PRIMARY TEACHERS' UNDERSTANDING OF THE INCLUSION RELATIONS OF QUADRILATERALS

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Abstract. The aim of this study was to investigate the primary teachers' understanding of inclusion relations of quadrilaterals, especially parallelogram. This descriptive study was conducted with 14 primary teachers in Kanisius Demangan Baru Primary School in Yogyakarta, Indonesia. Data were collected during a workshop that is was aimed to develop primary teachers' mathematics ability in geometry. Data were analysed using the framework of Van Hiele. Findings showed that in the beginning of the workshop most of the teachers are likely to recognise quadrilaterals primarily by prototypical examples. Therefore they got difficulty in understanding the inclusion relations of quadrilaterals. Based on the analyses of the written test gathered at the end of the workshop, it was found that many teachers are struggling in understanding the inclusion relation of quadrilaterals.

Keywords:
Inclusion relations of quadrilateral, primary teachers, geometrical thingking, Van Hiele

1. INTRODUCTION

Classification of quadrilaterals is one of the mathematics topics taught in elementary school. In classifying quadrilaterals, children and also teachers should comprehend the class inclusion of quadrilaterals. Understanding class inclusion is the ability to have an overview of relationships among figures and it is important to support students' deductive reasoning (Currie & Pegg, 1998).
In mathematics education, the Van Hiele theory describes the different levels of understanding through which students progress when learning geometry. The basic idea of the theory is that a student's growth in geometry takes place in terms of distinguishable levels of thinking. This study is attempting to answer the question: "how well do primary teachers understand the hierarchical classification of quadrilaterals?"

2. RELATED LITERATURE

Studies has shown that preservice teachers' content knowledge on quadrilaterals is not at the expected level (Paksu, Pakmak, & Iymen, 2012; Çontay & Paksu, 2012). Moreover, preservice teachers generally use partition classification while classifying quadrilaterals

(Turnuklu, Gundogdu Alayli & Akkas, 2013). These studies imply that it is important to investigate primary teachers' understanding of the inclusion relations of quadrilaterals.

Hierarchical class inclusion is the classification of a set of concepts in such a manner that the more particular concepts form subsets of the more general concepts (de Villiers, 1994). Moreover de Villiers (1994) stated that hierarchical classification helps people to define concepts or formulate theorems economically, simplifies the deductive systematization and derivation of the properties of more special concepts, contributes to problem solving process, and serves a global perspective. The hierarchy is built up, like a tree, from its trunk into branches that are special shapes of quadrilateral and as the tree grows, one finds greater specialization (Craine & Rubenstein, 1993). From the quadrilateral hierarchy, characteristics of quadrilaterals are inherited through the generations. de Villiers (1994) stated that hierarchical classification is more useful than a partition classification and he presented the parallelogram hierarchy below (Figure 1) to describe the mathematical processes of generalization and specialization.

