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Variational iteration method for solving the population dynamics model of two species

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Abstract. This paper applies the variational iteration method for solving systems of nonlinear ordinary differential equations. The model under consideration in this work is the population dynamics model of two species. Our results show that the variational iteration method provides formulas to approximate the exact solution at every time value with a very cheap computation.

1. Introduction

Systems of differential equations do not only have important roles in mathematics, but they also play essential roles in other fields of study, such as economics, physics, biology, computer sciences etc. [1-3]. Furthermore, non-linear phenomena often occurs in real problems. A system of non-linear ordinary differential equations are ordinary differential equations which satisfy that the unknown functions only rely on one independent variable and fulfill at least one of the following requirements: consists of dependent variables and/or derivatives to the power of except one, contains multiplication of dependent variable and/or its derivatives.

In biology, differential equations occur in the model of population growth. This paper solves systems of non-linear ordinary differential equations. In particluar, we solve the population dynamics model of two species with the variational iteration method.

Variational iteration method has been a well-known technique to solve mathematical equations [4-9]. It is an analytical approach to solving differential equations. Its greatest advantages are that the method is meshless, the solution is an explicit function, and the iterations are convergent to the exact solution very rapidly. Readers interested in other type of meshless method are referred to the Adomian decomposition method [10-12].

The paper is written in the following structure. In Section 2, we write the population dynamics model, which is the problem that we want to solve. The variational iteration method to solve the model is presented in Section 3. Computational results are provided in Section 4. We conclude the paper with some remarks in Section 5.

2. Population dynamics model

This section provides the general form of the model of the population dynamics of two species:

$$\frac{dx}{dt} = x(a_1 + b_1 x + c_1 y)\,, \tag{1}$$

$$\frac{dy}{dt} = y(a_2 + b_2 y + c_2 x)\,, \tag{2}$$

where:

- x represents the population of the first species,
- y represents the population of the second species,
- a_1 is constant denoting the intrinsic growth rate of species x,
- a_2 is constant denoting the intrinsic growth rate of species y,
- b_1 is constant denoting the rate of the declining in growth of species x due to the increase in the population of species x,
- b_2 is constant denoting the rate of the declining in growth of species y due to the increase in the population of species y,
- c_1 is constant denoting the growth rate of the species x due to interaction with species y,
- c_2 is constant denoting the growth rate of the species y due to interaction with species x.

The free variable is time t. A predator and prey model related to equations (1)-(2) can be found in the work of Sharma and Samanta [13].

3. Variation iteration method

Variational iteration method consists of three basic concepts, that is: the correction functionals, the restricted variations, and the Lagrange multipliers. For the variational iteration method, in this section we follow the work of Batiha *et al.* [4]. Details of the original method can be found in the work of Wazwaz [14].

As an illustration of the basic concept of the variational iteration method, we are given the following non-linear differential equations:

$$Lu + Nu = g(t)\,, \tag{3}$$

with L is linear operator, N is non-linear operator, and $g(t)$ is a function. Variational iteration method can be established and analysed using a correction functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^x \lambda(\xi)\,[Lu_n(\xi) + N\tilde{u}_n(\xi) - g(\xi)]d\xi \tag{4}$$

with λ is a Lagrange multiplier, u_n is an approximate solution at the n-th interation, $\tilde{u}_n$ is the restricted variation with $\delta\tilde{u}_n = 0$, and δ is a variational derivative [4].

System (1)-(2) can be rewritten as follows:

$$\frac{dx}{dt} = a_1 x + b_1 x^2 + c_1 xy, \tag{5}$$

$$\frac{dy}{dt} = a_2 y + b_2 y^2 + c_2 xy. \tag{6}$$

The correction functionals of the system (5)-(6) are

$$x_{n+1}(t) = x_n(t) + \int_0^t \lambda_1(s)\left[\frac{dx_n(s)}{ds} - a_1 x_n(s) - b_1\tilde{x}_n^2(s) - c_1\tilde{x}_n(s)\tilde{y}_n(s)\right]ds, \tag{7}$$

