concrete models in creating a meaningful understanding of decimals and taking into account the psychological development of primary school students in the teaching approaches.

One of the main changes in cycle 2 was to better reflect the guided reinvention principle. This was attempted in cycle 2 by exploring ways of partitioning one to record the measurement result in the initial activity of Set 1. However, responses to this change suggested that a majority of groups focussed on the practical issue of physically dividing the one pipe by using a smaller unit of measurement such as hand span, tile and pens. The refinement of partitioning, links to decimal notation, and ease of calculations were not easily perceived. Hence the aim of the initial activity to expose pre-service teachers with 'guided reinvention' experience process was not successful in this cycle. I can say that the activities in this study did not reflect the guided reinvention tenet well. In this respect, the issue of addressing the RME basic tenets in the design of activities will need further study.

Pre-service teachers' knowledge of density improved significantly, which showed a positive impact of the inclusion of this area in the activity in this cycle. At the same time, the task in locating decimals (including negative decimals), common fractions and whole numbers on the same number line revealed misconceptions of negative decimals and pointed out the important omission of addressing negative decimals in the activities. The researcher had not foreseen the extent of these difficulties in the design of the activities. This aspect needs to be improved in design of the activities for the future.

Similar to findings in cycle 1, pre-service teachers learnt to decompose decimals in multiple ways related to place value in Activity 16 (See Appendix A5). Moreover, this knowledge of decomposing decimals was translated well in teaching ideas for comparing decimals. Majority of pre-service teachers made reference to decomposing decimals in their teaching ideas for comparing decimals as recorded in the post-test and post-course interviews. The nature of learning activities which emphasized group work allowed pre-service teachers to explore multiple interpretations of decimals together and learnt flexible ways of interpreting decimals. Moreover, the format of activities encouraged flexible thinking by providing up to 8 ways to decompose a decimal number. This arrangement stimulated pre-service teachers to share and discuss their ideas together. Interview data and observation of group discussion documented pre-service teachers' improved flexibility in interpreting decimals which was something they did not have in the past.

However, this same activity (Activity 16 in Appendix A5) that seemed to be very successful showed lack of success in overcoming pre-service teacher's tendency to work in symbolic ways. For instance, many groups tended to apply arithmetic manipulations in decomposing decimals. Pre-service teachers' current knowledge was one of the reasons for their tendency to take shortcuts rather than reasoning with models that were offered in activities. Working with arithmetic operations were considered faster. This also suggested that nearly all pre-service teachers did not need to use concrete models as a reasoning tool. In teacher education context, the use of concrete models was usually linked with teaching ideas to assist children's learning. This was observed in pre-service teachers' general reflections of Set 3 activities which was probed in Activity 21 (See Appendix A5) in later stage but not in their own strategies for decomposing decimals. Fitting concrete models with pre-service teachers' patchy knowledge of knowledge in a way that put them in a situation to extend their knowledge was a challenging task.

The general overview of findings from both cycles and feed forward recommendations for future studies will be articulated in the next chapter.

# CHAPTER 6 OVERVIEW OF FINDINGS FROM THE TWO CYCLES

## 6.1 Introduction

This chapter presents the overview of the research findings from two cycles, which consists of the overview of outcomes of the teaching experiments on content knowledge (CK) and pedagogical content knowledge (PCK) of pre-service teachers from the two cycles. The discussion in this chapter will address the first two research questions of this study. The first research question looks into the impact of the designed activities on pre-service teachers' content knowledge, whereas the second research question examines the impact of the designed activities on pre-service teachers' pedagogical content knowledge. In Section 6.2, overview of improvements in various areas of content knowledge from the two cycles will be discussed. Similarly, Section 6.3 will present an overview of improvements in various areas of pedagogical content knowledge between two cycles. In Section 6.4, samples of four pre-service teachers' development in content and pedagogical content knowledge will be presented. These four cases illustrate the wide variety of pre-service teachers' knowledge and how the activities work on this non-uniform knowledge base. The answers to research question 1 will be discussed in Section 6.5 whereas the answers to research question 2 will be discussed in Section 6.6. Finally, Section 6.7 will present concluding remarks on the features of Indonesian pre-service teachers' content and pedagogical content knowledge.

## 6.2 Overview of Improvements on Content knowledge

In this section, the effects of the design experiment on content knowledge (CK) from the two cycles will be summarized and compared. Table 6.1 presents a summary of the outcomes on various content areas. For each of various content areas and for both cohorts, it records whether the activities and the items of the written tests remained the same or were refined between cycles. The table also reports whether students' achievement was judged to have improved on each content area during each cycle by comparing pre-test and post-test results. Note that in Table 6.1, improved and worse means statistically significant improvement or decline at the statistical tests reported in the relevant sections of Chapters 4 and 5. When the improvement or decline was not statistically significant, the outcome is labelled the same.

As can be observed in Table 6.1, there are more areas recording improved achievements in cycle 2 as compared to cycle 1. Overall, this pleasing result arises from two aspects of better design of activities in cycle 2. In some cases the activities used in cycle 1 were refined for cycle 2, and the outcomes were better. In other cases, cycle 1 did not address this area of content, but cycle 2 was able to do this, by better use of available time. (These instances are marked by not applicable (n/a) in the refinement column). For example, as will be discussed below, the important notions of density of decimals and relative magnitude of decimals on the number line were missed in cycle 1 but addressed in cycle 2, which resulted in the improved achievements in these areas. A further reason to be confident of the improved achievements in cycle 2 is that some of the written test items were also refined between the cycles, as summarised in Table 6.1. The new items measured improvement in a more probing fashion, giving additional weight to the improvements noted for cycle 2. The next sections will discuss the nature of these improvements for each content area in turn.

Table 6.1 also shows that both primary and secondary cohorts gained advantage from the activities and recorded improved achievements in some content areas. However, the improvements were not uniform across all content areas and showed the different traits of the two cohorts. This will be elaborated below in the discussion of the evaluation of improvements in different areas.

In general, content areas that were addressed in activities during the teaching experiments such as place value and decomposition of decimals documented improved achievements whereas areas that were not addressed in activities during the teaching experiment such as density of decimals in cycle 1 did not show significant improvements. There were two areas with exceptions in cycle 1 that showed decline in the post-test results, and this will be discussed in more detail in the following sub-sections.

Table 6.1: Outcome of teaching experiments on various content areas from two cycles

| Areas of content knowledge assessed in written tests | Cohort | Results of Post-test compared with Pre-test[a] | | Changes to activities between cycles | Changes to pre- & post-tests between cycles |
|---|---|---|---|---|---|
| | | Cycle 1 Cohort | Cycle 2 Cohort | | |
| Identifying place value names | Primary<br>Secondary | Improved<br>Same[b] | Improved<br>Improved | No change | Refined |
| Decomposition of decimals | Primary<br>Secondary | Improved<br>Improved | Improved<br>Improved | No change | Refined |
| Unitising decimal place value | Primary<br>Secondary | Not assessed<br>Not assessed | Same<br>Same | No change | New |
| Density of decimals | Primary<br>Secondary | Same<br>Same | Improved<br>Improved | New | Refined |
| Decimals on the number line | Primary<br>Secondary | Same<br>Same | Same<br>Improved | New | Refined |
| Sequencing decimals | Primary<br>Secondary | Declined<br>Declined | Same<br>Same | No activities | Refined |
| Ordering decimals | Primary<br>Secondary | Same<br>Same[b] | Not assessed<br>Not assessed | Not activities | Deleted |
| Closeness to a decimal | Primary<br>Secondary | Same<br>Same | Improved<br>Same | Deleted | Refined |

a: Improved or Declined indicates statistically significant at 0.05 level
b: no room for improvement

### 6.2.1 Evaluating improvements on place value and decomposition of decimals

Significant improvements in identifying place value names of a decimal digit and decomposition of decimals were recorded in both cycles (see row 1 and 2 of Table 6.1). As reported in Section 4.5, activities in cycle 1 addressed place value and decomposing decimals well, as indicated by the improved achievements in place value understanding of both cohorts. Hence, no refinement was made in these activities for cycle 2 and both cohorts again documented significant improvements in cycle 2 in these two areas. The written tests were refined in cycle 2 to address the limitation of the items in picking up the incorrect way of thinking of place value observed in cycle 1. The following paragraphs will expand on the refinement of the written tests.

The primary cohort demonstrated weak knowledge of place value in the pre-test as reflected by the lower facility of correct responses (facility of correct answer increased from 53% to 81%). As reported in Section 4.5, written test responses in cycle 1 documented a pattern of incorrect responses in identifying place value names of a decimal digit based on the length of the decimal digits. The results of the secondary cohort suggested there was not much room for improvement in identifying place value names of a decimal digit (facility of correct answer increased from 87% to 90.3%). The initial scores signified the importance of attending to place value, particularly in the primary cohort.

The lack of room for improvement of the secondary cohort in cycle 1 signalled the limitation of the written tests to pick up the incorrect thinking in identifying explicit place value names. Consequently, the written test items on this area were refined between cycles (see Section 5.2). Better items were able to reveal and confirm the predicted error pattern (i.e., identifying place value names based on the length of decimal digits) in cycle 2. Moreover, findings in cycle 2 documented significant improvements in identifying place value names of a decimal digit. There is additional confidence in the improved achievement observed in cycle 2 because this improvement was observed using the refined items.

Decomposition of decimals into place value related terms (see row 2 of Table 6.1) was recorded as the weakest area in the pre-test of both cycles but showed the highest improvements in the post-tests as reported in Section 4.5 and Section 5.4. Scant place value understanding was evident in some pre-service teachers re-ordering decimal digits as alternatives in the pre-tests (e.g., 0.375 as 5 ones + 7 tenths + 3 hundredths + 0 thousandths). Activities such as exploring different ways of constructing a decimal number using the concrete model LAB and illustrating their sketches in related place value terms allowed pre-service teachers to learn multiple ways of interpreting decimals. Interview excerpts and worksheet of activities in both cycles recorded pre-service teachers' comments about the novelty for them of interpreting decimals in various ways (see more details in Section 4.5 and Section 5.4). These tasks appeared new to them, which probably indicates low emphasis on this part of place value in their previous schooling. The strong emphasis placed on place value and decomposing decimals on activities in both cycles explained the significant improvements on this area.

Linking various ways of decomposing decimals in related place value terms and their standard decimal notation was an area of content knowledge overlooked in the written test of cycle 1. It was expected that the activity of decomposing decimals with number expanders would allow pre-service teachers to see the link between decomposing decimals and unitising decimals. To better inspect the knowledge of relations between expanded and standard decimal notation, new test items on unitising decimals were added as part of the refinement of test items in cycle 2 (see row 3 of Table 6.1).

As reported in Section 5.4, difficulties in linking decomposing and unitising decimals were recorded in the both pre and post-test of cycle 2 (e.g., to write 2 ones + 6 tenths + 15 hundredths + 3 thousandths as 2.6153). Row 3 of Table 6.1 showed that both cohorts recorded same results after the teaching experiments, which indicated non-significant improvements. The common incorrect responses in the pre-test on unitising of decimals, i.e., lining up the decimal digit and ignoring the decimal relations suggested scant knowledge of place value. The expectation that the link between the decomposing and unitising decimals could be made easily by pre-service teachers while working with number expander was not attained. This signified the compartmentalized nature of pre-service teachers' knowledge which constrained pre-service teachers to establish the link between decomposing and unitising decimals as indicated in the results of pre- and post-tests in cycle 2.

### 6.2.2 Evaluating improvements on density and relative magnitude of decimals on the number line

The advantage of addressing density of decimals and relative magnitude of decimals on the number line (row 4 and 5 of Table 6.1) in Set 2 activities was reflected in the significant improvements on these two areas in cycle 2 (see Table 5.6 and Table 5.7). These two areas were not addressed in cycle 1 activities due to lack of time and consequently pre-service teachers from both cohorts recorded no significant improvements on density of decimals. As reported in Section 4.5, interview data in cycle 1 revealed the limitation of test items to pick up incorrect thinking about density of decimals (i.e., thinking there are finitely many decimals in between two given decimals). The refined items used in cycle 2 gave more confidence in identifying pre-service teachers who have correct understanding of density of decimals.

Playing the 'Number Between' game at the start of Set 2 activities as a whole class in cycle 2 was successful resolving the incorrect thinking that there was no decimal in between some pairs of decimals. However, as reported in Section 5.3, a tendency to work with decimals of the same length by working with the equivalent common fractions with the same denominators tended to persist. This tendency was observed during group and whole class discussions, particularly in the primary cohort of cycle 2 (see Section 5.3 for more details). This approach reflected the curriculum sequence in approaching fractions and decimals, which encouraged students to work with decimals of the same length only. Working with pairs of decimals of different lengths in the 'Number Between' game was one way to encourage pre-service teachers to move away from this tendency.

The primary cohort in both cycles documented substantial difficulties with placing negative decimals on the number line in the written tests, particularly in the pre-tests. These difficulties reflected their weak knowledge of negative numbers, which was not attended during the teaching experiment in cycle 1 as well as confusion between decimals and negative numbers as has been reported by Stacey, Helme, & Steinle (2001). As reported in Section 5.3, worksheet of activities (Set 2- Activity 15) and observation of group discussions in cycle 2 confirmed difficulties of many primary cohort groups with negative decimals. In some cases, this confusion also affected their thinking about certain positive decimals. A whole class discussion was carried out with the primary cohort classes to address these difficulties. However, this short classroom discussion was not substantial enough to resolve difficulties with negative decimals. The fact that these difficulties were observed in the primary cohort in both cycles underscored the importance of attending and resolving these problems during pre-service teachers' training. Lack of attention to negative decimals and incorrect association of decimals with negative numbers in the activities was acknowledged as an important omission in the design of activities in this study particularly for the primary cohort. An improved set of activities should include attention to this aspect.

### 6.2.3 Evaluating improvements on sequencing, ordering decimals and closeness of decimals

The three content areas of sequencing of decimals, ordering of decimals, and finding the closest decimal to a given decimal (see row 6, 7 and 8 of Table 6.1) were not included in either set of activities in either cycles. Both sequencing and finding the closest decimal to a given decimal were assessed in the written tests of both cycles after some refinement of the test items. In cycle 1, both cohorts showed a decline in the post-test of sequencing decimals. This was due to a lack of comparability of pre- and post-test items, with higher cognitive load of the post-test items as reported in Section 4.5. However, both cohorts recorded non-significant improvement on this area after a refined set of test items was used in both the pre-test and post-test in cycle 2 (see more detail in Section 5.4). This might indicate that lack of attention to these content areas could be the explanation behind lack of improvement on these content areas. This raised a concern that the weak performance on sequencing decimals might be caused by lack of attention in the activities in both cycles on this area.

Similarly, both cohorts showed lack of significant improvement in finding a closest decimal to a given decimal in cycle 1 (see Section 4.5). The interview data in this cycle revealed tendency of inappropriate application of rounding and truncating rules. Hence, the written test items were refined between cycles to identify this behaviour more accurately. Results showed an improved achievement by the primary cohort but lack of significant improvement (same) of the secondary cohort in cycle 2. The low mean score of the primary cohort in the pre-test of cycle 1 explained the significant improvement of this cohort in cycle 2. Meanwhile, stronger reliance on rules such as rounding or truncating rules without understanding was particularly dominant in the secondary cohort and explained lack of significant improvement of this cohort on this area. However, despite lack of significant improvement by the secondary cohorts, they still outperformed the primary counterparts in both cycles.

The section above has shown there were content areas recording lack of improvements in one or both cycles which was affected by lack of attention to these content areas on the activities and by pre-service teachers' reliance on rules. In the next section, students' improvements in pedagogical content knowledge will be examined in a parallel fashion. The final recommended local instruction theory for decimals will be presented in the next chapter.

## 6.3 Overview of Improvements in Pedagogical Content knowledge

In this section, improvements on pedagogical content knowledge (PCK) from the two cycles will be summarized. In Section 6.3.1 and 6.3.2, trends in areas of improvements examining pedagogical content knowledge in both cycles will be discussed. Contributing factors to the improvements and explanations behind lack of success in areas with little improvement (recorded as same in Table 6.2) will be identified and discussed. Furthermore, common trends and differences in terms of improvement in various pedagogical content areas in the two cycles will be discussed.

Note that PCK were addressed in activities in both cycles by probing reflections on activities at the end of the set of activities for teaching ideas. As reported in Chapter 4 and 5, the nature of the reflections on teaching ideas tended to focus on general reflections of pre-service teachers' participation on activities for their teaching ideas. Hence there were not many direct links between activities which addressed particularly teaching ideas on areas of PCK addressed in the written tests as can be observed in Table 6.2.

Both cohorts initially showed weak pedagogical content knowledge as reported in Section 4.5 and Section 5.4. The poor performance on PCK items in cycle 1 was characterised by the high proportion of blank answers and lack of attention to teaching ideas. Weak knowledge on the link between decimals and the corresponding fractions as well as large proportion of blank answers led to poor performance on PCK items of the primary cohort in cycle 1. As discussed in Section 4.5, it is likely that the blank answers were due to students not being able to respond, rather than other factors such as having insufficient time to complete the times. Meanwhile, poor PCK performance of the secondary cohort was due to lack of articulation in teaching ideas. Hence, more emphasis on the articulation of teaching ideas was added in cycle 2 to better observe the pedagogical ideas (see Appendix B9 for detail of the refined written test instruments). Fewer blank responses and increased facilities on PCK items were recorded in cycle 2. As reported in Section 5.4, about half the pre-service teachers in cycle 2 were classified as having medium PCK. However, there remained a lack of satisfactory explanations in teaching ideas with strong reliance on rules and algorithms (sometimes incomplete and fragmented) and lack of the use of concrete models in teaching ideas.

As shown in Table 6.2, the primary cohort in both cycles recorded significant improvement in all PCK items, whereas the secondary cohorts' improvement was not significant except for teaching ideas in diagnosing students' error in ordering decimals and in resolving this error. These findings suggested that the primary cohorts in both cycles were more accommodating to the teaching ideas introduced during the teaching experiments. However, despite showing lack of significant improvements, the secondary cohort outperformed the primary cohort as indicated in their higher mean scores on PCK items. The following section will discuss the outcomes of teaching experiments from the two cycles in various pedagogical content knowledge areas.

Table 6.2: Outcome of teaching experiments on various pedagogical content knowledge areas between cycles

| PCK items assessed in written tests | Cohort | Results of Post-test compared with Pre-test[a] | | Changes to activities between cycles | Changes to pre- & post-tests between cycles |
|---|---|---|---|---|---|
| | | Cycle 1 Cohort | Cycle 2 Cohort | | |
| Teaching ideas for comparing pair of decimals | Primary | Improved | Improved | Deleted | Refined |
| | Secondary | Same | Same | | |
| Diagnosis of students' error in ordering decimals and teaching ideas to resolve it. | Primary | Improved | Improved | No activities | Refined |
| | Secondary | Improved | Improved | | |
| Teaching ideas on division of decimals by 100 | Primary | Same | Improved | No activities | Refined |
| | Secondary | Same | Same | | |
| Teaching ideas on the links between fraction and decimal notation | Primary | Improved | Improved | No activities | Refined |
| | Secondary | Same | Same | | |

a: Improved indicates statistically significant at 0.05 level

### 6.3.1 Evaluating improvements in teaching ideas for comparing decimals and diagnosing an error in ordering decimals

As revealed in Table 6.2, the two cohorts showed different outcomes in terms of improvements on teaching ideas for comparing a pair of decimals in both cycles (see row 1 of Table 6.2). The primary cohort recorded significant improvements on this teaching idea in both cycles. However, the pre-test responses documented how pre-service teachers' misconceptions were also revealed in their choice of models. For example, some pre-service teachers incorrectly extended the models commonly used for teaching fractions, i.e., fair-sharing context for teaching decimals. The interview data revealed that lack of experience with concrete models in learning decimals and weak knowledge on the links between fractions and decimals led to this inappropriate extension of fraction models for teaching decimals. For some pre-service teachers this inappropriate extension was related to and further confirmed their own misconception of decimals and reciprocals.

Interestingly the secondary cohort in both cycles recorded no significant improvement in teaching ideas for comparing a pair of decimals. In cycle 1, the secondary cohort showed a better performance in both pre- and post-tests. Teaching ideas incorporating the use of number line or subtraction for comparing decimals was commonly found in the pre-test responses of the secondary cohort. The post-test responses showed a tendency to provide similar teaching ideas as given in the pre-test with some pre-service teachers referred to the use of LAB models or decomposing decimals for comparing a pair of decimals. This explained lack of significant improvement in teaching ideas of this cohort in cycle 1. Meanwhile as reported in Section 5.4, there were cases indicating misinterpretation of the problem in teaching ideas to compare decimals which explained the low facility of Item 16 particularly in the pre-test of cycle 2 of the secondary cohort.

As reported in Section 4.5 and Section 5.4, the different traits between the two cohorts were depicted in the choice of models in teaching ideas for comparing a pair of decimals. The secondary cohort attended to more symbolic and formal teaching ideas such as number line or ruler whereas the primary cohort attended more to the use of concrete models such as LAB models in their teaching ideas.

Both cohorts in the two cycles recorded significant improvements in diagnosing an error in ordering decimals and in articulating teaching ideas to resolve this error in the two cycles (see row 2 Table 6.2). The initial teaching ideas for resolving an error in ordering decimals showed strong reliance on rules such as rounding rules, and annexing zeros. Reliance on a computational approach such as multiplying decimals by power of

10, subtraction, or comparing the equivalent common fractions was also common in the pre-test from both cohorts.

Only a small proportion of teaching ideas made reference to the place value notion. Moreover, the common models proposed in the pre-test reflected a more symbolic teaching approach such as using the ruler or number line. This tendency to use a symbolic teaching approach reflected the common teaching approach of decimals in Indonesia. In contrast, teaching ideas after the enactment of activities incorporated the use of concrete models such as LAB, and place value column charts (see Figure 5.13) to resolve an error in comparing decimals. Post-course interview data also confirmed the fact that concrete models introduced during the teaching experiments were useful for improving their teaching ideas particularly for teaching decimals in the primary school contexts. However, as reported in Section 4.5 and Section 5.4, both cycles showed the trend that the secondary cohort opted for a symbolic teaching approach in their teaching ideas.

Overall, both cycles recorded the shift of teaching ideas in the post-test with more reference to the place value notion for decomposing decimals to resolve students' error in ordering decimals. This shift in teaching ideas was in line with improvement on content areas, which recorded the highest improvement in decomposing decimals into related place value terms as discussed in Section 6.2.1. Moreover, this showed the connection between improvement in content and pedagogical content knowledge.

### 6.3.2 Evaluating improvements in teaching ideas on division of decimals and linking fractions with decimals

Teaching ideas on division of decimals by 100 (see row 3 of Table 6.2, Item 18 in Appendix B1 and B2 and Item 16 in Appendix B5 and B6) was the area least improved on PCK items in both cycles. Neither cohort showed a significant improvement in cycle 1. Only the primary cohort recorded a significant improvement in cycle 2 but despite this significant improvement, the performance indicated in the mean scores showed poor teaching ideas. Extending the use of concrete model of LAB for division of decimals was not addressed explicitly in activities of either cycle. It was expected that pre-service teachers could link their experience in exploring decimal relationships among various LAB pieces with teaching ideas for division by 100, but this proved to be a false assumption.

The teaching ideas to demonstrate division of decimals by 100 proposed by pre-service teachers in the pre-tests of both cycles showed a strong reliance on rules and standard algorithms such as invert and multiply and division algorithms. However, many of the explanations indicated pre-service teachers' incomplete and deficient knowledge on these algorithms. Moreover, teaching ideas in the post-tests still documented strong reliance on these two algorithms without much understanding. Only a small number of pre-service teachers were able to link their experience during the teaching experiment to make sense of division of decimals by 100 as reported in Section 4.5 and Section 5.4. This could also be due to the fact that the researcher has focussed on the meaning for the number but has not put much emphasis on the meaning for the operation such as division.

Similarly, teaching ideas to link fractions and decimals in both pre- and post-tests showed lack of meaningful understanding of division algorithm. The primary cohort from both cycles recorded significant improvements in teaching ideas to link fractions and decimals. One factor that explained the significant improvements of the primary cohort was the large proportion of blank answers in the pre-tests. As discussed in Section 4.5 and Section 5.4, only few pre-service teachers were able to extend their own learning experience to give meaningful interpretation of division algorithm to link fractions and decimals. .

In contrast, the secondary cohorts recorded no significant improvements despite their higher mean scores compared to the primary counterparts in both cycles. Stronger knowledge on procedures was observed in the secondary cohort teaching ideas. However, teaching ideas which overlooked the context of teaching decimals in the primary school such as the use of scientific notation in solving division of decimals mathematics were documented in the secondary cohort responses.

The sections above have shown the development of PCK in both cycles and also shown how it grew for the whole sample. In the next section, some case studies are presented in order to demonstrate what this growth looked like in practice.

## 6.4 Samples of development on CK and PCK

This section will present samples of four pre-service teachers from the two cycles with different initial levels of CK and PCK and the progress of their CK and PCK after the teaching experiments.

These four pre-service teachers were selected out of the total of 28 pre-service teachers who participated in both pre- and post-course interviews from two cycles. They were chosen because they provided insightful responses during the interviews to illuminate their problems and progress during the teaching experiments. A summary of their progress based on their responses on the pre- and post-tests, pre- and post-course interviews were included in these reports. A lot of cases of development of CK and PCK were observed but these four pre-service teachers were chosen because their developments not only represented the wide variety of pre-service teachers' knowledge but also depicted non-uniform levels of impacts of activities on pre-service teachers' CK and PCK.

***Case 1:*** Ayi is a secondary pre-service teacher with good content knowledge on decimals who took part in the cycle 1 teaching experiment. Her good content knowledge is reflected in her performance in both DCT3a and DCT3b and Part B of the written tests. Ayi was classified as A1 based on DCT3a and DCT3b in both tests. She scored 26 out of 27 in the pre-test and 27 out of 27 in the post-test of Part B. Ayi was also part of a video-recorded group from the secondary cohort in cycle 1. She was actively engaged in good collaborative work during the teaching experiment. In the pre-course interviews, she revealed that her knowledge relied on formal algorithm and rules and this was well reflected in her articulation of teaching ideas for Part C in the pre-test (scored 4 out of 9, classified as medium PCK). The following interview excerpt recorded her reliance on rules and algorithms in teaching ideas to find the equivalent decimals for 1/3 and an absence of reference to any explanatory models:

*Ayi:* **I had trouble working on that item but I have no other way to solve that except by using division algorithm so 1/3 equals to 0.333… From there we can talk about rounding. We can see that the result will not stop here but it depends on our consensus how many decimal digits we would like to round it to… From the division, students can see that it always has 1 as a remainder so the repeated decimal digit will always be 3.**

Ayi's evolving knowledge was apparent in her pedagogical content knowledge as she progressed from the score of 4 out of 9 (medium PCK) at the pre-test to score 8 out of 9 (high PCK) in Part C of the post-test. In the post-course interview, she commented that it was the first time for her to use concrete models in learning decimals and she was particularly impressed with LAB model. In fact, Ayi was able to use the LAB model in making sense the division process to find the equivalent decimals for 1/6 as documented in the following excerpt:

| | |
|---|---|
| *Researcher:* | Could you explain your ideas in assisting students to understand the decimal notation of 1/6? |
| *Ayi:* | If 1 is divided into 6, we use the LAB and then from 1 if we are to divide it into 6, in the first place it can't be done. Therefore, because we cannot do that, one is equal to ten tenths so now it can be divided into 6. We get one, so this one is one of the tenth. Then from ten tenths if we divide them into 6, we have each group consists of one tenth but we still have 4 tenths more and because 4 tenths can't be divided equally into 6. |
| *Researcher:* | Then what happened? |
| *Ayi:* | Then we use the hundredths. From there, we divide them into 6, and get 6 groups of 6 hundredths, I mean each has 6 hundredths. Then from here we get 4 as a remainder again, so the students will observe that it can never be evenly divided. |
| *Researcher:* | So how do you compare this teaching idea to the one you proposed in the pre-test to find the corresponding decimal for 1/3? |
| *Ayi:* | Obviously it is different. Before I used long division so it is purely symbolic and just works with numbers. |

Clearly, Ayi expanded her teaching knowledge from the symbolic approach based on her prior learning experience of decimals to incorporate the use of LAB models in understanding the division process and in linking fractions and decimals. Ayi is one example from a group of pre-service teachers with strong initial content knowledge who gained advantage from the teaching experiment in improving her pedagogical content knowledge.

***Case 2**:* Adrian is a primary pre-service teacher with weak content knowledge and pedagogical content knowledge on decimals from cycle 1 teaching experiment. He was classified as holding denominator focussed thinking (S1), which was indicated by his choosing the shorter decimals as the larger decimals in the pre-test DCT3a. During the pre-course interview, a confirmation for his S1 thinking was obtained as he explained his thinking in comparing 5.62 and 5.736 as follows:

> The larger one is 5.62 because this is in hundredths, it means 562/100 whereas this one s 5736/1000. I don't use number line for this one. Another way is to compare the equivalent fractions but it will be too long.

His S behaviour was confirmed in ordering decimals, such as 0.800001 < 0.7821 < 0.788 < 0.8 < 0 (Item 3a in Part B of pre-test). Note that he has also put all of these decimals as less than zero, indicating a sub-type of S1 behaviour.

