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Jin–Xin relaxation method for solving a traffic flow problem in one dimension

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Jin–Xin relaxation method for solving a traffic flow problem in one dimension

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Abstract. We test the performance of the Jin–Xin relaxation and Lax–Friedrichs finite volume numerical methods in solving a traffic flow problem. In particular, we focus on traffic flow at a traffic light turning from red to green. Numerical solutions are compared with the analytical solution to the mathematical model. We find that the Jin–Xin relaxation solution is more accurate than the Lax–Friedrichs finite volume solution.

1. Introduction

Mathematical models are either linear or nonlinear equations [1-4]. Mathematical models may be solved either analytically or numerically [5-7]. Many equations are difficult to solve analytically, so we have to solve it numerically [8-10]. Some examples of models given by partial differential equations are the traffic flow model for busy streets [11], blood flow model for an elastic artery [12], models for gas [13] and hydraulic dynamics [14-16], elasticity in heterogeneous media [17], etc.

Traffic flow and transportation have been studied by a number of authors in the literature. The traffic light is expected to overcome traffic jams on the road and accelerate traffic flow. In this paper we focus on simulations for the traffic flow at a traffic light which turns from red to green.

We consider the Lax–Friedrichs finite volume method and Jin-Xin relaxation method in this paper. We choose those methods because they are simple to implement. We shall compare the errors of the numerical solutions produced by these two methods.

The rest of this paper is organised as follows. We provide the problem to solve in Section 2. Numerical methods are written in Section 3. Numerical results and discussion are given in Section 4. Finally some remarks conclude the paper in Section 5.

2. Problem formulation

We consider the mathematical model for traffic flow:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u(\rho))}{\partial x} = 0\,, \tag{1}$$

where x is the free variable of the space, t is the free variable of time, $\rho = \rho(x,t)$ is the density of the traffic on the road, $u = u(x,t)$ is the vehicle velocity. The space domain is a closed interval $[a, b]$. In this paper we want to find the density of vehicles after the traffic light turns from the red to green (see Figure 1 for illustration).

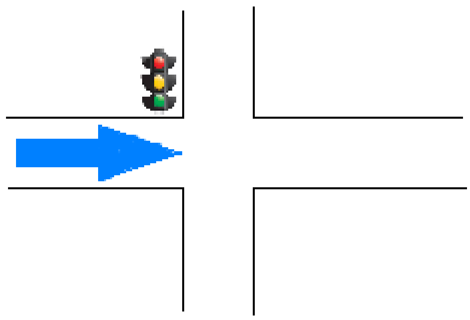

Figure 1. Illustration of the traffic light problem at a road intersection [11].

As used by Mattheij *et al.* [8], the velocity function is given as

$$u(\rho) = u_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right) \tag{2}$$

where $u_{\max}$ is the maximum vehicle velocity and $\rho_{\max}$ is the maximum density of the traffic on the road. If the velocity approaches zero, then the density tends to the maximum density. Conversely, if the density tends to zero, then the velocity approaches the maximum velocity.

3. Numerical method

In this section, we present the Lax–Friedrichs finite volume and Jin–Xin relaxation methods for solving the traffic flow model (1).

3.1. Lax–Friedrichs finite volume method

The traffic flow model (1) is a conservation law in the form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial f(\rho)}{\partial x} = 0\,. \tag{3}$$

The fully explicit finite volume method for conservation laws is

$$\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x}\left(F_{i+\frac{1}{2}}^n - F_{i-\frac{1}{2}}^n\right) \tag{4}$$

where $\rho_i^n \approx \rho(x_i, t^n)$ is the conserved density and $F_{i+1/2}^n \approx f\left(\rho\left(x_{i+1/2}, t^n\right)\right)$ is the flux function computed in the finite volume framework. Here Δt is the time step and Δx is the cell-width. $F_{i+1/2}^n$ is the flux at time t^n at the space point $x_{i+1/2}$.

The Lax–Friedrichs fluxes for equation (1) are

$$\begin{aligned} F_{i+\frac{1}{2}}^n &= \frac{1}{2}\left(f(\rho_{i+1}^n) + f(\rho_i^n)\right) - \frac{\Delta x}{2\Delta t}(\rho_{i+1}^n - \rho_i^n) \\ &= \frac{1}{2}\left(\rho_{i+1}^n u_{\max}\left(1 - \frac{\rho_{i+1}^n}{\rho_{\max}}\right) + \rho_i^n u_{\max}\left(1 - \frac{\rho_i^n}{\rho_{\max}}\right)\right) - \frac{\Delta x}{2\Delta t}(\rho_{i+1}^n - \rho_i^n)\,, \end{aligned} \tag{5}$$

and

$$\begin{aligned} F_{i-\frac{1}{2}}^n &= \frac{1}{2}\left(f(\rho_i^n) + f(\rho_{i-1}^n)\right) - \frac{\Delta x}{2\Delta t}(\rho_i^n - \rho_{i-1}^n) \\ &= \frac{1}{2}\left(\rho_i^n u_{\max}\left(1 - \frac{\rho_i^n}{\rho_{\max}}\right) + \rho_{i-1}^n u_{\max}\left(1 - \frac{\rho_{i-1}^n}{\rho_{\max}}\right)\right) - \frac{\Delta x}{2\Delta t}(\rho_i^n - \rho_{i-1}^n)\,. \end{aligned} \tag{6}$$

3.2. Jin–Xin relaxation method

Equation (3) can be modified to

$$\frac{\partial \rho}{\partial t} + \frac{\partial v}{\partial x} = 0\,, \tag{7}$$

where $v := f(\rho) = \rho\, u(\rho)$ and v is assumed to be a smooth function.