$$y_{n+1}(t) = y_n(t) + \int_0^t \lambda_2(s)\left[\frac{dy_n(s)}{ds} - a_2 y_n(s) - b_2\tilde{y}_n^2(s) - c_2\tilde{x}_n(s)\tilde{y}_n(s)\right]ds, \tag{8}$$

where $\tilde{x}_n$ and $\tilde{y}_n$ is the restricted variations with $\delta\tilde{x}_n = 0$ and $\delta\tilde{y}_n = 0$. From equations (7) and (8) we obtain

$$\begin{aligned}\delta x_{n+1}(t) &= \delta x_n(t) + \delta\int_0^t \lambda_1(s)\left[\frac{dx_n(s)}{ds} - a_1 x_n(s) - b_1\tilde{x}_n^2(s) - c_1\tilde{x}_n(s)\tilde{y}_n(s)\right]ds \\ &= \delta x_n(t) + \delta\int_0^t \lambda_1(s)\left[\frac{dx_n(s)}{ds} - a_1 x_n(s)\right]ds\,,\end{aligned} \tag{9}$$

$$\delta y_{n+1}(t) = \delta y_n(t) + \delta \int_0^t \lambda_2(s) \left[\frac{dy_n(s)}{ds} - a_2 y_n(s) - b_2 \tilde{y}_n^2(s) - c_2 \tilde{x}_n(s)\tilde{y}_n(s)\right] ds$$

$$= \delta y_n(t) + \delta \int_0^t \lambda_2(s) \left[\frac{dy_n(s)}{ds} - a_2 y_n(s)\right] ds \,. \tag{10}$$

Using integration by parts, equation (9) becomes

$$\delta x_{n+1}(t) = \delta x_n(t) + \delta \left(\lambda_1(s)x_n(s) - \int_0^t \lambda_1'(s)x_n(s)\,ds - \int_0^t \lambda_1(s)a_1 x_n(s)\,ds\right)$$

$$= \big(1+\lambda_1(t)\big)\delta x_n(t) - \delta \int_0^t [\lambda_1'(s)x_n(s) + a_1\lambda_1(s)x_n(s)]\,ds\,,$$

$$= \big(1+\lambda_1(t)\big)\delta x_n(t) - \delta \int_0^t \big[\big(\lambda_1'(s) + a_1\lambda_1(s)\big)x_n(s)\big]\,ds\,. \tag{11}$$

Using integration by parts, equation (10) becomes

$$\delta y_{n+1}(t) = \delta y_n(t) + \delta \left(\lambda_2(s)y_n(s) - \int_0^t \lambda_2'(s)y_n(s)\,ds - \int_0^t \lambda_2 a_2(s) y_n(s)\,ds\right)$$

$$= \big(1+\lambda_2(t)\big)\delta y_n(t) - \delta \int_0^t [\lambda_2'(s)y_n(s) + a_2\lambda_2(s)y_n(s)]\,ds$$

$$= \big(1+\lambda_2(t)\big)\delta y_n(t) - \delta \int_0^t \big[\big(\lambda_2'(s) + a_2\lambda_2(s)\big)y_n(s)\big]\,ds. \tag{12}$$

The Lagrange multipliers $\lambda_1(t)$ and $\lambda_2(t)$ can be obtained by solving the following system as the stationary conditions:

$$1+\lambda_1(t) = 0\,, \qquad \lambda_1'(s) + a_1\lambda_1(s)|_{s=t} = 0, \tag{13}$$

$$1+\lambda_2(t) = 0\,, \qquad \lambda_2'(s) + a_2\lambda_2(s)|_{s=t} = 0. \tag{14}$$