Adrian's teaching ideas also reflected and confirmed his S1 thinking as he suggested teaching "tenths are larger than hundredths and thousandths" to resolve students' error in ordering decimals as 0.3 < 0.34 < 0.33333 (Item 19 in the pre-test of Part C of cycle 1). Apparently his answer suggested that he might reverse the order presented in student's erroneous response.

Adrian's S1 thinking was also evident in his explanation on density of decimals. Adrian consistently answered that there is no decimal in between two decimals of the same length. His explanation in finding no decimal between 3.14 and 3.15 (Item 5 from Part B of the pre-test) was because "14 and 15 are two consecutive numbers". As an S1 thinker, Adrian would think that all the decimal tenths (3.1, 3.2, 3.3 etc are larger than all the decimal hundredths (3.01, 3.02, 303, etc) which are larger than all the decimal thousandths (3.001, 3.002, etc). For Adrian, the decimals 3.14 and 3.15 are indeed "consecutive". Adrian's weak content knowledge was reflected in his low score of Part B (score 13 out of 27 in the pre-test).

Adrian progressed on some areas of content knowledge in the post-test and he was classified as apparent expert (A1) on DCT3a. His improvement in content knowledge was apparent in his ability to decompose decimals and in density of decimals and reflected in his improved score of 17 out of 27 in the post-test. However, his S1 thinking was still retained in ordering decimals in Item 31 of Part B, i.e., ordering 0.40001<0.4421<0.444<0.4<0. Consistent with this, he showed difficulties with placing of negative decimals on the number line in the post-test. Despite his improvements on some areas of content knowledge, Adrian's content knowledge was still fragile and his improvements seemed to be isolated.

Adrian also showed low pedagogical content knowledge (score 2 out of 9 in the pre-test of Part C). He made some progress in his teaching ideas by progressing to medium PCK (score 5 out of 9 in the post-test of Part C) and attempted to accommodate the LAB model in his teaching ideas to link fraction 1/6 with decimals. However, as he admitted during the post-course interview, his idea to use LAB was limited to representing decimals after obtaining the corresponding decimals for 1/6 by the division algorithm. Adrian did not show a tendency of S1 thinking in his teaching ideas to resolve students' error in comparing decimals 0.66666<0.63<0.6 (Item 19 in the post-test of Part B) in the post-test. However, his suggestion to explain that "the longer does not necessarily indicate the smaller" did not show a meaningful teaching idea. Clearly Adrian's improvement in some areas of content knowledge which was isolated seemed to inhibit his progress in pedagogical content knowledge.

***Case 3:*** Marni is a primary pre-service teacher with medium knowledge on decimals who participated in the cycle 2 teaching experiment. She was classified as holding A2 thinking and scored 15 out of 28 in Part B of the pre-test. However, Marni's responses in the pre-test of DCT3a to type 4, type 4R items (i.e., 17.35>17.353, 4.45>4.4502, 3.7>3.7777, and type 8 (i.e., 0>0.6, and 0.00>0.7) items showed possible association of decimals with negative numbers. The post-course interview confirmed Marni's initial association of decimals with negative numbers in comparing decimals of certain types.

| | |
|---|---|
| *Researcher:* | So what do you think when comparing 3.7 and 3.7777? |
| *Marni:* | In the case like this, where the numbers are almost the same, in comparing them, I know that the difference between them will be 7/100, 7/1000, and 7/10000. |
| *Researcher:* | How did you think of solving it in the pre-test? |
| *Marni:* | Somehow, I used to think of decimals as if they were negatives, using the context of owing money. I think of comparing for example -4 and -1. I know that -4 is smaller because the value is getting further from 0. |

This trend was in line with the observation by Steinle (2004) in her longitudinal data which suggest the link between *S* behaviour in A2 which was expressed as below. (Note that Steinle's test used in the longitudinal study did not include comparisons of decimals with zero as in DCT3a, so Steinle inferred this link without the direct evidence of the test but from the interview data).

> ... there is an evidence to support the hypothesis that *A2* students are harbouring a latent misconception (*S behaviour*) to deal with the failure of their incomplete algorithm ... this provides additional evidence that *A2* students are using algorithms to compare decimals, but harbour an *S behaviour*. (p.208)

Marni's low pedagogical content knowledge in the pre-test reflected her confusion about decimals and reciprocals, which was also translated in confusing models for decimals and reciprocals (score 2 out of 9 in the pre-test). Her strong association of decimals with fractions and her lack of knowledge about models for teaching decimals led her to opt for fair sharing context for teaching decimals. She proposed an example of sharing a candy with 8 people to represent 0.8 and sharing a candy with 88 people to represent 0.88 in the pre-test. Clearly the chosen model of sharing a candy also highlighted her lack of exposure in the use of models in teaching decimals and this was confirmed during the pre-course interview.

Marni was classified as holding A1 thinking in the post-test. She recorded improvements in areas of content knowledge such as place value, decomposing and unitising, and density of decimals. Her improvement on some content areas was reflected in her post-test score of 22 out of 28 in part B. However, Marni still recorded difficulties in sequencing decimals and placing decimals on the number line.

Marni also showed some progress in her pedagogical content knowledge as evident in her improved score of 5 out of 9 (medium PCK) in the post-test of Part C. As illustrated in the above post-interview excerpt, Marni made reference to place value notions in her teaching ideas for comparing decimals and incorporated the LAB model in her teaching ideas for comparing a pair of decimals. However, her teaching ideas in division of decimals did not seem to change as she still made reference to the 'invert and multiply' algorithm in her teaching idea without giving much explanation. Her teaching idea in linking fractions and decimals also documented her fragile knowledge on the division algorithm. Hence, the improvements that she apparently made on content areas did not seem to translate well into teaching ideas.

***Case 4:*** Vivi is a pre-service primary teacher with weak content knowledge (score 9.5 out of 27) who participated in the cycle 2 teaching experiment. She was diagnosed as holding A2 thinking in the pre-test and indicated reliance on rounding rules as evident in her incorrect responses in DCT3a (i.e., noting that 17.35=17.353, 4.4502=4.45, and 3.7=3.7777). Vivi's reliance on rounding rules was confirmed in her responses in both Part B of the pre-test by utilising rounding rules inappropriately to find the closest decimal to a given decimal (Item 10, 11 of Part B). In her explanation, she rounded up the given decimal 8.0791 to two decimal places and hence chose 8.08 as the closest decimal instead of 8.079001.

Vivi also had scant knowledge of place value as evident in the way she reversed the order of decimal digits in decomposing and unitising decimals (e.g., she answered 0 ones + 7 tenths + 1 hundredth + 12 thousandths as 1.2170). Moreover, she showed lack of knowledge about density of decimals by noting there was no decimal in between two given decimals and confused the positions of negative decimals on the number line. The pre-course interview revealed that Vivi held confusion about decimals and reciprocals as illustrated in Figure 6.1a.

Vivi's teaching ideas in the pre-test confirmed her reliance on rounding rules without much understanding as reflected in her medium PCK (score 4 out of 9) in the pre-test). Reference to rounding was made in her teaching ideas to compare 0.8888 and 0.8 (Part C, item 15) and in teaching ideas to resolve an error in ordering decimals (Part C, item 17). Inappropriate use of the rounding rule, i.e., rounding 0.33333 = 0.3, was documented in her teaching idea to resolve an error of ordering decimals 0.3, 0.33333, and 0.34. Clearly the use of rounding rule in this case was not helpful in resolving an error in ordering decimals.

Figure 6.1: Vivi's notes made during the interview

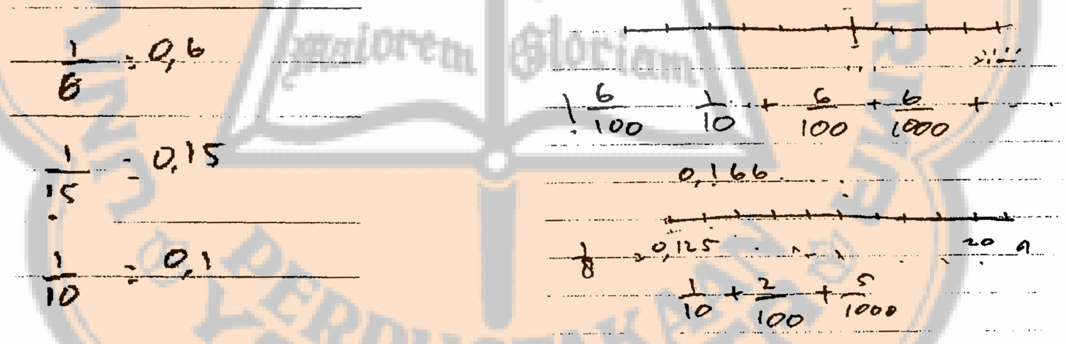


Figure 6.1a: Vivi's pre-course interview note Figure 6.1b: Vivi's post-course interview note

Vivi showed improvements in both content knowledge (score 23 out of 28 in Part B) and pedagogical content knowledge. She progressed to high PCK (score 8 out of 9 in Part C) and she was classified as holding A1 thinking using DCT3b in the post-test. Her improved place value knowledge was evident in her abilities to answer 3 different ways of decomposing decimals. She also showed improved knowledge of density of decimals. However, her difficulties with negative decimals were not resolved by the time of the post-test.

Improvement in pedagogical content knowledge was documented in Vivi's reference to place value notions by decomposing decimals for teaching ideas to compare and order decimals in the post-test (both Item 15 and Item 17). In the post-course interview, Vivi utilized LAB model in linking $\frac{1}{6}$ with its decimal notation as recorded in the following excerpt and illustrated in Figure 6.1 above:

| | |
|---|---|
| *Researcher:* | Could you explain how did you find the decimal for 1/6 using the LAB pieces? |
| *Vivi:* | First 1/6 means 1 piece divided into 6 parts. We need to divide the whole piece into 10 pieces and distribute them among 6 people |
| *Researcher:* | Then? |
| *Vivi:* | Because there were 10 parts and we use six parts so there were 4 parts remaining, we can divide each of them into 10 shorter parts. |
| *Researcher:* | So first, shall we go back and work out the names of each piece again? |
| *Vivi:* | First we start with one and then divide it into 6 parts, so each has 1 tenth and we have a remainder of 6 tenths… no that is wrong |
| *Researcher:* | Each part has how many of what? |
| *Vivi:* | Each has one tenth and there are 4 remaining tenths and we divide them again into 10, so we have 40 hundredths and divide it again into 6 parts, etc |
| *Researcher:* | So how do you find the decimal notation for 1/6? |
| *Vivi:* | 1/6 is the same as 0.166 |
| *Researcher:* | Will it stop? |
| *Vivi:* | No the 6 will repeat forever |
| *Researcher:* | Why do you think it will repeat forever? |
| *Vivi:* | Because there is always a remainder of one |

The above excerpt showed Vivi's progress in her understanding on relations between fractions and decimals. More importantly she was able to make a meaningful understanding of fractions and the division process by utilizing the concrete model LAB. It was encouraging to learn that a pre-service teacher with weak content knowledge was able to progress to develop a meaningful interpretation of links between fractions and decimals.

These four cases of pre-service teachers' development on content and pedagogical content knowledge depicted the wide variety of pre-service teachers' content and pedagogical content knowledge. These four examples show that pre-service teachers with different initial levels of content knowledge gained advantage from the activities. Pre-service teachers with weak initial content knowledge in both cycles, such as Adrian and Vivi, gained advantage from their participation in the teaching experiment. They still had more to learn and their new knowledge did not seem well integrated, but they had made progress. However, as explained above, the fragmented nature of their knowledge was resulted in isolated improvements in different content areas. This was evident in Adrian's case. Similarly, pre-service teachers with medium content knowledge, such as Marni, also showed progress after the teaching experiment. Pre-service teachers with strong initial content knowledge, such as Ayi, did not have much room for improving their content knowledge. However, the advantage of participating in the teaching experiment was clear on her evolved pedagogical content knowledge.

In line with the overall findings reported in Section 6.2 and 6.3, persistent difficulties in some areas such as negative decimals were observed in all cases except for Ayi. This is an area that was not adequately addressed in the activities. Further discussions about features and characteristics of pre-service teachers' content and pedagogical content knowledge will be addressed in the next two sections.

## 6.5 Answering Research Question 1 and sub-questions

Research question 1 (see section 1.4) asked "To what extent do the activities improve pre-service teachers' content knowledge (CK) on decimals?". Based on the evidence brought together from the two cycles in section 6.2, it was clear that the activities delivered during the teaching experiments contributed to the improvement of pre-service teachers' knowledge of decimals in both cycles. Discussion in Section 6.2 signified that in general areas that were well attended in the activities recorded significant improvements. Similarly content areas that were not addressed on the activities recorded non-significant improvements. Findings from both cycles recorded different areas of concerns on content areas from the primary and the secondary cohort. This implies that activities need to be modified to better attend different areas of concerns of the two cohorts by offering different programs to suit the characteristics of each cohort. This issue will be addressed later in Chapter 7.

Research question 1 also asked two sub-questions, which will now be discussed in turn.

**What is the current state of Indonesian pre-service teachers' content knowledge of decimals? (Research Question 1a)**

Before addressing this question, I will clarify the meaning of 'current state' of pre-service teachers' knowledge. As mentioned in Section 1.1, in general, decimal topics are not revisited and addressed in teacher education. Therefore, pre-service teachers' knowledge measured in the pre-test and pre-course interviews could be perceived as a reasonable indicator of the knowledge that pre-service teachers usually have. Hence, pre-service teachers' knowledge observed in the pre-tests is taken as representative of the current state of their knowledge in both content and pedagogical content knowledge.

As recorded in the pre-test results and illustrated by the four samples of pre-service teachers' development of content knowledge and pedagogical content knowledge, pre-service teachers' content knowledge at the beginning of both cycles was widely spread. Both cycles showed a trend of stronger content knowledge of pre-service teachers from the secondary cohort compared to the primary cohorts.

The current state of Indonesian pre-service teachers' content knowledge of decimals could be characterised as:

*fragmented;

*with strong reliance on rules or algorithms without understanding, and

*strong association of decimals with fractions.

Each of these characteristics will be explained in the following paragraphs.

Fragmented knowledge of pre-service teachers was evident in the practice of applying a mix of rules inappropriately without understanding to solve different problems about decimals. Cases where pre-service teachers were able to decompose a decimal 0.375 = 0 one + 3 tenths + 7 hundredths + 5 thousandths, but at the same time said that 17.353 < 17.35 illustrated fragmented knowledge of decimals.

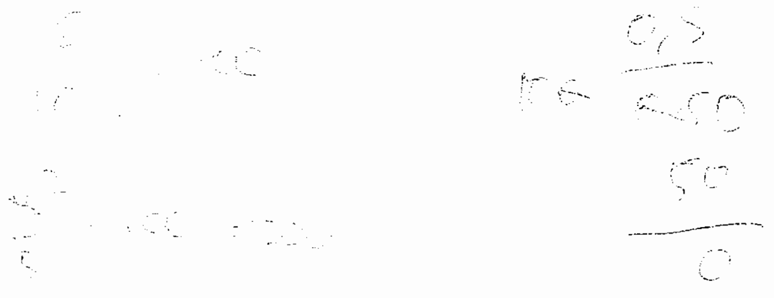
Strong association of decimals with fractions was evident in pre-service teachers' tendency to revert to fraction notation and computations in working with problems involving decimals. As reported in Section 4.5 and Section 5.4, the activity of finding the total lengths in measuring a table recorded a strong tendency to rely on fraction notation and operations. Similarly, in comparing and ordering decimals, many pre-service teachers converted decimals to equivalent fractions and operated on them rather than relied on place value notion. This strong association of decimals to fractions reflects the sequence of Indonesian curriculum in teaching fractions before decimals which emphasize on computational skills in working with fractions. One advantage of this approach is that pre-service teachers acquire knowledge on relations between decimals and fractions and certain degree of fluency in converting between fractions and decimals. However, heavy emphasis on a computational approach and deficient knowledge of fractions inhibit pre-service teachers from making meaningful links between fractions and decimals.

Reliance on rules or algorithms was observed in all parts of the written tests and recorded in pre-service teachers' explanations given during the interviews. When probed about the reason for using rounding, this rule was cited to simplify problems of working with decimals of longer decimal digits. Application of rounding or truncating rules was particularly dominant in comparing decimals with few repeated digits (such as comparing 3.7 with 3.777) on DCT3a and DCT3b in both cycles. A similar trend of adult students' reliance on incomplete algorithms without understanding was reported by Steinle and Pierce (2006) and Stacey and Steinle (2006) with nursing students.

Moreover, data in this study showed instances of inappropriate application of rounding rules, for example in finding the closest decimal to a given decimal (see Vivi as an example in the previous section). Teaching ideas for comparing a pair of decimals and for ordering decimals also documented evidence of some inappropriate application of rounding or truncating rules. For instance, whilst using rounding rules for comparing a pair of decimals such as 0.8888 and 0.8 will lead to correct answer, it overlooks the important aspect of developing students' understanding of the meaning of decimals.

Strong reference to algorithms such as the 'invert and multiply' algorithm was commonly found in teaching ideas for division of decimals. However, explanations of these algorithms recorded in the written test responses were limited to the procedural steps in carrying out the computations based on memorized facts which were often incomplete or incorrect, as documented in Figure 6.2.

Figure 6.2: Deficient knowledge of invert and multiply algorithm and the division algorithm



**What is the interplay between pre-service teachers' participation in the set of activities on decimals and their CK of decimals? (Research Question 1b)**

Pre-service teachers' current content knowledge shaped the nature of their participation and their responses on the activities. Their strong association of decimals with fractions based on a computational approach was evident in responses to the initial activities of measuring and recording the result of measurement in both cycles, as reported in Section 4.5 and Section 5.3.

Pre-service teachers' prior knowledge including their strong reliance on rules or algorithms without understanding explained the nature of their participation in activities. For instance, pre-service teachers utilized their knowledge of the metric system and their knowledge of rulers in finding the total lengths using the LAB models. This might indicate a positive sign that these pre-service teachers were aware of the similarities in base ten relations between metric systems and LAB models. However, the design of LAB models was not intended to directly link to the metric system. In fact, the use of LAB models with one unit of reference was expected to set the focus on exploring base ten relations. This showed an example where prior knowledge of pre-service teachers about decimals could be problematic in engaging with exploratory activity in this study. Further discussion on this issue will be discussed in Chapter 7.

The fragmented nature of pre-service teachers' knowledge was also evident in that improvement in content areas of decimals was often isolated as illustrated in the case of Adrian and Marni in Section 6.4. However, some pre-service teachers such as Vivi showed an 'aha' moment during the post-course interview. As reported in Section 6.4, Vivi was able to utilize LAB as a thinking tool to create a meaningful link between fractions and decimals. Her pre-course interview note (see Figure 6.1a) and transcripts documented her confusion of decimals and reciprocal. For pre-service teachers with good content knowledge such as Ayi, participation in the activities contributed to widening their pedagogical ideas particularly in the use of concrete models for teaching decimals. There was an indication that pre-service teachers with weaker content knowledge, represented by the primary cohort in this study gained more advantage from the activities to improve their content knowledge on decimals.

Clearly, pre-service teachers' participation in the set of activities resulted in improvement on some areas of content knowledge as summarized in Table 6.1. These improvements were not uniform across all areas of content knowledge but reflected how well the content areas were addressed in the activities as has been explicated in the overview of improvements in various areas in Section 6.2. Areas such as place value, and decomposition of decimals took up a significant portion of the activities and resulted in significant improvements on these areas. Moreover, significant improvement on density of decimals in cycle 2 showed that addressing density in the activities resulted in improved knowledge of pre-service teachers on density of decimals. Similarly, areas that were not addressed in the activities of both cycles such as sequencing decimals and finding the closest decimals to a given decimal showed lack of significant improvements in both cycles. This trend indicated that addressing content areas in the activities during the teaching experiment made a difference in pre-service teachers' knowledge in the corresponding content areas.

## 6.6 Answering Research Question 2 and sub-questions.

Research question 2 asked "To what extent do the activities improve pre-service teachers' pedagogical content knowledge (PCK) on decimals?" This has been discussed above in section 6.3, which documented the considerable gains made by many students.

More reference to the basic notion of place value in the teaching ideas after enactment of activities indicated a positive impact of activities on pre-service teachers' PCK on decimals. Incorporating the use of concrete models for teaching decimals to create more meaningful understanding was clearly documented in pre-service teachers' teaching ideas after the teaching experiments. It was also found that strong reliance on rules and fragmented nature of pre-service teachers' content knowledge in some areas constrained the uptake of new teaching ideas.

In answering research question 2 about the impact of the designed activities on pre-service teachers' pedagogical content knowledge, there are also two sub-questions to answer, which are now treated in turn.

**What is the current state of Indonesian pre-service teachers' PCK of teaching decimals? (Research question 2a)**

The current state of Indonesian pre-service teachers' PCK reflected the pre-service teachers' current content knowledge, and their prior learning experiences on decimals, which also reflected the curriculum sequence of how decimals were approached in primary school.

Strong reliance on rules and algorithms such as rounding and truncating rules, division and invert and multiply algorithms were featured in the proposed teaching ideas in the pre-tests of both cycles. In the pre-tests, pre-service teachers generally offered rules as explanations, rather than explanations based on reasoning from place value, or reasoning from models. This reliance on rules was further confirmed during the pre-course interviews. Lack of reference to the use of models as a pedagogical tool for learning and teaching decimals was also evident. Based on these facts, the current state of pre-service teachers' PCK could be characterised by teaching ideas relied on rules and computational approach with lack of inclusion of concrete models (see Section 4.5 and Section 5.4 for more detail).

Interestingly, an attempt to extend models for teaching fractions such as fair sharing situation for teaching ideas for decimals was found in both cycles. This also revealed misconceptions. For instance, a model of sharing a cake with 8 people to represent decimal 0.8 was commonly observed in both cycles. This attempt was based on knowledge that fractions and decimals were related. However, weak knowledge on the link between fractions and decimals and lack of knowledge on models for teaching decimals resulted in teaching ideas which confused models for decimals with models for reciprocals as reported in Section 4.5 and Section 5.4. Moreover, common model proposed in the pre-test such as ruler and number line revealed pre-service teachers' limited knowledge of the use of concrete models for teaching decimals. This fact was also recorded in the interview data as reported in Section 4.5 and Section 5.4. Clearly pre-service teachers' knowledge of concrete models for teaching decimals was improved by the end of the activities.

Lack of alternatives for less symbolic teaching ideas was clearly expressed during the interviews and inhibited pre-service teachers to propose alternative ways for teaching decimals. Meanwhile, some pre-service secondary teachers noted that they have not been taught about teaching ideas for decimals. However, the majority of pre-service teachers realized the limitations of this teaching approach for decimals, which was dominated by symbolic and mechanistic approach during the pre-course interviews. Interview excerpts by Ayi, Vita, and Ana (see Section 4.5.3 and Section 5.4.2) showed that teaching approach on decimals was symbolic with reliance on rules and operations with no reference to the use of concrete models.

**What is the interplay between pre-service teachers' participation in the set of activities on decimals and their PCK of decimals? (Research question 2b)**

The incorporation of concrete models for learning decimals was a new experience for almost all pre-service teachers involved in both cycles of the teaching experiment. This experience has expanded pre-service teachers' knowledge of alternative ways of teaching decimals. Pre-service teachers' reflections notes recorded in the worksheets of activities of cycle 1 and cycle 2 documented the positive impact of models introduced in the teaching experiment such as LAB and the number expander. However, both cycles showed that for the majority of pre-service teachers, the use of models were seen more as representational tools rather than a thinking tool. Many pre-service teachers were able to represent the numbers using the models but unable to use their actions on the models to make sense of arithmetic operation and algorithm.

Teaching ideas related to this appears to require further work. Justification of algorithms in terms of the models showed the least improvements in both cycles.

Despite attending to place value notion and making use of concrete models such as LAB and number expanders for comparing and ordering decimals in some of the post-tests, only a small proportion of pre-service teachers in both cycles were able to utilize concrete models to make meaningful sense of algorithms or rules in their teaching ideas (see Section 4.5 and Section 5.4). The fact that pre-service teachers had difficulties to link their improved knowledge on content areas of decimals to teaching ideas indicated the fragmented nature of pre-service teachers' knowledge. The strong reliance of pre-service teachers' content knowledge on rules and algorithms without understanding explained their struggles to utilize models in creating meaningful understanding of decimals. Moreover, this highlighted the need for activities to enable pre-service teachers in making use of models in creating more meaningful interpretation of decimals and their operations.

Both cycles documented different traits of the two cohorts which seemed to reflect different stages of their training and influenced their preference for models and teaching approaches. Pre-service teachers from the primary cohort with some practical teaching experience in schools demonstrated more awareness to the use of concrete models in their teaching ideas and linked their use to the need in providing primary school children with more hands-on learning experience. In contrast, many of the secondary pre-service teachers revealed lack of thought about teaching ideas at this early stage of their training. The fact that the secondary pre-service teachers had done no practical teaching experience at schools might explain this different pedagogical awareness.

## 6.7 Concluding Remark

The overview of findings from the two cycles showed that the activities in both cycles were successful, particularly in improving pre-service teachers' knowledge of decimal place value and in expanding meaningful interpretation of decimals. The activities in both cycles also contributed to improvement on pre-service teachers' pedagogical content knowledge, particularly on knowledge of the use of concrete models for teaching decimals. However, it was observed that incorporating concrete models in teaching ideas were still limited to representational tools, rather than as the thinking tool. Interestingly, pre-service teachers from the primary cohort in both cycles seemed to be more accommodative to the use of concrete models in teaching ideas as compared to the secondary cohort counterparts.

Weak and fragmented content knowledge indeed affected the nature of pre-service teachers' participation in the designed activities. Despite the uptake of new teaching ideas to incorporate the use concrete models for teaching decimals in more engaging and meaningful way, reliance on old teaching approach based on rules was still observed in both cycles. Similarly, pre-service teachers' existing fragments of knowledge influenced the way in which they engaged with the exploratory activities which aim to reflect the guided reinvention tenet. This resistance to take up new approach was commonly reported in any reform effort studies. Further discussions on the design of activities and researcher's reflections on the extent to which the basic tenets of RME fitting for adaption in teacher education and the role of teacher education in promoting the teaching tenet of RME (answering research question 3) will be taken up in the next chapter.

# CHAPTER 7 CONCLUSION

## 7.1 Introduction

This chapter starts by presenting in Section 7.2 the proposed LIT on decimals for pre-service teachers based on the teaching experiments in two cycles. The proposed LIT comprises goals of the designed activities, the rationale for selecting the activities and the conjectured learning paths of the pre-service teachers. In Section 7.3, the researcher's reflections on the implementation of the basic tenets of RME and the RME teaching principles based on the teaching experiments in two cycles will be discussed. These reflections serve as a basis for articulating the role of teacher education to adapt RME basic tenets in teaching and learning mathematics in Indonesia. This will provide answers to research question 3. The strengths and limitations of this study will be discussed in Section 7.4. Finally, directions and recommendations for further research on design of activities on decimals in teacher education in Indonesian context will be articulated in Section 7.5.

## 7.2 Proposed LIT for decimals

This section will present the summary of the proposed LIT on decimals in teacher education based on the empirical experience of carrying out the activities in two cycles. Following Graveimejer's (2004) notion, the proposed LIT comprises learning goals, planned activities and tools, and conjectures of pre-service teachers' learning paths in achieving the learning goals. As pointed out by Graveimeijer, van Galen & Keijzer (2005), re-designing the activities based on analyses of the actual learning processes after the trial of activities in the classroom is part of the LIT development. This has been carried out at the end of both the first and second cycles.

In Table 7.1, a summary of the links among content areas, goals and sub-goals and the proposed activities are offered. In Table 7.1, activities refer to either the earlier trialled activities or the modification of activities trialled in either cycle 1 or cycle 2. The earlier trialled activities are as given in the Appendices. The modified activities are not given in this thesis, but section 7.2.1 describes the modifications that should be made in the future. Conjectured learning paths of pre-service teachers will be discussed along with the modified activities in Section 7.2.1. General features of the proposed LIT will be presented in Section 7.2.2.

Table 7.1 gives the program for primary pre-service teachers. For secondary teachers, the same activities can be used but they can be expected to complete them in a shorter time.