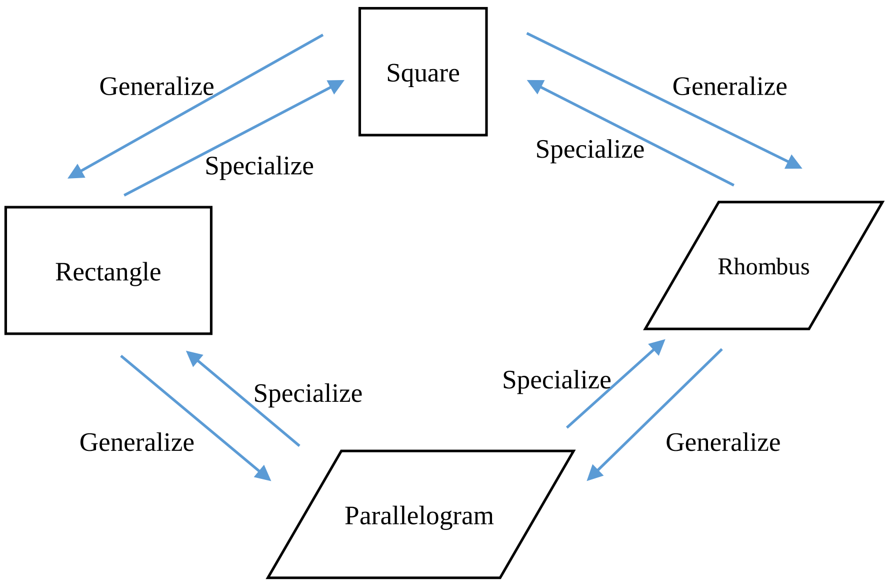

Figure 1. Hierarchical Classification of Parallelogram

Form the classification, people could start with the most special shape, a square, and generalize rectangle and parallelogram consecutively. For example, the rectangle can be generalized from the square by deleting the properties that all sides must be equal. People can also start from the more general shape and specialize to a new concept by demanding additional properties. For example, the square can be specialized from the rhombus by requiring the additional property of equal angles.

In contrast to hierarchical classification of quadrilaterals, there is a possibility to classify quadrilaterals with a partition classification (de Villiers, 1994). In the partition classification, squares are not rhombi, and rectangles and rhombi are not parallelogram (illustrated in Figure 2).

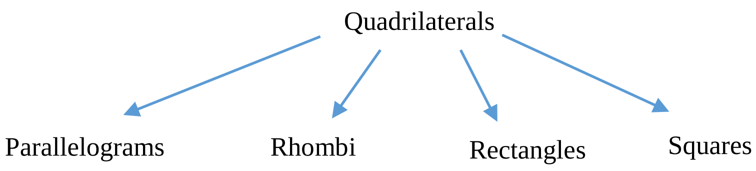

Figure 2. Partition Classification of Parallelogram

de Villiers (1994) stated that many teachers prefer to use the conventional hierarchical classification rather than the partition classification and ignore discussing the reason for the conventional preference for the hierarchical classification. Therefore, students have little or no functional understanding of the hierarchical classification.

According to the Van Hiele theory, students should have a meaningful learning by exploring rich experiences of geometric ideas to move to a higher level of sophistication which correspond to the Van Hiele Levels. The Van Hiele levels describe the way that students reason about geometric ideas. Below the descriptions of Van Hiele levels based on Fuys, Geddes & Tischler (1988) and Çontay & Paksu (2012):

Level 0. Visualization	:	Students identify, name, compare and operate on geometric figures based on their appearance. At this stage, the students solve a problem by operating on shape visually and they cannot make generalizations.
Level 1. Analysis	:	Students analyze figures in terms of their components and relationships among components. At this stage, the students are able to discover properties/rules of shapes empirically, but they cannot explain the relationship among properties of a figure.
Level 2. Abstraction	:	Students are able to explain the relationship of previously discovered properties/rules using informal deductive arguments. However, at this stage they cannot understand the meaning of these deductions in axiomatic sense.
Level 3. Deduction	:	Students prove theorems deductively and establish interrelationships among networks of theorem. At this stage, they can prove the axiomatic relations by giving formal deductive arguments.
Level 4. Rigor	:	Student establishes theorems in different axiomatic systems and analyzes/compares these system.

From the Van Hiele levels, it is clear to see that the development of class inclusion occur at Van Hiele Level 2 (Fuys, Geddes & Tischler, 1988; Çontay & Paksu, 2012).

3. METHODS

This study was a descriptive study investigating the primary teachers' understanding of inclusion relations of quadrilaterals, especially parallelogram. This study was conducted with 14 primary teachers in Kanisius Demangan Baru Primary School in Yogyakarta, Indonesia.

Data were collected during a workshop that is was aimed to develop primary teachers' mathematics ability in geometry. In the workshop, the researchers posed many questions related with properties of square, rectangle, rhombus, and parallelogram. Through these activities, the teachers were expected to relate the properties of a figure and also among figures.