The Jin–Xin relaxation system for equation (7) is

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial v}{\partial x} = 0\,, \\ \dfrac{\partial v}{\partial t} + a\dfrac{\partial \rho}{\partial x} = -\dfrac{1}{\varepsilon}\left(v - f(\rho)\right)\,, \end{cases} \tag{8}$$

in which we can take $a = f'(\rho)^2$ and ε is a small positive constant. The iteration for ρ is given by

$$\frac{\rho_j^{n+1} - \rho_j^n}{\Delta t} + \frac{1}{\Delta x}\left(v_{j+\frac{1}{2}}^n - v_{j-\frac{1}{2}}^n\right) = 0\,, \tag{9}$$

or

$$\rho_j^{n+1} = \rho_j^n - \frac{\Delta t}{2\Delta x}\left(v_{j+1}^n - v_{j-1}^n\right) + \frac{\sqrt{a}\,\Delta t}{2\Delta x}\left(\rho_{j+1}^n - 2\rho_j^n + \rho_{j-1}^n\right). \tag{10}$$

The iteration for v is given by

$$\frac{v_j^{n+1} - v_j^n}{\Delta t} + \frac{a}{\Delta x}\left(\rho_{j+\frac{1}{2}}^n - \rho_{j-\frac{1}{2}}^n\right) = -\frac{1}{\varepsilon}\left(v_j^n - f(\rho_j^n)\right), \tag{11}$$

or

$$v_j^{n+1} = v_j^n - \frac{a\,\Delta t}{2\Delta x}\left(\rho_{j+1}^n - \rho_{j-1}^n\right) + \frac{\sqrt{a}\,\Delta t}{2\Delta x}\left(v_{j+1}^n - 2v_j^n + v_{j-1}^n\right) - \frac{1}{\varepsilon}\left(v_j^n - f(\rho_j^n)\right). \tag{12}$$

In other words, if the Jin–Xin relaxation method is written in the fully discrete explicit finite volume method, then the fluxes are given by

$$\rho_{j+\frac{1}{2}}^n = \frac{1}{2}\left(\rho_{j+1}^n + \rho_j^n\right) - \frac{1}{2\sqrt{a}}\left(v_{j+1}^n - v_j^n\right), \tag{13}$$

$$\rho_{j-\frac{1}{2}}^n = \frac{1}{2}\left(\rho_j^n + \rho_{j-1}^n\right) - \frac{1}{2\sqrt{a}}\left(v_j^n - v_{j-1}^n\right), \tag{14}$$

and

$$v_{j+\frac{1}{2}}^n = \frac{1}{2}\left(v_{j+1}^n + v_j^n\right) - \frac{\sqrt{a}}{2}\left(\rho_{j+1}^n - \rho_j^n\right), \tag{15}$$

$$v_{j-\frac{1}{2}}^n = \frac{1}{2}\left(v_j^n + v_{j-1}^n\right) - \frac{\sqrt{a}}{2}\left(\rho_j^n - \rho_{j-1}^n\right). \tag{16}$$

Once again, here ρ_i^m is an approximation of ρ at time t^m at the space point x_i. The notation Δt gives the time step. The notation Δx is the cell width of the discretised space domain.

4. Numerical results

In this section we present our numerical results.

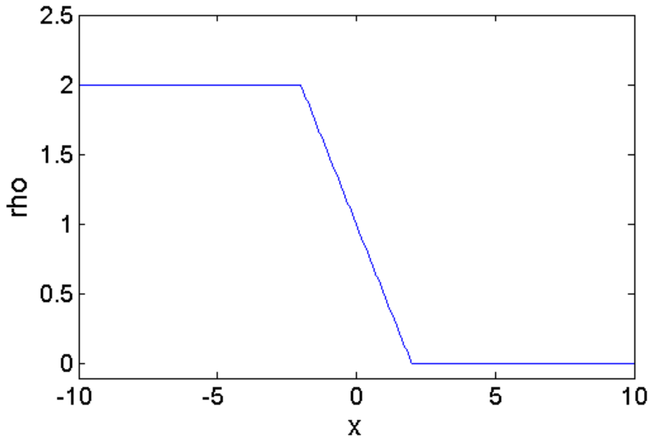

Figure 2. Analytical results for 1 second after a traffic light turns from red to green.