Table 7.1: Proposed LIT on decimals in teacher education

| Content areas in the activities | Goals and sub-goals | Activities | Time for primary |
|---|---|---|---|
| Place value of decimals | PSTs* develop meaningful understanding of decimals based on place value.<br>▪ PSTs can express base ten relations in decimal place value digits. | ▪ Revisiting decimal place value relations using length as a model.(Modification of Set 1- Activity 1 of cycle 2)<br>▪ Exploring ways of refining the tenths in the context of getting more accurate measurement (Modification of Set 1- Activity 2 of cycle 2) | 1 meeting of 100 minutes |
| | ▪ PSTs can use the base ten relations observed in establishing the names for LAB pieces and in linking them with the formal notation. | ▪ Establishing the name reflecting the base ten relations in various LAB pieces (Set 1- Activity 3 of cycle 1).<br>▪ Reasoning about the choice of subsequent refining of one into ten (Modification of Set 1- Activity 3 of cycle 2). | |
| | ▪ PSTs can explore the additive and multiplicative structures in the context of measuring length and recording the result in formal notation. | ▪ Measuring by iterating different pieces of LAB and recording the result of measurement using decimal notation (Set 1- Activity 4 of cycle 2). | |
| | ▪ PSTs can apply their knowledge of place value for interpreting and comparing decimals. | ▪ Articulating teaching ideas for interpreting decimals and comparing pairs of decimals. (Set 1- Activity 7 of cycle 1) | |
| Decomposing and unitising decimal place value | ▪ PSTs can decompose decimals in various ways | ▪ Finding multiple ways of decomposing the same decimals (Set 3- Activity 16 of cycle 2).<br>▪ Decomposing decimals in symbolic form (Set 3- Activity 17 of cycle 2). | 1 meeting of 100 minutes |

| Content areas in the activities | Goals and sub-goals | Activities | Time for primary |
|---|---|---|---|
| | ▪ PSTs can link decomposing and unitising decimals. | ▪ Linking various ways of decomposing decimals and unitising using number expander, and the symbolic representations (Modification of Set 3 – Activity 18 of cycle 2). | |
| | ▪ PSTs can reflect on multiple ways of decomposing and unitising decimals for their teaching ideas | ▪ Linking decomposing and unitising decimals and articulating teaching ideas (Modification of Set 3- Activity 21 of cycle 2). | |
| Density of decimals and relative magnitude of decimals on the number line | PSTs understand that there are infinitely many decimals in between any two decimals<br>▪ PSTs can find decimals in between two given decimals.<br>▪ PSTs can apply their knowledge of density for teaching properties of decimals<br>PSTs have sense of relative magnitude of decimals in relation to other numbers such as fractions and whole numbers.<br>▪ PSTs can link concrete and symbolic representations of decimals.<br>▪ PSTs can locate the positions of decimals and other numbers on the number line. | ▪ Playing Number in between game as whole class activity.<br>▪ Explaining the property of decimals observed from playing 'Number Between' game (Set 2- Activity 10 of cycle 2).<br>▪ Articulating teaching ideas for decimals about property of decimals (Set 2- Activity 11 of cycle 2).<br>▪ Representing and comparing different numbers using concrete model LAB before placing numbers on the number line (Modification of Set 2- Activity 15 of cycle 2).<br>▪ Placing fractions, whole numbers and decimals (including negative decimals) on the same number line. (Modification of Set 2- Activity 21 of cycle 2, with more attention to resolve misconceptions about negative decimals, particularly with the primary cohort) | 1 meeting of 100 minutes |

* PSTs : pre-service teachers

### 7.2.1 Modified activities, the rationale, and a conjectured learning path

**Modification of Set 1 Activities, the rationale and a conjectured learning path**

Findings from the initial activity of cycle 2 showed that partitioning one into ten equal parts in the context of measuring length was not easily perceived by pre-service teachers. Instead of exploring the conceptual partitioning which leads to revisiting the decimal partitioning, pre-service teachers showed tendency to focus on the practical aspects of physical partitioning as reported in Section 5.4. This finding concurs with a study by Keijzer, van Galen, & Oosterwall (2004) who reported difficulties in perceiving the choice for a decimal system in partitioning one shown by the primary school children in the Netherlands. Indeed the rationale for choosing a decimal system of repeated partitioning into ten is based on simplicity of operating with decimals by extending whole number algorithms. This is very sophisticated and not evident in a measuring context, unless operations with the numbers are called for. The history of decimal notation showed that the invention of decimal notation by Simon Stevin was made quite late in 1585 (see Figure 7.1).

Figure 7.1: History of decimal notation (Steinle et al., 2006)

| Author | Time | Notation |
|---|---|---|
| Before Simon Stevin | | $37\frac{245}{1000}$ |
| Simon Stevin | 1585 | $37\ 2^{(1)}\ 4^{(2)}\ 5^{(3)}$ |
| Trigonometric Tables | 1593 | first decimal point |
| Franciscus Viete | 1600 | $37\vert_{245}$ $37,\frac{245}{1000}$ **37**, 245 |
| John Kepler | 1616 | 37 (245 |
| John Napier | 1617 | $37:2^{I}\ 4^{II}\ 5^{III}$ |
| Henry Briggs | 1624 | $37^{\underline{245}}$ |
| William Oughtred | 1631 | 37 \| $\underline{245}$ |
| Balam | 1653 | 37 : 245 |
| Ozanam | 1691 | $37 \cdot \overset{(1)}{2}\ \overset{(2)}{4}\ \overset{(3)}{5}$ |
| Modern | | 37.245 |

Based on the above facts, the modified initial activity in the proposed LIT will focus on having pre-service teachers revisit the decimal place value structure, whereas the rationale of partitioning one into ten will be included as whole class discussion, rather than individual or group guided discovery. Both the longest piece of LAB (representing one) and one tenth of the one piece of LAB are given at the start to guide pre-service teachers in revisiting the decimal place value structures in the context of measuring length and refining the measurement unit. It is expected that pre-service teachers are able to observe and capitalize on this one tenth relation for the next refinement for measuring and recording the result of shorter lengths. Note that, this initial activity differed from the activity in cycle 1 when all the LAB pieces were presented at the same time.

The modified activities are considered as 'less structured' because the longest piece of LAB and the 'one tenth' of the longest LAB piece will be given but not the relationship between them. Instead, pre-service teachers are expected to explore this relationship in the context of refining the result of measurement. By extending the 'one tenth' relation for the next refinement, the need for 'one hundredth' piece of LAB will surface. Similarly, the need for further refinement in the context of measuring shorter lengths for greater accuracy, will lead to reinvention of a thousandth, and so on.

This activity is then followed by an activity to establish the names of the LAB pieces and to link them with the formal notation as trialled in Set 1- Activity 3 of cycle 1 (see Appendix A2). Discussion about the rationale for successive partitioning into 10 as the basis for the decimal system will be carried out as a whole class discussion. Learning about the notion of successive partitioning into ten will provide insights for pre-service teachers on the common approach of applying whole number rules in operating with decimals, particularly in addition and subtraction. In achieving this aim, an explicit task to explore the link between whole number place value and the endless base ten chain will be provided to guide pre-service teachers in making this link. This serves as a modification of Set 1- Activity 3 from cycle 2 in Appendix A4.

In the next sequence, Set 1- Activity 4 of cycle 1 (see Appendix A2) entails pre-service teachers capitalizing on decimal relations and notation to record the result of measuring lengths by iterating different pieces of the LAB model (e.g., placing 3 tenths together and recording as 0.3). Finally, Set 1 activities resume with teaching ideas to interpret decimals and compare the size of decimals, which was the same as Set 1- Activity 7 of cycle 1 in Appendix A2. It is expected that pre-service teachers will draw on base ten relations explored in the previous activities as well as their new experience with concrete models in learning decimals in their teaching ideas for interpreting and comparing decimals. Moreover, it is expected that multiplicative and additive relations will be explored in the teaching ideas for interpreting and comparing pairs of decimals.

**Modification of Set 2 Activities, the rationale and a conjectured learning path**

Set 2 of the proposed LIT starts with an activity to explore various ways of decomposing decimals into place value terms (Set 3- Activity 16 of cycle 2 in Appendix A4). In this activity, pre-service teachers are encouraged to explore different ways of decomposing decimals. Findings from both cycles (see Section 4.5 and Section 5.4) documented that the majority of pre-service teachers lack of knowledge about multiple ways of interpreting decimals. Moreover, the concrete model LAB was noted as useful in helping them to create a meaningful interpretation of decimals. Following this, Set 3 - Activity 17 of cycle 2 (see Appendix A4) to decompose decimals in symbolic forms is carried out. Establishing the link between decomposing and unitising decimals in related place value is explored using the number expander model designed in Set 3- Activity 18 of cycle 2 (see Appendix A4). However, findings in both cycles showed that a majority of pre-service teachers failed to notice the link between decomposing and unitising of decimals. Therefore in the proposed LIT, the reflection task places a stronger emphasis on place value knowledge in linking decomposing and unitising of decimals in modification of Set 3- Activity 21 of cycle 2.

**Modification of Set 3 Activities, the rationale and a conjectured learning path**

Set 3 activities in the proposed LIT started with playing the 'Number Between' game as a whole class activity, which is followed by a task to articulate the property of decimals learnt from this game as contained in Set 2- Activity 10 of cycle 2 (see Appendix A4). Findings reported in Chapter 5 indicated that this activity was useful in acquainting pre-service teachers about density of decimals. Moreover, it confronts pre-service teachers' misconceptions about decimals such as thinking that decimals form a discrete system evident in responses such as 'there is no decimal in between 3.14 and 3.15.

Building a sense of relative magnitude of decimals among other numbers such as whole numbers and fractions is a key part of number sense. Findings in both cycles showed a strong reliance on the algorithm and lack of meaningful understanding of the link between decimals and fractions. However, the fact that a few pre-service teachers were able to make a meaningful link between decimals and fractions indicated a promising result. Therefore in the proposed LIT, Set 2- Activity 15 of cycle 2 (see Appendix A4) is modified by linking decimals and fractions using the concrete model of LAB as whole class discussion before placing the numbers on the number line.

Both cycles also revealed a trend of difficulties with negative decimals, particularly the primary cohort when they undertook Set 2, Activity 21 of cycle 2 containing a task of placing decimals, whole numbers and fractions on the same number line (see Appendix A4). Taking into account this trend of difficulties, more time will be spent to elicit and to resolve any misconception about negative decimals during whole class and group discussions, particularly in the primary cohort.

Based on experience in both cycles, the realistic time recommendation to carry out the whole set activities in the proposed LIT is 3 meetings of approximately 100 minutes for the primary cohort. As reported in Section 4.5 and Section 5.4, the primary cohort in both cycles showed weaker knowledge of decimals. For instance, pre-service teachers in both cycles tended to have difficulties with negative decimals. For the secondary cohort, the activity of placing decimals on the number line (Set 2- Activity 21 of cycle 2) could be abbreviated. The data in both cycles suggested that the secondary cohort show that this activity did not show many difficulties with negative decimals. Hence, it is considered realistic to carry out the whole set of activities in 2 meetings of approximately 100 minutes for the secondary cohort.

This proposed LIT addresses the meaning of decimal notation and its basic properties. However, the evidence from the written post-test and interviews showed that pre-service teachers could use LAB to help them think about these aspects of decimals, but many of them were not able to transfer this knowledge to a meaningful understanding of decimal algorithms (e.g., finding a decimal expansion of 1/6). This needs to be addressed in teacher education, following this LIT or in conjunction with instruction on whole number place value.

### 7.2.2 General features of activities in the proposed LIT

The general features of the recommended activities can be summarized as follows:

- Reasoning about base ten relations in place value using length as a model;
- Promoting flexible thinking by exploring different ways of interpreting or representing decimals;
- Utilizing concrete models in creating meaningful understanding;
- Interactive modes of learning through group and classroom discussion.

These features were perceived as important components of revisiting the notion and properties of decimals for pre-service teachers considering the nature of their knowledge about decimals in this study as discussed earlier in Chapter 6. The enactment of these features of activities about decimals with pre-service teachers is affected by the role of teacher education in Indonesia in adapting the RME. The following section will present the researcher's reflections on the attempts to interpret and to accommodate the basic tenets of RME and the teaching principles of RME in the activities.

## 7.3 Reflections on the role of teacher education in Indonesia in adapting RME (Answering Research Question 3)

As noted in Chapter 1, the reflections on basic tenets of RME and the role of teacher education in Indonesia to adapt these tenets formed an underlying concern in this study. Research question 3 stated: "How can teacher education assist Indonesian schools to adapt RME principles?". Assisting the Indonesian schools to adapt the RME principles is the underlying goal of this thesis, but it is only indirectly addressed. The thesis has assumed that RME in schools will be promoted by introducing pre-service teachers to activities designed to reflect RME teaching principles. It would also be necessary to introduce pre-service teachers to the RME theory but this has not been part of the experimental work. However, two aspects of teacher education role will be explored in this thesis. The first aspect is the extent to which the RME tenets are suitable for adaption by Indonesian teacher education in the context of revisiting and improving pre-service teachers' content and pedagogical content knowledge. Another role is the role of teacher education in acquainting pre-service teachers with RME teaching principles.

The researcher's reflections were based on the empirical work carried out in the two cycles of teaching experiments. In this respect, the interpretations of the basic tenets of RME in the design of activities of this study were limited by the early stage of adapting RME in the Indonesian context through the PMRI project and the researcher's limited experience in designing RME based teaching activities.

Section 7.3.1 will start with reflections on the extent to which the basic tenets of RME are reflected in the activities of this study and how appropriate the tenets for adaption by teacher education in Indonesia. In Section 7.3.2, reflections on the teaching principles of RME recorded in pre-service teachers' responses during the teaching experiments will be reported.

### 7.3.1 Reflections on the basic tenets of RME in activities from two cycles

This section will review the researchers' reflections on efforts to accommodate the basic tenets of RME in the design of activities and the enactment of the activities and the challenges faced during the teaching experiment over the two cycles.

- **Guided reinvention tenet**

The initial activity (Set 1 activities) in both cycles attempted to reflect the guided reinvention tenet particularly in the initial activity as reported in Chapter 4 and Chapter 5. The initial activity in cycle 1 tried to address the guided reinvention tenet for revisiting decimal place value by exploring decimal relations using the concrete model LAB and establishing the names for different pieces of LAB model based on their length. However, it was observed that partitioning into 10 shorter parts as already 'embedded' in various pieces of LAB left not much room for the interpretation of guided reinvention tenet in cycle 1. This initial activity in Set 1- Activity 1 of cycle 1 was perceived as 'too structured' from the RME perspective and did not leave much room for guided reinvention in the activities to take place.

The initial activity in cycle 1 was refined by giving more attention to better reflect the guided reinvention tenet in cycle 2. It was expected that exploring ways of partitioning one in the context of refining the unit of measurement would lead to an exploration of repeated refinement of one into ten parts, the salient characteristic of decimal system pertaining both decimals and whole numbers. However, the initial activities of cycle 2 were 'too open' and not engineered well enough. Consequently, little success was recorded with respect to the choice of partitioning one into ten.

However, this lack of success was also due to the fact that the rationale for choosing a decimal system was rather sophisticated as discussed in Section 7.2.1 and hence may not be suitable for guided reinvention. Similar difficulty in grasping the choice of decimal system was reported in Keijzer et al. (2004) study with the primary school children in the Netherlands. This demonstrates that guided reinvention route for decimals is indeed difficult for both the primary school children and pre-service teachers.

Besides the limitation of design activities in reflecting the guided reinvention tenet in this study, interpreting the guided reinvention tenet for design activities in teacher education level has a problematic face. Pre-service teachers' prior knowledge about decimals and the nature of their knowledge (which was characterised in section 6.5 as fragmented and strongly reliant on rules) impeded pre-service teachers from engaging in the activities with a fresh perspective as intended by the design. So, for example, instead of logically thinking through the situations with the intention of solving the problem presented, the pre-service teachers often tried to patch together a solution from their partially remembered rules. This presents a challenge for interpretation of guided reinvention tenet in teacher education and calls for revisiting the interpretation of the guided reinvention tenet in teacher education level in general.

- **Didactical phenomenology tenet**

The review of literature about RME in Section 2.4 has pointed out several phenomenological aspects for decimals including system of measurement (metric measures), and money in some countries. However, prior studies (see Brekke, 1996; Brousseau, 1997) pointed out the fact that the use of money context and metric measures (e.g., m and cm) might reinforce the idea of decimals as a pair of whole numbers.

Both cycles of this study employed the context of measuring length (not in metric units) as the didactical phenomenon to explore the basic notion of repeated refinement into ten (see Section 4.2.3). In this study, the phenomenon of measuring length and refining the measurement tools was explored using a linear concrete model based on length, called Linear Arithmetic Blocks (LAB) instead of the standard metric ruler. The choice of linear concrete model LAB was perceived to fit with the context of measuring length in the design of activities for this study. Moreover, the linear nature of LAB model was seen to bridge the link to the more symbolic model of the number line.

As reported in Section 4.5 and Section 5.3, pre-service teachers' prior knowledge affected the way they responded to the problems presented in the activities. For instance, instead of creating measurements using the LAB, some students measured in

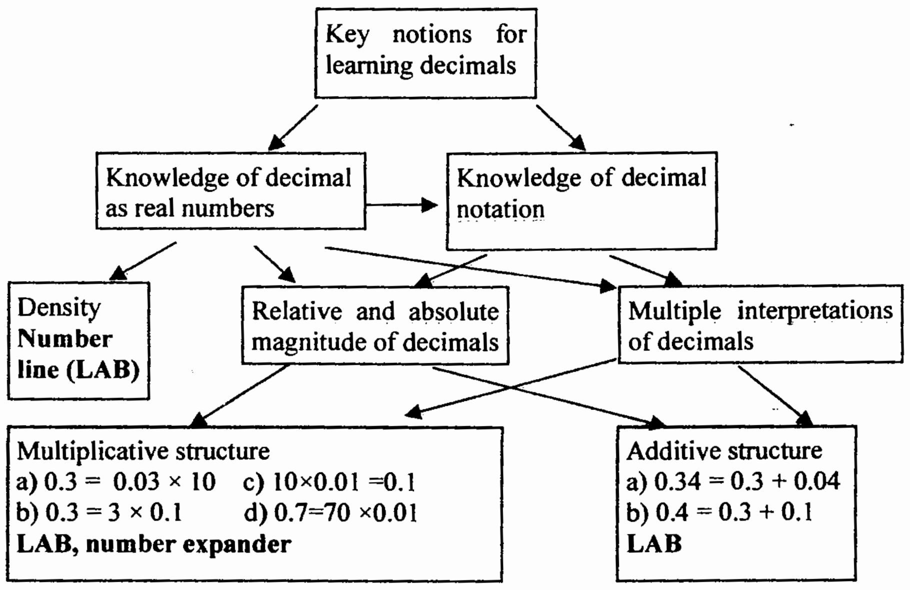
LAB, then measured LAB pieces in centimetres and then presented the requested length in centimetres. These students did not engage in the measurement task as presented in the didactic situation, because of their prior knowledge. The fragmented knowledge of pre-service teachers discussed in Chapter 6 explained the reason for pre-service teachers' tendency to jump into the conclusion for the story, rather than to engage with the reasoning processes which were intended to promote in the activities.

- **Mediating model tenet**

The mediating model of RME functions to link informal to formal knowledge. Gravemeijer (1997; 1998) differentiates a progression in the levels of use of models, (see Section 2.4.1 for detail). Students begin working in the concrete situation of the model, and they progress to using the model to answer mathematical questions, and finally they achieve the goal of working with the mathematical objects only, unattached to the model.

In this study, various models were employed based on teaching ideas adapted from earlier studies on decimals (Condon & Archer, 1999; Gravemeijer, 1998; Steinle et al., 2006). Figure 7.2 shows the main uses of the three main models: LAB, number line and number expander. As illustrated in Figure 7.2, the LAB model was useful to introduce the additive structure of decimal notation (e.g., showing 0.23 as 2 tenths placed alongside 3 hundredths pieces), and the multiplicative structure (e.g., replacing one tenth by ten hundredths). The strength of the number expander was to highlight the multiple interpretations of decimals related to unitising (e.g., by showing that 0.684 is equal to 6 tenths + 8 hundredths + 4 thousandths, but also 68 hundredths + 4 thousandths. The number line was used as the main model for thinking about density of decimals, although the concept of successive partitioning of LAB pieces into ten (beyond the physical limits of the concrete objects) is also relevant. The positive comments from the pre-service teachers reported in Chapters 4 and 5 showed that the use of these models were beneficial.

Figure 7.2: Main uses of models for learning decimals




It was realized after the reflections of cycle 2 that the use of models in both cycles of the study had not examined transitions of models at different levels well as proposed by Gravemeijer (1997; 1998) and that that the study had not purposively gathered evidence about the transitions occurring. However, some relevant observations can be made.

However, outcomes of the activities, responses in the post-tests and post-interviews of both cycles showed that the LAB models were useful in mediating the abstract notion of decimals and its interpretations. In both cycles, pre-service teachers employed the LAB models in explaining their ideas for comparing decimals and finding multiple ways of interpreting decimals. As reported in Chapter 4 and Chapter 5, few pre-service teachers who were able to relate the LAB models for making a meaningful interpretation of division of decimals by 100 (e.g., solving a division of 0.3 by 100 by replacing 3 tenths of LAB pieces with 3 thousandths pieces of LAB). Moreover, both cycles recorded a few pre-service teachers who were able to extend the use of concrete model LAB with formal algorithm such as division algorithm. However, this trend is understandable as the activities in this study did not attend to decimal operation and standard procedures such as division algorithm.

The other two models, i.e., the number expander and the number line model were perceived as more symbolic models. The number expander model was found most useful in showing various ways of interpreting decimals, which was based on the multiplicative structure of decimals. Pre-service teachers observed the advantage of the number expander model in that the number expander model was seen as a more symbolic compared to the LAB model, and required some knowledge of decimal place value. As reported in Chapter 4 and Chapter 5, the number line model was useful in revealing pre-service teachers' knowledge and difficulties with decimals particularly with negative decimals in the primary cohort. In cycle 2, activities using the number line model in the 'Number Between' game was successful in addressing density of decimals.

The fact that pre-service teachers already have acquired some knowledge of decimals at the 'formal level' leads to a tendency of pre-service teachers to skip the process and to fit in their formal knowledge of decimals, particularly based on computational skill. This presented a challenge for interpreting the mediating model tenet in the activities, and possibly more generally in teacher education. Similar difficulties were also reported by Barnes (2004) who designed an intervention activities on place value, fractions, and decimals for low attainers (Grade 10, 11, and 12) in South Africa.

### 7.3.2 Reflection on the role of RME teaching principles

Overall pre-service teachers were quite positive about the new teaching and learning style as observed in the reflection notes and anecdotal comments during the teaching experiments in the two cycles. There were two aspects of teaching and learning style in line with the instructional principles of RME (see Section 2.4.2) particularly evident in pre-service teachers' reflections as will be explained below.

**Students' contribution**

Students' active contribution in constructing their knowledge is one of the essential features of the RME teaching principles (see principle 3 in Section 2.4.2). The designed activities in this study reflected this teaching principle by placing a heavy emphasis on engaging pre-service teachers in group activities and calling for pre-service teachers to reflect on their learning process for their future teaching. The use of concrete models and hands on activities were designed to encourage pre-service teachers' active construction and contribution to revisit and improve their knowledge of decimals and its basic properties. Reflection notes reported in Section 4.5 and Section 5.4 documented that the approach in carrying out the activities was perceived as engaging and meaningful approach. Pre-service teachers' contribution was observed during the group and whole class discussion in both cycles to be at a medium to high level and more than the researcher expected. During the whole class discussion, representative of groups presented their responses to the activities in front of the classes. Other groups contributed to the whole class discussion by asking questions or presenting different solutions. However, in a few occasion, the lecturers had to facilitate pre-service teachers' contribution by encouraging them to participate in the whole class discussion. Considering previous learning style of the majority of pre-service teachers which was dominated by a passive role for students, this level of encouragement from the lecturer is reasonable and in fact necessary. With a determined effort to encourage pre-service teachers' active contribution in the learning process, the researcher believes that this principle of having students' contribution in the learning will benefit pre-service teachers and their future students.

**Interactivity**

Treffers (1987) defines interactive instruction as "instruction where is the opportunity to consult, to participate, to negotiate, to cooperate, with review afterwards and where the teacher holds back from providing explanations." (p.261). Therefore, the interactivity of learning process in the RME teaching principle (see principle 4 in Section 2.4.2) was characterized by the presence of explicit negotiation through discussion and cooperation in learning. These characteristics of learning were exercised in this study through the use of group and whole class discussions as the main mode of carrying out the activities, which promotes cooperation and negotiation of ideas in learning process. The role of lecturers in this study which was mainly as facilitator for delivering the activities and whole class discussion (see Chapter 3) served the purpose to accommodate the interactive learning.

Positive responses about the interactive nature of activities were articulated by pre-service teachers in both cycles. Moreover, pre-service teachers contrasted favourably the interactive learning approach on decimals introduced in this study with their prior learning approach, which focussed on symbolic operation with decimals and 'spoon-feeding' teaching. As documented in pre-service teachers' reflections on the activities in Section 4.5 and Section 5.3, the interactive way of learning was one part of a learning approach which they appreciated.

Overall, pre-service teachers' positive comments on less symbolic and more interactive teaching styles in line with the RME teaching and learning principles, in my opinion, indicated a promising impact of implementing the RME teaching principles in teacher education in Indonesia. This was in line with findings from prior studies (Armanto, 2002; Hadi, 2002; Widjaja & Heck, 2003) on the impact of PMRI which has been attributed in creating an active and engaging learning atmosphere in mathematics classrooms. The lecturers who participated in the study also expressed positive comments about the more interactive teaching approach and increased role of pre-service teachers' contribution. One of the practical issues for implementing learning through discussion is the size of the classes in Indonesia. In general, the class size is quite big (over 40 students per class), which might be challenging for ensuring that the intended learning goals can be achieved through group and whole class discussions. Clearly, the change of classroom and learning culture require a determined effort from both lectures and pre-service teachers involved.

This relates to the increased role of teacher education in establishing and disseminating RME in Indonesian context through PMRI project. As shown in this study, the process of adapting RME tenets in activities for teacher education faces some challenges. The efforts to interpret the basic tenets of RME in activities for pre-service teachers show that the design process to accommodate RME tenets well in the activities is quite complex, although promising in many respects.

Teacher education has played a key role to introduce and to implement RME teaching approach to primary schools in Indonesia through PMRI project (Sembiring, 2007; Zulkardi & Ilma, 2007). Overall the testing showed that attention to basic concepts of decimal numbers is needed in teacher education and that the activities assisted pre-service teachers to make good progress. This study accepts some unresolved issues regarding on how to translate and interpret some aspects of the basic tenets of RME into the design of activities on decimals in teacher education, particularly the guided reinvention tenet. However, based on the positive outcome of the activities designed in this study as reported in Chapter 6, the LIT proposed as a result of this study is recommended for revisiting decimals in teacher education. Further attention to the interpretation of guided reinvention tenet in teacher education will need special attention in the future studies.

## 7.4 Strengths and Limitations of the study

The strengths of this study can be summarized in the following points:

- This study offers insights about design and enactment of activities on decimals based on empirical studies in teacher education in Indonesia over two cycles. It revealed the nature of pre-service teachers' content knowledge and pedagogical content knowledge. Accounting for both pre-service teachers' nature of content and pedagogical content knowledge enables better design of activities in teacher education. This study was able to analyse students' thinking about decimals very deeply because it was built on extensive prior research in the area.
- Close observations in both cycles with the researcher acting as a participant observer enabled detailed monitoring of the success and lack of success of activities. This methodological aspect allowed the researcher to observe the learning and document responses to refinement of activities and research instruments between cycles and to reflect on the findings based on the observations.
- Various data sources (i.e., written tests, observations, and interviews) enable the researcher to triangulate the findings and examine the phenomena from a variety of perspectives. The availability of various data involving 258 pre-service teachers over two cycles to examine the issues investigated in this study added confidence on the results obtained from this study.
- The fact that this study operated within the time constraints of teacher education could be perceived as both limitation and strength of this study. The limited number of meeting (4-5 meetings in each class) inhibited the researcher from addressing some of limitations and difficulties experienced by pre-service

teachers observed during the teaching experiment. However, the limited number of meetings could be considered as the strength of this study because it meant that the activities were tested within the real constraints of teacher education. The fact that the lecturers in charge of the classes carried out the learning activities in both cycles was also the strength of this study as it showed that the activities have been trialled and tested in the practical setting which design researchers call "the crucible of practice" (Dede, Nelson, Ketelhut, Clarke, & Bowman, 2004; Shavelson, Phillips, Towne, & Feuer, 2003).

- Looking at RME in designing LIT on decimals in the context of teacher education rather than school education, adds a new area to the research literature and developments of RME.

The researcher acknowledges the following limitations of this study:

- Lack of success in attempts to reflect some of the RME basic tenets in this study signified the complexity of designing and engineering the RME activities in teacher education level. However, this study also has highlighted a problematic aspect of interpreting the guided reinvention tenet for designing activities on decimals in teacher education, which warrants further research and theorising.
- The fact that this study was carried out in one teacher education institution, which might not be a representative case of teacher education in Indonesia was one of the limitations of this study. Apart from its involvement with PMRI project, the researcher knows of no characteristic of this institution that would influence the results.
- The measurement of PCK in activities and only 4 items of the written tests is another limitation of this study. The data obtained from pre-service teachers' reflections on activities was not specific enough to pick up the intended aspect of PCK. Nonetheless, the fact that observation of PCK measures in this study was based on extensive data was expected to compensate on this limitation.
- The activities designed in this study focus just on decimals are not typical of RME activities, which seem to often link several areas of mathematics. In this respect, the sole focus on decimals might be considered as a limitation of the study in reflecting the teaching tenet of RME. On the other hand, the sole focus

on decimals enabled the researcher to have a more comprehensive understanding and difficulties of pre-service teachers' CK and PCK on decimals. This study has demonstrated that decimal is an important area that needed attention, which might have been missed because of the incorrect perception that decimals are easy. Decimal number is a very central topic, which has characteristics that make it applicable to other areas of number (e.g., whole numbers etc). Therefore having an LIT on decimals as a framework of reference for pre-service teachers is useful.

## 7.5 Directions and Recommendations for further study

Findings of this study signified the importance of revisiting and addressing the incomplete and fragmented knowledge of pre-service teachers about decimals. In contrary to the common perception of decimals as a simple and easy topic, this study showed that many pre-service teachers had weak knowledge of decimals which limited their teaching ideas. Positive comments on the use of concrete models and the interactive mode of learning about decimals were voiced by pre-service teachers in both cycles. Moreover, the experience of learning decimals in a less symbolic way has enriched pre-service teachers' teaching ideas about decimals for future teaching ideas in the primary school. Hence it is recommended that teacher education capitalises on teaching activities that incorporates the use of concrete models in revisiting pre-service teachers' knowledge about decimals.