In this study, the teachers were given a written task about class inclusion of parallelogram and the data were analysed using the framework of Van Hiele. Mainly in this study, the teachers were faced with the following questions:

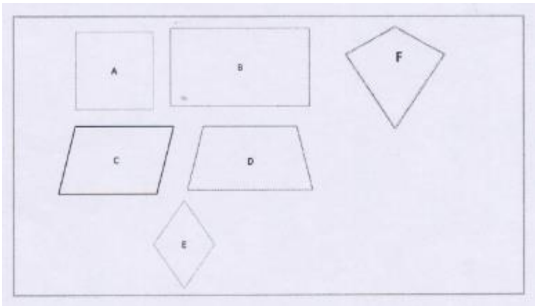

Figure 3. Quadrilaterals in written task

1. What figures are parallelograms? (give your reason)
2. Is a square a rectangle? (give your reason)
3. Do the interrelationships among quadrilaterals need to be taught to students?

4. FINDINGS AND DISCUSSION

4.1.Identification sufficient conditions for a parallelogram

Based on their written answer, it is found that 9 (64%) of the participants could identify that figures A, B, C, and E (figures in number 1) are parallelograms since they have two pairs of parallel sides. Moreover they can identify that figures F and D are not parallelograms since the opposite sides are not parallel. 1 (7%) of the teachers stated that all of the figures are parallelograms except trapezium. 3 (22%) of the primary teachers claimed that all figures are parallelograms because satisfy the properties of parallelogram. The remaining 1 (7%) did not give a clear answer, the teacher only stated that not all of the figures are parallelograms, without mentioning the figures which are not included parallelogram.

These findings show that many of the teachers used the properties of having two pairs of parallel sides which are the sufficient conditions of a parallelogram. That means these primary teachers reached 2^{nd} Van Hiele geometric thinking level. However, there are four teachers who put kite or trapezium into parallelogram group were at Van Hiele Level 0 to 1.

4.2.Relationships between squares and rectangles

From the teachers' answer on the second question, it is found that 4 (29%) of the teachers could identify that a square is a rectangle because a square is a special form of a rectangle in which all sides are equal. This answers show teachers' understanding about inclusion relations of quadrilaterals, especially rectangles. The phrase 'a square is a special form of a rectangle' indicates that these teachers understand that squares are subset of rectangles. Moreover, 1 (7%) of the teacher stated that a square is a rectangle because square has four right angles. This answer indicates that this teacher can identify the necessary and sufficient characteristic of a rectangle, that is has four right angles. Therefore, these teachers can reach 2nd Van Hiele geometric thinking level.

5 (36%) of the teachers said that a square is a rectangle, but there are several different types of reason. First, a square is a rectangle because square has equal sides and 4 right angles. Second, a square is a rectangle because square has right angles and the opposite sides are equal, and the last, a square is a rectangle due to the right angles and two pairs of parallel sides. These teachers could mention properties of a square but the properties mentioned are more than necessary to claim a quadrilateral as a rectangle. It shows that the teachers could not identify minimum sets of properties of a square and a rectangle. Therefore, these teachers could not reach Level 2 of Van Hiele levels.

Furthermore, 1 (7%) of the teacher stated that a square is a rectangle because square has equal sides. This characteristic is true for square but it is cannot be used to claim a quadrilateral as a rectangle. The remaining, 3 (21%) of the teachers could not give reasons why a square is a rectangle. These answers show that these teachers were at Van Hiele Level 0 to 1.

4.3.Inclusion relations in learning geometry

All teachers stated that the interrelationships among quadrilaterals need to be introduced to students at the primary school. Most of the teachers opine that by analyzing the interrelationships among quadrilaterals, students can have a deep understanding of the properties of each type of quadrilaterals and a global perspective about quadrilateral.

5. CONCLUSION

This study has revealed that primary teachers' content knowledge about inclusion relations of quadrilaterals is not at the expected level. Many of them (64% of the teachers) are good at classifying parallelogram, however only 36% of the teachers are good at class inclusion relations of square and rectangle. It is because many teachers gave answer just based on the physical appearance of the figures.

This result implies that the teachers might get much difficulty when they were faced with class inclusion of kite and rhombus which is much harder than square and rectangle since its complex nature of kite and rhombus. Therefore, it is important to improve their knowledge of geometry, especially about inclusion relations of quadrilaterals.

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