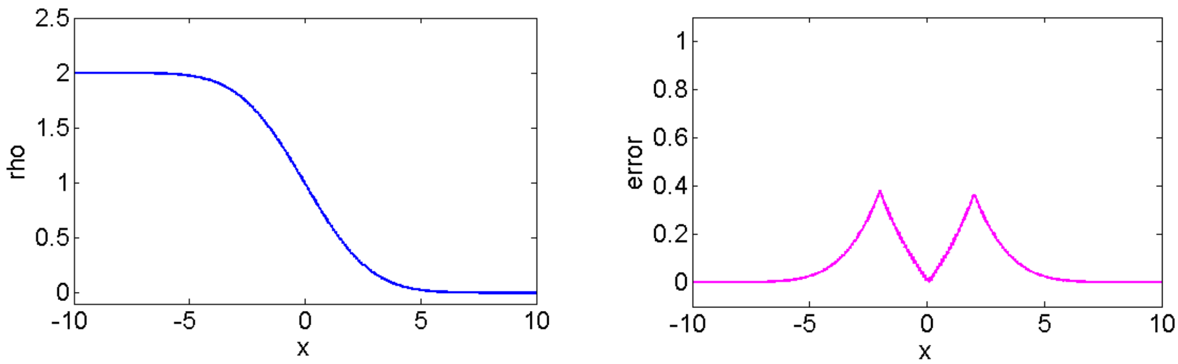

Figure 3. Lax–Friedrichs results for 1 second after a traffic light turns from red to green with $\Delta x = 0.05$ and $\Delta t = 0.01\Delta x$. The figure on the left is the Lax–Friedrichs solution. The figure on the right is its numerical error.

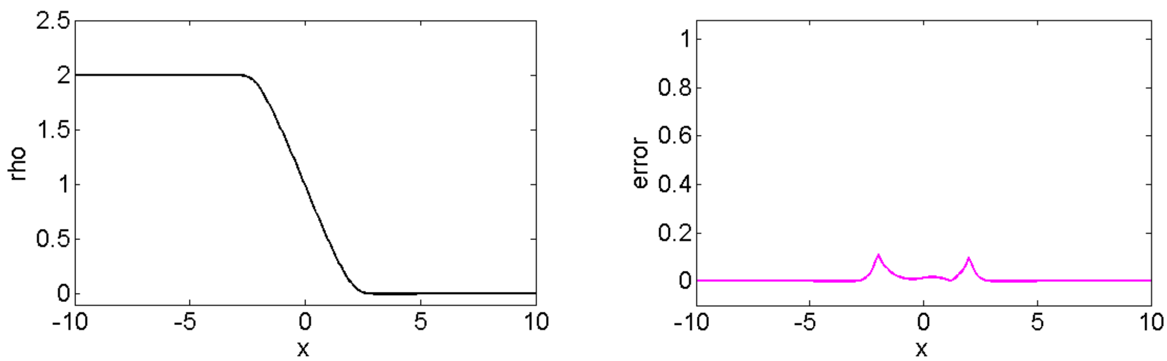

Figure 4. Jin–Xin relaxation results for 1 second after a traffic light turns from red to green with $\Delta x = 0.05$, $\Delta t = 0.01\Delta x$ and $\varepsilon = 10^{-2}$. The figure on the left is the Jin–Xin relaxation solution. The figure on the right is its numerical error.

Following Gunawan [11], we consider the space domain $-10 \leq x \leq 10$ and the time domain $t \geq 0$. The initial condition is

$$\rho(x,0) = \begin{cases} 2 & \text{if } x < 0, \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

The boundary condition is $\rho(-10,t) = 2$ and $\rho(10,t) = 0$ for all t. We assume that $\rho_{\max} = 2$ and $u_{\max} = 2$. With the initial condition (17), the analytical solution to this problem (see Mattheij *et al.* [8] and Gunawan [11]) is:

$$\rho(x,t) = \begin{cases} 2 & \text{if } \quad \frac{x}{t} < f'(2), \\ \frac{1}{2}\rho_{\max}\left(1 - \frac{x}{u_{\max}t}\right) & \text{if } \quad f'(2) \leq \frac{x}{t} < f'(0), \\ 0 & \text{if } \quad \frac{x}{t} \geq f'(0). \end{cases} \tag{18}$$

The results are shown in Figures 2, 3, and 4. The analytical solution is shown in Figure 2. The numerical solutions are shown in Figures 3 and 4. Figure 3 plots the Lax–Friedrichs solution and its numerical error for 1 second after a traffic light turns from red to green with $\Delta x = 0.05$ and $\Delta t = 0.01\Delta x$. Using the same discretisations, Figure 4 plots the Jin–Xin relaxation solution and its numerical error.

From the numerical results compared to the analytical solution, we obtain that the numerical results have the same behaviour as the physical real problem. The density spreads to the right direction for positive time value. In addition, the Jin–Xin relaxation method performs better than the Lax–Friedrichs method. From Figures 3 and 4, we observe that the numerical error produced by the Jin–Xin relaxation method is much smaller than the error produced by the Jin–Xin relaxation method.

5. Conclusion

Our numerical results show a physically correct behaviour of the traffic flow, after the traffic light turns from red to green. We obtain that the traffic moves from left to right. Furthermore, the density at the left side of the traffic light decreases over time. In addition, the Jin–Xin relaxation solution is more accurate than the Lax–Friedrichs finite volume solution.

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