One of the challenges in designing activities on decimals for teacher education is the need to attend to both content and pedagogical content knowledge. The researcher is aware of the fact that designing activities for pre-service teachers that they can directly use in schools provides a direct way to improve pre-service teachers' repertoire of teaching ideas. However, the activities that are suitable for children are often not those that will engage pre-service teachers in the experience of learning mathematics and challenge their underlying ideas. Consequently, activities that are devised to fit with pre-service teachers' knowledge such as the one designed in this study, could not be considered as the only vehicle to improve pre-service teachers' teaching ideas in the classroom. In the researcher's opinion, it would be more realistic for pre-service teachers to engage in the activities that challenge their own ideas. Moreover, it is important for pre-service teachers to learn how to adapt and interpret their own learning experience for their future teaching. Some discussion on how the experiences that the pre-service teachers have experienced would be modified for children would enhance the teacher education course. Whether it should follow the work proposed in the LIT or be part of another course needs further investigation.

The RME instructional principle that emphasizes and encourages the interactive learning process and active role of the learners is a promising approach for learning mathematics. Introducing the new style of teaching to pre-service teachers is important for the success of this national initiative. Teacher education is playing a key role in the current dissemination of PMRI because the strategy for dissemination has been through a program of strengthening the teacher education. This indicates that further research into the design of RME-inspired LITs on other topics for pre-service teachers will be useful.

The proposed LIT on decimals from this study is at an initial stage and open for adjustments, particularly with respect to reflection of the RME tenets in teacher education level. The accumulated research literature and practical experience of introducing educational reform in Indonesia and other countries should be seriously considered in planning for the widespread introduction of teaching according to RME.

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# Appendix A

Appendix A1: Trial version of Activities

Appendix A2: Conjectured LIT for cycle 1

Appendix A3: Sets of Activities in cycle 1 (Indonesian version)

Appendix A4: Refined LIT in cycle 2

Appendix A5: Sets of Activities in cycle 2 (Indonesian version)

# Appendix A1: Trial version of Activities

## Set 1
*Understanding of decimal notation*

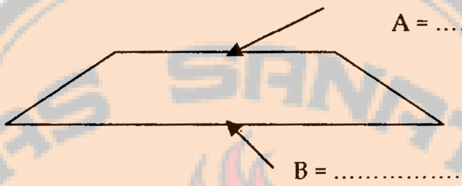
Name: ______________________

**Activities:**

1. Call the length of the longest unit as "a rod".
   Observe and explore the relationships of other pieces of LAB, and establish the labels of the other pieces.

| *Piece* | *Label* |
|---|---|
|  | a rod |
|  |  |
|  |  |
|  |  |

2. Use LAB to measure the length of the sides of the table as appear in the picture below:



A = ..............................

B = ..........................................

If you are to find the difference between the lengths of the two sides using the rods, what strategy will you use?

..........................................................................................................

..........................................................................................................

3. Match the cards with the LAB pieces and fill in the table with corresponding labels, decimal and fractional notations. From hereon, we will focus on the use of decimal notation and its verbal names

a tenth | $\frac{1}{100}$ | a hundredth | one | 0.001 | 1

0.01 | a thousandth | $\frac{1}{10}$ | 0.1 | $\frac{1}{1000}$

| ***Sketch of the pieces*** | ***Verbal Names*** | ***Decimal& Fractional Notations*** |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

4. Now look back at task 3 and express the result of your measurement in task 3 again by employing the decimal notation and its labels.

   The length of side A is ………………………..……………………. rods

   The length of side B is …………………... ……………………… rods

5. Illustrate the construction of the following length by drawing the sketch of LAB representations.

| | Sketch of representation of the decimals (need not be scaled) |
|---|---|
| 0.206 rods | |
| 0.26 rods | |
| 0.260 rods | |

6. What can you conclude from the construction process above?

………………………………………………………………………………………………………
………………………………………………………………………………………………………
………………………………………………………………………………………………………

What is the length that 6 represent in 0.26 rods?

……………………………………………………………………………………………

Is it the same length that 6 has in 2.06 rods? .
YES / NO
Why?

Is it the same that 6 has in 0.260 rods?
YES / NO
Why?

7. Rounding to the nearest

What length made only of tenths is 0.57 rods closest to?

……………………………………………………………………………………………

What length made only of tenths is 0.53 rods closest to?

……………………………………………………………………………………………

What length made only of tenths is 0.55 rods closest to?

……………………………………………………………………………………………

Imagine if you divide the thousandth of a rod into ten equal pieces, then what will you call that piece? How do you write it using decimal notation?

……………………………………………………………………………………………
……………………………………………………………………………………………

If you divide it again into ten equal smaller pieces, what will you call that piece and how do you write it using decimal notation?

……………………………………………………………………………………………
……………………………………………………………………………………………

8. Now if we want to measure things with certain length by using only hundredths pieces, please answer the following questions.

What length made only of hundredths is 0.666 closest to?

What length made only of thousandths is 1.55569 closest to?

9. Compare the following decimals and express the relations by using the sign > (larger than), < (smaller than), or = (equal). You may use the LAB to find the answers if you need to.

| | |
|---|---|
| 0.5 | 0.51 |
| 0.9 | 0.90 |
| 0 | 0.6 |

| | |
|---|---|
| 4.66 | 4.6666 |
| 2.56 | 2.56123 |
| 1.91211999 | 1.991212 |

What strategy did you use when you compared and ordered the length represented by the above decimals?

Please justify your strategy.

Name: ____________________

# Set 2

***Understanding of equivalent decimals***
***Understanding of the additive structure of decimals***
***Understanding the unitising and re-unitising***

**Activities:**

10. Use the LAB to construct a decimal 0.213. Answering the following questions might be helpful.

| |
|---|
| 0.213 = 2 tenths + ………. hundredths + ………. thousandths |
| 0.213 = 2 tenths + 0 hundredths + ………. thousandths |
| 0.213 = 0 tenths + ……… hundredths + ……… thousandths |
| 0.213 = 1 tenth + ……….. hundredths + …….. thousandths |
| 0.213 = ………. tenths + ……….. thousandths |
| 0.213 = ……… hundredths + ……….. thousandths |
| 0.213 = …………..hundredths |
| 0.213 = ………… thousandths |

11. Now can you find more ways to construct 0.213? What can you conclude?

………………………………………………………………………………………………………
………………………………………………………………………………………………………
………………………………………………………………………………………………………
…………………………………………………………………………………………………

12. Use the number expander to check your answers above.

Name: ____________________________

# Set 3

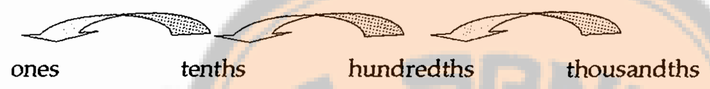
***Understanding of the multiplicative structure of decimals***

**Activities:**

13. Observe the relationships between
(a) 2 and 2 tenths
(b) 2 tenths and 2 hundredths
(c) 2 hundredths and 2 thousandths

| | | |
|---|---|---|
| 2 ones = ……... tenths .<br><br>Therefore,<br>ones ↶ tenths | 2 tenths =…….. hundredths.<br><br>Therefore,<br>tenths ↶ hundredths | 2 hundredths = …….thousandths.<br><br>Therefore,<br>hundredths ↶ thousandths |

What pattern of relationships can you find? Express that relation in the following diagram.

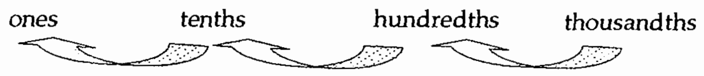



Do you think this pattern will continue the same for 2 thousandths and 2 ten-thousandths?

*Conversely,*

| | | |
|---|---|---|
| 2 tenths = ....... ones.<br><br>Therefore,<br>ones tenths | 2 hundredths = .......... tenths.<br><br>Therefore,<br>tenths hundredths | 2 thousandths = .......hundredths<br><br>Therefore,<br>hundredths thousandths |

What pattern of relationships can you find? Express that in the following diagram.




Do you think this pattern will continue the same for 2 thousandths and 2 ten-thousandths?

14. Using the observed pattern, fill in the following gaps.

| |
|---|
| 0.1 = …….. tenths = ………..hundredths = ……… thousandths |
| …… ones = …….. tenths = ………. hundredths = 11 thousandths |
| 3.07 = ……. tenths = ………. hundredths = ……… thousandths |

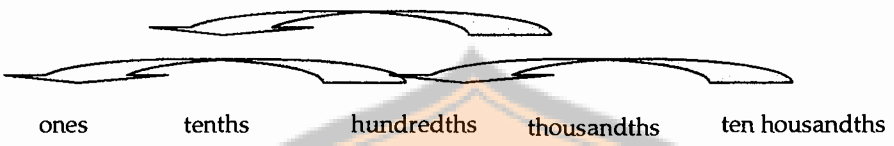
15. Observe the relationships between 2 and 2 hundredths, 2 tenths and 2 thousandths.

How many hundredths equal to 2? …… hundredths = 2.
How many thousandths equal to 2 tenths ? … thousandths = 2 tenths.
How many ten-thousandths equal to 2 hundredths? …. ten-thousandths = 2 hundredths

Therefore 2 = ………… hundredths
2 tenths = ………… thousandths
2 hundredths = ………… ten-thousandths

What pattern do you find?

- Complete this chart with the corresponding relations.

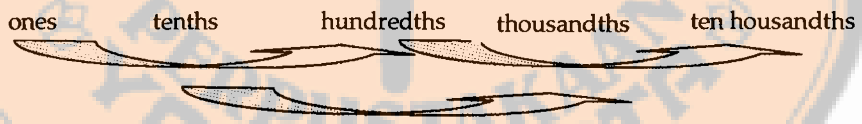



How many ones equal to 2 hundredths? …… ones = 2 hundredths.
How many tenths equal to 2 thousandths? …… tenths = 2 thousandths.
How many hundredths equal to 2 ten-thousandths? … hundredths= 2 ten-thousandths.

Therefore ………… ones = 2 hundredths
………… tenths = 2 thousandths
……………. hundredths = 2 ten-thousandths

What pattern do you find?

- Complete this chart with the corresponding relations



16. Fill out this table to express the relationships above.

| = | ones | tenths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|---|---|
| 2 ones | | | | | |
| 2 tenths | | | | | |
| 2 hundredths | | | | | |
| 2 thousandths | | | | | |
| 2 ten-thousandths | | | | | |

Name: ______________________

# Set 4

***Understanding density of decimals***

Activities:

17. Using the LAB to construct pairs of the given decimal numbers in Set A below. For each pair, please check whether you could find decimal numbers in between the pair of numbers. If yes, please write the number/s in the middle boxes.

| | |
|---|---|
| 0.9 | 1 |
| 0.1 | 0.11 |
| 0.66 | 0.666 |
| 1.21 | 1.23 |
| 1.5 | 1.51 |
| 1 | 1.001 |

Set A

18. Use the number line to locate the pair of decimal numbers given in the Set B, and discuss whether it is possible to find any number in between a given pair:

| | |
|---|---|
| 0.7501 | 0.75011 |
| 0.600 | 0.60001 |
| 2.2452 | 2.245201 |
| 0.366666 | 0.36666001 |

Set B

19. What can you conclude from working with the problems in Set A and Set B?

.......................................................................................................

.......................................................................................................

20. Can you find any decimal number that is bigger than 0.36666001? If yes, how many can you find? Name a few examples.

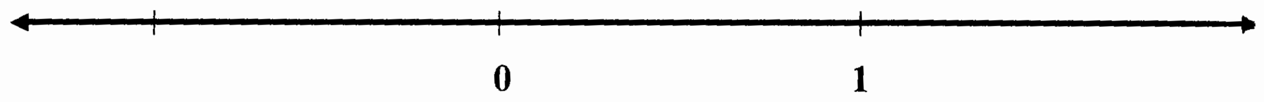
.......................................................................................................

.......................................................................................................

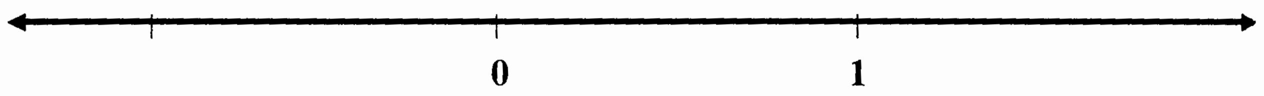
Appendix A1

**21. Locate the following numbers in the number line:**

a) 2, 2 ¼, -1 , ⅓ , 0.3333333333, 0.3334 , 2.25




b) 1.5, ⅕ , 0.21 , 1/10, 0.1, 0.010, 0.100




c) -1.65, ½ , 0.6, -0.9999, 0.9999, 0.501


Appendix A1

# Appendix A2 – Conjectured LIT for Cycle 1

**Decimals notation, additive and multiplicative structures, equivalent decimals, density of decimals**

## Goals

- **To construct meaningful understanding of decimals** by seeing the connection between decimal, fraction notation, verbal words and concrete models.
- **To understand different ways of interpreting decimals** that is to recognise the equivalence of decimals using unitising
- **Understanding the magnitude of decimals** that is to determine the order of decimals based on the understanding of place value concept and not based on the whole number rules.
- **Understanding the additive and multiplicative structures of decimals** that is to recognise that a decimal can be represented as a linear combination of powers of 10 and to recognise the base ten multiplicative structure of decimals.
- **Understanding the density of decimals** that is to recognise that there is infinitely many decimals in between a pair of decimals.

**Resources needed to teach this unit:**

- Set 1
- Set 2
- Set 3
- Linear Arithmetic Blocks (LAB)
- Number expander
- Black board
- Number line

| Planning sheet Set 1 | Time: 2 lesson meeting 200 minutes | |
|---|---|---|
| **Conjectured LIT** | | |
| **Goals & Subgoals** | **Conjectured learning paths** | **Activities** |
| 1. To understand decimals as part of a whole<br><br>To explore the decimal relationships in establishing the names for different pieces of concrete model (LAB).<br><br>To link representation of LAB pieces with its the verbal names and decimals | - Students will be able to find that the smaller pieces from partitioning the one piece into ten smaller pieces and use these decimal relationships to one in establishing the name for the LAB pieces.<br>- Important to emphasise about the unit of reference in establishing the name and notation for each piece of the model.<br>- Students will be able to match and link different notations with no difficulty. Discussion about the reasoning on how they link the decimal and fraction notation with concrete model might reveal if the process help them to create meaningful interpretation of those notation.<br>- Important to note the links between different pieces that reflects base ten chain relations (that one hundredth can be found from 1divided by 100 but also 0.1 divided by 10. | 1. Work together in a group of 4-5 to discuss and solve the following problems.<br>• We assign the longest LAB piece as a one and agree to name the length of the longest piece as one. Explore the relationships between different pieces of LAB and establish the names of the pieces (verbally).<br>(table 1 below)<br>2. Match up the following cards with the LAB pieces and fill out the following table with corresponding verbal names, fraction and decimals. From now on we will focus on the use of the decimals.<br>Cards: a tenth · one · $\frac{1}{10}$ · 0.1 · $\frac{1}{100}$ · a hundredth · 0.01 · 1 · $\frac{1}{1000}$ · 0.001 · a thousandth<br>• Discuss the relationships between different pieces of LAB that you find from your exploration.<br>(table 2 below) |

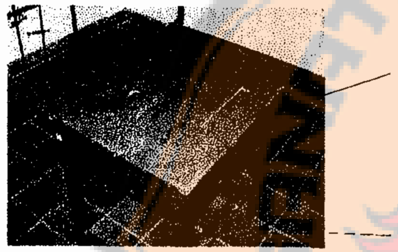
Table 1

| Pieces | Name |
|---|---|
| | one |
| | ..... |
| | ..... |
| | ...... |

Table 2

| Pieces | name | Decimal notation | Fraction notation |
|---|---|---|---|
| | .... | ..... | ... |
| | .... | ..... | ..... |
| | ..... | .... | ...... |
| | ..... | ...... | ...... |

Appendix A2

<table>
<tr>
<td>To use the notation establish in previous activity in recording the result of the measurement.<br><br>To observe and to use the extended notation of decimal and link it with the conventional decimals.</td>
<td>- In discussing the idea to use the LAB pieces to measure dimension of a table or a chair, it is expected that students will explore the additive structure of decimals as well as multiplicative structure. The multiplicative structure.<br>- Based on the finding of the first cycle, it is possible that some groups find the conversion of the length of each LAB pieces in metric measures using a ruler. If this happen then there is a need to discuss and emphasize that both approach utilize the repeated partitioning into ten smaller units.</td>
<td>3. Working in the same group, now<br>Discuss with your ideas to measure the length and width of your table using the LAB pieces with good accuracy? Explain your ideas.<br><br>4. Explain why do you choose the above approach in measuring the table and record the answers in decimals. Discuss what concepts or properties did you apply when you record the result of your measurement in decimals.<br><br><br>A = ..........................................................<br><br>B = ..........................................................</td>
</tr>
<tr>
<td>To observe the importance of place value in decimals.<br><br>To observe the additive and multiplicative structure in decimals.</td>
<td>-By sketching the LAB model that represent the decimals, students are expected to observe the additive and multiplicative structure in decimals<br>- The sketch of construction aims to encourage reflection on the activity.<br>- Discussion should emphasize on decimal place value in decimals.<br>- The teacher leads the discussion about the role of a decimal comma to note the</td>
<td>5. Sketch the representation of the following decimals. Please note that there is no need to have a precise scale. Do you think there is a unique way to sketch a decimal number? What can you conclude from this process?<br><br><table><tr><th>Decimals</th><th>Sketch of decimals</th></tr><tr><td>2.06</td><td></td></tr><tr><td>0.26</td><td></td></tr><tr><td>0.206</td><td></td></tr></table><br>• What is the value of 6 in 0.26? ..........<br>Is it the same as the value of 6 in 2.06? Why?<br>Is it the same as the value of 6 in 0.206? Why?</td>
</tr>
</table>

| | | |
|---|---|---|
| | position of a unit/one.<br>- The discussion needs to highlight the pattern between 2.06 and 0.206. | What can you tell about the role of decimal point in the decimal number?<br>• Do you observe any interesting pattern in sketching out the decimals 2.06 and 0.206? Explain |
| To use rounding to hundredths in the context of measurement using LAB model. | - By answering this question, idea about rounding to a certain unit (hundredths) can be explored. | 6. If you are asked to measure something with only the hundredth piece of LAB, how would you respond to the following questions:<br>• How many hundredth pieces are needed to find a length closest to 0.666?<br>• How many hundredth pieces are needed to find a length closest to 1.55569? |
| To explore ideas of how to help students understanding and comparing the magnitude of decimals. | - The linear nature of LAB model might help students to represent and thus compare two decimals based on length.<br>- Discuss how the students extend the model in comparing decimals with more than 3 decimal digits. Whether the models are extended or just leave behind to help students in solving this problem. | 7. What is your idea to use the LAB model to help your students in comparing the following pairs of decimals. Explain your reasons.<br>**0.9** — **0.90**<br>**0** — **0.6**<br>**1.666** — **1.66**<br>**1.912999** — **1.9912** |
| To reflect on their learning experiences and accommodate new learning experiences in their ideas for future teaching | - Focus on what new things the students learn from their experience. What do they find meaningful or less meaningful.<br>- Can they translate their experiences into their ideas for future teaching in giving meaning to decimals? | 8. Based on what you learnt last week, write your new experience about decimals? Explain your reasons.<br>9. Explain your ideas based on your experience about how to introduce decimals to primary school students. |

| Planning sheet Set 2 | Time: 1 lesson meeting 100 minutes | |
|---|---|---|
| **Conjectured LIT** | | |
| **Goals & Subgoals** | **Conjectured learning paths HLP** | **Activities** |
| To promote an understanding of different interpretation of decimals | Students are asked to explore different ways of constructing equivalent decimals. The practical limitation of the model (limited number of pieces) may not allow students to construct 0.213 as 213 thousandths. However, this relationship is important to observe. Therefore, questions that help student observe the pattern and allow them to generalize are posed. | 10. Using the LAB pieces, show how you will help students to give meaning to a decimal number 0.123 and 1.23. Try to find as many alternatives as possible and sketch your constructions. |

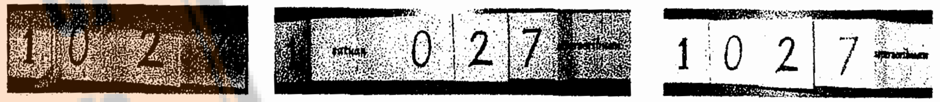
| | Sketches | How many ones | How many tenths | How many hundredths | How many thousandths |
|---|---|---|---|---|---|
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |

| | Sketches | How many ones | How many tenths | How many hundredths | How many thousandths |
|---|---|---|---|---|---|
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |

- What concepts or properties of decimals did you learn when doing the above activity?
- Is there any new teaching idea for decimal that you learn from this activity?

Appendix A2

Appendix A2

| | | |
|---|---|---|
| To promote an understanding of the additive and multiplicative structure of decimals | It is hypothesized that students might explore the non-canonical expansion, i.e., knowing for example that 0.213 = 1 tenth + 11 hundredths + 3 thousandths. | 11. Based on the above activity and your sketches, now fill out the gaps in the following problem:<br>**0.213 = ... ones+ 2 tenths + ...... hundredths + ....... thousandths**<br>**0.213 = ... ones + 2 tenth + 0 hundredths + ....... thousandths**<br>**0.213 = ... ones + 0 tenths + ... hundredths + ..... thousandths**<br>**0.213 = ... ones + 1 tenth + .... hundredths + .... thousandths**<br>**0.213 = ..... hundredths + .... thousandths**<br>**0.213 = .... hundredths**<br>**0.213 = .... thousandths** |
| | To provide links between the activity of constructing the decimal number using LAB and the symbolic representations. | <br>**Number expander model of decimal number 1.027 (Indonesian version)**<br>12. Use a number expander model to check your answers above!<br>13. What is the relationship between number expander model and the LAB model? |
| To reflect on their learning experience and build up ideas for their future teaching. | To reflect on the learning experience with the group.<br><br>To gain ideas for future teaching from reflection on their activities. | 14. Write your new learning experience about decimals in these learning activities with the group?<br>15. Write what did you learn about decimals from these activities that you could use later to teach decimals in primary school. Provide specific examples that your group can find. |

<table>
<tr><td>Planning sheet Set 3</td><td colspan="2">Time: 1 lesson meeting<br>100 minutes</td></tr>
<tr><td colspan="3">Conjectured LIT</td></tr>
<tr><td>Goals & Subgoals</td><td>Conjectured learning paths</td><td>Activities</td></tr>
<tr><td>To promote an understanding of the density of decimals.</td><td>The use of the number line to locate decimals is meant to overcome the practical limitation of concrete model. Students can be asked to first determine the length of one unit and then locate the decimals on the number line.<br><br>Discussion needs to help students understand that there are infinite numbers of decimals in between a pair of decimals.</td><td>16. Use the LAB to construct pairs of the given decimals in Set A (See below). For each pair, please check whether you could find decimals in between the pair of numbers. If yes, please name the number, and explain how you find the number or numbers.<br><br>
<table><tr><td>0.9</td><td>1</td></tr><tr><td>0.66</td><td>0.666</td></tr><tr><td>1.21</td><td>1.23</td></tr><tr><td>1.5</td><td>1.51</td></tr><tr><td>1</td><td>1.001</td></tr></table>
Set A<br><br>
17. Use the number line to locate the pairs of decimals given in the Set B, and discuss whether it is possible to find any number in between a given pair<br><br>
<table><tr><td>0.1</td><td>0.11</td></tr><tr><td>0.7501</td><td>0.7501</td></tr><tr><td>0.600</td><td>0.60001</td></tr><tr><td>2.2452</td><td>2.245201</td></tr><tr><td>0.366666</td><td>0.3666601</td></tr></table>
Set B<br><br>
18. What can you learn from working with the problems above?<br>
19. Can you find any decimal that is bigger than 0.36666601?<br>
20. Can you find any decimal that is bigger than 99.999999?</td></tr>
</table>



Appendix A2

| | | |
|---|---|---|
| To link decimals, fractions, and whole numbers on a number line. | • It is expected that in this activity confusion and misconception involving common fractions and decimals can be confronted and resolved in the discussions.<br>• It is expected that any confusion or difficulty will be captured in their reflection notes. | 21. Locate the following numbers on the same number line.<br>**a)** 2 ; 2¼ ; -1 ; $\frac{1}{3}$ ; 0.333333333 ; 0.3334 ; 2.25<br>**b)** 1.5 ; $\frac{1}{5}$ ; 0.21 ; $\frac{1}{10}$ ; 0.1 ; 0.010 ; 0.100<br>**c)** -1 ; $\frac{1}{2}$ ; 0.6 ; -0.9999 ; 0.501 ; 0 |

# Appendix A3- Sets of Activities in cycle 1 (Indonesian version)

## Kegiatan Belajar 1

Nama: ______________________
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1. Disepakati bahwa batangan "*Linear Arithmetic Blocks*" (LAB) yang terpanjang kita sebut satu. Sekarang, perhatikan hubungan antara unit-unit LAB dengan ukuran yang berbeda. Tuliskan hubungan antara batangan-batangan LAB yang berbeda ukuran tersebut satu dengan yang lain dengan memberi nama pada batangan yang lain.

| Batangan | *Nama* |
|---|---|
| | Satu |
| | |
| | |
| | |

2. Sekarang pasangkan kartu-kartu berikut dengan batangan LAB dan isilah tabel berikut dengan nama, notasi pecahan dan desimal yang bersesuaian. Mulai sekarang, kita akan memusatkan perhatian pada penggunaan notasi desimal dan label yang bersesuaian.

Seperseputuh | $\frac{1}{100}$ | Seperseratus | satu | 0,001 | 1

0,01 | Seperseribu | $\frac{1}{10}$ | 0,1 | $\frac{1}{1000}$

| *Sketsa dari batang-batang LAB* | *Nama* | *Notasi desimal* | *Notasi pecahan* |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |

3. Diskusikan dalam kelompok bagaimana gagasan anda untuk mengukur meja di dekat anda dengan menggunakan batangan tersebut sehingga diperoleh hasil yang cukup akurat? Jelaskan gagasan yang anda diskusikan tersebut.

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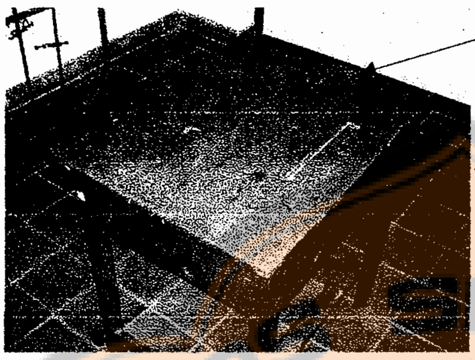
4. Ceritakan mengapa anda memilih cara pengukuran yang demikian dan tuliskan hasil pengukuran anda berdasarkan cara yang dipilih.



A = ..............................................

B = ......................................

Penjelasan

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..............................................................................................................
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5. Gambarkan konstruksi batang-batang LAB yang bersesuaian dengan notasi desimal yang diberikan di bawah ini.

| | ***Sketsa representasi dari bilangan desimal (tidak perlu sesuai skala yang akurat)*** |
|---|---|
| **2,06** | |
| **0,26** | |
| **0,206** | |

Apa yang dapat kamu simpulkan dari proses konstruksi di atas?

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Berapakah nilai dari 6 pada 0,26 ......................................................................................

Apakah sama dengan nilai dari 6 pada 2,06?
Ya/Tidak.
Mengapa?

Apakah sama dengan nilai dari 6 pada 0,206?
Ya/Tidak
Mengapa?

6. Jika kamu mengukur suatu benda dengan panjang tertentu hanya menggunakan batangan dengan ukuran seper-seratusan, jawablah pertanyaan-pertanyaan berikut ini.

Berapa panjang yang disusun dari batangan seper-seratusan yang terdekat dengan 0,666?
.........................

Berapa panjang yang disusun dari batangan seper-seratusan yang terdekat dengan 1,55569 ?
.....................

Appendix A3

7. Bagaimana gagasanmu untuk menggunakan batangan LAB untuk membantu siswa anda membandingkan desimal berikut yaitu untuk menentukan mana bilangan desimal yang terbesar atau memutuskan bahwa keduanya sama besar.

| | | |
|---|---|---|
| 0,9 | | 0,90 |
| 0 | | 0,6 |
| 1,666 | | 1,66 |
| 1,912999 | | 1,9912 |

Penjelasan:

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8. Berdasarkan pengalaman belajar minggu lalu, tuliskan pengalaman belajar baru yang anda peroleh tentang desimal!

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9. Tuliskan gagasanmu bagaimana membelajarkan tentang desimal kepada siswa SD berdasarkan pengalamanmu menyelesaikan soal minggu yang lalu.