Therefore, the Lagrange multipliers are $\lambda_1(t) = -e^{-a_1(s-t)}$ and $\lambda_2(t) = -e^{-a_2(s-t)}$. The solution to system (1)-(2) in linearized forms (taking $b_1 = b_2 = c_1 = c_2 = 0$) is as follows:

$$x(t) = C_1 e^{a_1 t}\,, \tag{15}$$

$$y(t) = C_2 e^{a_2 t}\,. \tag{16}$$

The variational iterations for system (1)-(2) with $\lambda_1(t) = -e^{-a_1(s-t)}$ and $\lambda_2(t) = -e^{-a_2(s-t)}$ is given by:

$$x_{n+1}(t) = x_n(t) + \int_0^t -e^{-a_1(s-t)} \left[\frac{dx_n(s)}{ds} - a_1 x_n(s) - b_1 x_n^2(s) - c_1 x_n(s) y_n(s)\right] ds\,, \tag{17}$$

$$y_{n+1}(t) = y_n(t) + \int_0^t -e^{-a_2(s-t)} \left[\frac{dy_n(s)}{ds} - a_2 y_n(s) - b_2 y_n^2(s) - c_2 x_n(s) y_n(s)\right] ds\,. \tag{18}$$

Equations (17) and (18) compute the series of solutions to the population dynamics model of two species. The series converges to the exact solution.

4. Computational results

From the previous section, values of C_1 and C_2 can be obtained from the initial value. We assume that $x(0) = 4$ and $y(0) = 10$. We obtain $C_1 = 4$ and $C_2 = 10$. Therefore, $x_0(t) = 4e^{a_1 t}$ and $y_0(t) = 10e^{a_2 t}$.

In this section, we provide variational iteration solutions to examples of mutualism, parasitism, and competition of two species.

4.1. Mutualism model

Below are given the solution of the system (1)-(2) the model of mutualism using the variational iteration method. We assume that $a_1 = 0.1$; $a_2 = 0.08$; $b_1 = -0.0014$; $b_2 = -0.001$; $c_1 = 0.0012$; $c_2 = 0.0009$.

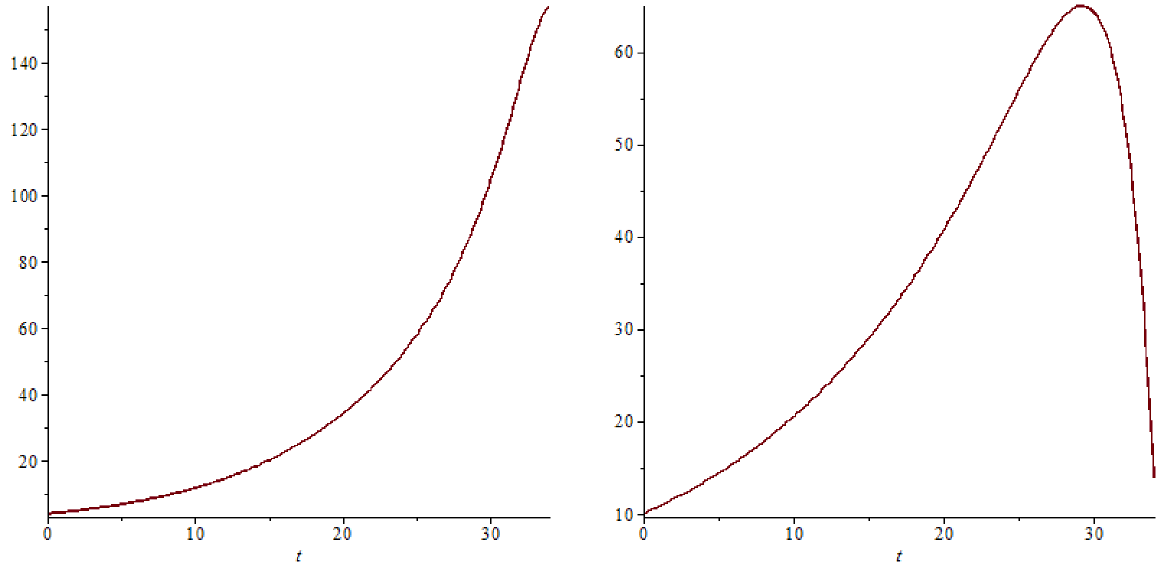
Then the variational iteration solutions up to $x_2(t)$ and $y_2(t)$ are as follows:

$$x_1(t) = 3.624e^{0.1t} - 0.224e^{0.2t} + 0.6e^{0.18t}, \quad (19)$$

$$y_1(t) = 10.89e^{0.08t} + 0.36e^{0.18t} - 1.25e^{0.16t}, \quad (20)$$

$$\begin{aligned} x_2(t) = {} & 3.609515785e^{0.1t} - 0.183867264e^{0.2t} + 0.5919804e^{0.18t} \\ & - 0.04138879997e^{0.28t} + 0.0009983999997e^{0.38t} + 0.01503e^{0.26t} \\ & - 0.00375e^{0.34t} + 0.0003507692307e^{0.36t} - 0.0002341546667e^{0.4t} \\ & + 0.011364864e^{0.3t}, \end{aligned} \quad (21)$$

$$\begin{aligned} y_2(t) = {} & 11.00045852e^{0.08t} + 0.35518824e^{0.18t} - 1.48240125e^{0.16t} \\ & + 0.17015625e^{0.24t} - 0.006510416667e^{0.32t} - 0.00510624e^{0.28t} \\ & - 0.00024192e^{0.38t} - 0.03354000001e^{0.26t} \\ & + 0.0008653846153e^{0.34t} + 0.001131428571e^{0.36t}. \end{aligned} \quad (22)$$



(a). Solution for the first species of the mutualism model.

(b). Solution for the second species of the mutualism model.

Figure 1. Graphics of solutions to the mutualism model: (a). $x_3(t)$, (b). $y_3(t)$ with $0 \le t \le 34$.

Representatives of the solutions for the mutualism model are plotted in Figure 1 for $x_3(t)$ and $y_3(t)$. We do not write $x_3(t)$ and $y_3(t)$ in this paper, because the formulas are too long. In Figure 1, we observe that due to mutualism, populations of both species increase with respect to time at initial stages of the interaction.

4.2. Parasitism model

Below are given the solution of the system (1)-(2) for parasitism model using the variational iteration method. We assume that $a_1 = 0.1$; $a_2 = 0.08$; $b_1 = -0.0014$; $b_2 = -0.001$; $c_1 = 0.0012$; $c_2 = -0.0009$.

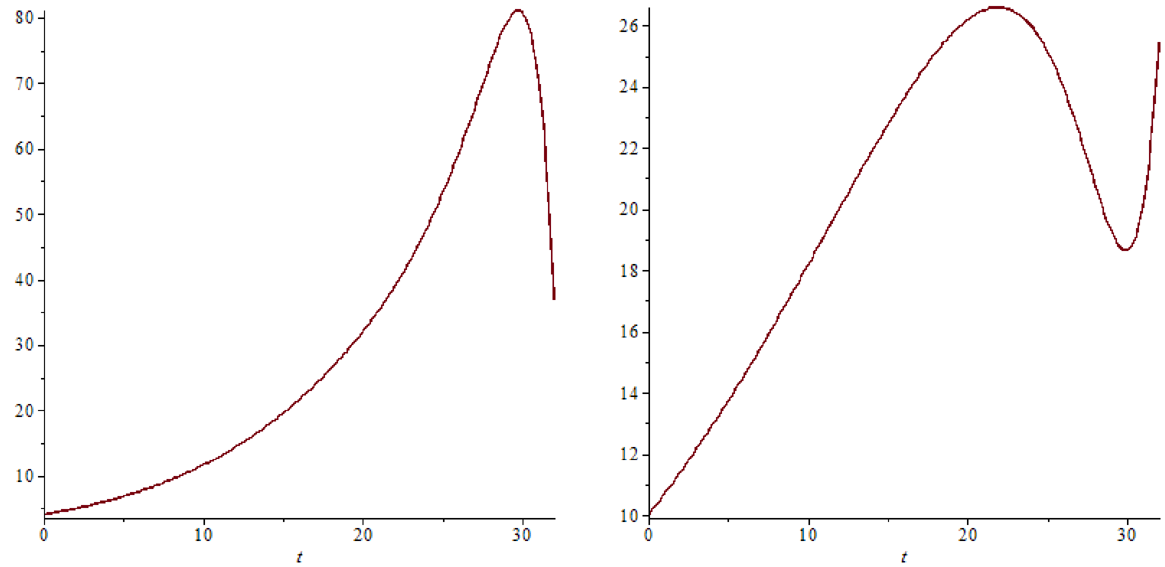
Representatives of the series of variational iteration solutions are:

$$x_1(t) = 3.624e^{0.1t} - 0.224e^{0.2t} + 0.6e^{0.18t}, \quad (23)$$

$$y_1(t) = 11.61e^{0.08t} - 0.36e^{0.18t} - 1.25e^{0.16t}, \quad (24)$$

$$x_2(t) = 3.586909632e^{0.1t} - 0.183867264e^{0.2t} + 0.6311196e^{0.18t} - 0.001643076923e^{0.36t} - 0.0598592e^{0.28t} + 0.0016896e^{0.38t} + 0.01827e^{0.26t} - 0.00375e^{0.34t} - 0.0002341546667e^{0.4t} + 0.01136486399e^{0.3t}, \tag{25}$$

$$y_2(t) = 11.83861929e^{0.08t} - 0.37867176e^{0.18t} - 1.68490125e^{0.16t} + 0.18140625e^{0.24t} - 0.006510416667e^{0.32t} + 0.0006685714285e^{0.36t} + 0.01757376e^{0.28t} - 0.00024192e^{0.38t} + 0.03425999999e^{0.26t} + 0.0008653846153e^{0.34t}. \tag{26}$$



(a). Solution for the first species of the parasitism model.

(b). Solution for the second species of the parasitism model.

Figure 2. Graphics of solutions to the parasitism model: (a). $x_3(t)$, (b). $y_3(t)$ with $0 \le t \le 32$.

Representatives of the solutions to the parasitism model are plotted in Figure 2 for $x_3(t)$ and $y_3(t)$. In this figure we observe that due to parasitism, one of the population decreases respect to time. This then is followed by the other population. Again we do not write $x_3(t)$ and $y_3(t)$ in this paper, because the formulas are too long.

4.3. Competition model

Below are given the solution to the system (1)-(2) for the competition model using the variational iteration method. We assume that $a_1 = 0.1$; $a_2 = 0.08$; $b_1 = -0.0014$; $b_2 = -0.001$; $c_1 = -0.0012$; $c_2 = -0.0009$.

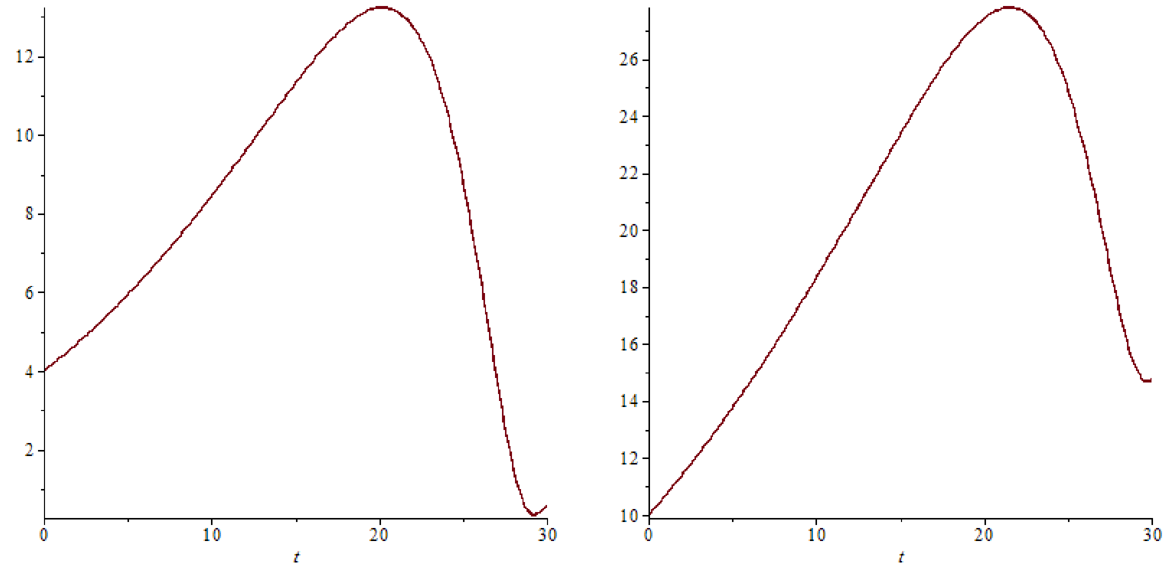
Representatives of the series of the variational iteration solutions are:

$$x_1(t) = 4.824e^{0.1t} - 0.224e^{0.2t} - 0.6e^{0.18t}, \tag{27}$$

$$y_1(t) = 11.61e^{0.08t} - 0.36e^{0.18t} - 1.25e^{0.16t}, \tag{28}$$

$$x_2(t) = 4.989257447e^{0.1t} - 0.325793664e^{0.2t} - 0.8400995999e^{0.18t} - 0.00375e^{0.34t} - 0.004227692307e^{0.36t} + 0.07393919997e^{0.28t} - 0.0016896e^{0.38t} + 0.015128064e^{0.3t} - 0.0002341546667e^{0.4t} + 0.09747e^{0.26t}, \tag{29}$$

$$y_2(t) = 11.89148417e^{0.08t} - 0.50405976e^{0.18t} - 1.68490125e^{0.16t} + 0.18140625e^{0.24t} - 0.006510416667e^{0.32t} - 0.006057692307e^{0.34t} - 0.002057142857e^{0.36t} + 0.01951776e^{0.28t} - 0.00024192e^{0.38t} + 0.11142e^{0.26t}. \quad (30)$$



(a). Solution for the first species of the competition model.

(b). Solution for the second species of the competition model.

Figure 3. Graphics of solutions to the competition model: (a). $x_3(t)$, (b). $y_3(t)$ with $0 \leq t \leq 30$.

Representatives of the solutions to the competition model are plotted in Figure 3 for $x_3(t)$ and $y_3(t)$. Once again, we do not write $x_3(t)$ and $y_3(t)$ in this paper, because the formulas are too long. For small time, both populations increase with respect to time.

Numerical computations for all three cases (mutualism, parasitism, and competition) are given in Table 1. We have compared with these results with the second order Runge-Kutta numerical method. These results are very accurate, as the discrepancy is less than 10^{-6}.

Table 1. Numerical results from the variational iteration method based on the given examples.

t	Mutualism model		Parasitism model		Competition model	
	$x_2(t)$	$y_2(t)$	$x_2(t)$	$y_2(t)$	$x_2(t)$	$y_2(t)$
0.0	4.000000000	10.00000000	4.000000000	10.00000000	4.000000000	10.00000000
0.1	4.042793029	10.07385374	4.042791267	10.06656813	4.033070759	10.06657252
0.2	4.086055587	10.14821700	4.086048406	10.13347257	4.066363553	10.13349044
0.3	4.129792919	10.22309301	4.129776450	10.20071348	4.099879151	10.20075432
0.4	4.174010333	10.29848492	4.173980487	10.26829093	4.133618314	10.26836475
0.5	4.218713190	10.37439597	4.218665655	10.33620501	4.167581809	10.33632224
0.6	4.263906918	10.45082936	4.263837141	10.40445571	4.201770394	10.40462736
0.7	4.309596990	10.52778838	4.309500171	10.47304310	4.236184811	10.47328059
0.8	4.355788954	10.60527627	4.355660046	10.54196712	4.270825824	10.54228248
0.9	4.402488408	10.68329631	4.402322086	10.61122779	4.305694169	10.61163354
1.0	4.449701012	10.76185184	4.449491688	10.68082503	4.340790588	10.68133427

5. Conclusion

Based on the research that has been done, it can be concluded that the variational iteration method can be used to find the solution of the population dynamics model of two species accurately. The population dynamics model of two species being researched is a system of non-linear ordinary differential equations of the first order with initial values. The variational iteration method gives approximate solutions at every time value without any discretisation of the time domain. The iteration formulas are simple. Therefore, they are easy to compute in solving population dynamics models.

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