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Nama:

## Kegiatan Belajar 2

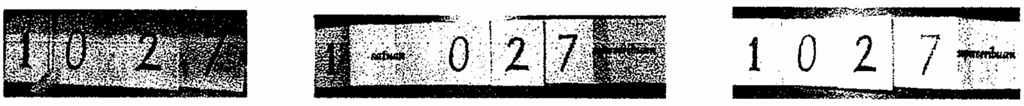
10. Dengan menggunakan batangan LAB, perlihatkan bagaimana anda membantu siswa anda untuk memberi makna pada bilangan desimal 0,123 dan 1,230. *Cobalah untuk menemukan beberapa alternatif jawaban dan gambarkan sketsa* konstruksi anda!

| | Sketsa | Berapa banyak satuan | Berapa banyak seper-sepuluhan | Berapa banyak seper-seratusan | Berapa banyak seper-seribuan |
|---|---|---|---|---|---|
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |

| | Sketsa | Berapa banyak satuan | Berapa banyak seper-sepuluhan | Berapa banyak seper-seratusan | Berapa banyak seper-seribuan |
|---|---|---|---|---|---|
| 1,230 | | | | | |
| 1,230 | | | | | |
| 1,230 | | | | | |
| 1,230 | | | | | |
| 1,230 | | | | | |
| 1,230 | | | | | |
| 1,230 | | | | | |
| 1,230 | | | | | |

11. Berdasarkan kegiatan sebelumnya (Kegiatan 1), sekarang isilah titik-titik berikut ini:

**0,213** = ….. satuan + **2** seper-sepuluhan + ….seper-seratusan +….seper-seribuan
**0,213** = …. satuan + **2** seper-sepuluhan + **0** seper-seratusan + …… seper-seribuan
**0,213** = ….. satuan + **0** seper-sepuluhan + …..seper-seratusan + ……seper-seribuan
**0,213** = …. satuan + **1** seper-sepuluhan + …… seper-seratusan + ……seper-seribuan
**0,213** = ……… seper-sepuluhan + …….. seper-seribuan
**0,213** = ………. seper-seratusan + ……… seper-seribuan
**0,213** = ………… seper-seratusan
**0,213** = ………. seper-seribuan



Pengekspansi bilangan (number expander)

12. Gunakan "number expander" (pengekspansi bilangan) untuk memeriksa jawaban anda! Bagaimana anda merespon siswa anda yang menanyakan prinsip matematis dari alat ini?

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13. Menurut anda bagaimana hubungan alat ini, pengekspansi bilangan dengan model LAB?

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……………………………………………………………………………………………
……………………………………………………………………………………………
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14. Tuliskan pengalaman anda tentang desimal yang baru dari kegiatan belajar ini bersama kelompok.

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15. Tuliskan apa yang anda pelajari tentang desimal dari kegiatan ini yang selanjutnya dapat anda gunakan untuk mengajar siswa SD belajar tentang desimal di kelas? Berikan contoh khusus yang anda temukan dalam diskusi kelompok.

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Nama:

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## Kegiatan Belajar 3

16. Untuk setiap pasangan bilangan desimal dalam Tabel A yang diberikan, carilah bilangan desimal di antara pasangan tersebut jika ada. Diskusikan dalam kelompok bagaimana cara menemukan bilangan tersebut dan berikan beberapa contoh serta gambarkan contoh yang anda temukan pada garis bilangan.

| | |
|---|---|
| 0,1 | 0,11 |
| 0,66 | 0,666 |
| 1,5 | 1,51 |
| 1 | 1,001 |

Tabel A

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Tempat untuk menggambar garis bilangan

Appendix A3

17. Sekarang gunakan garis bilangan untuk menentukan letak dari bilangan desimal yang diberikan pada tabel B. Diskusikan dengan kelompok anda, apakah mungkin mencari bilangan desimal di antara pasangan bilangan desimal tersebut.

| | |
|---|---|
| 0,7501 | 0,75011 |
| 0,600 | 0,60001 |
| 2,2452 | 2,245201 |
| 0,366666 | 0,36666001 |

Tabel B

Tempat untuk menggambar garis bilangan

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18. Apa yang dapat anda simpulkan dari soal 1 dan 2?

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19. Dapatkah anda menemukan bilangan desimal yang lebih besar dari 0,36666001? Jika ya, ada berapa banyak bilangan desimal yang anda temukan? Sebutkan beberapa contoh dari yang anda temukan!

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20. Dapatkah anda menemukan bilangan desimal yang lebih besar dari 99,9999999? Jika ya, ada berapa banyak bilangan desimal yang anda temukan? Sebutkan beberapa contoh dari yang anda temukan!

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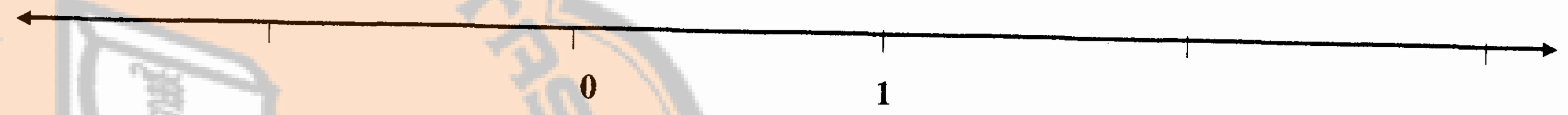
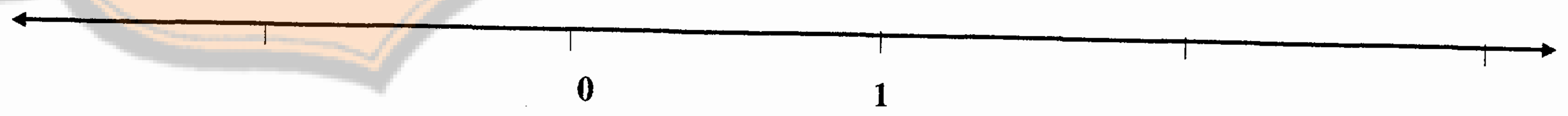
21. Tentukan letak dari bilangan berikut pada garis bilangan di bawah ini:

a) 2 ; 2¼ ; -1 ; ⅓ ; 0,3333333333 ; 0,3334 ; 2,25




b) 1,5 ; ⅕ ; 0,21 ; 1/10 ; 0,1 ; 0,010 ; 0,100


c) -1,65 ; ½ ; 0,6 ; -0,9999 ; 0,9999 ; 0,501



# Appendix A4 – Refinement of LIT from Cycle 1 to Cycle 2

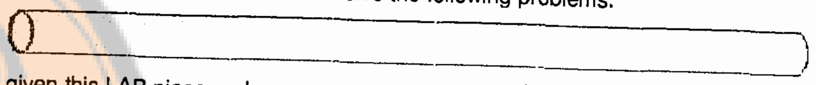
**Decimals notation, additive and multiplicative structures, equivalent decimals, density of decimals**

## Goals

- **To construct meaningful understanding of decimal** notation by seeing the connection between decimal, fraction notation, verbal words and concrete models.
- **To understand different ways of interpreting decimals** that is to recognise the equivalence of decimals using unitising.
- **Understanding the magnitude of decimals** that is to determine the order of decimals based on the understanding of place value concept and not based on the whole number rules.
- **Understanding the additive and multiplicative structures of decimals** that is to recognise that a decimal can be represented as a linear combination of powers of 10 and to recognise the base ten multiplicative structure of decimals.
- **Understanding the density of decimals** that is to recognise that there is infinitely many decimals in between a pair of decimals.

**Resources needed to teach this unit:**

- Set 1
- Set 2
- Set 3
- Linear Arithmetic Blocks (LAB)
- Number expander
- Black board
- Strings

| **Planning sheet Set 1** | **Time: 2 lesson meeting 200 minutes** | |
|---|---|---|
| **Goals & Subgoals** | **Conjectured learning paths** | **Activities** |
| 1. To understand decimals as part of a whole<br>• To explore the decimal relationships in establishing the names for different pieces of concrete model (LAB).<br>• To link representation of LAB pieces with its the verbal names and decimal notation. | - Students will be able to find that the smaller pieces from partitioning the one piece into ten smaller pieces and use these decimal relationships to one in establishing the name for the LAB pieces.<br>- Important to emphasise about the unit of reference in establishing the name and notation for each piece of the model.<br>- Students will be able to match and link different notations with no difficulty. Discussion about the reasoning on how they link the decimal and fraction notation with concrete model might reveal if the process help them to create meaningful interpretation of those notation.<br>- Important to note the links between different pieces that reflects base ten chain relations (that one hundredth can be found from 1divided by | Work together in a group of 4-5 to discuss and solve the following problems.<br><br>1. You are given this LAB piece and we agree to call it one. If you were to measure the length and width of your table with the piece, how many parts would you divide the piece into? Explain why do you choose this? What is the advantage and limitation of your choice?<br>If you want to get a more accurate result compared to the one before, what do you need to do?<br>2. Now if you need to measure the length of an eraser, how would you divide the piece?<br>3. Write the relationships among different LAB pieces and establish the names which reflect the relation to the longest LAB piece, called one. |

| Pieces | name | Decimal notation | Fraction notation |
|---|---|---|---|
| | ...... | ...... | .... |
| | ...... | ...... | ..... |
| | ...... | .... | ...... |
| | ...... | ...... | ...... |

Appendix A4

<table>
<tr><td></td><td>100 but also 0.1 divided by 10.</td><td></td></tr>
<tr><td>
<ul><li>To use the notation establish in previous activity in recording the result of the measurement.</li>
<li>To observe and to use the extended notation of decimal and link it with the conventional decimal notation.</li></ul>
</td><td>
- In discussing the idea to use the LAB pieces to measure dimension of a table or a chair, it is expected that students will explore the additive structure of decimals as well as multiplicative structure.<br><br>
- The finding of the first cycle suggests that some groups find the conversion of the length of each LAB pieces in metric measures using a ruler. If this happens then there is a need to discuss and emphasize that both approach utilize the repeated partitioning into ten smaller units.
</td><td>
4. Working in the same group, now<br>
• Discuss your ideas to measure the length and width of your table using the LAB pieces with good accuracy.<br>
• Record the result of your measurement in decimal notation and explain how did you get the result.<br><br>
<br>
A = ..........................................................<br>
B = ..........................................................
</td></tr>
<tr><td>
<ul><li>To observe the importance of place value in decimal notation.</li>
<li>To observe the additive and multiplicative structure in decimal notation.</li></ul>
</td><td>
-By sketching the LAB model that represent the decimal numbers, students are expected to observe the additive and multiplicative structure in decimal notation<br><br>
- The sketch of construction aims to encourage reflection on the activity.<br><br>
- Discussion should emphasize on decimal place value in decimal notation.
</td><td>
5. Sketch the representation of the following decimal numbers. Please note that there is no need to have a precise scale. Do you think there is a unique way to sketch a decimal number? What can you conclude from this process?<br><br>
<table>
<tr><th>Decimal numbers</th><th>Sketch of decimal numbers</th></tr>
<tr><td>2.06</td><td></td></tr>
<tr><td>0.26</td><td></td></tr>
<tr><td>0.206</td><td></td></tr>
</table><br>
6. What is the interesting pattern that you observe from constructing these decimals?<br>
7. What can you conclude from the construction activity?
</td></tr>
</table>

Appendix A4

| | | |
|---|---|---|
| | | |
| • To reflect on their learning experiences and accommodate new learning experiences in their ideas for future teaching | - Focus on what new things the students learn from their experience. What do they find meaningful or less meaningful.<br>- Can they translate their experiences into their ideas for future teaching in giving meaning to decimal notation? | 8. After your experience with LAB model in representing decimals, discuss in your group your interpretation of decimals such as 1.23456?<br>9. Do you have new ideas about how to introduce decimals in the primary schools? Include examples when possible. |

<table>
<tr><td>Planning sheet</td><td>Set 2 Activities</td><td colspan="2">Understanding density of decimals</td><td>Time: 1 lesson meeting 100 minutes</td></tr>
<tr><td>Goals & Subgoals</td><td colspan="2">Conjectured learning paths</td><td colspan="2">Activities</td></tr>
<tr><td>• To promote an understanding of the density of decimals.</td><td colspan="2">▪ The use of the number line to locate decimals is meant to overcome the practical limitation of concrete model. Students can be asked to first determine the length of one unit and then locate the decimals on the number line.<br>▪ Discussion needs to help students understand that there are infinite numbers of decimals in between a pair of decimals.</td><td colspan="2">Play Number in between games in front as a whole class activity. First, students are asked to nominate two numbers and volunteer students are asked to come in front and locate a decimal number in between the two starting numbers. (It can be played with the number line drawn on the black board or using a rope and some piece of papers with numbers proposed written on it). The same activity is repeated several times until students get an idea that they can always find a decimal number in between two decimal numbers. Later in the group they will be asked to further deepen the discussion by thinking about the number of decimal numbers in between two decimals.<br><br>10. Explain what property of decimals that your learn from playing the 'Number Between' game?<br><br>11. What are your ideas for teaching decimals based on your experience in playing the 'Number Between' game?<br><br>12. For each pair of decimals in Table A below, discuss in your group whether it is possible to find decimals in between the pairs. Explain how did you find that number, give some examples and locate them on the number line.<br><br><table><tr><td>1.5</td><td>1.51</td></tr><tr><td>0.99</td><td>0.999</td></tr><tr><td>1.7501</td><td>1.75011</td></tr></table><br>13. What can you conclude from Activity 12 above?<br><br>14. Can you find a decimal number that is larger than 0.366666001? If so, how many decimals can you find? Give few examples.</td></tr>
</table>

| | | |
|---|---|---|
| • To link decimals, fractions, and whole numbers on a number line. | • It is expected that in this activity confusion and misconception involving common fractions and decimals can be confronted and resolved in the discussions.<br>• It is expected that any confusion or difficulty will be captured in their reflection notes. | 15. Locate the following numbers on the same number line.<br><br>Discuss and explain your strategy in locating those numbers also note any confusion or difficulties that you face in solving these problems and how you resolve that confusion.<br><br>**a)** 2 ; 2¼ ; -1 ; $\frac{1}{3}$ ; 0,333333333 ; 0,3334 ; 2,25<br><br>**b)** 1,5 ; $\frac{1}{5}$ ; 0,21 ; $\frac{1}{10}$ ; 0,1 ; 0,010 ; 0,100<br><br>**c)** -1 ; $\frac{1}{2}$ ; 0,6 ; -0,9999 ; 0,501 ; 0 |

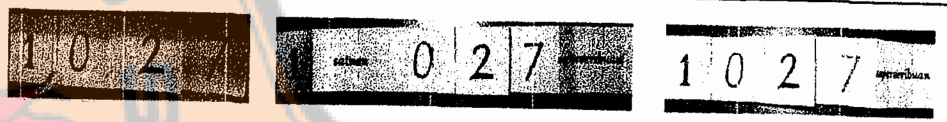
| Planning sheet | Set 3 Activities | *Understanding of equivalent decimals*<br>*Understanding of different ways of decomposing decimals* | Time: 1 lesson meeting 100 minutes |
|---|---|---|---|
| **Goals & Subgoals** | **Conjectured learning paths** | **Activities** | |
| • To promote an understanding of different interpretation of decimal notation | Students are asked to explore different ways of constructing equivalent decimals. The practical limitation of the model (limited number of pieces) may not allow students to construct 0.213 as 213 thousandths. However, this relationship is important to observe. Therefore, questions that help student observe the pattern and allow them to generalize are posed. | 16. Using the LAB pieces, show how you will help students to give meaning to a decimal number 0.123 and 1.23. Try to find as many alternatives as possible and sketch your constructions. | |

| | Sketches | How many ones | How many tenths | How many hundredths | How many thousandths |
|---|---|---|---|---|---|
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |
| 0.123 | | | | | |

| | Sketches | How many ones | How many tenths | How many hundredths | How many thousandths |
|---|---|---|---|---|---|
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |
| 1.230 | | | | | |



Appendix A4

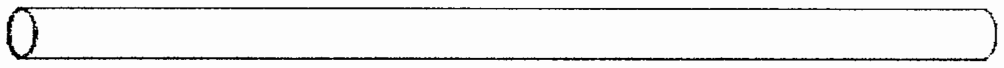
| | | |
|---|---|---|
| • To promote an understanding of the additive and multiplicative structure of decimals | It is hypothesized that students might explore the non-canonical expansion, i.e., knowing for example that 0.213 = 1 tenth + 11 hundredths + 3 thousandths. | 17. Based on the above activity and your sketches, now fill out the gaps in the following problem:<br>**Q 0.213 = ... ones+ 2 tenths + ...... hundredths + ....... thousandths**<br>**0.213 = ... ones + 2 tenth + 0 hundredths + ....... thousandths**<br>**0.213 = ... ones + 0 tenths + ... hundredths + ..... thousandths**<br>**0.213 = ... ones + 1 tenth + .... hundredths + .... thousandths**<br>**0.213 = ..... hundredths + .... thousandths**<br>**0.213 = .... hundredths**<br>**0.213 = .... thousandths** |
| • | • To provide links between the activity of constructing the decimal number using LAB and the symbolic representations. | <br>**Number expander model of decimal number 1.027 (Indonesian version)**<br>18. Explore different ways of decomposing decimal numbers using the number expander for 0.213. List all of the different ways you found.<br>19. Did you find any pattern from expanding the same decimal number? What is your conclusion? |
| • To reflect on their learning experience and build up ideas for their future teaching. | • To reflect on the learning experience with the group.<br>• To gain ideas for future teaching from reflection on their activities. | 20. Can you use LAB to illustrate the expansion of decimals as you find using the number expander?<br>21. Do you have new teaching ideas for decimals in the primary school? |

# Appendix A5 – Cycle 2 Activities (Indonesian version)

## Kegiatan Belajar 1

Nama: ______________________

1. Tuliskan gagasan kelompok anda untuk membagi batang satu sehingga anda dapat membandingkan dan menuliskan hasil pengukuran seakurat mungkin.



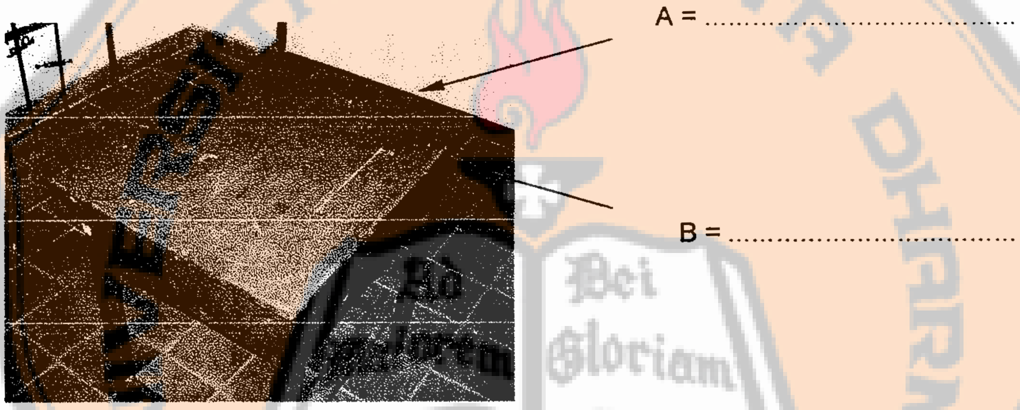
Jelaskan mengapa anda memilih pembagian yang demikian? Dalam penjelasan ini, sebutkan keunggulan dan kelemahan dari pembagian yang anda pilih.

2. Diskusikan gagasan anda jika masalah pengukuran di atas diperluas menjadi masalah pengukuran benda yang lebih pendek seperti mengukur dimensi panjang dan lebar dari karet penghapus anda. Bagaimana pembagian selanjutnya dari batangan satu yang dipakai di awal?

3. Tuliskan hubungan antara batangan-batangan LAB yang berbeda ukuran tersebut satu dengan yang lain dengan memberi nama pada batangan yang lain.

| *Batangan* | *Nama* | *Notasi* | *Hubungan* |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |

4. Sekarang tuliskan hasil dari pengukuran meja yang anda lakukan dengan menggunakan notasi desimal. Tuliskan bagaimana hasil tersebut anda dapatkan.




5. Gambarkan konstruksi batang-batang LAB yang bersesuaian denagn notasi desimal yang diberikan di bawah ini.

| Bilangan desimal | *Sketsa batang (tidak perlu sesuai skala yang akurat)* |
|---|---|
| 2,06 | |
| 0,26 | |
| 0,206 | |

6. Adakan pola menarik yang kamu temukan dalam proses konstruksi ketiga bilangan tersebut?

7. Apa yang dapat kamu simpulkan dari proses konstruksi ini?

**Refleksi**

8. *Dari proses konstruksi bilangan desimal dengan LAB, bagaimana* pemahaman atau interpretasi anda tentang bilangan desimal, misalnya 1,23456?

9. Gagasan baru apakah yang dapat anda gunakan untuk memberikan pemahaman akan bilangan desimal pada siswa SD? Berikan contoh konkrit yang muncul dalam proses diskusi kelompok anda.

Nama:

______________________

______________________

______________________

______________________

______________________

## Kegiatan Belajar 2

10. Jelaskan sifat atau konsep bilangan decimal apakah yang anda pelajari dan amati dari permainan 'Number Between' tadi?

11. Bagaimana gagasan anda untuk mengembangkan pembelajaran decimal di Sekolah Dasar berdasarkan permainan 'Number Between' tadi?

12. Untuk setiap pasangan bilangan desimal dalam Tabel A, carilah bilangan desimal di antara pasangan tersebut jika ada. Jelaskan cara anda menemukan bilangan tersebut, berikan contoh-contoh dan gambarkan pada garis bilangan.

| | |
|---|---|
| 1,5 | 1,51 |
| 0,99 | 0,999 |
| 1,7501 | 1,75011 |

Tabel A

Adakah model atau alat peraga atau permainan yang dapat pakai untuk membantu siswa menyelesaikan masalah ini?



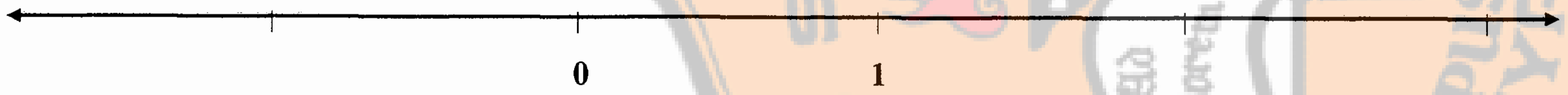
13. Apa yang dapat kamu simpulkan dari soal 12 di atas?

14. Dapatkah anda menemukan bilangand desimal yang lebih besar dari 0,36666001? Jika ya, ada berapa banyak bilangan desimal yang anda temukan? Sebutkan beberapa contoh dari yang anda temukan!

15. Tentukan letak dari bilangan berikut pada garis bilangan di bawah ini:

a) 2 ; $2\frac{1}{4}$ ; -1 ; $\frac{1}{3}$ ; 0,3333333333 ; 0,3334 ; 2,25

b) 1,5 ; $\frac{1}{5}$ ; 0,21 ; 1/10 ; 0,1 ; 0,010 ; 0,100



c) -1,65 ; ½ ; 0,6 ; -0,9999 ; 0,9999 ; 0,501

Nama: ____________________

# Kegiatan Belajar 3

16. Dengan menggunakan batangan LAB, carilah berbagai kemungkinan *mengkonstruksi dan memberi makna pada dua bilangan berikut.*

| | Sketsa | Berapa banyak satuan | Berapa banyak per-sepuluhan | Berapa banyak per-seratusan | Berapa banyak per-seribuan |
|---|---|---|---|---|---|
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |
| 0,123 | | | | | |

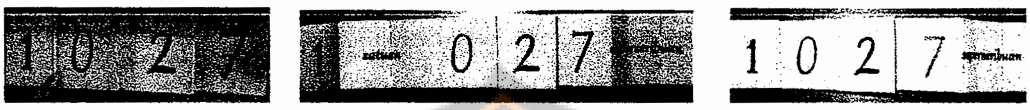
| | Sketsa | Berapa banyak satuan | Berapa banyak per-sepuluhan | Berapa banyak per-seratusan | Berapa banyak per-seribuan |
|---|---|---|---|---|---|
| 1,23 | | | | | |
| 1,23 | | | | | |
| 1,23 | | | | | |
| 1,23 | | | | | |
| 1,23 | | | | | |
| 1,23 | | | | | |
| 1,23 | | | | | |
| 1,23 | | | | | |

Appendix A5

17. Berdasarkan kegiatan sebelumnya, isilah titik-titik berikut ini:

**0,213 =** **...** satuan + 2 per-sepuluhan + ....per-seratusan +... per-seribuan
**0,213 =** **...** satuan + 2 per-sepuluhan + **0** per-seratusan + ... per-seribuan
**0,213 =** **...** satuan + **0** per-sepuluhan + .... per-seratusan +.... per-seribuan
**0,213 =** ... *satuan* + 1 *per-sepuluhan* + ... *per-seratusan* + .... *per-seribuan*
**0,213 =** ... per-sepuluhan + .... per-seribuan
**0,213 =** ... per-seratusan + ..... per-seribuan
**0,213 =** ..... per-seratusan
**0,213 =** .... *per-seribuan*

Model pengekspansi bilangan menunjukkan berbagai ekspansi bilangan desimal dalam berbagai nilai tempat yang berbeda.



Pengekspansi bilangan (number expander)

18. *Tuliskan berbagai ekspansi dari bilangan desimal* 0,213 *yang kamu temukan* dengan menggunakan model pengekspansi bilangan.

19. Apakah kamu dapat melihat pola tertentu dalam mengekspansi bilangan desimal yang sama? Apa yang dapat kamu simpulkan?

20. Dapatkah anda menggunakan model LAB untuk mengilustrasikan hasil ekspansi suatu bilangan desimal dengan menggunakan pengekspansi bilangan? Berikan contoh.

21. Adakah gagasan baru yang dapat anda pakai untuk mengajar di Sekolah Dasar?

# Appendix B

Appendix B1: Pre-test cycle 1 Part A, B, and C (English version)
Appendix B2: Post-test cycle 1 Part A, B, and C (English version)
Appendix B3: Pre-course interview cycle 1 (English version)
Appendix B4: Post-course interview cycle 1 (English version)
Appendix B5: Pre-test cycle 2 Part A, B, and C (Indonesian version)
Appendix B6: Post-test cycle 2 Part A, B, and C (Indonesian version)
Appendix B7: Pre-course interview cycle 2 (English version)
Appendix B8: Post-course interview cycle 2 (English version)
Appendix B9 Record of changes of the written tests from cycle 1 to cycle 2

# Appendix B1: Pre-test Cycle 1 Part A, B, and C (English version)

## Part A

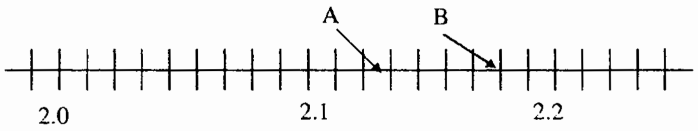
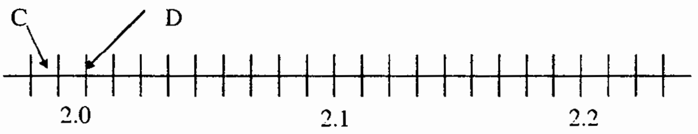
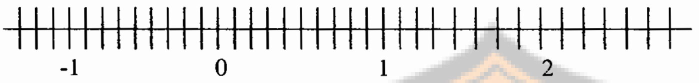
*Name:* ______________________ *Student number:*______________ *Date:* ________

**For each pair of decimals below, circle the larger decimal or write = in between!**

| | | | | | |
|---|---|---|---|---|---|
| 4.8 | 4.63 | 2.681 | 2.94 | 3.92 | 3.4813 |
| 0.74 | 0.8 | 0.41 | 0.362 | 0.374 | 0.2165 |
| 2.6 | 2.83 | 5.62 | 5.736 | 7.942 | 7.63 |
| 0.6 | 0.73 | 0.426 | 0.37 | 0.62 | 0.827 |
| 1.86 | 1.87 | 0.3 | 0.4 | 2.4 | 2.3 |
| 3.0 | 3 | 0.0 | 0 | 0.8 | 0.80000 |
| 4.08 | 4.7 | 3.72 | 3.073 | 0.3 | 0.03 |
| 3.72 | 3.07 | 8.052 | 8.514 | 0.0004 | 0.4 |
| 17.35 | 17.353 | 4.4502 | 4.45 | 0 | 0.6 |
| 4.666 | 4.66 | 3.7 | 3.77777 | 0.7 | 0.00 |

**Part B**

| Written test items | Rationale for item | Concept |
|---|---|---|
| 1. Tick the boxes that indicate the value of digit 1 in the following decimal numbers:<br>a) 9.31?<br>□ a tenth □ a hundredth □ a thousandth □ a ten thousandth<br>b) 23.001?<br>□ a tenth □ a hundredth □ a thousandth □ a ten thousandth<br>c) 5.1064?<br>□ a tenth □ a hundredth □ a thousandth □ a ten thousandth | Recognise the place value names of a decimal digit | Place value |
| 2. Fill in the gaps with as many options as you can to make this a correct statement.<br>b) 0.375 = …ones+…tenths+… hundredths +…thousandths<br>= …ones+…tenths+… hundredths +…thousandths<br>= …ones+…tenths+.. .hundredths +…thousandths<br>= …ones+…tenths+… hundredths +…thousandths<br>c) 1.025 = …ones+…tenths+… hundredths +…thousandths<br>= …ones+…tenths+… hundredths +…thousandths<br>= …ones+…tenths+….hundredths +…thousandths<br>= …ones+…tenths+… hundredths +…thousandths | Recognise the different ways of decomposing decimals | Place value. Additive and multiplicative structures of decimals |
| 3. Order the following decimals from smallest to largest:<br>a) 0 0.788 0.7821 0.8 0.80000<br>b) 0.375 0 0.0025 0.125000 0.3752 | Recognise the relative magnitude of decimals | Sequencing decimals |
| 4. Write three numbers that follows for each of sequence:<br>a) 1.5; 1; 0.5; …. ; …… ; ……<br>b) 0.092 ; 0.094 ; 0.096 ; …… ; …… ; ……. | Recognise the magnitude of decimals and sequence of decimals | Sequencing of decimal numbers |
| 5. How many decimal numbers can you find in between 3.14 and 3.15? Tick one of the options and explain briefly your reasoning<br>☐ None, because……………………………………………<br>☐ 1, namely ……………………………………………….<br>☐ Less than 200, because………………………………….<br>☐ More than 200, because ……………………………….<br><br>6. How many decimal numbers can you find in between 0.799 and 0.80? Tick one of the options and explain briefly your reasoning<br>☐ None, because……………………………………………<br>☐ 1, namely ……………………………………………….<br>☐ Less than 200, because………………………………….<br>☐ More than 200, because ………………………………. | Observe density of decimals. i.e. there are infinitely many decimals in between two decimals | Density of decimals |

| Written test items | Rationale for item | Concept |
|---|---|---|
| 7. Write the decimal numbers pointed by the arrows A and B in the corresponding boxes. Explain briefly how do you find the answers.   A = ☐ Explanation.......... B = ☐ .......... | Observe knowledge of relative magnitude of decimals on the number line | Decimals on the number line |
| 8. Write the decimal numbers pointed by the arrows A and B in the corresponding boxes. Explain briefly how do you find the answers.   C = ☐ Explanation.......... D = ☐ .......... | | |
| 9. Mark the position of the following decimal numbers in the number line: -1.1 ; -0.35 ; 1.6 ; 0.25 ; 1.05   | Recognise the position of decimals (including negative decimals) on the number line | Decimals on the number line |
| 10. Tick the decimal number which is closest to 8.0791 ☐ 8.08 ☐ 8.0917 ☐ 8.709 ☐ 8.079001 | Recognise place value in determining the closeness of two decimals | Place value. Relative magnitude of decimals |
| 11. Tick the decimal number which is closest to 0.10793 ☐ 0.11793 ☐ 0.10693 ☐ 0.10783 ☐ 0.10795 | | |
| 12. Ratna needs 0.25 hours to go to school by bus. If Ratna leaves home at 7.15 a.m. what time Ratna will arrive at school? | Recognise decimals in units of time | Multiplication and addition of decimals |
| 13. Ani wants to bake a cake for Rina's birthday. Based on the recipe, she will need 1.25 kg of flour. Because the store Ani went to shop only sell the flour in small package of 100 grams, how many packages will Ani need to buy? | Recognise rounding in up in contextual problems involving metric measurements | Multiplication and division of decimals in daily life contexts |
| 14. A truck can carry 3.8 tonnes of rice. How many kilograms of rice can the truck carry? Hint: 1 tonne = 1000 kg | Recognise decimals in metric measurements | Multiplication of decimals by 1000 |
| 15. Bayu bought 10 bottles of 1.25 litres coke for a picnic with his friends. How many litres of coke altogether did Bayu buy? | Recognise base ten chain in decimals in the metric context | Multiplication of decimals by 10 |
| 16. The distance between two cities is 12.5 km. What is the distance of those two cities on the map whose scale is 1: 100.000? With this scale. 1 distance unit represents 100.000 on the ground. | Recognise base ten chain in decimals in the metric context | Division of decimals by power of 10 |

**Part C**

| | | |
|---|---|---|
| 17. What is your idea of teaching your students to find the larger number between 0 and 0.6? | Observe teaching ideas on the meaning of decimals and their magnitude | Place value and magnitude of decimals |
| 18. How will you teach your students to divide 0.5 by 100? | Observe teaching idea on division of decimals | Division of decimals by 100 |
| 19. Responding to a problem of ordering decimal numbers 0.34; 0.33333. and 0.3; a student answered 0.3 < 0.34 < 0.33333. In your opinion, what is this student need to understand? How would you help this student to understand it? | Diagnose misconceptions in ordering decimals and teaching ideas to resolve it | Place value and relative magnitude of decimals |
| 20. How will assist your students in converting 1/3 into decimal number? | Observe ideas in linking fractions and decimals | Link between fractions and decimals |

# Appendix B2: Post-test Cycle 1 Part A, B and C (English version)

## Part A

*Name:* ____________________ *Student number:* ____________ *Date:* ________

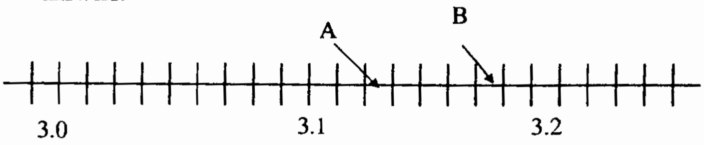
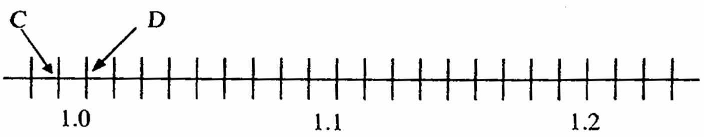
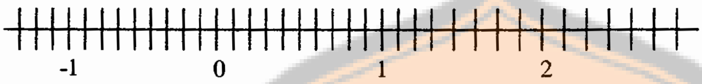
**For each pair of decimals below, circle the larger decimal or write = in between!**

| | | | | | |
|---|---|---|---|---|---|
| 1.9 | 1.74 | 1.1503 | 1.15 | 0 | 0.7 |
| 0.81 | 0.9 | 3.792 | 3.81 | 0.8 | 0.00 |
| 3.7 | 3.94 | 0.72 | 0.673 | 4.63 | 4.1924 |
| 0.7 | 0.84 | 5.73 | 5.847 | 0.481 | 0.3275 |
| 1.777 | 1.77 | 0.737 | 0.68 | 8.813 | 8.74 |
| 2.97 | 2.98 | 4.8 | 4.88888 | 0.73 | 0.938 |
| 4.0 | 4 | 0.7 | 0.6 | 0.9 | 0.90000 |
| 1.09 | 1.8 | 0.00 | 0 | 3.7 | 3.6 |
| 4.83 | 4.08 | 4.83 | 4.084 | 0.4 | 0.04 |
| 28.45 | 28.454 | 9.053 | 9.521 | 0.0006 | 0.6 |

**Part B**

| Written Test item | Rationale for item | Concept |
|---|---|---|
| 1. Tick the boxes that indicate the value of digit 3 in the following decimal numbers:<br>a) 9.31?<br>□ a tenth □ a hundredth □ a thousandth □ a ten thousandth<br>b) 23.001?<br>□ a tenth □ a hundredth □ a thousandth □ a ten thousandth<br>c) 5.1063?<br>□ a tenth □ a hundredth □ a thousandth □ a ten thousandth | Recognise the place value of each digits in decimal numbers | Place value |
| 2. Fill in the gaps with as many options as you can to make this a correct statement.<br>a) 0.753 = …ones+...tenths+... hundredths +…thousandths<br>= …ones+...tenths+… hundredths +…thousandths<br>= …ones+...tenths+.. .hundredths +…thousandths<br>= …ones+...tenths+… hundredths +…thousandths<br>b) 2.051 = …ones+...tenths+… hundredths +…thousandths<br>= …ones+...tenths+… hundredths +…thousandths<br>= ...ones+...tenths+….hundredths +…thousandths<br>= ...ones+...tenths+….hundredths +…thousandths | Recognise the different ways of representing a decimal number by using unitising and re-unitising | Place value, additive and multiplicative structures in decimals |
| 3. *Order the following decimals from smallest to largest:*<br>a) 0.40001 0.444 0.4421 0.4 0<br>b) 0.273 0 0.0013 0.11300 0.2731 | Recognise the magnitude of decimals and sequence of decimals | Sequencing of decimal numbers |
| 4. Write three numbers that follows for each of sequence:<br>a) 1.4; 1; 0.6; …. ; …… ; ……<br>b) 0.0125 ; 0.025 ; 0.0375; …… ; …… ; ……. | | |
| 5. How many decimal numbers can you find in between 2.18 and 2.19? Tick one of the options and explain briefly your reasoning<br>□ None, because…………………………………………..<br>□ 1, namely ……………………………………………..<br>□ Less than 200, because……………………………..<br>□ More than 200, because ………………………….. | Observe density property of decimals, i.e. there are infinitely many decimals in between two decimals | Density of decimals |
| 6. How many decimal numbers can you find in between 0.899 and 0.90? Tick one of the options and explain briefly your reasoning<br>□ None, because…………………………………………..<br>□ 1, namely ……………………………………………..<br>□ Less than 200, because……………………………..<br>□ More than 200, because ………………………….. | | |

Appendix B2

| Written Test item | Rationale for item | Concept |
|---|---|---|
| 7. Write the decimal numbers pointed by the arrows A and B in the corresponding boxes. Explain briefly how do you find the answers.<br><br><br>A = ☐ Explanation……………………………<br>B = ☐ ……………………………… | Locate the decimals on the number line and gain information about the thinking process | Decimals on the number line |
| 8. Write the decimal numbers pointed by the arrows A and B in the corresponding boxes. Explain briefly how do you find the answers.<br><br><br>C = ☐ Explanation……………………………<br>D = ☐ ……………………………… | | |
| 9. Mark the position of the following decimal numbers in the number line: -0.9 ; -0.45 ; 0.15; 1.4<br><br> | Recognise the position of decimals (including negative decimals) on the number line | Decimals on the number line |
| 10. Tick the decimal number which is closest to 3.0751<br>☐ 3.075001 ☐ 3.0715 ☐ 3.075 ☐ 3.751 | Recognise place value in determining the closeness of two decimals | Place value |
| 11. Tick the decimal number which is closest to 0.10793<br>☐ 0.10691 ☐ 0.107 ☐ 0.10693 ☐ 0.1069 | | |
| 12. Santi were asked by her mother to buy some 1 kg milk powder for her baby sister. The store only sell the milk in package of 400 grams, so how many packages does Santi need buy to fulfil her mother request? | Recognise decimals in time units. | Multiplication and addition of decimals |
| 13. To travel by plane from Jakarta to Yogyakarta takes 1.5 hours. If the plane leaves Soekarno Hatta airport in Jakarta at 17:45, what time will the plane arrive in Yogyakarta, assuming there is no delay in the flight schedule? | Recognise the conversion of decimals involving metric measurements | Multiplication and division of decimals in daily life contexts |
| 14. Tanti bought a package of 0.250 kg red beans in the supermarket. If one kilogram red bean costs 9900 rupiahs, how much did Tanti need to pay? | Recognise the conversion of decimals involving metric measurements | Multiplication of decimals by 1000 |
| 15. Hani bought 10 cans of 330 mills coke of different flavours for a farewell party with friends. How many litres coke in total did Hani buy? | Recognise the conversion of decimals involving | Multiplication of decimals by 10 |

Appendix B2

| Written Test item | Rationale for item | Concept |
|---|---|---|
| | metric measurements | |
| 16. The distance between two cities is 21.7 km. What is the distance of those two cities on the map whose scale is 1: 100,000? With this scale, 1 distance unit represents 100,000 on the ground. | Recognise the conversion of decimals involving metric measurements | Division of decimals by power of 10 |

## Part C

| | | |
|---|---|---|
| 17. What is your idea of teaching your students to find the larger number between 0.7777 and 0.770? | Observe ideas for teaching comparison of decimals | Place value and magnitude of decimals |
| 18. How will you teach your students to divide 0.3 by 100? | Ideas to link division of decimals with meaningful ideas for teaching it | Division of decimals by 100 |
| 19. Responding to a problem of ordering decimal numbers 0.63 ; 0.66666, and 0.6; a student answered 0.66666 < 0.63 < 0.6. In your opinion, what is the thinking behind this student's answer? How would you help this student to understand it? | Diagnose misconceptions in ordering decimals and find ideas to resolve it | Place value and magnitude of decimals |
| 20. How will assist your students in converting 1/6 into decimal number? | Observe ideas in explaining the relationship between common fractions and decimals | Common fractions and infinite recurring decimals |

# Appendix B3: Pre-course Interview Protocols Cycle 1

| *Pre-course interview items* | Rationale |
|---|---|
| 1. How did your teacher introduce the decimal number to you, for example to introduce a decimal number 0.6? Were you given examples from real life? So if you are teaching in primary school, what is your idea of teaching/introducing decimal numbers such as 0.6? Do you have any idea of using a model to help children learn decimals? | To learn pre-service teachers' prior learning experience, understanding of decimal notation and ideas for introducing decimals in their future teaching including ideas for using models. |
| 2. How were you taught on how to multiply decimals numbers by 10, for instance to do 2.05 × 10? How would you teach 2.05 × 10 to your students? If a student does not understand your explanation, how would you help them? If your students still have problems in understanding that, what will you do? | To gain insights about prior schooling experience shapes pre-service teachers' teaching ideas on multiplication of decimals. It allows the researcher to learn if knowledge on multiplication of decimals is based on place value concept or more on following standard procedures. Observe ideas on the same topic for teaching. |
| 3. How was your experience in learning addition of two decimals, for example how did your teacher teach you to add 1.8 and 1.31? Is the reason for lining up the decimal comma explained to you? What are your ideas to teach addition of decimals in the future? | To gain insights about pre-service teachers' knowledge on addition of decimals, whether their knowledge of lining up the decimal comma is related to underlying notion of place value. Observe teaching ideas on addition of decimals. |
| 4. How did your teacher teach about rounding of decimal numbers for example to round the number 23.4128 (or 23.4189) to two digits decimal numbers? How would you teach rounding for your future students? Do you have any new idea or use any model to teach rounding? | To gain insights into pre-service teachers' knowledge about rounding rules. Observe pre-service teachers' teaching ideas on rounding.. |
| 5. Could you identify three models that you can or may use in helping students learn about decimals in primary school? Explain how each model assists you to understand a particular concept of decimals better? | To inspect pre-service teachers ideas about the role of models in teaching and learning decimals. |
| 6. Would you explain how did you get this answer? Can you think of another way to solve this problem? (ask as follow up questions to items with error in the pre-test. | These questions will confirm/investigate further the thinking involved in solving problems. Question will vary for different interviewees depend on their responses in the pre-test. |

# Appendix B4: Post-course Interview Protocols Cycle 1

| Post-course interview items | Rationale |
|---|---|
| 1. Could you identify three models that you can use in teaching and learning decimals? How would you rank those models according to their level of difficulty in helping students to understand decimals from the easiest to the most difficult one? | To inspect pre-service teachers ideas about the role of models in teaching and learning decimals. |
| 2. Which part of the learning activities did you find difficult to understand? Explain! | To gain feedback about the learning activities and ideas for improvement. |
| 3. Is there any concept in decimals that you have a problem or difficulty with before and get a clearer picture afterwards or the opposite (some concepts that become less clear or you get confused after following the learning activities)? | To encourage self-evaluation & reflection about their evolution of understanding on aspects of decimal numeration. |
| 4. Did you find any change from participating in the pre and post-test after the learning activities? Any particular difficulty in solving any post-test item? | To encourage self-diagnostic of own progress in performance in pre to post-test. Need to elucidate which factors/ learning activities contribute to their progress. |
| 5. Do you have a new idea of models to help students learn decimals in the primary school? | To explore more ideas for the role of models in learning and teaching decimals. |
| 6. Do you have any feedback to improve the learning activities or the test items? | To gain feedbacks on the learning activities and test items |
| 7. What is your idea to help children find the decimals of 1/6? | To gain indication of ideas to link and translate their understanding of decimal notation and models to standard algorithm (long division). |
| 8. Would you explain how did you get this answer? *Can you think of another way to solve this problem?* (ask as follow up questions to items with error in their responses in the post-test. | To confirm/ investigate further the thinking involved *in solving problems. Question will vary for different* interviewees depend on their responses in the post-test. |

# Appendix B5 - Pre-test cycle2 (Indonesian version)

**Bagian A**

*Nama:* ____________ *Nomor mahasiswa:* ____________ *Tanggal:* ____________

**Petunjuk: Untuk setiap pasangan bilangan di bawah ini, lingkarilah bilangan yang terbesar atau tuliskan tanda = diantaranya!**

| | | | | | |
|---|---|---|---|---|---|
| 4,8 | 4,63 | 2,681 | 2,94 | 3,92 | 3,4813 |
| 0,74 | 0,8 | 0,41 | 0,362 | 0,374 | 0,2165 |
| 2,6 | 2,83 | 5,62 | 5,736 | 7,942 | 7,63 |
| 0,6 | 0,73 | 0,426 | 0,37 | 0,62 | 0,827 |
| 1,86 | 1,87 | 0,3 | 0,4 | 2,4 | 2,3 |
| 3,0 | 3 | 0,0 | 0 | 0,8 | 0,80000 |
| 4,08 | 4,7 | 3,72 | 3,073 | 0,3 | 0,03 |
| 3,72 | 3,07 | 8,052 | 8,514 | 0,0004 | 0,4 |
| 17,35 | 17,353 | 4,4502 | 4,45 | 0 | 0,6 |
| 4,666 | 4,66 | 3,7 | 3,77777 | 0,7 | 0,00 |

# Bagian B

*Nama:* ________________ __________________________ *Nomor mahasiswa:*______________

*Program Studi:* ______________________________________________________________

1. Berilah tanda silang pada kotak yang tersedia untuk menandakan nilai dari 1 pada bilangan desimal berikut:

| | | | | | |
|---|---|---|---|---|---|
| a) | 9,31? | ☐ seper-sepuluh | ☐ seper-seratus | ☐ seper-seribu | ☐ seper-sepuluhribu |
| b) | 23,001? | ☐ seper-sepuluh | ☐ seper-seratus | ☐ seper-seribu | ☐ seper-sepuluhribu |
| c) | 5,1064? | ☐ seper-sepuluh | ☐ seper-seratus | ☐ seper-seribu | ☐ seper-sepuluhribu |
| d) | 2,318? | ☐ seper-sepuluh | ☐ seper-seratus | ☐ seper-seribu | ☐ seper-sepuluhribu |

2. Isilah titik-titik di bawah sehingga didapat pernyataan yang benar sebanyak mungkin anda bisa:

0,375 = ........ satu + ......... sepersepuluh + ......... seperseratus + ........ seperseribu
0,375 = ........ satu + ......... sepersepuluh + ......... seperseratus + ........ seperseribu
0,375 = ........ satu + ......... sepersepuluh + ......... seperseratus + ........ seperseribu
0,375 = ........ satu + ......... sepersepuluh + ......... seperseratus + ........ seperseribu

3. Tuliskan notasi desimal untuk masing-masing bilangan berikut:

a) 2 satuan + 6 persepuluhan + 15 perseratusan + 3 perseribuan = .......... ..................
b) 0 satuan + 7 persepuluhan + 1 perseratusan + 12 perseribuan = .......... ..................

4. Tuliskan tiga bilangan selanjutnya untuk setiap barisan bilangan berikut:

a) 1,092 ; 1,094 ; 1,096 ; ............... ; .................. ; ..............
2. 1,125 ; 1,25 ; 1,375 ; ............. ; .................. ; ...............

5. Berapa banyak bilangan desimal antara 3,14 dan 3,15? Berilah tanda silang pada salah satu pilihan berikut dan jelaskan secara singkat alasan anda.

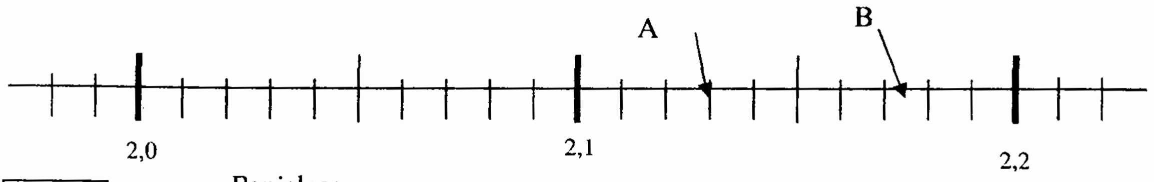
☐ Tidak ada, karena.............................................................................................
☐ 1, yaitu ..........................................................................................................
☐ Kurang dari 200, karena ...................................................................................
☐ Lebih dari 200 tapi berhingga banyaknya karena.................................................
☐ Tak berhingga banyaknya ..................................................................................

6. Berapa banyak bilangan desimal di antara 0,799 dan 0,80? Berilah tanda silang pada salah satu pilihan berikut dan jelaskan secara singkat alasan anda

☐ Tidak ada, karena ............................................................................................
☐ 1, yaitu ..........................................................................................................
☐ Kurang dari 200, karena ...................................................................................
☐ Lebih dari 200 tapi berhingga banyaknya karena ................................................
☐ Tak berhingga banyaknya....................................................................................

Appendix B5

7. Tuliskan bilangan desimal yang bersesuaian pada kotak yang tersedia.




A = ☐

B = ☐

Penjelasan

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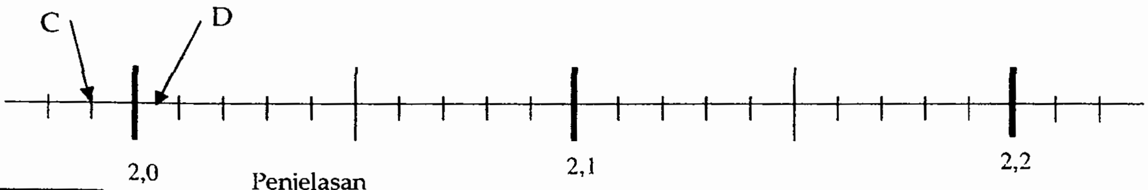
8. Tuliskan bilangan desimal yang bersesuaian pada kotak yang tersedia.




Penjelasan

C = ☐

D = ☐

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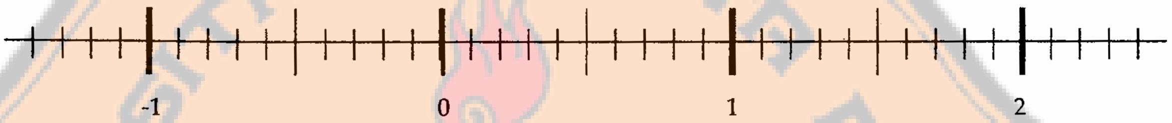
9. Berilah tanda panah untuk menunjukkan posisi bilangan desimal berikut pada garis bilangan: -1,2 ; -0,5 ; 1,6 ; 0,25.




10. Berilah tanda silang pada bilangan desimal yang terdekat dengan 8,0791

☐ 8,08 ☐ 8,0917 ☐ 8,709 ☐ 8,079001

Jelaskan bagaimana anda mendapat jawaban ini!

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11. Berilah tanda silang pada bilangan desimal yang terdekat dengan 0,55

☐ 0,56 ☐ 0,551 ☐ 0,6 ☐ 0,5511

Jelaskan bagaimana anda mendapat jawaban ini!

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Appendix B5

12. Ratna membutuhkan waktu 0,25 jam untuk berangkat ke sekolah dengan menggunakan bis. Jika Ratna berangkat dari rumah pukul 7:45 pagi, pukul berapakah Ratna akan tiba di sekolah?

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.......................................................................................................
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13. Ani ingin membuat kue untuk ulang tahun sahabatnya Nina. Berdasarkan buku resep yang dipakainya, untuk membuat satu loyang kue, Ani memerlukan 1,25 kilogram tepung. Karena toko tempat Ani berbelanja hanya menjual tepung dalam kemasan 100 grams, berapa banyak kemasan yang perlu dibeli oleh Ani?

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14. Bayu membeli 10 botol coca-cola kemasan 1,25 liter untuk acara piknik bersama teman-teman. Berapa liter keseluruhan coca-cola yang dibeli oleh Bayu?

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## Bagian C

15. Jelaskan gagasan anda mengajarkan siswa di Sekolah Dasar menemukan bilangan *yang lebih besar dari 0.8 dan 0.8888. Sebutkan alat peraga atau model yang dapat anda* gunakan dalam gagasan anda (jika ada).

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16. Saat ditanya bagaimana membagi 0,5 dengan 100, seorang siswa menjelaskan dengan menggeser koma sebanyak dua angka desimal sehingga mendapat 0,005.
   a) Apakah menurut anda ide siswa tersebut benar? Mengapa?
   b) Jika anda berpikir bahwa penting bagi siswa untuk dapat menalar gagasan daripada sekedar mengingat rumus, bagaimana gagasan anda untuk menjelaskan 0,5 dibagi 100 dengan cara yang lain?

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.................................................................................................................................
.................................................................................................................................

17. Menanggapi permasalahan mengurutkan bilangan desimal 0,34 ; 0,33333 dan 0,3 ; seorang siswa menjawab demikian $0,3 < 0,34 < 0,33333$.
   a) Menurut pendapatmu, gagasan apa yang membuat siswa menjawab demikian?
   b) Bagaimana idemu untuk membantu siswa ini untuk mengerti hal tersebut?

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18. Ketika diminta untuk menyelesaikan $\frac{1}{3} \times 100000$, Tata menjawab $\frac{1}{3} \times 100000 = 33000$.
   a) Apakah anda setuju dengan jawaban Tata?
   b) Bagaimana gagasan anda sebagai guru membantu Tata?

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.................................................................................................................................

*Terima kasih*

Appendix B5

## Appendix B6: Post-test cycle 2 Part A, B and C (Indonesian version)

### Bagian A

*Nama:* ____________ *Nomor mahasiswa:* ____________ *Tanggal:* ________

**Petunjuk: Untuk setiap pasangan bilangan di bawah ini, lingkarilah bilangan yang terbesar atau tuliskan tanda = diantaranya!**

| | | | | | |
|---|---|---|---|---|---|
| 1,9 | 1,74 | 1,1503 | 1,15 | 0 | 0,7 |
| 0,81 | 0,9 | 3,792 | 3,81 | 0,8 | 0,00 |
| 3,7 | 3,94 | 0,72 | 0,673 | 4,63 | 4,1924 |
| 0,7 | 0,84 | 5,73 | 5,847 | 0,481 | 0,3275 |
| 1,777 | 1,77 | 0,737 | 0,68 | 8,813 | 8,74 |
| 2,97 | 2,98 | 4,8 | 4,88888 | 0,73 | 0,938 |
| 4,0 | 4 | 0,7 | 0,6 | 0,9 | 0,90000 |
| 1,09 | 1,8 | 0,00 | 0 | 3,7 | 3,6 |
| 4,83 | 4,08 | 4,83 | 4,084 | 0,4 | 0,04 |
| 28,45 | 28,454 | 9,053 | 9,521 | 0,0006 | 0,6 |

# Bagian B

*Nama:* ________________ __________________________ *Nomor mahasiswa:*______________

*Program Studi:* ____________________________________________________________________

1. Berilah tanda silang pada kotak yang tersedia untuk menandakan nilai dari 1 pada bilangan desimal berikut:

| | | | | | |
|---|---|---|---|---|---|
| a) | 9,31? | ☐ seper-sepuluh | ☐ seper-seratus | ☐ seper-seribu | ☐ seper-sepuluhribu |
| b) | 23,001? | ☐ seper-sepuluh | ☐ seper-seratus | ☐ seper-seribu | ☐ seper-sepuluhribu |
| c) | 5,1064? | ☐ seper-sepuluh | ☐ seper-seratus | ☐ seper-seribu | ☐ seper-sepuluhribu |
| d) | 2,318? | ☐ seper-sepuluh | ☐ seper-seratus | ☐ seper-seribu | ☐ seper-sepuluhribu |

2. Isilah titik-titik di bawah sehingga didapat pernyataan yang benar sebanyak mungkin anda bisa:

   0,375 = ........ satu + ......... sepersepuluh + ......... seperseratus + ........ seperseribu
   0,375 = ........ satu + ......... sepersepuluh + ......... seperseratus + ........ seperseribu
   0,375 = ........ satu + ......... sepersepuluh + ......... seperseratus + ........ seperseribu
   0,375 = ........ satu + ......... sepersepuluh + ......... seperseratus + ........ seperseribu

3. Tuliskan notasi desimal untuk masing-masing bilangan berikut:

   a) 2 satuan + 6 persepuluhan + 15 perseratusan + 3 perseribuan = .......... ................
   b) 0 satuan + 7 persepuluhan + 1 perseratusan + 12 perseribuan = .......... ................

4. Tuliskan tiga bilangan selanjutnya untuk setiap barisan bilangan berikut:

   a) 1,092 ; 1,094 ; 1,096 ; ............... ; .................. ; ..............
   b) 1,125 ; 1,25 ; 1,375 ; ............. ; ................... ; ..............

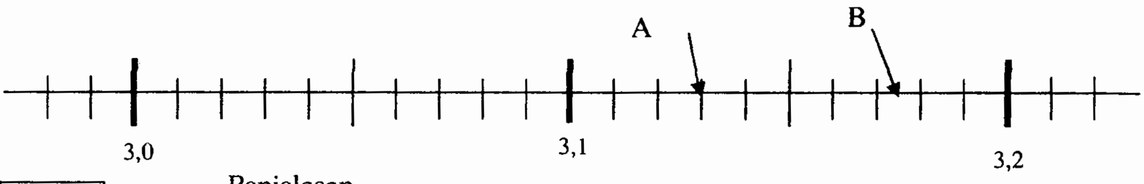
5. Berapa banyak bilangan desimal antara 3,14 dan 3,15? Berilah tanda silang pada salah satu pilihan berikut dan jelaskan secara singkat alasan anda.

   ☐ Tidak ada, karena.........................................................................................
   ☐ 1, yaitu ....................................................................................................
   ☐ Kurang dari 200, karena ..................................................................................
   ☐ Lebih dari 200 tapi berhingga banyaknya karena.................................................
   ☐ Tak berhingga banyaknya ...................................................................................

6. Berapa banyak bilangan desimal di antara 0,799 dan 0,80? Berilah tanda silang pada salah satu pilihan berikut dan jelaskan secara singkat alasan anda

   ☐ Tidak ada, karena ........................................................................................
   ☐ 1, yaitu ....................................................................................................
   ☐ Kurang dari 200, karena ..................................................................................
   ☐ Lebih dari 200 tapi berhingga banyaknya karena .................................................
   ☐ Tak berhingga banyaknya...................................................................................

7. Tuliskan bilangan desimal yang bersesuaian pada kotak yang tersedia.




A = ☐

Penjelasan

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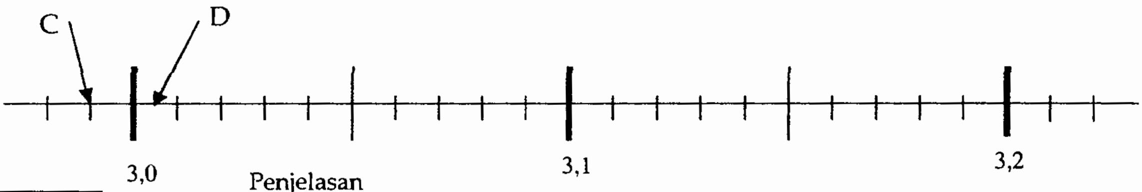
B = ☐

8. Tuliskan bilangan desimal yang bersesuaian pada kotak yang tersedia.




Penjelasan

C = ☐

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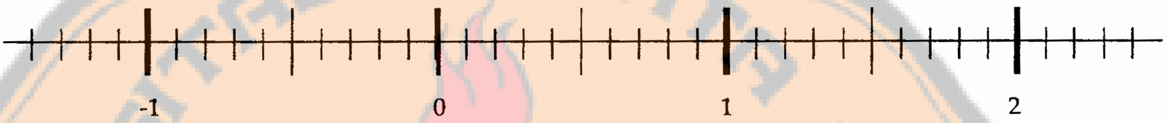
D = ☐

9. Berilah tanda panah untuk menunjukkan posisi bilangan desimal berikut pada garis bilangan: -1,3 ; -0,35 ; 1,4 ; 0,75.




10. Berilah tanda silang pada bilangan desimal yang terdekat dengan 3,0751

☐ 3,075001 ☐ 3,0715 ☐ 3,075 ☐ 3,751

Jelaskan bagaimana anda mendapat jawaban ini!

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11. Berilah tanda silang pada bilangan desimal yang terdekat dengan 0,10692

☐ 0,10691 ☐ 0,107 ☐ 0,10693 ☐ 0,1069

Jelaskan bagaimana anda mendapat jawaban ini!

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Appendix B6

12. Santi diminta ibu membeli susu bubuk formula bayi Entamil untuk adiknya sebanyak 1 kg. Kebetulan kemasan susu 1 kg habis dan hanya tersedia kemasan 400 gram, berapa banyak kemasan susu yang harus dibeli Santi agar cukup sesuai pesanan ibu?
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..........................................................................................................
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13. Diketahui lama perjalanan dari Jakarta ke Yogyakarta memerlukan waktu selama 1,25 jam. *Jika pesawat meninggalkan bandara Soekarno Hatta Jakarta pada pukul 17:45,* pukul berapakah pesawat tersebut akan tiba di Yogyakarta?

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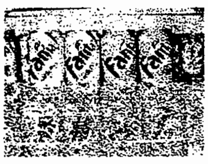

14. Hani membeli 100 kaleng fanta kemasan 330 mililiter dengan aneka rasa untuk pesta perpisahan bersama teman-teman. Berapa liter keseluruhan fanta yang dibeli oleh Hani jika diketahui 1 liter = 1000 mililiter.
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Appendix B6

## Bagian C

15. Jelaskan gagasan anda mengajarkan siswa di Sekolah Dasar menemukan bilangan ynag lebih besar dari 0,7777 dan 0,770. Sebutkan alat peraga atau model yang dapat anda gunakan dalam gagasan anda (jika ada).

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16. Saat ditanya bagaimana membagi 0,3 dengan 100, seorang siswa menjelaskan dengan menggeser koma sebanyak dua angka desimal sehingga mendapat 0,003.
    a) Apakah menurut anda ide siswa tersebut benar? Mengapa?
    b) Jika anda berpikir bahwa penting bagi siswa untuk dapat menalar gagasan daripada sekedar mengingat rumus, bagaimana gagasan anda untuk menjelaskan 0,3 dibagi 100 dengan cara yang lain?

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17. Menanggapi permasalahan mengurutkan bilangan desimal 0,63 ; 0,66666 dan 0,6 ; seorang siswa menjawab demikian 0,66666 < 0,63 < 0,6.
    a) Menurut pendapatmu, gagasan apa yang membuat siswa menjawab demikian?
    b) Bagaimana idemu untuk membantu siswa ini untuk mengerti hal tersebut?

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18. Ketika diminta untuk menyelesaikan $\frac{1}{7} \times 100000$, Tata menjawab $\frac{1}{7} \times 100000 = 14000$.
    a) Apakah anda setuju dengan jawaban Tata?
    b) Bagaimana gagasan anda sebagai guru membantu Tata?

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# Appendix B7: Pre-course Interview Protocols Cycle 2

| Pre-course interview items | Rationale |
|---|---|
| 1. How do you understand a decimal number, for instance 1.05? What is your idea to introduce decimal topic to your students? Do you have any idea of models for teaching decimals? | To gain insights about the meaning of decimal notation and ideas for teaching decimal numbers. It also stress on ideas about models in teaching decimals. |
| 2. In answering a question to compare two decimals 1.66666 and 1.66, a student said that 1.66666 = 1.66. Do you agree with this answer? Why do you think she answer that way? What is your idea to resolve this problem? Can you think of any model that will be helpful in addressing/resolving this problem? | Reveal and challenge rounding/truncating misconception. Also address the pedagogical aspect of resolving misconception involving repeating digits. |
| 3. Could you identify three models that you can or may use in helping students learn about decimals in primary school? Explain how each model assists you to understand a particular concept of decimals better? | To inspect pre-service teachers ideas about the role of models in teaching and learning decimals. |
| 4. Would you explain how did you get this answer? Can you think of another way to solve this problem? (Asking questions to probe thinking in incorrect solutions or interesting solutions in the pre-test) | These questions will confirm/investigate further the thinking involved in solving problems. Questions will vary for different interviewees depend on their responses in the pre-test. |

# Appendix B8: Post-course Interview Protocols Cycle 2

| Post-course interview items | Rationale |
|---|---|
| 1. Could you identify three models that you can use in teaching and learning decimals? How would you rank those models according to their level of difficulty in helping students to understand decimals from the easiest to the most difficult one? | To inspect pre-service teachers ideas about the role of models in teaching and learning decimals. |
| 2. Is there any concept in decimals that you have a problem or difficulty with before and get a clearer picture afterwards or the opposite (some concepts that become less clear or you get confused after following the learning activities)? | To encourage self-evaluation & reflection about their evolution of understanding on aspects of decimal numeration. |
| 3. Did you find any change from participating in the pre and post-test after the learning activities? Any particular difficulty in solving any post-test item? | To encourage self-diagnostic of own progress in performance in pre to post-test. Need to elucidate which factors/ learning activities contribute to their progress. |
| 4. In answering a question to compare two decimals 1.66666 and 1.66, a student said that 1.66666 = 1.66. Do you agree with this answer? Why do you think she answer that way? What is your idea to resolve this problem? Can you think of any model that will be helpful in addressing/resolving this problem? | To reveal and challenge rounding/truncating misconception<br>To address the pedagogical aspect of resolving misconceptions involving decimals with repeated digits. |
| 5. Would you explain how did you get this answer? Can you think of another way to solve this problem? (Asking questions to probe thinking in incorrect solutions or interesting solutions in the post-test). | To confirm/ investigate further the thinking involved in solving problems. Question will vary for different interviewees depend on their answers in the post-test. |

# Appendix C

Appendix C1: Excerpts from trial of Set 1 activities with volunteer pre-service teachers in Melbourne

Appendix C2: Samples of pre-course interview from cycle 1

Appendix C3: Samples of post-course interview from cycle 1

Appendix C4: Table of responses to Initial Activity Set 1 Activity 1 in cycle 2

# Appendix C1: Excerpts from Trial of Activities with Volunteer Pre-service Teachers in Melbourne on exploring the way to label LAB pieces

## Activity 1 - Wednesday March 30, 2005

If we agree to name this longest piece a rod, how would you label the other pieces to reflect the relationships among those pieces to a rod.

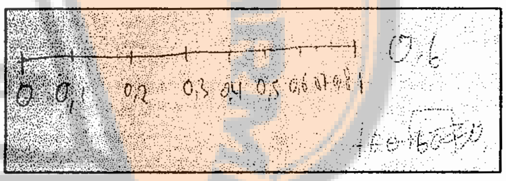
| Students | |
|---|---|
| J | Do we have to name the other pieces? |
| M | We have to make up the names in relation to the rod. |
| R | It is not exactly one meter, yeah we don't know. Just call this a rod |
| M | A deci |
| J | But doesn't go with a rod, right? |
| M | Ehm, yeah |
| J | A straw… a rod.. a… a |
| M | So what we are going to. So these will be a little, mini littles aren't they? |
| | We need to observe the relationships |
| J | Can we write on this? That's okay. So I'm going to write one tenth of a rod. |
| R | Yeap, that's good. |
| J | And how many are |
| M | There'll be a tenth of this, so a hundred of a rod |
| L | I think a hundredth , a hundredth of the smaller pieces |
| J | So a hundredth |
| M | This will be a thousandth |
| J | Thousandths |
| M | That's why they make up |
| J | We need to give them names ….a rod… a length |
| R | I think that can be considered a name, just like this one yeah |
| J | Oh that can be considered name, yeah |
| L | Can be considered name yeah |
| M | So we don't have to think of decis, milis, so we don't have to worry |
| R | If you want to write it like that is also another option |
| M | I said, a milli, decis, and …what did I say for hundredths? |
| R | Decis, millis, I miss the other one. |
| M | Decis, milli, and a centi |
| J | I don't understand your thinking but that's okay. No yeah |
| M | I am just trying to work what's for a hundredth |
| R | So okay |
| J | We could call the smallest one a splinter. |
| R | Okay, the next thing, we are going to use these pieces to measure the length of the sides of the table. |
| J | Oh okay |
| L | This A, this is B right |
| R | You can also use this one as a connector |
| J | Which table are we going to measure? That one |
| M | May be you want to measure the other side |
| J | Which table are we going to measure? |
| L | This one |

| | |
|---|---|
| R | May be that one |
| L | It is the same |
| R | Yeah, it is the same, you are right |
| J | Okay, sorry…. one rod, we've got we need to gave them a name |
| L | A name |
| J | We'll give the second one , a stick |
| M | Or a deci, I've got deci, centi and mili |
| J | a stick |
| L | Yeah, This one can be |
| M | Eh Just said what she said a tenth, just call it a tenth, |
| J | Makes it easier if every piece has a name |
| L | Yeah |
| J | We call the smallest one a splinter, what should the middle one be? A stick |
| L | Looks like pasta. What do you call for that… the snack for breakfast. |
| J | The snackfood. A chessle, we can call it a chessle |
| L | How do you spell it |
| J | CHEESS |
| M | No CHEEZLE |
| J | CHEEZLE. Okay, so |
| J | The last one is a SPLINTER |
| L | So how do you spell it…. a SPLINTER |
| J | Okay so we've got |
| J | It is a rod plus a stick |

# Appendix C2:
# Samples of Pre-course Interviews cycle 1 grouped by questions

Question 1 — How did your teacher introduce the decimals to you for the first time, for instance in introducing the decimals 0.6? Were you given examples from real life?

| Student | class | Responses |
|---|---|---|
| susilo | prim A | So if you are teaching in primary school, what is your idea of teaching/ introducing decimal numbers such as 0.6 or 0.9? Do you have an idea of using a model to help children learn decimals? |
| hery | prim A | Yes, when I was in high school, I was introduced first introduced to fractions and then decimals. For example, 0.6 is one eh..six over ten. No model was used, directly from fraction, we were taught the concept first but I can't remember |
| yulius | prim A | Frankly, I was not good in mathematics so when the teacher taught me, I didn't pay attention. I was neither good nor interested in mathematics. As far as I can remember, we were learning slowly by doing exercises from the book which introduce the decimals. Then we were asked to memorize them. After that we were given exercises and asked to solve the exercises in front of the class. |
| adrian | prim A | We started from zero, and then after zero we continued first with fractions, for instance what is the tenth, and then the hundredth, and the thousandth. Directly, that a tenth is zero point one but I forgot whether the process of getting to zero point one was explained or not. |
| nita | prim C | It was by dividing the numerator with the denominator. Before we learnt multiplication and division an+A9d it was linked to fractions. I forgot whether we had models or pictures to learn decimals. |
| susi | prim C | As far as I remember the teacher often did not come to the class and we were taught only for instance how to add 0.25 and other numbers, there was no explanation about the process, and no manipulatives so for me it was abstract and not clear. Meanwhile we were in the pre-operational concrete stage so if we were given models or manipulatives, perhaps I would be able to understand it better or faster the topics. |
| ismi | prim C | I suppose when the teacher taught me decimals, I understood it well and if I had questions, I would ask him. For instance for 0.6, 0.6 is six tenths. If we have one digit after the decimal comma, it means that the number is tenths, and if there are two decimal digits it means that the number is hundredths, that is what I understand. |
| dian | prim C | Decimals is a number that is divided by ten or powers of 10, multiplied by 10 and powers of 10. Not sure of I understood, because I used to memorized it, like in doing multiplication. Well.... I don't really understand it but I remember the rules. |



| | | |
|---|---|---|
| aris | prim C | decimals are numbers with a comma. 0.6, decimals is a number which is divided by a hundred, right? (to my knowledge) the decimal is linked with division. Yes, use division.Using a ruler, this is zero and this is one for example. One... zero point one... oh it does not fit, change the interval to two (0.2) ... like this. |
| nana | sec | we can get 0.6 from 6/10. Back then, it was taught just by dividing 6 into 10. Because this is less than 10, then 0 is added and then this becomes zero and a comma then divide it by 6. Yes, I had difficulty for example with bigger number, there must be a problem. In the past, we were only taught that because we add a zero here then we need to add a zero here too. |
| yaya | sec | not sure if I remember, I think I forget, as far as I remember it is only 6 divided by 10 or 6/10, that is all. We write the number into tenths. As I recalled, the teacher never use any model in teaching decimals. In my memory, we were told that 6/10 equals to 0.6 and 6/100 would be 0.06, it depends on the divisors.b |
| novo | sec | For 0.6, my teacher used to teach starting from fractions. To find the decimal notation of 6/10 we just divide 6 by 10. For instance this is 6 and this is 10, because we cannot divide this, then we add 0 and then this is 6 so we get 0.6. In fact, I was not quite clear from fraction 6/10 and then because the comment that 6/10 cannot be divided. Decimals 0.6, what is it like, was difficult for me. It is still difficult to draw how long it is, is still difficult. (the model) using a ruler for instance, a ruler of 30 cm long and for example here is 2.6. Perhaps at first 6/10, since we already have a calculator, we can use a calculator and to find out what is 6 divided by 10 and the calculator will show 0.6 |
| ayi | sec | As I remember decimals are the continuation of fractions. From fractions to convert to decimals, we need to bring the denominator into 10 or 100 when we started to introduce simple fractions such as 0.5. It is always linked with fractions. No, we have some word problems but I forgot how was it like.. difficult, but seems to me it is still related to fractions. For example in the word problems |
| | | |
| Question 2 | | So if you are teaching in primary school, what is your idea of teaching/ introducing decimal numbers such as 0.6 or 0.9? Do you have an idea of using a model to help children learn decimals? |
| Student | class | Responses |
| nana | sec | perhaps through fractions and then using division but I am also confused why we can add a zero here and a zero there because it was given like that. Perhaps also with telling a story, yeah perhaps like before. A story like mom has one loaf of bread which she would like to divide to four kids so each will get a quarter. So if the bread is divided for four kids meaning that the bread need to be sliced into four parts. |

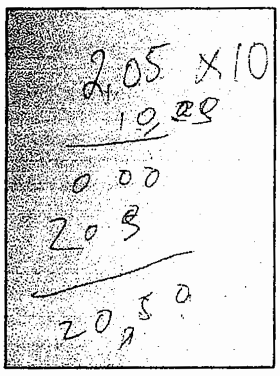
| | | |
|---|---|---|
| novo | sec | For me, I prefer to use a ruler. I think it is more easy for children to comprehend, like this is 2 centimeter and 6 milimetre for children it will be easier to understand that using a ruler. (for you, why do you think the division algorithm is difficult?) because when the child first divide 6 by 10, it is said that it can't then we do this (note: adding a zero after 1). The we have 6 becomes 60 nad this is 0 and then we add a comma, the student might have a problem in understanding that. The teacher used to teach that because 6 can't be divided into 10 then we add 0 here and then we add a comma here. The from 6 to multiply 6 by 10 to get 60 so now it can be divided by 10 and get 6. The teacher only told us that that is the way so I just follow the teachers' way. |
| ayi | sec | to be honest, for me when I answered it I knew the answers but to explain it to students, I don't know. First I will introduce what is decmials. First by using fractions, based on my understanding, we need to start with fractions then I will introduce decimals |
| nita | prim C | for me, mathematics is difficult and I don't quite understand decimal either. What can I say? I don't really know how to convert from decimals to fractions but from fractions to decimals, for instance 0.6 is six tenths with long division perhaps. Divide both the numerator and denominatro by 10 so 6 can't be divided by 10 therefore we add 0, so the answer will have a comma, 60 divided by 10 is 6 and 6 times 10 is 60. Perhaps like this, 6 parts divided into 10 people, how was that? Before, it is divided into 6 and then divided again into 10 parts. |
| ismi | prim C | to be honest, for me when I answered it I knew the answers but to explain it to students, I don't know. First I will introduce what is decmials. First by using fractions, based on my understanding, we need to start with fractions then I will introduce decimals and then combine those two. Perhaps we can use their daily experiences, for primary school children, using word problems will be easier. For example sharing a cake. One cake is divided for several students for example there is one cake and there are 10 children then we just need to divide one cake into 10. One part is a tenth so if we are asked .. if each child gets a tenth of one, then how many are .. for instance there are four children. Four children means.. eh.. it means that there are ten cakes, then if each child gets 2 cakes, then the total is 8 and we have 2 left over. Then we divide 2 into 4 children and each gets two and a half. |

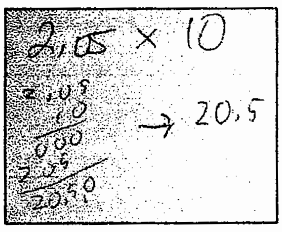
| | | |
|---|---|---|
| dian | prim C | Well.. Clearly not using memorization but to date I haven't been able to explain it to students, I am afraid of sharing misconceptions. Using concrete materilas, and posing questions until the students get to their own understanding, discovering themselves. For instance, using a cake as an example...but a cake is more to help learn with fractions and not decimals. For decimals, we can multply the numbers by tenths. (With the cake) well 0.6 means we have to find.. like if we need to find percents.. how much is 0.6 of 1 so we divide the cake into equal pieces.. from one.. multiply it by six tenths of it, so... how many parts right... probably six parts... I don't know, I weak at mathematics |
| aris | prim C | with division of whole numbers. |
| susilo | prim A | Clearly I won't use the same approach my teacher used before because I feel that my teacher gave lots of examples but we don't really know how we can apply them in real life. So I will try to use method different than what my teacher used to apply in the classusioom. Possibly I will try to make students build an understanding of fractions first because decimals is related to fractions. Students need to understand what fractions are, how to do operations with fractions. for instance, a fraction 1/2, what 1/2 looks like? I will give them a model, divided into two and then how to convert a fraction 1/2 into decimal and how to do it. |
| hery | primA | well, perhaps using another media so that stundets can understand more, not just know the concept. I haven't had an idea yet. Perhaps introduce the concept and then assist them using media but haven't exactly have a fixed media in my mind |
| yulius | prim A | teaching decimals by writing it in a small piece of paper to be introduced to students, and then ask students to come up in the front of the class and choose which one of the numbers. Using a piece of paper and then cut them into few parts from 0.1 to 0.4 and then ask students to infroduce the decimals from the smallest to the largest. |
| adrian | prim A | perhaps like what I said before, for instance 0.6 means 6/10. Because this is 10 then we have one zero so directly we will have 0.6. For fraction I have an idea about models like half a circle can represent a half but for decimals, it was introduced directly (in symbolic forms). We can use half a circle and it is 0.5 Yes, it is the same. |

Appendix C2

Question 3 — How were you taught to multiply decimals with 10, for instance to multiply 2.05 and 10? Did your teacher explain why do you have to count and move the decimal comma?

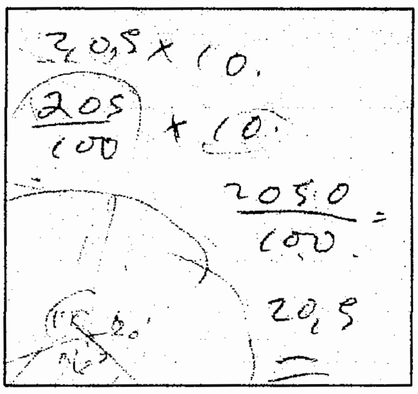
| Student | class | Responses |
|---|---|---|
| susilo | prim A | directly using an algorithm like this, we mutliply .. One … then the comma depends on the number of digits after the decimal comma. Because 10 does not have a decimal comma … and if there are 2 digits after the decimal comma in this decimal, then to count, we count one, two from the back. Because we have been taught about multiplication of whole numbers with algorithm so it is easy to multiply. It is only because there are decimal commas, .. the teacher directly taught that if there are two digits after the decimal comma, then the answer will also have two digits after the decimal comma. We do the multiplication the same as we did for whole numbers and then just count the digits after the decimal comma. For instance if we have two here, zero zero then we add two and two so we have four. No, we were not explained the reasons, just follow teachers' way. |
| hery | prim A | In the past, it was easy, because this is 10, then… we just need to move the comma in front one step. Perhaps based on calculation only, we will get this but manually we can just move the decimal comma if we multiply. If we are dividing then we move the decimal comma in front. No, we are not told the reason because we are only in elementary school but in highschool it was not explained because we already knew decimals. |
| aris | prim C | 2.05 multiply by 10, you just multiply them like whole numbers.. It is the same as 205 times 10 yeah the whole numbers mutliplication but here you can find the comma from here. There are two digits so one, two, here it is. No I wasn't told the reason just like that. |
| dian | prim C | direct way, just multiply 10 by 2.05. two point zero five, we multiply the digit one by one and then order them and write the number then we figure out how many digits after decimal comma. My teacher only told me that the decimal digits in the result of the multiplication is the same as the decimal digits in the number we multiply. |
| susi | prim C | I can't remember |
| ismi | prim C | for me, mostly I used a direct way… but to explain it to students, I am not sure.. My teacher used to teach me that if we multiply by 10, just move the decimal comma one. For instance 1.05 multiply by 10 meaning the 0 is moved to this. If for example, it is multiplied by 100 then we move the decimal comma here. The explanation is 1.05 equals to 105 hundredths, because it has two digits after a decimal comma, we just need to multiply it by 10, it means that we can cross this which means that the result is 105 tenths and 105 tenths is the same as 10.5 because it is tenths. Tenths means that we |




Appendix C2

| | | |
|---|---|---|
| | | move the decimal comma in this direction. |
| nana | sec | with multiplication algorithm. It is similar to addition of decimals but here we don't have to line up the decimal comma then to determine where the decimal comma is, we count how many digits are there after the decimal comma and correspond it with the decimal comma of the result. My teacher explained the reason but I forgot. In principle, if we multiply 2.05 by 1.5 for example, we don't need to line up the decimal comma but the important thing is to count the number of digits after the decimal comma. I am still confused why we need to line up the decimal comma when for addition but not need to do it for multiplication? |
| novo | sec | it is like this, this is the same as 205 divided by 100 and multplied by 10 so if we multiply 205 and 10, we get 2050 and then divided by 100, the answer will be 20.5. So first I convert the number into fraction, multiply it and then divide it using division algorithm. I have no problem with this approach because before I already learnt about fractions. It starts with fractions and then how to convert fractions into decimals. From decimals, we possibly can.. of course we can convert from decimals into fractions. When we study fractions, we learn about operation with fractions and other whole numbers. That 10 is whole number so we can understand it easily. |
| ayi | sec | the simplest way that I can remember is by moving the decimal comma. If we mutliply by 10 then the value wuill increase by 10 imes so we just need to move the decimal comma for one place. I think it is related to multiplication algorithm but I used to have problems mutliplying with zeros so I had problems with multiplying numbers with commas and zeros but then I was taught this way and the one that I remembered the most is moving decimal comma strategy. I think the explanation is given because for instance, 2.05 equals to 205 hundredths and if it is multiplied by 10 then the simpler way is to conver it to 205/10 and two hundred and five tenths equals to 20.5. |



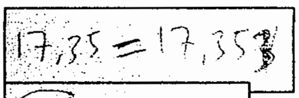
Question 4 What is your own idea of teaching multiplication of decimal with 10?

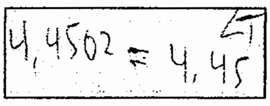
| Student | class | Responses |
|---|---|---|
| susilo | prim A | perhaps because I don't know decimals in real life, perhaps I just use the same way as my teachers. |
| yulius | prim A | Probably I will use th same approach as my teahers' ecause that is the only way I know. Perhaps one day if I am teaching my own students.. I will try my best to seek assistance except this way, because I should be able to help children so that can learn better than me. Try to learn harder. Now I still had obstacles. Last semester I used to ask my lecturer and friends but up to now, I am still having problems, haven't resolved my lack of knowledge |

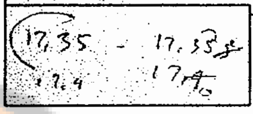
Appendix C2

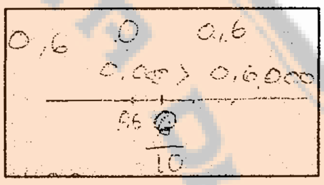
PreQ9 — pretest questions - different items asked for different students depend on their responses to the tests and try to explicate their thoughts

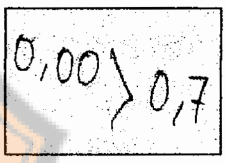
| Student | class | DCT3a Comparison Test |
|---|---|---|
| yulius | prim A | perhaps my principle last time was if 0.0 means that it has other digit behind and 0 is a whole number so one possibility that I can think of is that 0 is bigger than 0.6 because it has no decimal digit, just like that. The larger one is the one without a comma. That was what I remember. |
| yuliusus | prim A | because this one has more digits so I choose 4.45 as the larger |
| hery | prim A | 0 is the larger because 0.6 is six tenths and it tenths smaller than 0 because probably the value is smaller than 0. |
| hery | prim A | perhaps the theory is the same, so I round 4.4502 to two decimal places to 4.45 so they are the same but if not using rounding then they are different, of course 4.45 is larger because 4.4502 is 4 and 1/4502 whereas 4.45 is 4 and 1/45. So when we are dividing 1/4502 will be smaller |
| adrian | prim A | the larger one is 5.62 because this is in hundredths, it means 562/100 whereas this one is 5736/1000. I don't use the number line for this one. The other ones can use fractions equivalent as well but it will be long. |
| aris | prim C | 17.35 and 17.353 are the same because we can truncate the 3 in 17.353 as it is less than 5. Similarly 4.4502 = 4.45 and 1.86 = 1.87 because they all can rounded to 1.9. |
| nita | prim C | because both 17.35 and 17.353 are rounded to 17.4 so they are the same also 4.4502 is the same as 4.45 by rounding |
| susi | prim C | I answer 4.6666 = 4.6; 3.7= 3.77777 because I remember that my teacher taught me that we can discard the other digits so that is what remember. I don't really know if that is correct. I was told to discard the digits after the comma so at the end they are the same and I wasn't told how many digits that can be ignored so I say they are the same |
| ismi | prim C | The way I solved this is by using a number line, 0 is here and 0.6 is around here. Hence comparing 0.6 and 0, then 0 is larger because it is more in the right side. If not, it is the same, here is 0 and here is 0.6 and 0 is the same as 0.000 and 0.6 is the same as 0.600. So this way is the same, this (0) is closer to the right. 0.00 and 0.7 perhaps it is the same like this one. |
| hery | prim A | perhaps the theory is the same, so I round the number to two decimal places to 4.45 so they are the same. If we don't use rounding then 4.45 is larger because 4.4502 is 4 and 1/4502 whereas 4.45 is 4 and 1/45 so when we are dividing so 4.4502 will be smaller |







| | | |
|---|---|---|
| adrian | prim A | yeah the same like this. If we have two decimal digits meaning that we have hundredths, just like that. . To help children who have difficulty, we can explain that 2.05 means that 205 hundredths and when we multiply it by 10, then we cross this one and this one and then get 205/10 or 20.5. Explain it again if the students still don't understand. |
| nita | prim C | perhaps just by moving the decimal comma. If we use this (refer to mutliplication algorithm) the students might find it difficult. What I mean by moving the decimal comma is we move the decimal comma to the right for multiplication and to the left for division. Because if we are multiplying 2.05 and 10, if we multiply by 10 it means that 2.05 ten times so it will get bigger whereas if we divide it, it will get smaller. Therefore if we multiply by 10, 100, or 1000 , the value will get bigger and the comma should be moved to the right. |
| susi | prim C | I have no idea yet, still difficult for me |
| ismi | prim C | the students need to understand fractions very well. I still have a vague idea how to helo students. I am still struggling when we are asked to explain it to students. |
| nana | sec | When I teach, since I don't really know how to explain why the rules apply. I am confused what is exactly the reason. No idea yet. |
| ani | sec | perhaps using this, because children might not understand why the comma is moved. With moving decimal comma rules, we can use it if we still have many problems to solve and we are in the rush. My understanding is perhaps we start from this hundred, because for example this 0 is a hundredth then it follows from that. |
| yaya | sec | Ehm… how should I do it… the same perhaps. For instance, this is 2.00 if it is multiplied by 10 then it cannot be 2.00 or 200 that would be wrong. So the comma needs to be moved only one place behind. |
| novo | sec | for instance if we go for the easy way, if we multiply 2.05 by 10 then we can tell the students that if we multiply the number by 10 , we need to move the decimal comma one place to the right, that is all. How to give meaning to this rule, well I don't know, it is difficult. My teacher didn't set the rules, no but first ask to convert decimals into fractions and then multiply it by 10. After students have enough practice then they are given tips or methods to solve the problems faster by moving the decimal comma. By decomposing 2.05 into 2 plus 0.0 plus 0.05 and then multiply them by 10. This becomes 2 multiplied by 10 plus 0.0 multiplied by 10 plus 0.05 multiplied by 10. |
| ayi | sec | No idea. From the process at that time, perhaps because I am still confused with division algorithm using simple numbers so this becomes difficult. As soon as I understand the multiplication algorithm then eventhough there are zeros in the middle before the comma but I can understand it, so that is the basic. |

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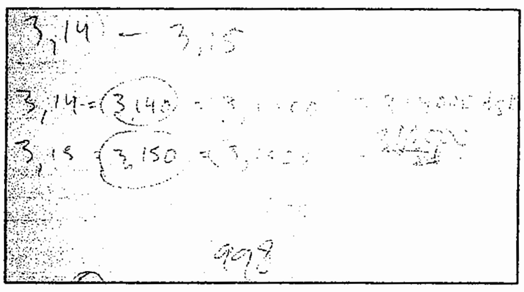
| | | |
|---|---|---|
| nana | sec | 3.142, 3.143, 3.144, 3.145 and so on until it it get close to 3.15. I think they are finite numbers in between because there is a boundary. For example if this is 2.1 then after this will be 2.10 and then this one is 2.11, 2.12, 2.13.. Because tis is 2.0 then possibly here are 2.01, 2.03, 2.03 etc. |
| yaya | sec | I just think that between 3.14 and 3.15 because there is a difference, like between 2 and 3 there is a difference. Therefore here there must be 2.1, 2.2, 2.3 so I think for 3.14 there must be 3.141, 3.142, 3.143 and so on until 3.15. How many are there, well last time I counted perhaps there were nine? |
| novo | sec | first I thought that since 3.14 equals to 3.140 equals to 3.140000 and so on. Similarly 3.15 is the same as 3.150 and 3.15000 and so on. Then the interval between them can be 10 and between this one and this one there are hundreds (hundredths?) interval and then for this case .. 3.1500 the interval is thousands so there should be more than 200 because thousandths implies that there will be more than 200. I personally think that there are 998 but because the option is only more than 200 then I choose the one with more than 200 as an option. |
| ayi | sec | at that time I remember that behind 3.14 there are infinitely many zeros, it can go very long so I imagine that from 0 we can make infinite digits, it can start from units to tens so clearly there are infinitely many numbers in between. |




Part C Item 17 How to help students to divide 0.5 by 100?

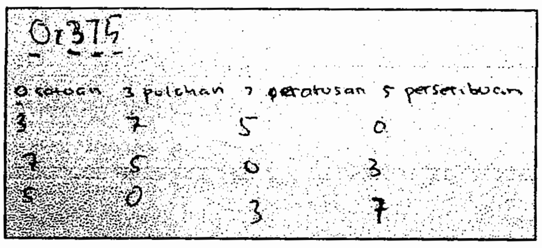
| Student | class | DCT3a Comparison Test |
|---|---|---|
| aris | prim C | I was wrong when I did this test, because a comma is gone then we could just cross this out. Cross these out again and yes, so 0.5 divided by 100, to get rid off the comma, automatically this will be gone. So we have 5 divide by 10. To explain it because this comma is gone then we can just corss this out and add, add this here. So we will have five tenths .. oh no that was wrong. |
| ismi | prim C | for this problem, I use reverse multiplication principle but I was a bit confused when I solved this one, may be just use fractions like before that 0.5 is the same as 5/10 and divide this by 100. So the fraction will be 5/100. So the answer is 200, that was how I taught in junior highschool. that division is the same as reverse multiplication. |

Part C Item 19 Why do you think the student ordering 0.3<0.34< 0.333333? How would you help studetns who answer this? Would you explain your thinking process when asnwering this problem?

| Student | class | DCT3a Comparison Test |
|---|---|---|
| susilo | prim A | Because the longer the digits in tenths, hundredths, thousandths, ten thousandths and etc. does not mean that the number will be larger but it becomes smaller. In this case perrhaps because hundredths is smaller than tenths and thousandths is |

Appendix C2

| Part B | Item 2a | decomposing of decimal 0.375 |
| --- | --- | --- |
| Student | class | DCT3a Comparison Test |
| yulius | prim A | from my opinion, this zero comma means that it is one over something. For instance 0.37 has two digits (decimal). Because this (0.375) has three digits, then it has tenths, hundredths, and thousandths. That 0.3 is 3 thousandths, that is what I remember. Last time, I just change the order of the digits. |
| nita | prim C | this is 0 times1 and then 3 divided by 10 is 0.3 and we have 0.7 divided by 100 equals to 0.07. This is 3 tenths so 3 tenths is 0.3. 0.7 times a hundredth, ...0.7 divided by a hundred, so we move the decimal comma twice and get 0.007 . No this is wrong I add one too many |
| susi | prim C | I could not do that part because I haven't learnt it before. |
| yaya | sec | I just think that since we need to decompose this decimals and there are 4 places, this one is 0 units, 3 tenths, 7 hundredths, and 5 thousandths. I don't exactly how to work it out correctly but I just order the digits and then move the digits in different places, move the 3 in front and then 7, 5, 0. I think it was spontaneous and I was a bit confused what did the question mean so I just tried to answer like this but I have no idea whether this works. |
| ayi | sec | first I didn't understand the question but then it cross my mind while working on other item that 0.375 does not have to come from 0.3 times 1 and then I find the other alternatives. That is actually the last problem I was working on. |
| | 3 | density item (Part B 5 or 6) Would you explain why do you choose this option (>200 or <200) |
| Student | class | DCT3a Comparison Test |
| susilo | prim A | Because this is 3.14 and there are 3.1401, 3.1402 and 3.140021, thousandths, hundredth. If we cound up until 3.15 there will be so many of them. It is not countable because this is 3.14 so it is in hundredths, if we add one more digit, it will be thousandhts, 3.1401 and if we add more 3.1402 until perhaps hundred thousandths and millionths there will be so many and they are infinitely many. |
| hery | prim A | there is, because this is 14 and 15 (referring to decimal digits of 3.14 and 3.15). It is difficult.... I think there is none because their difference is only 1. |
| nita | prim C | I could not answer that one, perhaps there are numbers in between those numbers but I cannot tell, don't know. |
| ismi | prim C | I chose more than 200 because if we can add more zeros digits after 4 and the difference between them is 0.01. So after 1 we can add zero zero zero and those places we can fill in with 1, 2, or 3 and we can continue further to the left. So there are infinite places that we can continue to fill. |





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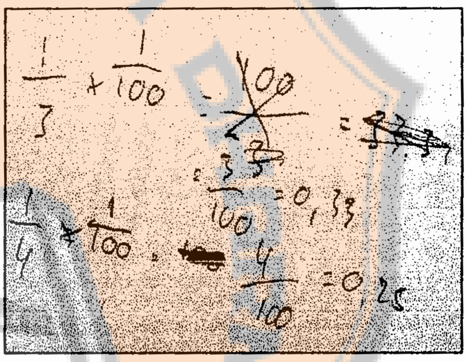
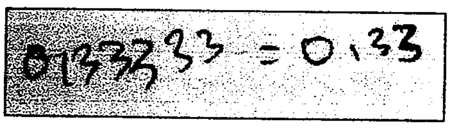
|---|---|---|
| | | smaller han tenths as well but this does not always work. |
| yulius | prim A | I just copied my friends' work and still have no idea, I am still confused. |
| susi | prim C | *no idea yet* |
| nana | sec | because here is 1 so they think that 1 ehm.. Because this one is 3 divided by 10 it has to be 0.30 but it is0.34 so it is larger then it means that the number is larger so each part is bigger than this 0.04 part. So 0.30 is larger than 0.34 because 0.34 has 0.04 which means that if we have a fruit and we need to get this 0.04, the result will be smaller. I think 0.333333 is the same as 0.33 because I am rounding it and these digits are less than 5 so we can just consider two decimal digits. |
| ani | sec | perhaps the student think like this: 0.34 has more digits than 0.3 whereas after 0.3 there is no more digit so then think that 0.3 is smaller than 0.34. In the case of thinking that 0.34 is smaller than 0.333333 perhaps he/she uses rounding. Perhaps because it is longer so the student think that it is larger. I will ask to subtract, for exmaple 0.3 - 0.34. If the answer is not a negative then it means that 0.3 is larger. |
| ayi | sec | oh in the test, I multiply those numbers first by 10 so they become whole numbers to make it easier so 0.5 becomes 5 and 100 becomes 1000. |

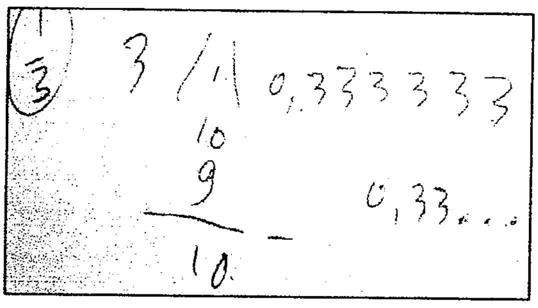
Part C Item 20 How would help your students to find the decimal notation of 1/3?

| Student | class | DCT3a Comparison Test |
|---|---|---|
| susilo | prim A | first we mutliply 1/3 by 100 and get 100/3 and then 100 divided by 3 is 33.3 hunderdths and it is the same as 0.333. I haven't had any idea how to help students actually. Perhaps because it used to have *difficulties in converting 1/3 into decimals* because in the past my teacher did not explain in detail how to convert from fraction to decimals so when I solved this, I was not sure what was the right answer. |
| aris | prim C | as far as I remember 1 divided by 3 is 0.33 and it stops there because it has one as a remainder and we cannot round it. Ussually we only deal with decimals with 2 digits after the comma. |
| ismi | prim C | oh that one is difficult and I don't remember that very well but if I remember correctly that we need to bring the denominator to tenths, hundredths, or thousandths. To find decimals for 1/3 is a bit difficult. For 1/4 if we change it to tenths then it is still in fraction. So for primary school it will be better if we start from the easier ones... For instance to bring 1/4 to tenths then we need to multiply it by 2.5 and also multiply the numerator by 2.5. So this becomes 2/5/10 or if we can change it to 25 then it would be easier so the result is 25. But then it is easier to change 1/4 to hundredths so we multiply both the numerator and denominator by 25 to get 25/100 and in notation, if we have two zeros then we just write it like this, it means there will be two digits so it will be 0.25. For 1/3 I tried it like this with the same principle but I need to multiply it by |



| | | |
|---|---|---|
| | | thousandhts and ten thousandths |
| yaya | sec | I answered it wrong last time. I divided it by 10 to convert to decimal right? Tes, perhaps we can teach it like this, to convert 1/3 to decimal, well.. It was difficult to answer this one. I always get it wrong. If we divide 1 by 3 just do it in direct way using division algorithm. 1 divided by 3 we will get 0.333. I memorized that because it is taught that way. The digits are repeating infinitely but I wasn't told the reason just know that it is the same as 0.33333 and the 3 will repeat forever. I could not explain the reason to myself so how possibly can I help the students? |
| novo | sec | I tend to use the division algorithm. 1 divided by 3 if we want to write all the 3s, it will be forever so we just need to write 0.33. If we want to show that the 3 will repeat forever, we just need to add three dots at the end. I have no other idea except using division algorithm to solve this, haven't thought about models. |
| ayi | sec | I had trouble working on that item but I have no other way to solve that except by using division algorith so 1/3 equals to 0.33… from there we can talk about rounding. We can see that the result won't stop here but it depends on our consensus how many decimal digits we would like to round it to. My sense is, in this case since the repeated digits are the same (3) the student might be able to see it easily but for instance if the repeated digits are 625 (meaning 0.625625625...) then they might find it difficult. Here from the division, the student can see that it always has 1 as remainde so the repeated decimal digits will always be 3 but I am a bit nervous when the digits are repeating after few digits. |




Appendix C2

# Appendix C3
# Samples of Post-course Interviews cycle 1 grouped by questions

PostQ1 Based on your learning experience, could you come up with three models or more to help primary school kids learn about decimals? Would you also please rate the models based on their level of difficulty to help students from the easiest to the most difficult one?

| Student | class | Responses |
|---|---|---|
| aris | prim C | the LAB, using fruits: water melon to be cut into ten pieces, a circular model, a paper with ones, tenths, hundredhts, thousandths (referring to the number expander), food. The easiest one is the LAB because based on my own learning experience, it is easier using the LAB than using the paper that we used last time (referring to the number expander), I understood better using LAB because the one is divided into ten, hundredths, and thousandths. With the circular model, the division into tenths, hundredths, and thousandths is yet to be done whereas in the LAB those relationships can be observed. With the number expanders, the students need to fill in the numbers but this can be considered as easy as well. |
| nita | prim C | using the LAB, the other alternative is using a piece of paper, also we can use bamboo sticks which we cut according to the scale. We can also use buttons, but the cannot be used to measure things whereas the LAB can be used to measure the length of something but buttons can be used for counting and expressing the different digit values: ones, tenths, hundredths, and thousandths. Food such as water melon also can be used but it is too difficult to measure and compare them but bing cut, a water melon can be used as a model. According to me for the higher grades such as 4 or 5, the LAB model is easier but for the lower grade, using buttons will be easier because they are more practical, it is like using an abacus to count. However, we need to realize that the buttons can only represent the different place value but the proportion among the different size of the buttons don't express the decimal relationships as like in the LAB pieces but they are more practical and affordable. |
| nita | prim C | A piece of paper is similar to the LAB model, they are both based on a scale, the students use the ruler to divide the paper into smaller pieces. In fact, the LAB is easier but a piece of paper is more available around the students to find so it is more practical. We can also use a bamboo sticks that hold similar principle as the LAB. we collect the bamboo sticks and measure the length of the one piece and then measure the length of a tenth stick, the way we divide them into tenths is similar to the LAB. |
| susi | prim C | using the LAB, using the number expander. For me, they are interesting because at the end, I understand which one is the one, which one is the tenths, which one is the hundredth and the thousandth. from there, it is clear, so it is suitable for children because children will not count on the memorization but they understand the ones, and they can find out from measuring that the tenth is shorter, so they can compare. Another model that is not discussed in our group is the circular model using a hard paper but it is difficult to diffentiate the tenths, hundreths,.. It is also because it is a circle, it might be difficult to understood by students. |
| ismi | prim C | using the LABs, and also using paper strips. For instance to model one we can use a paper strip of 10 cm long and 2 cm wide to represent a tenth. Also we can use the paper, the folded paper, but I think it is not easy to understand, I am afraid that the primary school students will be confused with the number expander model particularly that they have different options/ways of representing the same decimal number. For instance, given 2.059, they might come up with an answer of 2 ones, 0 tenths. Then 5 is 5 hundredths, so it can be reunitised again,right. For instance to 2 hundreths and then the remainder will be 39 eh.. See if there are different possible answers like this, it is for students to perceive. It is easier using the LAB. I myself was also confused at the beginning but my friends in my group helped me. After seeing many examples discussed in the classroom, I got a clearer picture of it works. With LAB, I came to an understanding of why we call these as tenths because there are ten tenths in one, similarly a hundred consists of ten tenths, and a thousand consist of ten hundredths. |
| nana | sec | The first is to use the LAB, using the LAB, the longest piece is a one and then to represent the tenth, we divide the one piece into ten parts. Another model is a piece of paper, if we consider that as one and to get a quarter, we divide one into four parts. To get a tenth, we divide one into ten equal parts, each part is a tenth. Then to get a hundred, we divide one tenths into ten parts. The easiest one is a piece of paper because the students have it and they can easily draw and cut them into smaller pieces by themselves whereas the [illegible] but we can change the LAB with the bamboo sticks, they share similar principle. For the paper model, the teacher need to make sure that the size is easier to divide, for instance 10 by 10 cm so that each of the piece is 1 cm length. The students can use a ruler to divide the paper. |

| | | |
|---|---|---|
| ayi | sec | from our group, we just think to change the materials of the LAB with a more simple and more general stuff for students which are more available and affordable such as straws but the principle are the same (based on division by ten). We can also try with the three dimensional model such as a cube. We think that if they already understand the fractions concept, it would be good if they don't count only on the LAB which is based on length but the students can also learn a more complicated model. So if they master the linear model, they should have been able to understand the more difficult one, i.e., the spatial model. It is difficult to come up with a completely new mode, because amongst us, this is the first time we find a model for decimal. It is very rare to make a model for decimals so we only have this experience as a benchmark. Based on our group discussion, and what we did in the classes, clearly it is easier to use the LAB. Because, eventhough the LAB looks like a three dimensional model but the prominent feature is the length whereas it is more difficult to divide the spatial model, besides we need to consider also the volume. From Mr. Marpaung task we also learn about the paper model. |
| novo | sec | Using the LAB, I'm quite interesetd in using the LAB. According to me, it is very helpful especially for the primary school students to understand the decimal numbers. It is helpful in a sense that if the longest piece of the LAB is one meter, then how long is the tenth? The if we arrange them, we can learn how many tenths are in one. Another way is to use the ruler for example, if this is 0.1 and 0.2, then to explain how big is 0.13, we need to divide the interval into ten. For example, here is 0.13. We can also use the number expander, that is also useful in helpin the kids to understand place value because often we confuse the place value for tenths and hundredths and add them together. For me, I prefer to use the number line because for primary school kids, they will have problem to understand how big is 0.13 and where is located between what numbers? With the number line, it is helpful because 0.1 is divided into ten and there are the hundredths, so 3 here represents the hundredth so this is 0.13. The LAB is limited for instance to represent 1.39283 because we only have tenths, hundredths and thousandths pieces so we don't know how to represent this number with the LAB. |
| ani | sec | We have learnt using the LAB so perhaps the first model will be the LAB. The second one, we can use a plastic rope to show the fraction 1/3. The last one is to use a piece of paper like in Mr. Marpaung's task which we divide one piece of paper into ten smaller parts and then ask what is one part of the smaller piece. So we divide one piece of paper into ten smaller parts, then one of the part is one tenth. If we divide it into one hundred, we divide one tenth into ten smaller parts again. Then to get one thousandth, we divide the one hundredth into ten smaller parts. We cut the paper based on their area, their size. We can also use a cube as a model. Amongst the four models, I think the LAB is the easiest one because the model clearly show how many of the smaller units are in the bigger unit. We can check easily by lining them up with the connector and check the answer. It will be faster and easier. However, the difficulty with using the LAB in primary school is we need plenty of them. For instance, like yesterday we only have limited number of the smaller units so if we have to represent a certain number, we need to have more of them. |
| adrian | primA | Yeah, we can use the model like the LAB but we can change the materials using woods. According to me the easiest model should be the one that the students are familiar with, those that they can find easily everywhere. The model I proposed using the woods are similar in principle with the LAB model, it is the material that are different. |
| susilo | primA | The first model is the LAB, with the learning experience that we had, I might be able to implement that in the primary school later. The second one is using grid paper which might be used to replace the LAB. The third one is using the floor tiles. with the paper strips, eh.. first with the LAB, we can explain the value of tenths, ones, hundredths of the decimals. We can represent the value of those units with the LAB,.. I mean we can tell the how long the unit is using the LAB, the same thing works with the paper strips. For instance, to represent 0.1 or a tenth we can use one of the smaller box, whereas we represent 1 with 10 grid blocks (long pause to draw the pictures of the model). For example, for this one we use 20 block papers whereas for 0.1, this means that 2 of these are combined to become 0.1. Whereas for 0.01, we can it smaller like this. It is divided into 10 again and again and the principle is similar to the LAB. with the floor tiles, it is the same, if we represent 20 tiles as 1, then 2 of the tiles will be 0.1. I chose 20 because if we use 10, there will be to small when we get to the hundredths. |
| | | |
| PostQ3 | | is there any form concept in decimals that you have a problem or difficulty with before you followed the learning activities but after that you get a clearer picture or the opposite, if you feel like there are some concepts that become less clear or you get confused after following the learning activities? |
| Student | class | Responses |
| nita | prim C | The difference that I felt is before when I saw decimal numbers, if the difference is small then I thought that they are all the same, but know I know that they are different. |

Appendix C3

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|---|---|---|
| novo | sec | perhaps from my friends in a group, we have a new viewpoint for instance about the model, we find that we can use models in learning decimals and it is quite helpful, for instance if we are going to teach in elementary school then it is very helpful. |
| ani | sec | Yes, there are many new things, before I didn't know how to use the LAB, now I know. I come to an understanding what is 0.1, how to get 0.1, 0.01 . Before, I was just taught by the teacher, we already had 0.1. I didn't know where it comes from. |
| nana | sec | Yes, item 2, this one is the one then 7 is the tenth, tthen 5 is the hundredths and 3 is the thousandhs. For instance this one, this is 0 one, and then I take 75 hundredths wihch means that this is 75 hundredths and this one is 753 thousandths. I studied it in the group discussion in the activities with the number expander. We can also use the LAB, in which one is divided into 10 to get 0.1 and then divided again to get 0.01 and the smallest piece is 0.001. |
| | | |
| PostQ4 | | Did you find any change from participating in the pre and the posttest after the learning activities? / Any particular difficulty in solving any post-test item? |
| Student | class | Responses |
| ayi | sec | I felt more ease in the posttest. Before, it is very focussed on the algorithm but now I started to understand the concept. If 1 is divided into 6, we use the LAB and then from 1 if we are to divide it into 6, in the first place it can't be done. Therefore, because we cannot do that, one is equal to ten tenths so now it can be divided into 6. We get one, so this one is one of the tenth. Then from ten tenths if we divide them into 6, we have each group consists of one tenth but we still have 4 more and because 4 can't be divided equally into 6, then we use the hundredths. From there, we divide them into 6, and get 6 groups of 6 hundredths, I mean each has 6 hundredths. Then from here we get 4 as a remainder again, so the students will observe that it can never be evenly divided. |
| novo | sec | about finding the decimals of 1/3, before I could not solve that problem, but now I have an idea. In the learning activities, we are asked to think how to represent 1/3 using the LAB, then I get an idea, based on the fact that 1/3 is equal to 0.333 continue forever, we can represent it by arranging 3 tenths, 3 hundredths, and 3 thousandths together. We did it after we find the decimal of 1/3 using the division algorithm. using model.. difficult... if we link it with a mode. For instance using the longest one of the LAB (the one), if we are finding 1/3 meaning three equal parts of one then each part will be less than one so.. zero comma something. to find the next decimal number in a sequence (Question 4b), the trouble is to connect, for instance 0.1 and 0.25. They are both divisible by5, divisible by 5, ... right or wrong?... It is difficult, I tend to divide both numbers because when I see 1.25, I think the next one should be found by division. |
| ani | sec | the test item that ask how many numbers in between two decimal numbers, I do not understand why one of the choices is more than 200? Why using 200? |
| adrian | | perhaps, the problem of finding new ideas or new ways to explain things to children. |
| ismi | prim C | for me the difficult part is in comparing 0.81 and 0.9, trying to find which one is close to 0. For this one because they are decimals, so if we draw them, they should be on the left side of 0, so I am looking for the one which is closer to 0 or positive. .. here . but oops, this is negative, I was thinking like that before the left side because it is closer to zero. |
| nita | prim C | I also did not understand this question (Posttest item 4a, 4b), for me it is unclear. I didn't understand Question 5 as well, I didn't understand how many decimal numbers are in between two decimals. In my experience, the group work has been quite helpful because we can discuss and solve the problems together. For instance asking "how did we find this answer, it is explained or perhaps if I have an idea but the other friends think differently then I will consider again which one is right. |
| | | |
| PostQ5 | | So do you have a new idea of models to help students learn decimals in the primary school? |
| Student | class | Responses |
| ismi | prim C | No, not yet, I am still having trouble myself when I had to help students in my teaching rounds. |

Appendix C3

| PostQ6 | | Do you have any feedbacks for to improve the learning activities or the test items? |
|---|---|---|
| Student | class | Responses |
| ayi | sec | the time provided for the tests is too short especially because few last questions require us to think. Also items from this part (word problems), I don't know what are the goals of these questions, because before and after the learning activities, there is no change in the way we answer these questions. So what concepts being targeted by these questions for me are not clear, I cannot tell whether my answers are based on concepts or they are still the same as before (procedural). Also, the learning activities should be carried out more relax, it is a bit too intense including the presentation of the group discussions. We think that the design of the learning activity is good, it started from the beginning so it builds the concept in a more holistic way, from the start, when the students didn't know what is fraction, from the simplest to a more difficult ones. Yet, we think that it takes a lot of time to undestand from the beginning till the end whereas if we teach later we need to find something that works for a more limited time. |
| novo | sec | to connect what we do with models with the algorithm is still difficult (referring to his process of finding 1/3 using the model and connect it with the steps in the division algorithm) |
| ani | sec | to make students more active, you can add more group problems because if we are listening to lectures, we get bored but if we are having group discussions, we have no choice except of participating in the discussions. |
| adrian | primA | it is already good. I was having a problem at the beginning but because we are working in groups so a friend can help me. By communicating each other own thoughts, then I come to know |
| | | |
| Part C | Item 20 | What is your idea to help children find the decimals of 1/6? |
| Student | class | Responses |
| novo | | Oh right.. That one I am still a bit confuse, how we can represent that so that the primary school kids can understand. If we are using the LAB, for 1/6, 1 is divided by 6, it can be difficult to find it but for us, we can divide 1 by 6 and get the results. For the primary school students I am not sure how we can help them to find 1/6. |
| susilo | | I was quite mixed up, I just answered by using the LAB but I am not very optimistic that the idea will work out well with primary school student. This is one so we divide it into 6, by first measuring the length and then divide it into 6. Same with this one, we also divide this into 6. That's how I think but when I came to the smallest pieces, I had problems to divide it into 6 then I got stuck so I just left it like that. Perhaps I can, with this one, we just divide thisinto 10 equal parts... so this is one, there are 10 so we connect these 10 and then we take only 6, and because there are not enough, we use the smaller pieces and connect them together. .. For instance, here are 5, then add 5 more here and here add 5 more until they have the same length. |
| ismi | prim C | Usually I try to bring it to the equivalent fraction first ... but for 1/6 I think it is still difficult. If I start with examples, I will start with the simpler one, for instance 1/4, this is 1 so 1/4 means that 1 is divided by 4, whereas 1 consists of 10 so we will find the result of 2.5. First 1 is divided into 2 frist, and then theose two are divided into 2 again. ..perhaps so we will have 5 but we need 5 more because this is 1 right and I didn't think directly using 3. I started to find 1/2 first, and then divide this part into 3 as well as the other part into 3. Yes, we can.. here we have 5 parts to be divided into 3, one, one and then we have 2 more which then those 2 parts have to be divided into thre but then it will take a very long steps to get the answer. ... For me the process is too long. From my understanding, to get the decimals we need to find decimal fractions in tenths, hundredths, or thousandths first, so in the case of 1/6, we should bring the denominator into tenths, hundredths or thousandths. |
| novo | sec | the idea is to give the students models to see what is 1 and then how to model 1 divided by 3, then from that we can teach them the division algorithm to get the answers faster. |

Appendix C3

## Appendix C4

### Table of responses to Initial Activity Set 1 Activity 1 in cycle 2

| Ways of partitioning | Number of cases | Frequency | Cases in primary | Cases in secondary |
|---|---|---|---|---|
| Halving includes partitioning into 2, 4, 8, 16, etc. | 13 | 38.2% | 12 | 1 |
| Decimating | 7 | 20.6% | 5 | 2 |
| Using hands' span, partitioning into 6 | 7 | 20.6% | 3 | 4 |
| Into 5 | 3 | 8.8% | 3 | 0 |
| Others, (using hands span or pen's length) | 3 | 8.8% | 0 | 3 |
| TOTAL | 34 | 100% | 24 | 10 |